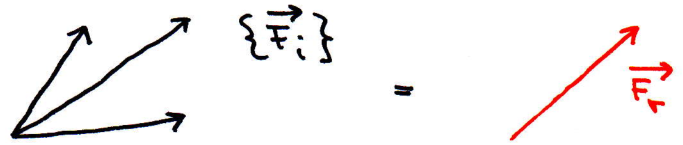
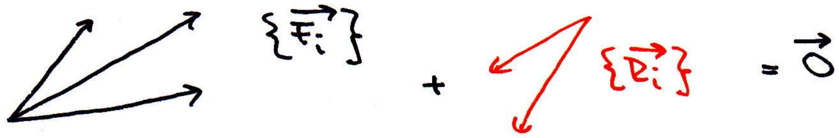
Teoretické okénko (a částečné opakování z minula)

Typy úloh:

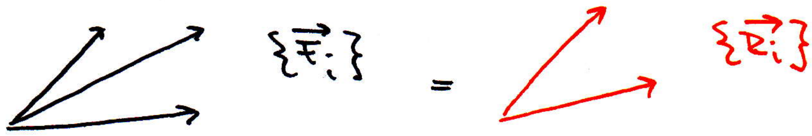
a) **Výsledný účinek** - nahrazení svazku sil jedinou silou se stejným účinkem (výslednice)



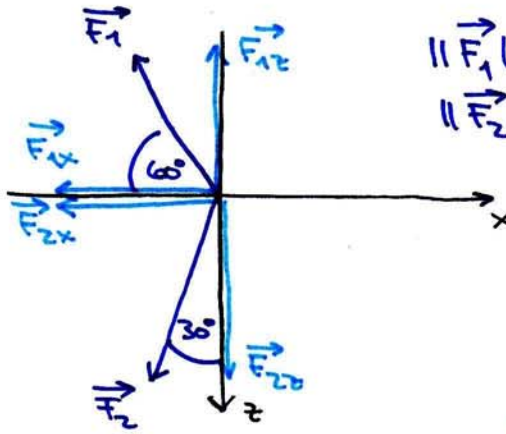
b) **Úloha rovnováhy** - vyrušení účinku svazku sil jiným svazkem sil, celkový účinek (na bod / těleso / atp.) je nulový.



c) **Úloha ekvivalence** - nahrazení účinku svazku sil jiným svazkem sil.



Př. Naleznete výslednici zadaného svazku sil.



$\|\vec{F_1}\| = 7\,kN$

$\|\vec{F_2}\| = 14\,kN$

$\|\vec{F_{1x}}\| = \|\vec{F_1}\| \cdot \cos 60° = 7 \cdot \frac{1}{2} = 3{,}5\,kN$

$\|\vec{F_{1z}}\| = \|\vec{F_1}\| \cdot \sin 60° = 7 \cdot \frac{\sqrt{3}}{2} = 3{,}5 \cdot \sqrt{3}\,kN$

