

# Comparison of optimization algorithms

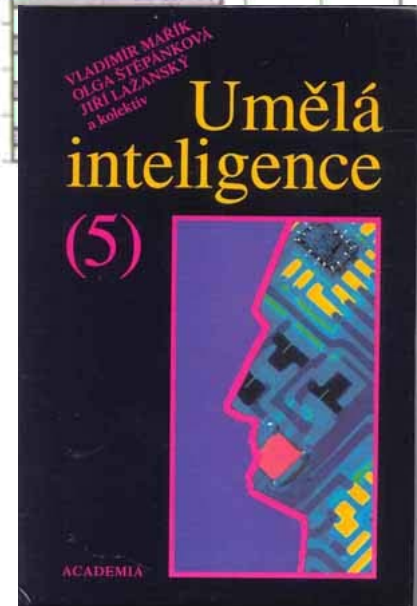
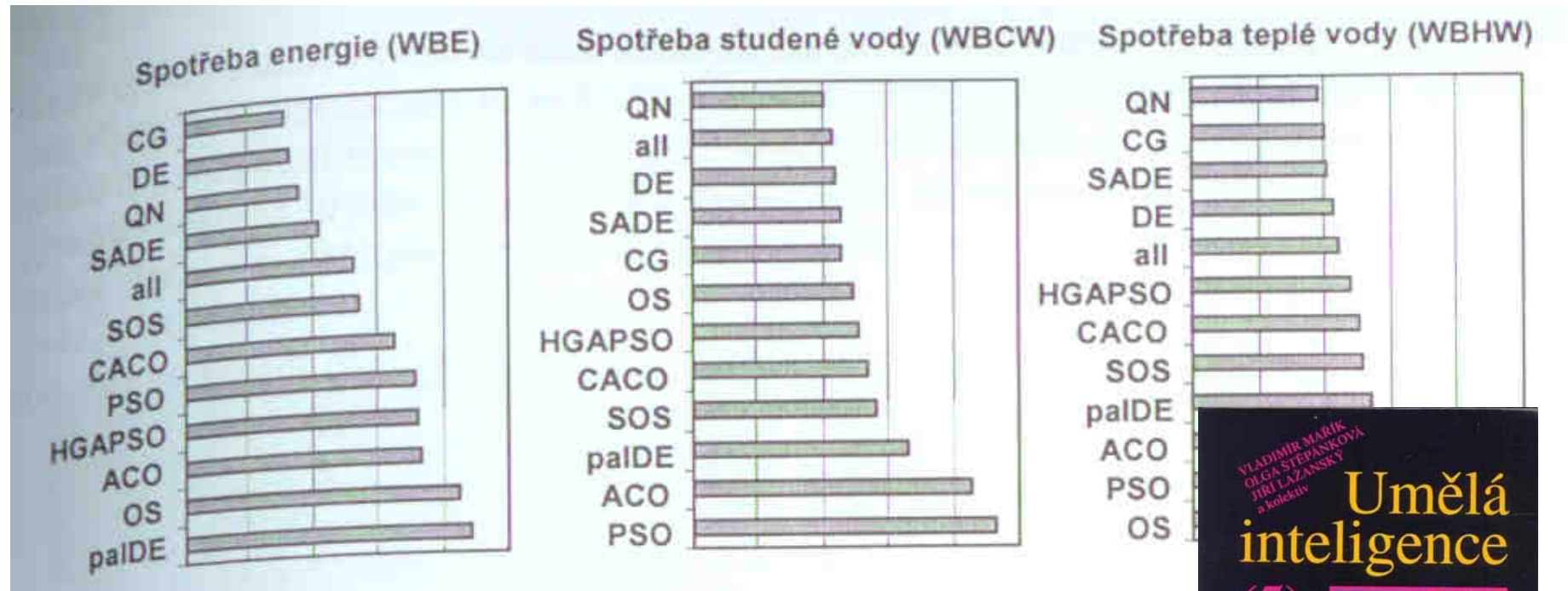
# Overview of single-objective algorithms

- With one solution in given time:
  - Gradient methods
  - Hill-climbing
  - Simulated annealing
  - TABU search
  - (1+1)-ES
- Advantages: small number of evaluations, fast convergence

# Overview of single-objective algorithms

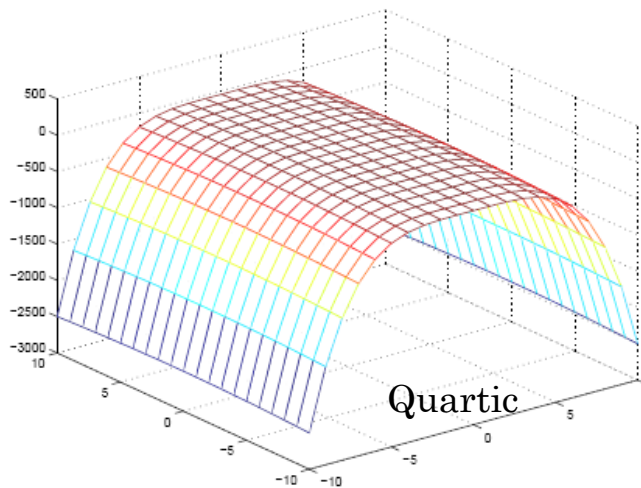
- With a set of solutions:
  - Binary Genetic Algorithms
  - Evolution Strategies
  - Differential Evolution
  - SADE/GRADE + CERAFA
  
- Advantage: Robustness

# Example of comparison

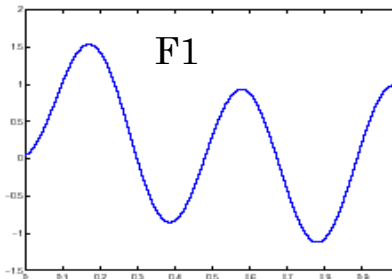


# Benchmarks

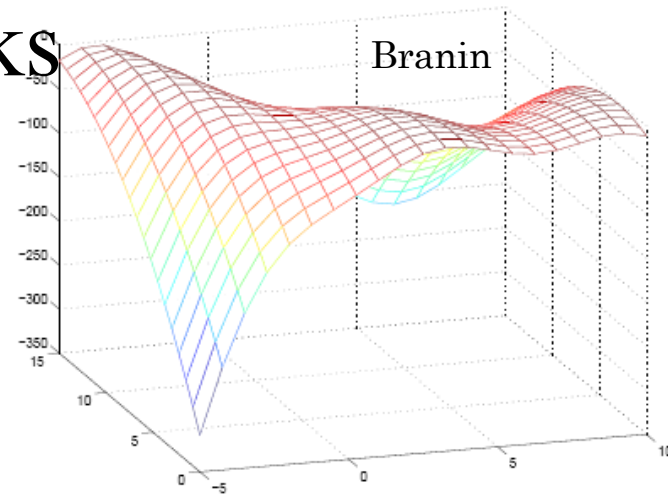
- 20 mathematical functions from 1 to 20 variables
- In Matlab and C++



