Constrained Optimization

- To solve practical problems we need to know how to solve *constrained* problems

	With constraints	Without constraints
Objective function	Constrained optimization	Unconstrained optimization
Without objective function	Constraint satisfactory problem	No problem

- How to use good solutions that violate some constraints?
- Several methodologies, usually from mathematical programming area

Single-objective methods

- **Death penalty approach**
  - Solution violating constraint is withdrawn from search
  - Simplest solution
  - Can be very ineffective => sometimes it is a problem to satisfy constraints
  - Often combined with repair operators [5] or problem-dependent solution [4]

Single-objective methods

- **Penalty functions**
  - Changes value of objective function based on satisfying constraints (± Lagrange multipliers)
  - Division:
    - *Inner*: penalizes valid solutions closed to the constraints' boundary => worst solution that death penalty
    - *Outer*: penalizes only violating of constraints

Penalty functions

- **General description:**

$$f_f(\mathbf{x}) = f(\mathbf{x}) + Q(\mathbf{x}),$$

where

$$Q(\mathbf{x}) = \lambda(\tau) \sum_{j=1}^{ne} g_j(\mathbf{x})^{\alpha} + \lambda(\tau) \sum_{j=ne+1}^{ne+ni} \min[0, g_j(\mathbf{x})^{\beta}] \ ,$$

- α and β are usually 1 or 2
- $\lambda(\tau)$ is usually increasing with time τ => so-called *dynamic* penalty (oppositely to *static*)
- See [2] and [3] for more information

Multi-objective methods

- Modern method to solve constrained problems proposed in [1]
- *Does not change* value of objective function
- Constraints are added as another objectives:

  optimization problem with constraints =>

  multi-objective problem without constraints
- Thanks to Pareto dominance – solutions are compared based on constraints violation
- Often much easier formulation of the problem
- Computational difficulty => actual research

References

[1] C. A. C. Coello, *Constraint-handling using an evolutionary multiobjective optimization technique,* Civil Engineering and Environmental Systems 17 (2000), 319-346.

[2] C. A. C. Coello, *Treating Constraints as Objectives for Single-Objective Evolutionary Optimization,* Engineering Optimization 32 (2000), no. 3, 275-308.

[3] M. Lepš and Z. Bittnar, *Optimization of RC beams using genetic algorithm,* Advances in Computational Engineering & Sciences (S. N. Atluri, T. Nishioka, and M. Kikuchi, eds.), Tech Science Press, 2001.

References

[4] Z. Michalewicz and G. Nazhiyath, *Genocop III: A co-evolutionary algorithm for numerical optimization problems with nonlinear constraints,* Proceedings of the Second IEEE International Conference on Evolutionary Computation (D. B. Fogel, ed.), IEEE Press, 1995, pp. 647-651.

[5] M. Schoenauer and Z. Michalewicz, *Boundary operators for constrained parameter optimization problems,* Proceedings of the *7th* International Conference on Genetic Algorithms (East Lansing, Michigan), July 19-23,1997, pp. 320-329.

A humble plea. Please feel free to e-mail any suggestions, errors and typos to **matej.leps@fsv.cvut.cz**.

Date of the last version: 15.4.2008
Version: 001