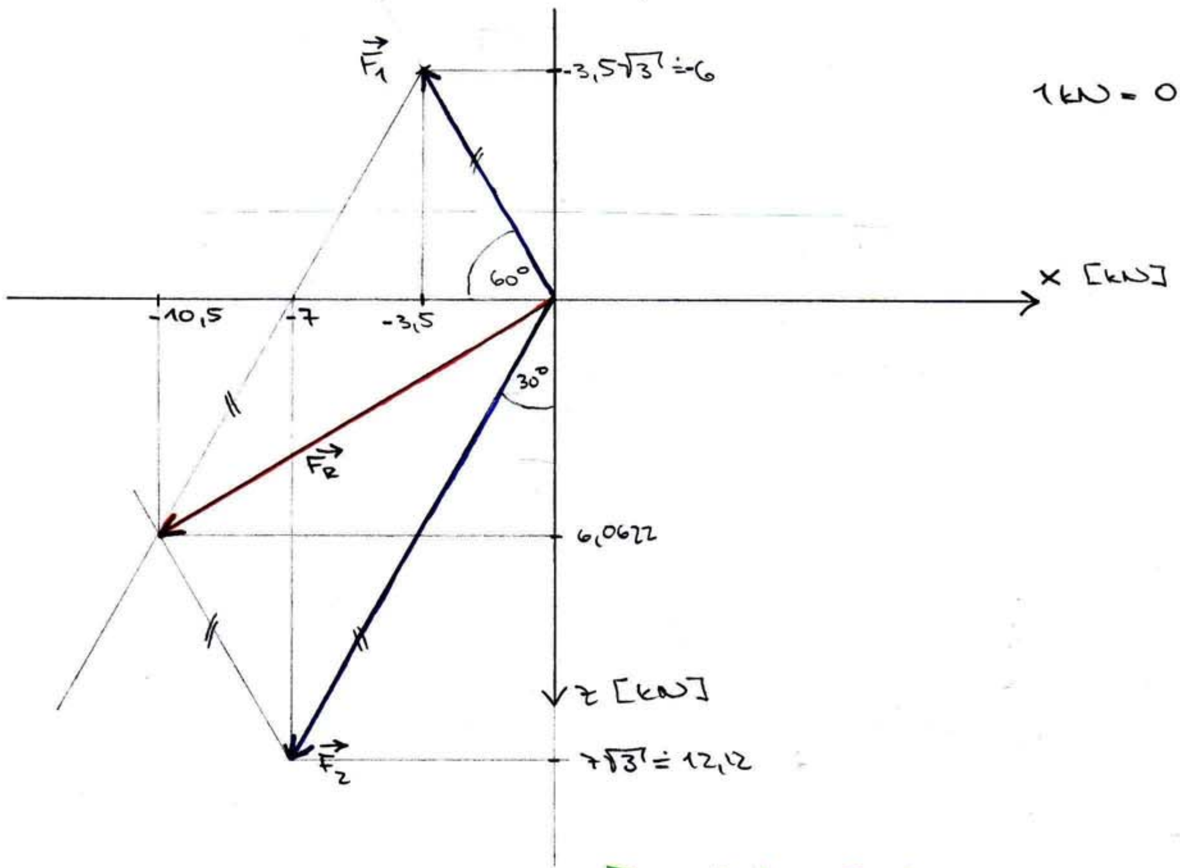
poznámka: selským rozumem zkontroluji, že $\|\vec{F_{1z}}\|$ je jasně větší než $\|\vec{F_{1x}}\|$ ($\vec{F_1}$ se více přimyká k ose z)

$\|\vec{F_{2x}}\| = \|\vec{F_2}\| \cdot \sin 30° = 14 \cdot \frac{1}{2} = 7\,kN$

$\|\vec{F_{2z}}\| = \|\vec{F_2}\| \cdot \cos 30° = 14 \cdot \frac{\sqrt{3}}{2} = 7\sqrt{3}\,kN$

$\rightarrow$: $F_{Rx} = -3{,}5 - 7 = -10{,}5\,kN$

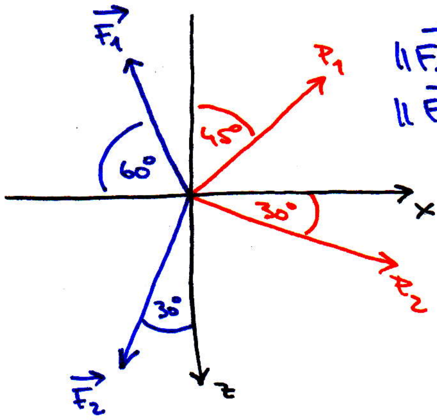
$\downarrow$: $F_{Rz} = -3{,}5\sqrt{3} + 7\sqrt{3} = 6{,}0622\,kN$



1kN = 0,5cm

Poznámka: Směry, ve kterých nasčítávám výslednici, si mohu zvolit. Nesmím ale zapomenout na orientaci souřadného systému při vynášení výslednice!

Př. **Nahraďte svazek sil $\vec{F_i}$ jiným svazkem sil $\vec{P_i}$.**



$\|\vec{F_1}\| = 7\,kN$

$\|\vec{F_2}\| = 14\,kN$

$$\boxed{\vec{F_i} = \vec{P_i}}$$

$\leftrightarrow$: $-F_{1x} - F_{2x} = P_{1x} + P_{2x}$ (rovnice ve směru osy x)

$$-F_1 \cdot \cos 60° - F_2 \cdot \sin 30° = P_1 \cdot \sin 45° + P_2 \cdot \cos 30°$$

$$-7 \cdot 0{,}5 - 14 \cdot 0{,}5 = P_1 \cdot \frac{\sqrt{2}}{2} + P_2 \cdot \frac{\sqrt{3}}{2}$$