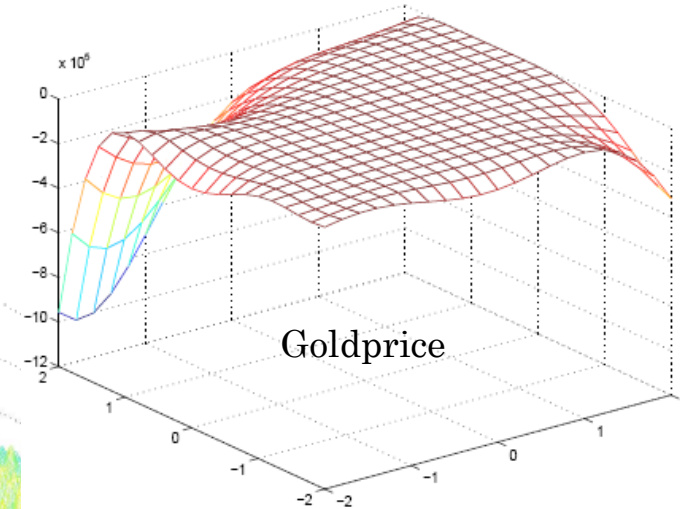
Quartic



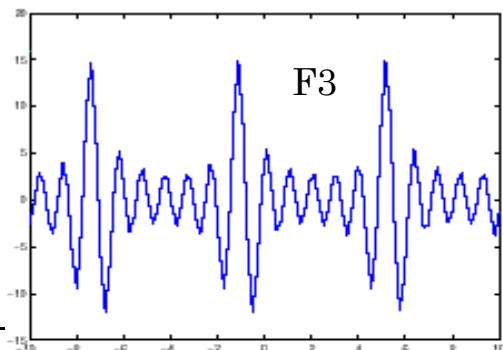
F1



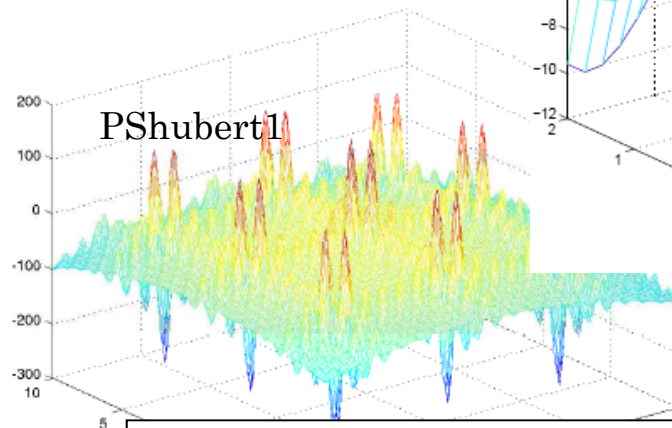
Branin



Goldprice



F3



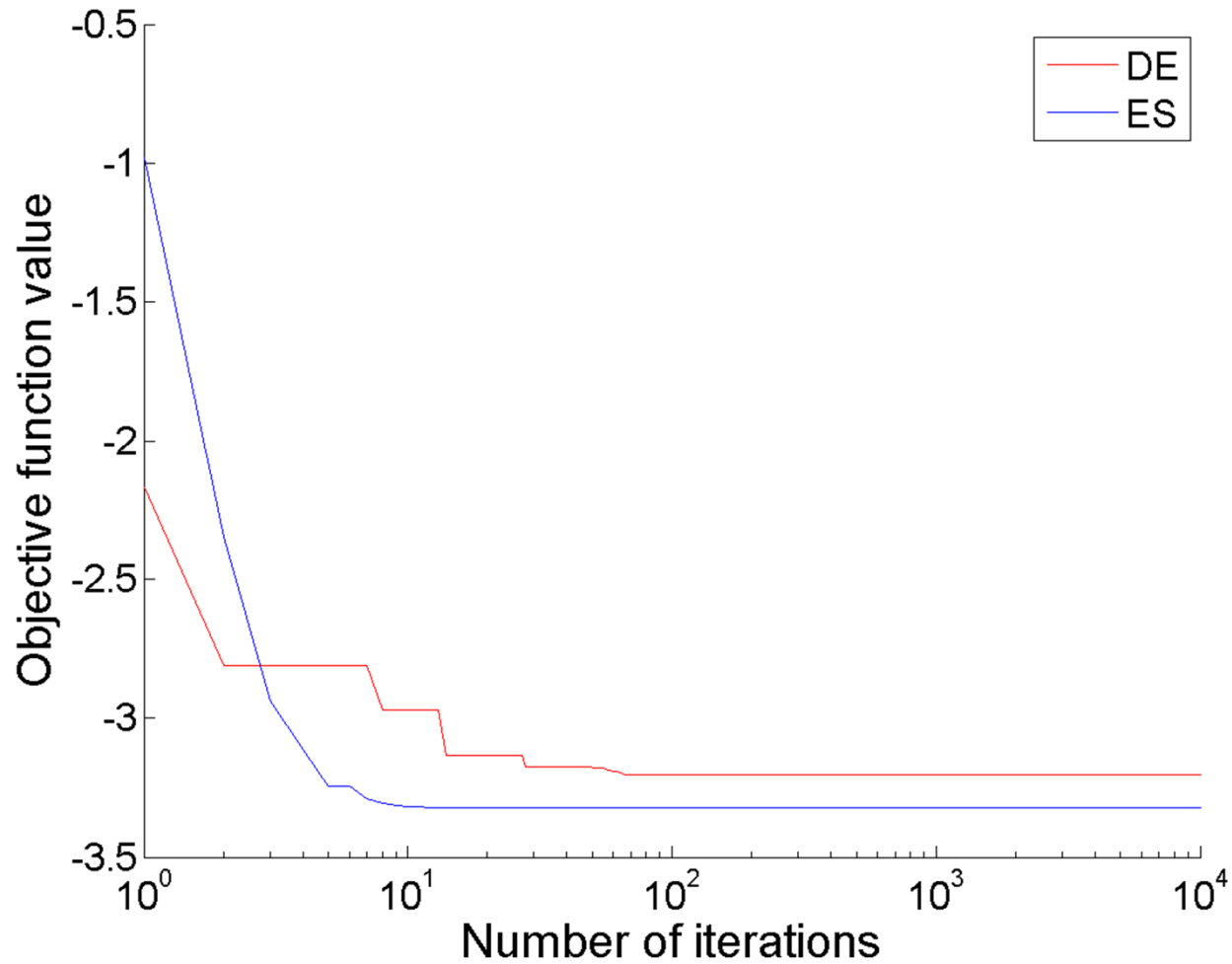
PShubert1

<http://klobouk.fsv.cvut.cz/>

