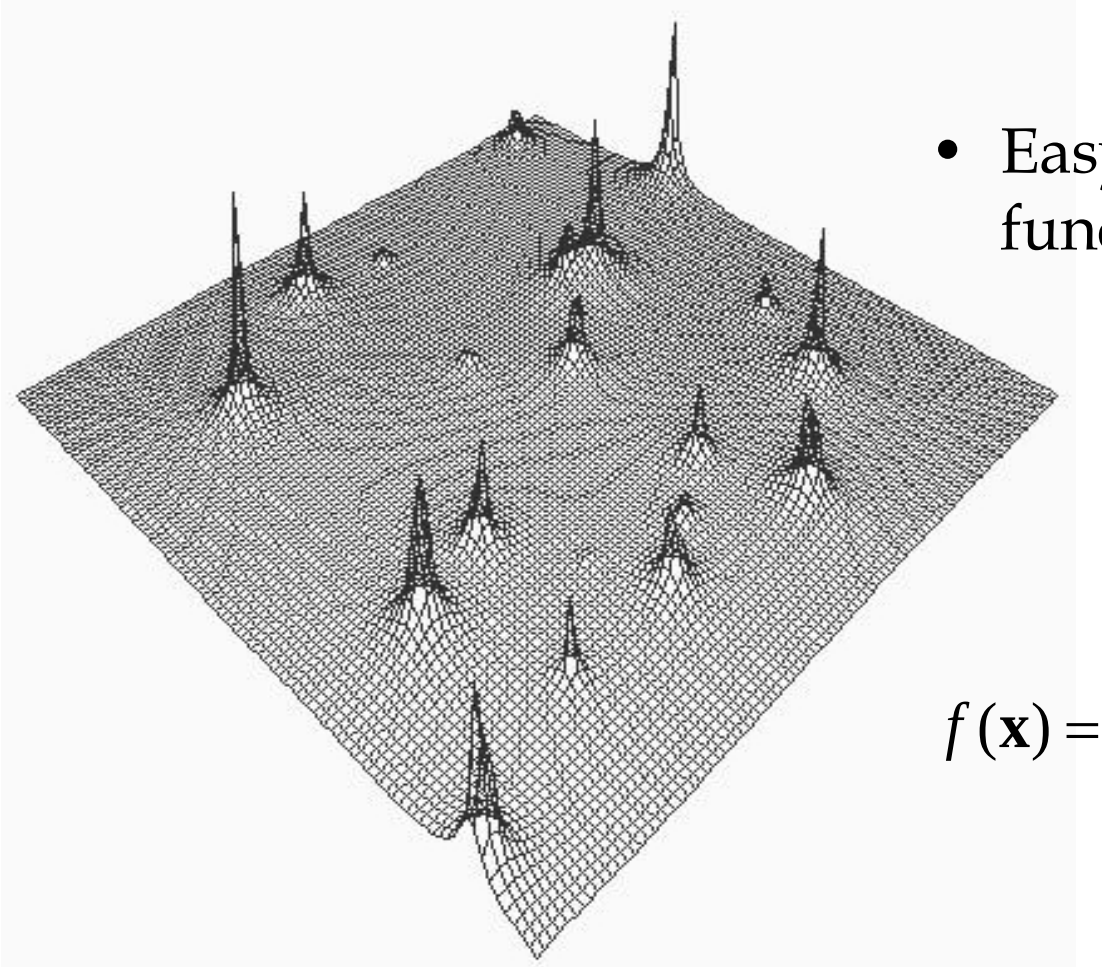
Multimodal optimization

- **Optimization of function with several local minima**
- **Identification: several independent runs of optimization algorithm lead to different results**
- **Closely coupled with premature convergence**

Multimodal problems ①

- Example: “Type 0” function



- Easy random multimodal function:

$$f(\mathbf{x}) = \sum_{i}^{n} y_i \left(\frac{\pi}{2} - \arctan \frac{\left\| \mathbf{x} - \mathbf{x}_i \right\|}{r_i} \right)$$