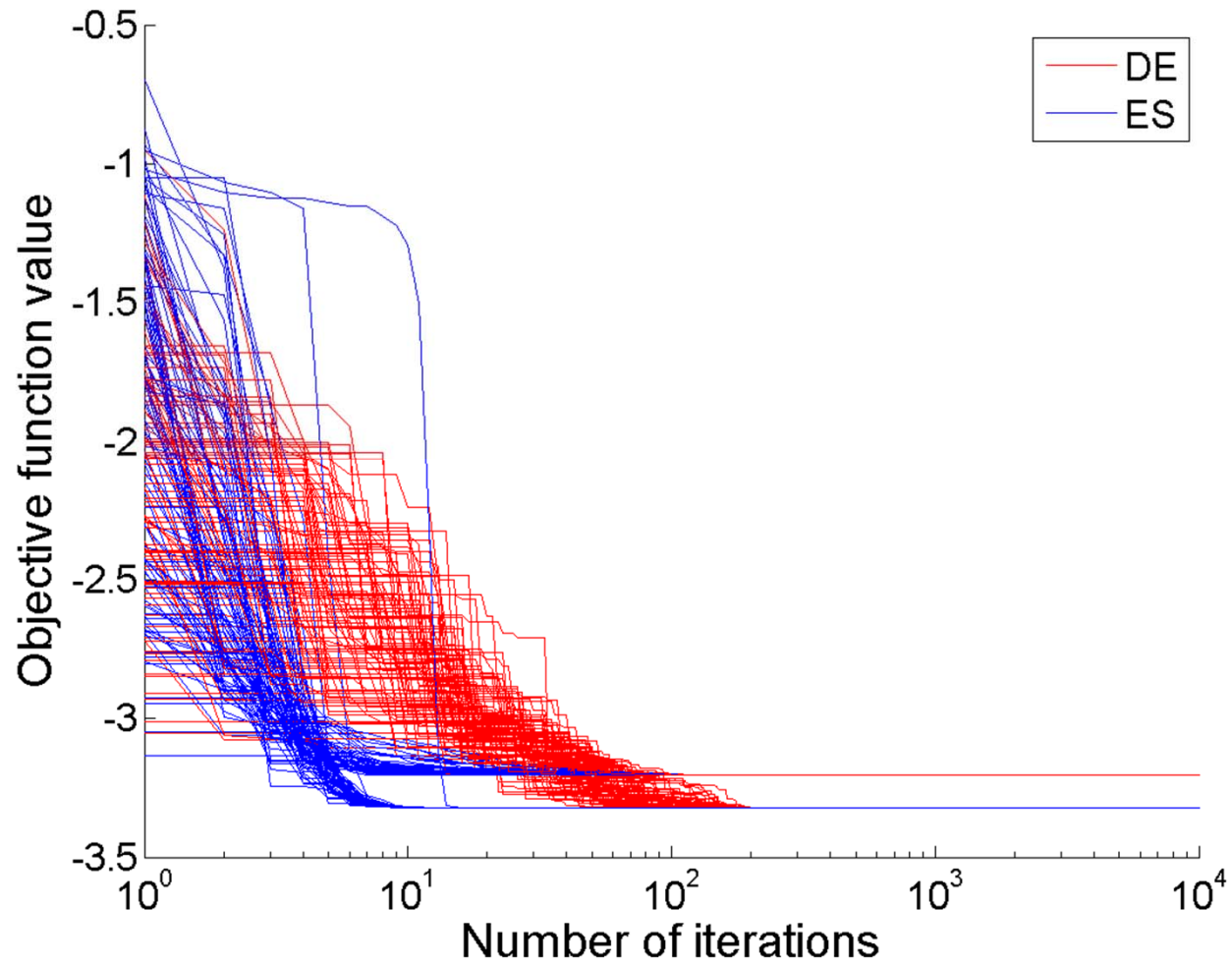
[~anicka/testfunc/testfunc.html](http://klobouk.fsv.cvut.cz/~anicka/testfunc/testfunc.html)

Modern optimization methods

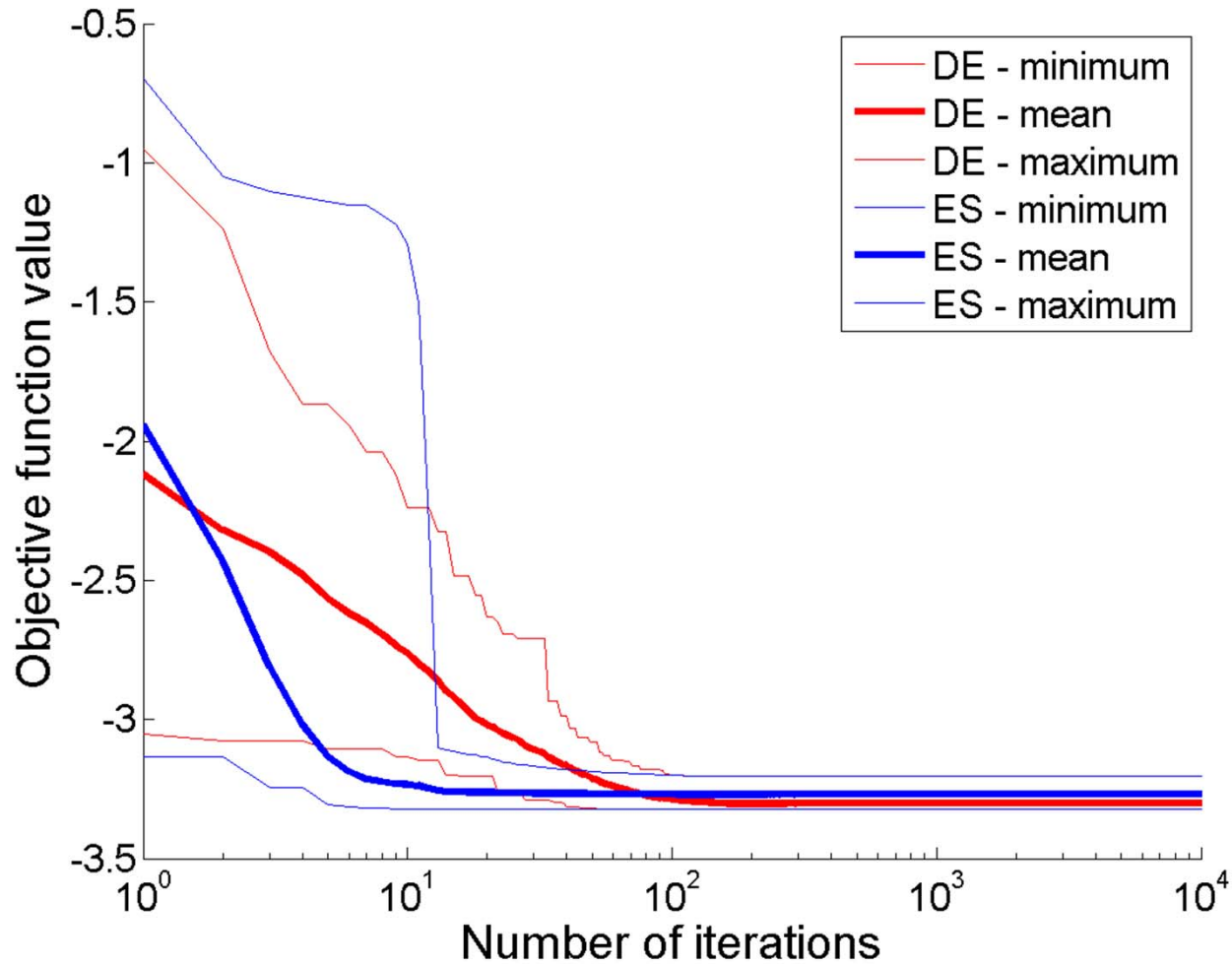
# One run comparison: “Progress plots”



# One hundred runs comparison

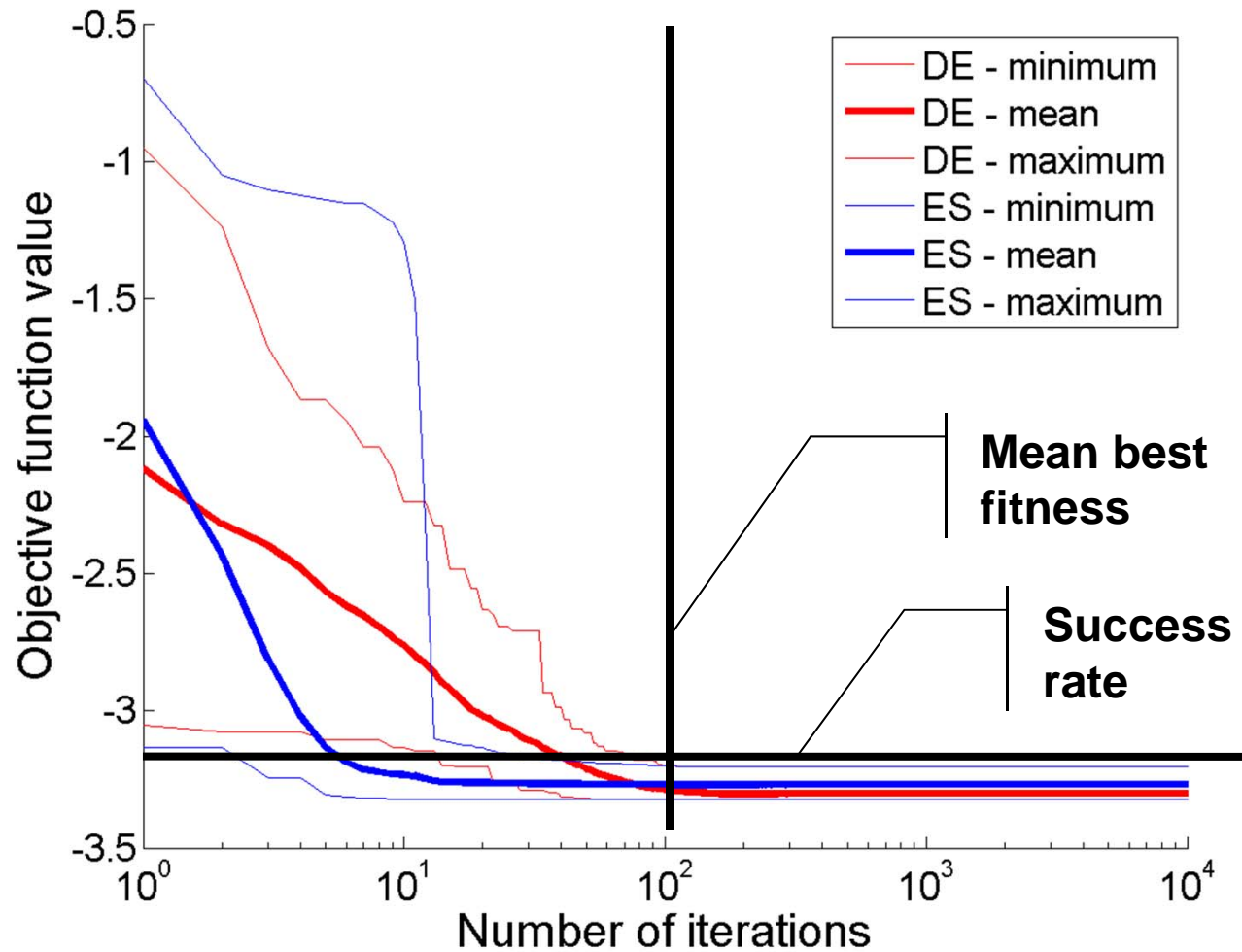


# One hundred runs comparison





# Traditional measures



# Comparison on reliability

Function	Dim	Fmincon	GRADE	GRADE+CERAF
F1	1	100	100	100
F3	1	100	100	100
Branin	2	100	100	100
Camelback	2	100	100	100
Goldprice	2	100	100	100
PShubert1	2	100	100	100
PShubert2	2	100	100	100
Quartic	2	100	100	100
Shubert	2	100	100	100
Hartman1	3	100	100	100
Shekel1	4	100	100	100
Shekel2	4	100	100	100
Shekel3	4	100	100	100
Hartman2	6	100	59	100
Hosc45	10	100	100	100
Brown1	20	100	100	100
Brown3	20	100	100	100
F5n	20	100	100	100
F10n	20	0	78	100
F15n	20	1	100	100

Modern optimization methods

# Comparison on convergence speed

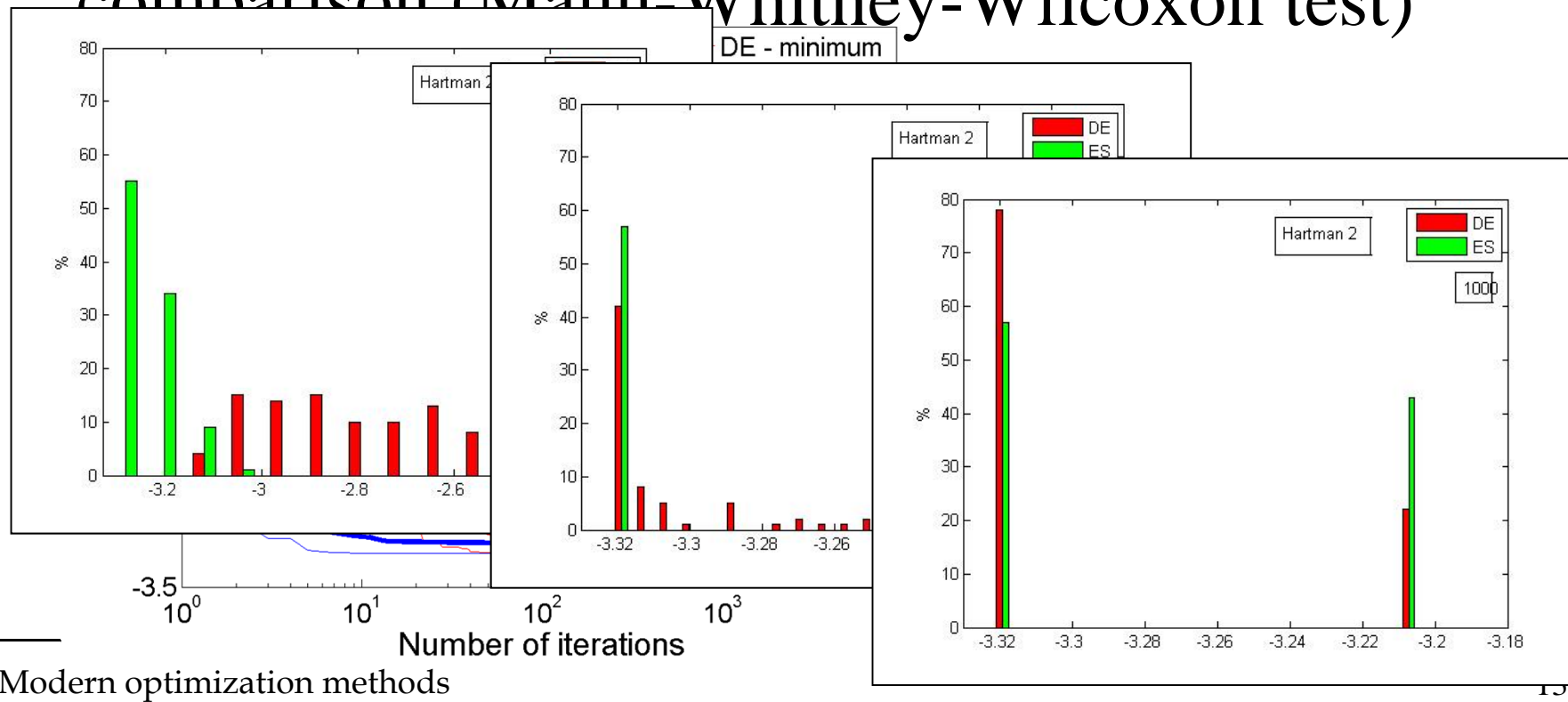
Function	Dim	Fmincon	GRADE	GRADE+CERAF
F1	1	27	55	55
F3	1	57	95	95
Branin	2	24	348	348
Camelback	2	40	198	198
Goldprice	2	63	337	337
PShubert1	2	2097	3879	1402
PShubert2	2	1615	2333	896
Quartic	2	56	320	331
Shubert	2	375	606	603
Hartman1	3	63	284	292
Shekel1	4	335	47577	4078
Shekel2	4	255	15356	2686
Shekel3	4	284	7310	2496
Hartman2	6	200	123727	9881
Hosc45	10	264	2147	2096
Brown1	20	286979	176628	182390
Brown3	20	5660	36568	36090
F5n	20	15838	6734	7284
F10n	20	-----	89715	226374
F15n	20	374110	22378	25528

# Summary

- Disadvantages of traditional measures:
  - Unpractical setting of functions calls limit for MBF type of measures
  - Need of optimum value knowledge for SR based measures
- Result:
  - Whole progress plot is of importance
- Disadvantage:

# Proposed solution for two methods

- Store only  $10 \times \text{Dim}$  results (like generations)
- Use statistical test to judge the result of comparison (Mann-Whitney-Wilcoxon test)