Multimodal problems ②

- Example: Optimization of RC beam

Řez		Rozměry		Parametry smykové výztuže							Cena
$A-A$	$B-B$	b [mm]	h [mm]	pr_w	l_{w1}	s_{w1}	l_{w2}	s_{w2}	l_{w3}	s_{w3}	[Kč]
4 ø 6.0	9 ø 5.5	150	550	4.0	9	400	3	75	-	-	593.00
4 ø 6.5	9 ø 5.5	175	550	4.0	10	425	2	100	-	-	660.44
5 ø 5.5	9 ø 5.5	150	550	4.0	9	400	3	75	-	-	618.92
6 ø 5.5	8 ø 6.5	150	450	6.0	9	400	3	125	-	-	616.77
5 ø 5.5	9 ø 5.5	150	525	4.0	9	425	3	75	-	-	583.83
4 ø 6.0	8 ø 6.0	150	550	4.0	9	425	3	75	-	-	590.34
4 ø 6.0	10 ø 5.0	150	550	4.0	9	425	3	75	-	-	618.13
6 ø 6.0	8 ø 7.0	150	400	6.0	9	425	3	100	-	-	618.66
4 ø 6.0	8 ø 6.0	150	525	4.0	9	425	3	75	-	-	582.58
4 ø 6.0	9 ø 5.5	150	525	4.0	9	425	3	75	-	-	578.81

Tab. 4.7: Výsledná řešení bez metody scaling a s $p_m = 0.9$

[Lepš, Master thesis, 2000]

Multimodal problems ②

Example: Optimization of RC beam cont.

9 007 199 254 740 992	**Possible solutions**
100 000 000	**MC points, where:**
24 698 222	**Valid solutions**
722	**With price under 1000 Kč**
55	**With price under 900 Kč**
0	**With price under 800 Kč**
	Minimum 573,50 Kč

Solutions

- **Re-start**
  - **Repeated runs from different starting points**
  - **Mostly often used with mathematical programming methods**
- **Remembering of already visited points**
  - **See e.g. TABU Search method, in other words, usage of „memory“**

Solutions

- **Re-annealing**
  - **Rapid change of temperature in Simulated Annealing**
- **Methods of preserving diversity within population-based algorithms**
  - **For instance by division of population into disjoint parts, see parallel EAs**
  - **or by reducing elitism in selection**

Solutions

- **Fitness Sharing**
  - **Change of objective function based on the number of solutions sharing same local minimum**

$$F'(i) = \frac{F(i)}{\sum_j sh(d(i,j))},$$

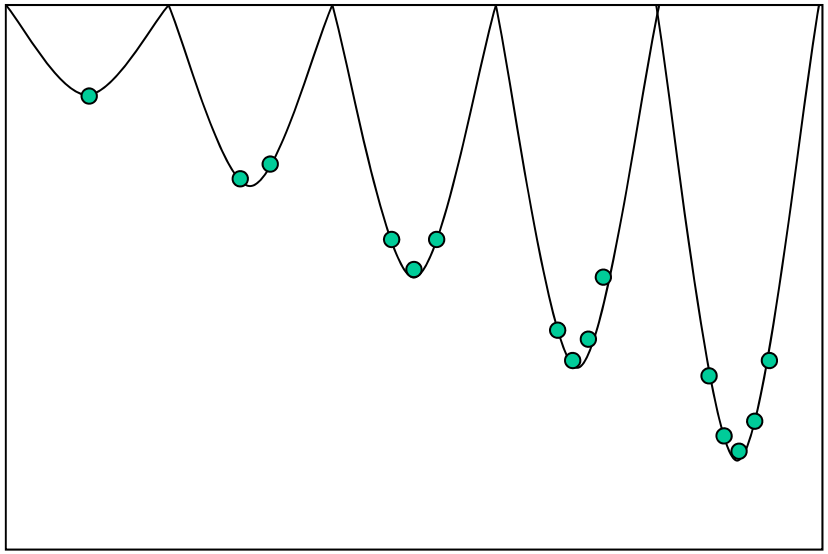
$$sh(d) = \begin{cases} 1-(d/\sigma_{share})^{\alpha} & \text{if } d \le \sigma_{share}, \\ 0 \end{cases}$$