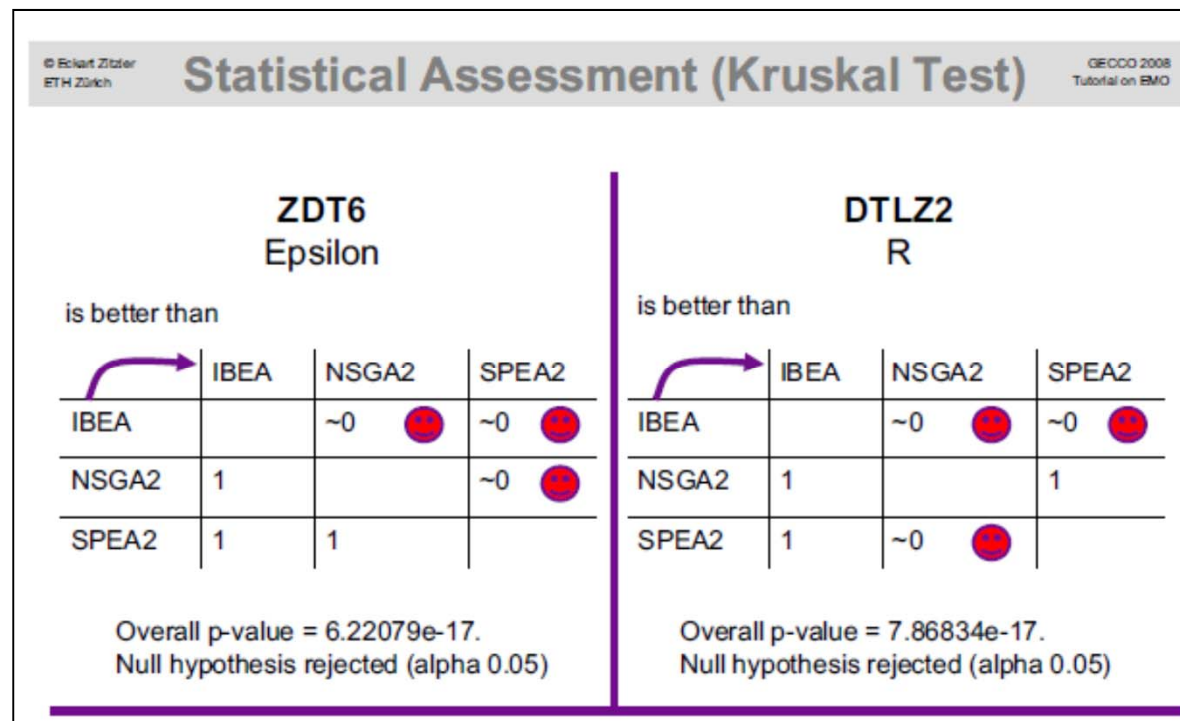


# Proposed solution cont.

	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000
F1	1	1	1	1	1	1	0	0	0	0	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
F3	1	1	1	0	0	0	0	0	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Branin	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Camelback	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Goldprice	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Pshubert 1	-1	-1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
Pshubert 2	-1	-1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Quartic	-1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Shubert	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Hartman 1	0	0	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Shekel 1	-1	-1	-1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Shekel 2	-1	-1	-1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Shekel 3	-1	-1	-1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Hartman 2	-1	-1	-1	-1	-1	-1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Hosc 45	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0
Brown 1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
Brown 3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	0	1	1	1	1	1	1	1	1	1
F5n	-1	-1	-1	-1	-1	-1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
F10n	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
F15n	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

# Proposed solution for more methods

- How to graphically compare more than two methods?
  - From multi-objective domain: Pair-wise comparison table



# Proposed solution for more methods

- How to graphically compare more than two methods?
  - From single-objective domain: Partial ordering?

**10 nodes:**

*C B E F H I G D J K A L*

**25 nodes:**

*C B F G I E H D L A J K*

**50 nodes:**

*C B I G F D H E L A K J*

[Carrano et. al.: GECCO'08]

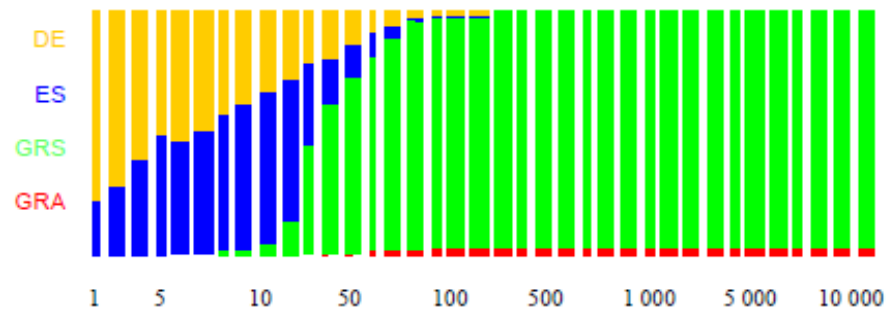
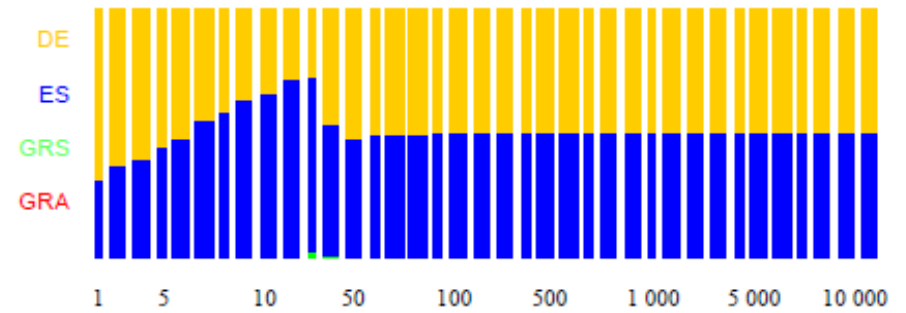
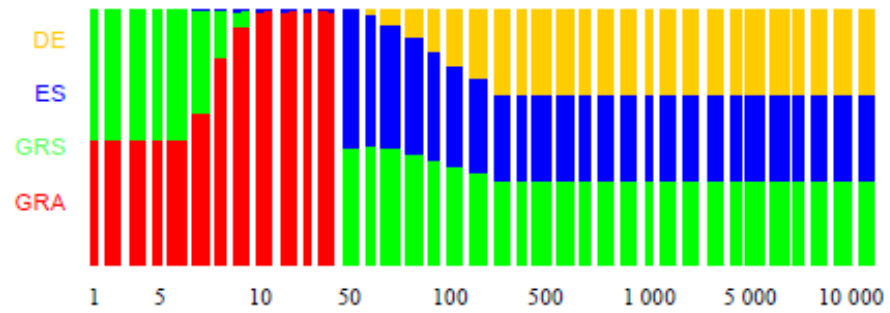
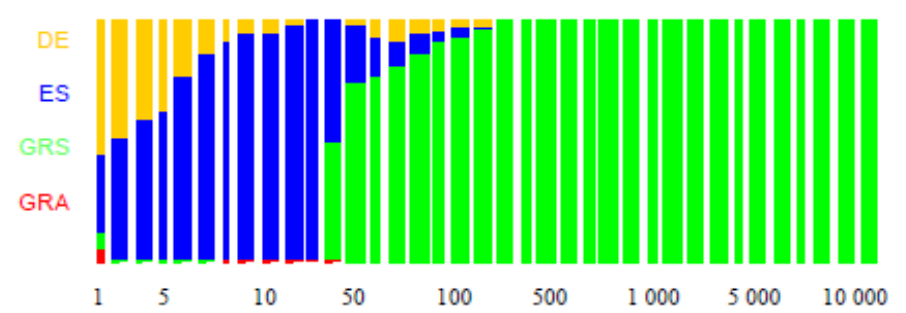
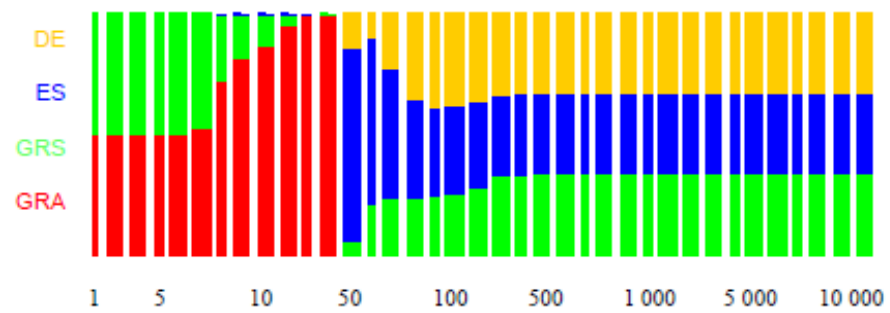
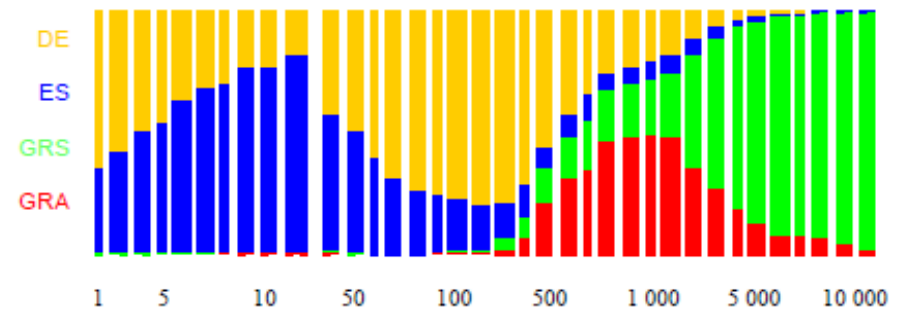