  - **Two parameters:** α **- shape**

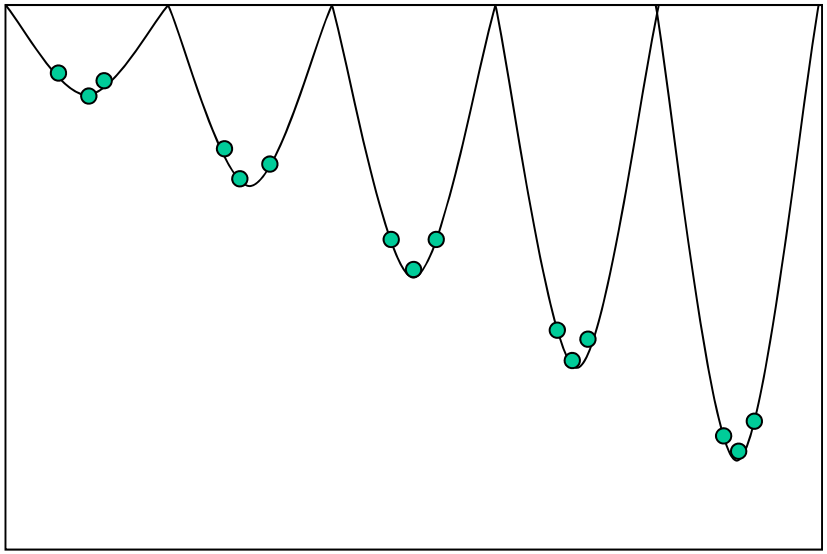
    σ_{share} **– neighborhood of minimum**

Solutions

- **Crowding**
  - **Steady-state selection**
  - **Children replace only their parents**



Fitness Sharing



Crowding

CERAF Method

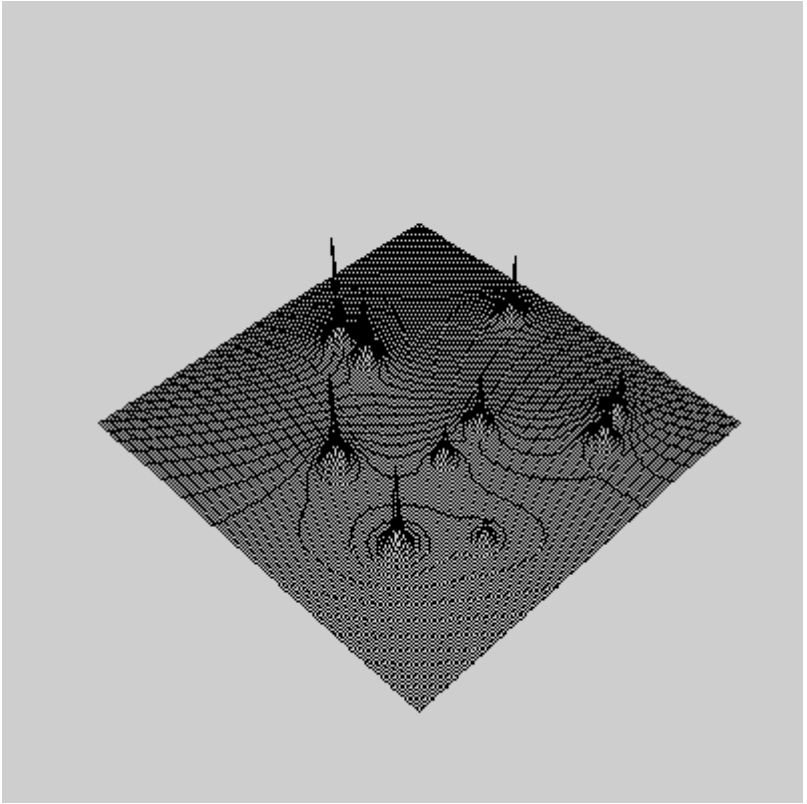
- **CERAF** (La méthode de CentrE Radio-ActiF)
- Developed by O. Hrstka and A. Kučerová, 2003
- Independent enhancement for the population based algorithm
- Re-starting algorithm with „memory“
- Stores already visited minima
- Forbids optimization algorithm to fall again into visited minima
- Rapidly increases robustness of algorithm
- User can selected solution from list of found minima

CERAF Method

- If the best-so-far value dos not change within given range for given number of iterations, the best point is selected as a minimum.
- Algorithm is restarted with random population.
- Around local minima, elliptical „radioactive“ zone is created.
- Every solution created in zone is “killed” during selection and replaced by new one outside zone.
- If such solution was created by crossover, the zone is decreased to allow search for local minima hidden in zone.

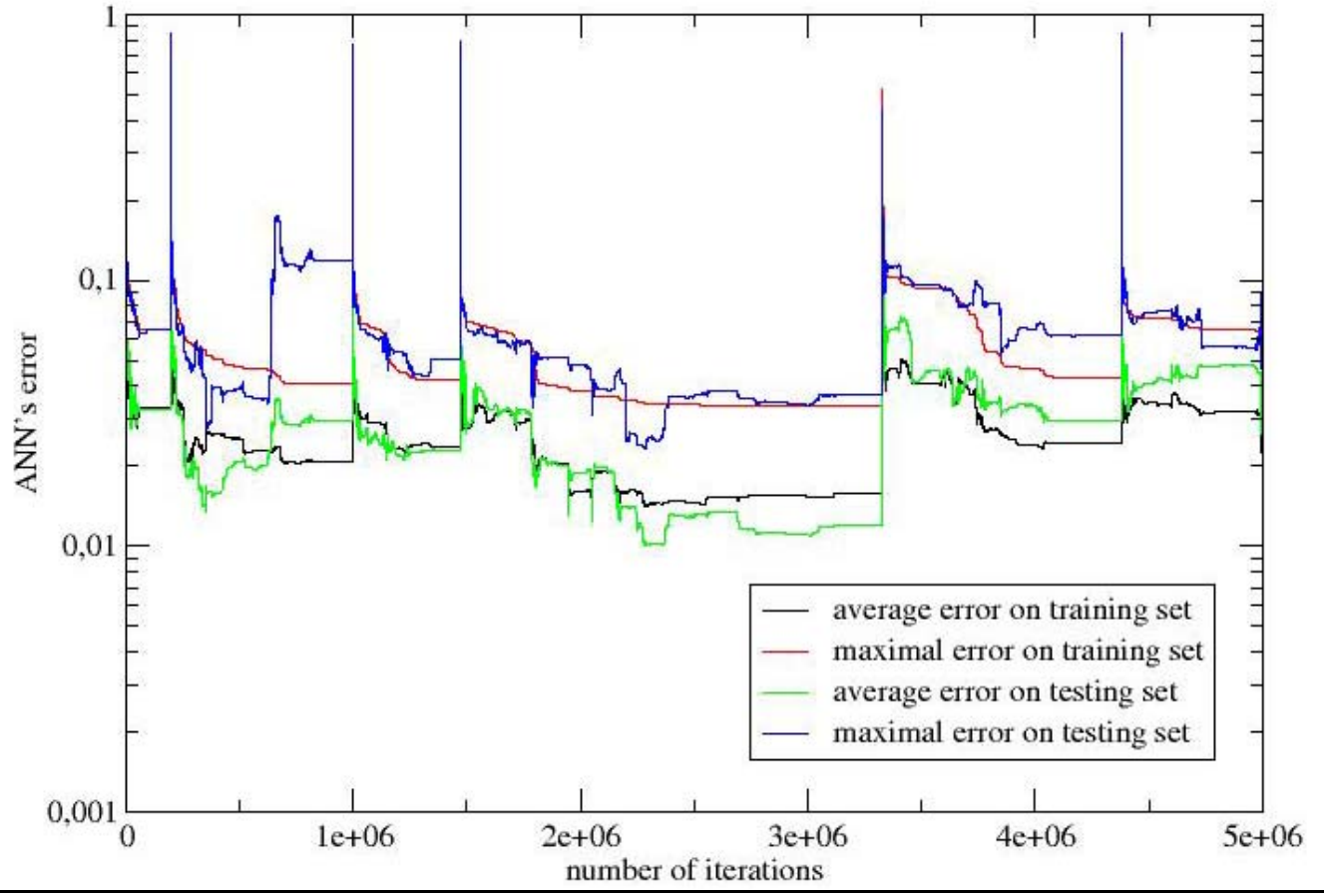
Usage of CERAF Method ①

- Type 0 function optimized with SADE algorithm and CERAF method



Usage of CERAF Method ③

- Optimization of neural network weigts



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[2] Mahfoud, S. W. (1995). Niching methods for genetic algorithms. PhD thesis, University of Illinois at Urbana-Champaign, Urbana, IL, USA.

[3] Hrstka, O. and Kučerová, A. (2004). Improvements of real coded genetic algorithms based on differential operators preventing the premature convergence. Advances in Engineering Software, 35(3–4):237–246.

[4] A. E. Eiben, J. E. Smith, Agoston E. Eiben, J. D. Smith (2003). Introduction to Evolutionary Computing. Springer.

A humble plea. Please feel free to e-mail any suggestions, errors and typos to **matej.leps@fsv.cvut.cz**.

Date of the last version: 23.11.2011
Version: 002