# Proposed solution for more methods

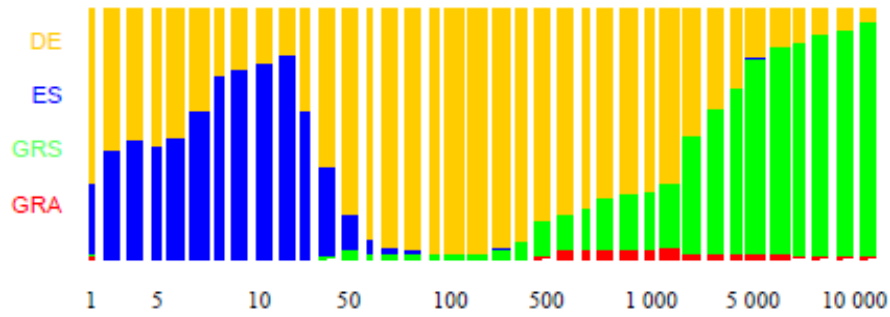
- Relative Winning Score

$$RWS_i = \frac{\sum \text{No. cases } i \text{ is winner}}{\sum \text{of scores}}$$

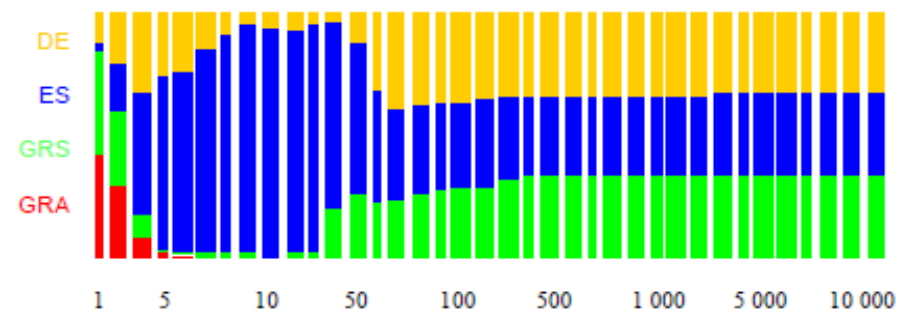
$$RWS_i \in \langle 0,1 \rangle \quad \sum RWS_i = 1$$

**F1****F3****Branin****Camelback****Goldprice****PShubert1**

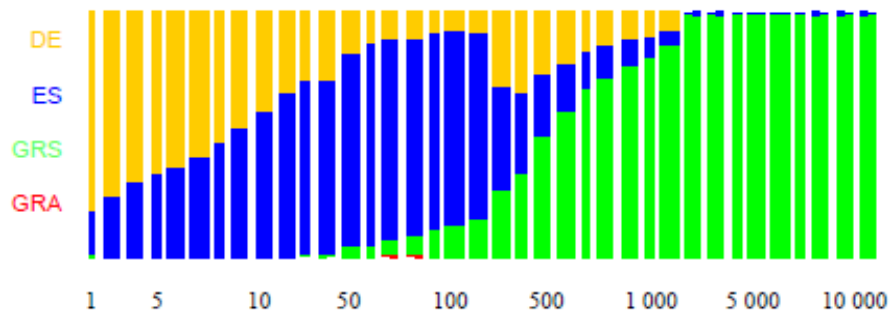
**PShubert2**



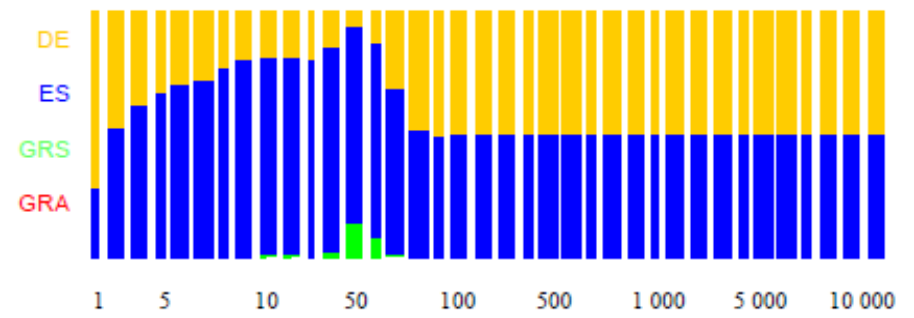
**Quartic**



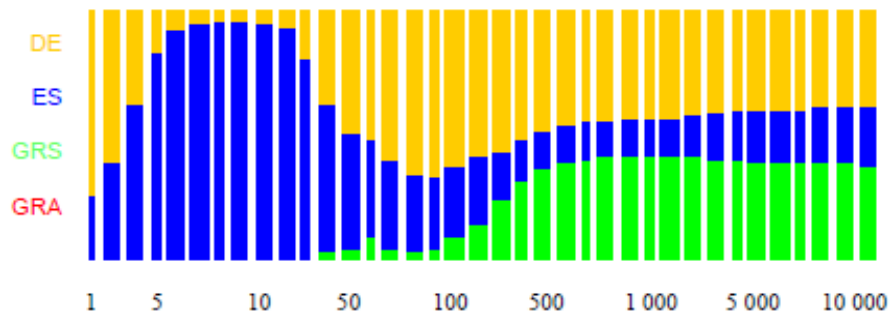
**Shubert**



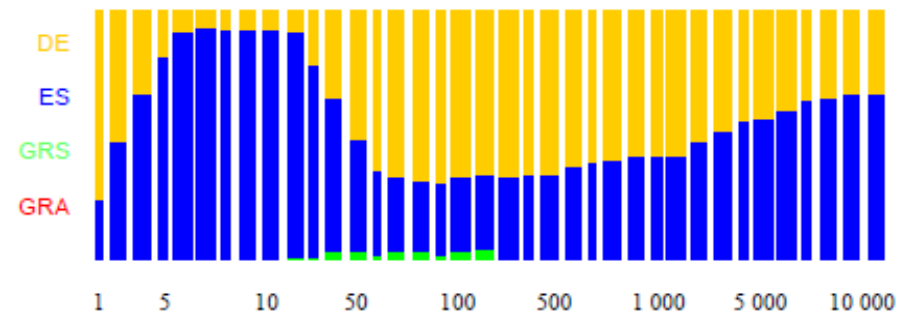
**Hartman1**



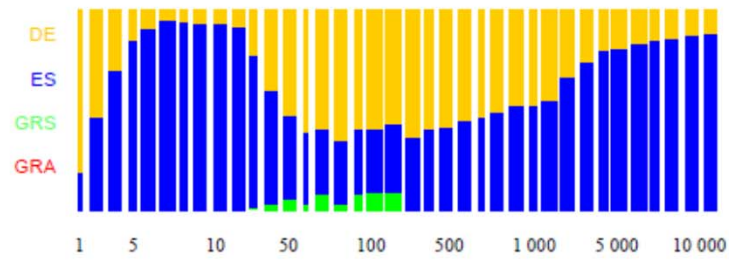
**Shekel1**



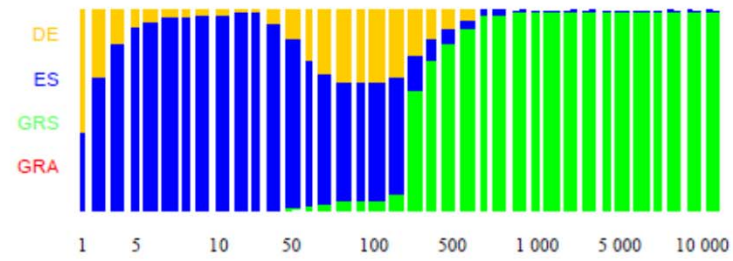
**Shekel2**



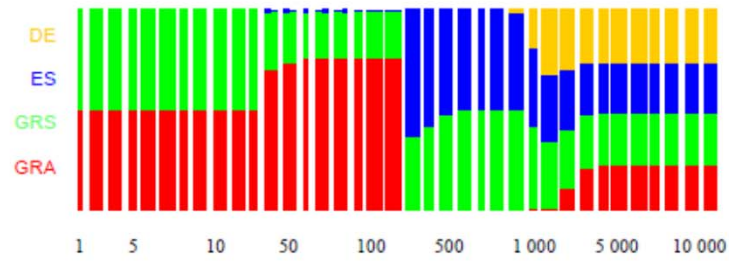
Shekel3



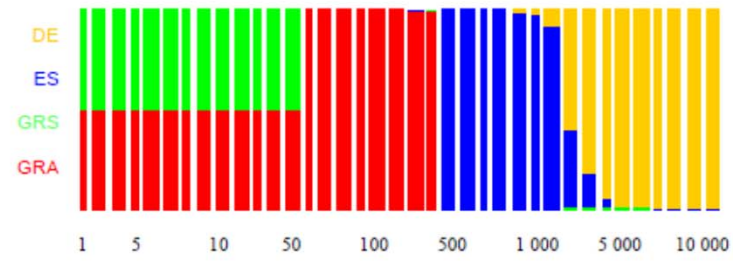
Hartman2



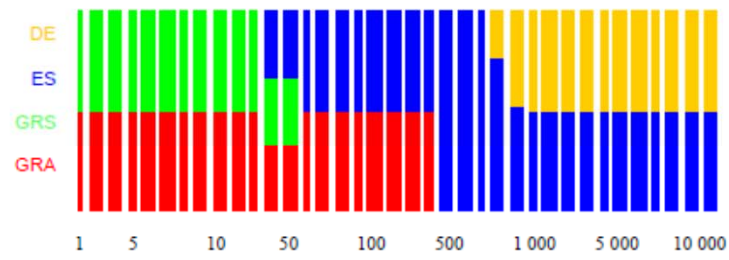
Hosc45



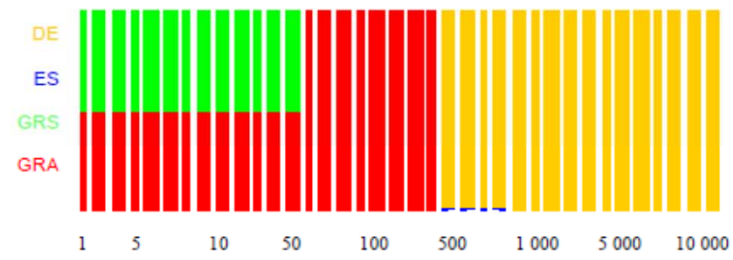
Brown1



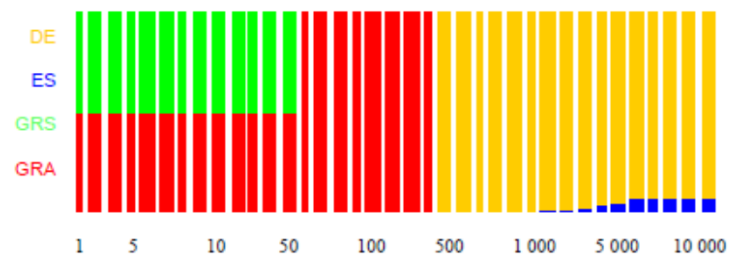
Brown3



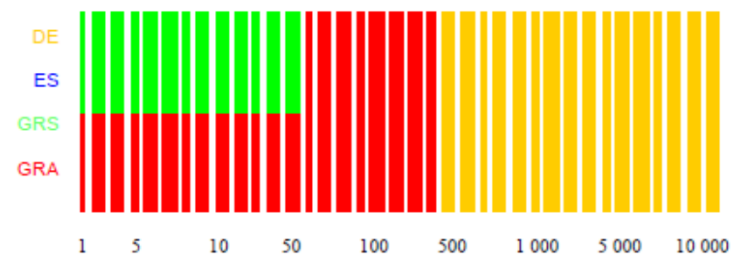
F5n



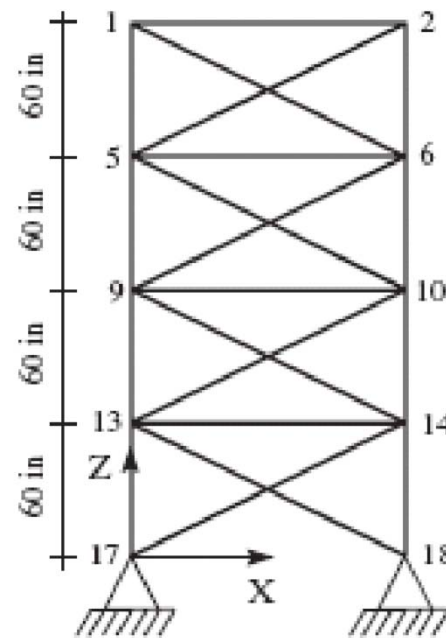
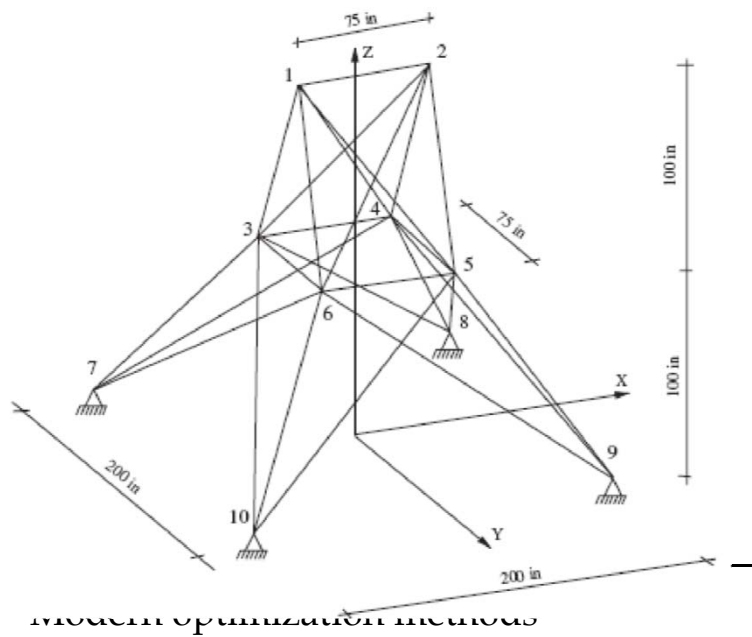
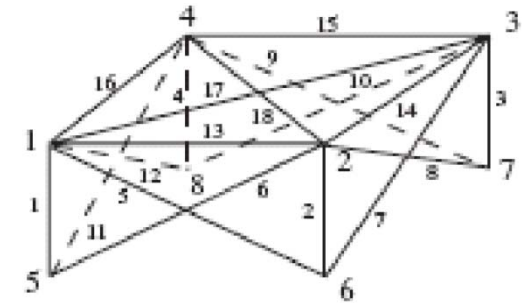
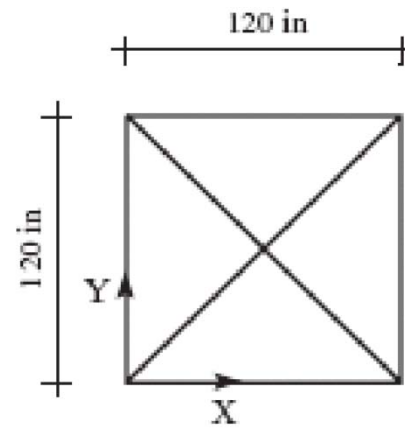
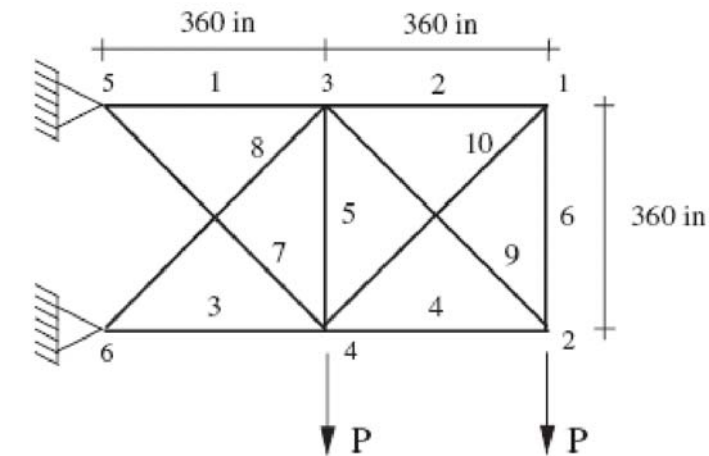
F10n



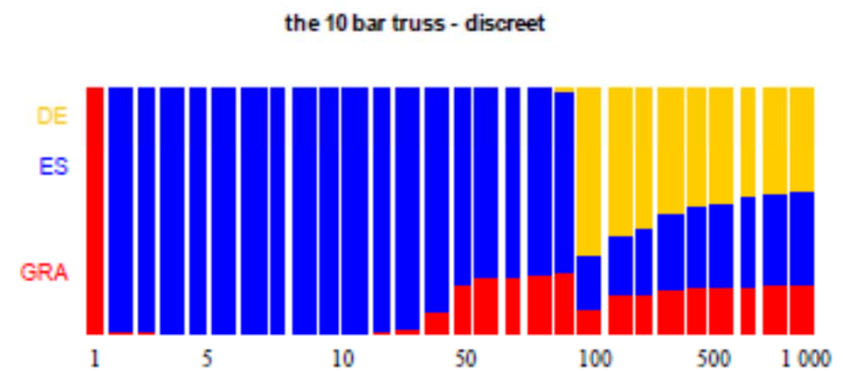
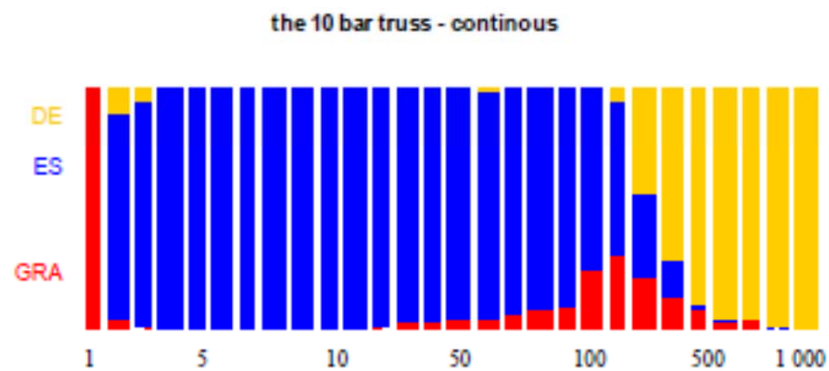
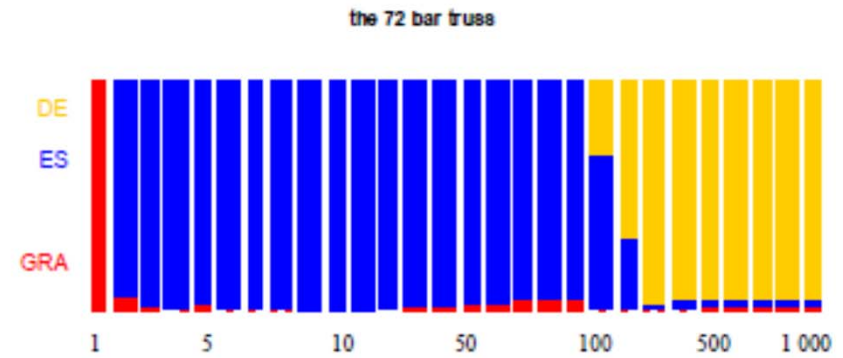
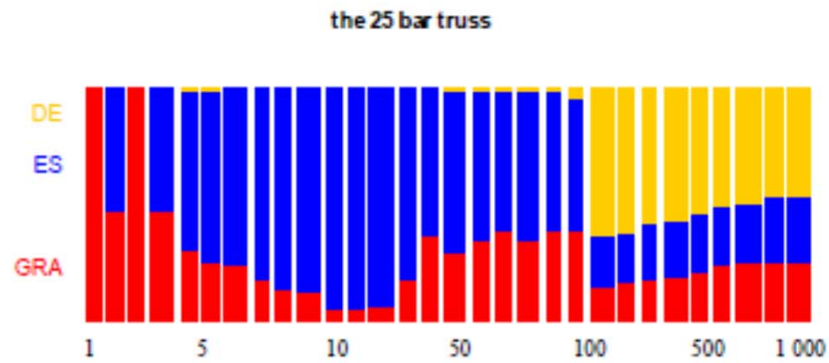
F15n



# Traditional sizing problems



# Proposed solution for more methods



# References on Traditional measures

- A.E. Eiben, J.E. Smith: Introduction to Evolutionary Computing, Springer (2008).
- A.E. Eiben, M. Jelasity: A critical note on experimental research methodology in EC, Proceedings of the 2002 Congress on Evolutionary Computation, 2002. CEC '02.
- Thomas Bartz-Beielstein: Experimental Research in Evolutionary Computation - The New Experimentalism. Springer, Berlin, 2006.
- Thomas Bartz-Beielstein web-page:  
<http://ls11-www.informatik.uni-dortmund.de/people/tom/>



**A humble plea.** Please feel free to e-mail any suggestions, errors and typos to **`matej.leps@fsv.cvut.cz`**.

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