$$\boxed{0{,}707\,P_1 + 0{,}866\,P_2 = -10{,}5}$$

$\updownarrow$: $-F_{1z} + F_{2z} = -P_{1z} + P_{2z}$ (rovnice ve směru osy z)

$$-F_1 \cdot \sin 60° + F_2 \cdot \cos 30° = -P_1 \cdot \cos 45° + P_2 \cdot \sin 30°$$

$$-7 \cdot \frac{\sqrt{3}}{2} + 14 \cdot \frac{\sqrt{3}}{2} = -P_1 \cdot \frac{\sqrt{2}}{2} + P_2 \cdot \frac{1}{2}$$

$$\boxed{-0{,}707\,P_1 + 0{,}5\,P_2 = 6{,}062}$$

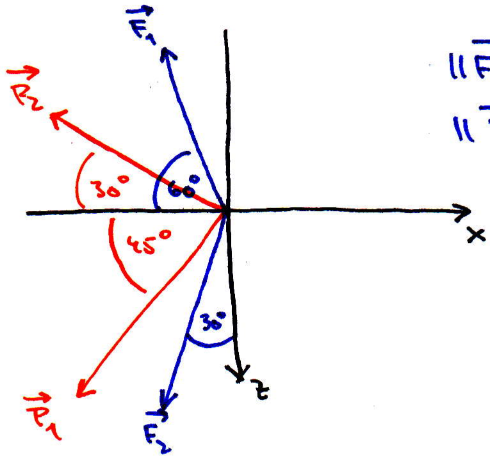
soustava 2 rovnic o 2 neznámých

$$0{,}707\,P_1 + 0{,}866\,P_2 = -10{,}5$$
$$-0{,}707\,P_1 + 0{,}5\,P_2 = 6{,}062$$
$$1{,}366\,P_2 = -4{,}438$$
$$P_2 = -\frac{4{,}438}{1{,}366} = \underline{\underline{-3{,}249\,kN}}$$

$$0{,}707\,P_1 + 0{,}866 \cdot (-3{,}249) = -10{,}5$$
$$0{,}707\,P_1 = -10{,}5 + 3{,}249 \cdot 0{,}866$$
$$P_1 = -\frac{7{,}686}{0{,}707} = \underline{\underline{-10{,}871\,kN}}$$

Kontrola: dosadím do rovnic vypočtené P_1 a P_2 a zkontroluji, zda-li mi na obou stranách rovnic vychází stejná čísla. To mi ověří správnost řešení soustavy rovnic.

Př. Uveďte zadaný svazek sil $\vec{F_i}$ svazkem sil $\vec{P_i}$ do rovnováhy.



$\|\vec{F_1}\| = 7\,kN$

$\|\vec{F_2}\| = 14\,kN$

$$\boxed{\vec{F_i} + \vec{P_i} = \vec{0}}$$

$\rightarrow: -F_1 \cdot \cos 60° - F_2 \cdot \sin 30° - P_2 \cdot \cos 30° - P_1 \cdot \cos 45° = 0$

$-7 \cdot \frac{1}{2} - 14 \cdot \frac{1}{2} - P_2 \cdot \frac{\sqrt{3}}{2} - P_1 \cdot \frac{\sqrt{2}}{2} = 0$

$$\boxed{0{,}707\, P_1 + 0{,}866\, P_2 = -10{,}5}$$

$\downarrow\; -F_1 \cdot \sin 60° + F_2 \cdot \cos 30° + P_1 \cdot \sin 45° - P_2 \cdot \sin 30° = 0$

$-7 \cdot \frac{\sqrt{3}}{2} + 14 \cdot \frac{\sqrt{3}}{2} + P_1 \cdot \frac{\sqrt{2}}{2} - P_2 \cdot \frac{1}{2} = 0$

$$\boxed{-0{,}707\, P_1 + 0{,}5\, P_2 = 6{,}062}$$

$0{,}707\, P_1 + 0{,}866\, P_2 = -10{,}5$

$\underline{-0{,}707\, P_1 + 0{,}5\, P_2 = 6{,}062}$

$1{,}366\, P_2 = -4{,}438$

$P_2 = \frac{-4{,}438}{1{,}366} = \underline{\underline{-3{,}249\, kN}}$

$0{,}707\, P_1 + 0{,}866\,(-3{,}249) = -10{,}5$

$0{,}707\, P_1 = -10{,}5 + 3{,}249 \cdot 0{,}866$

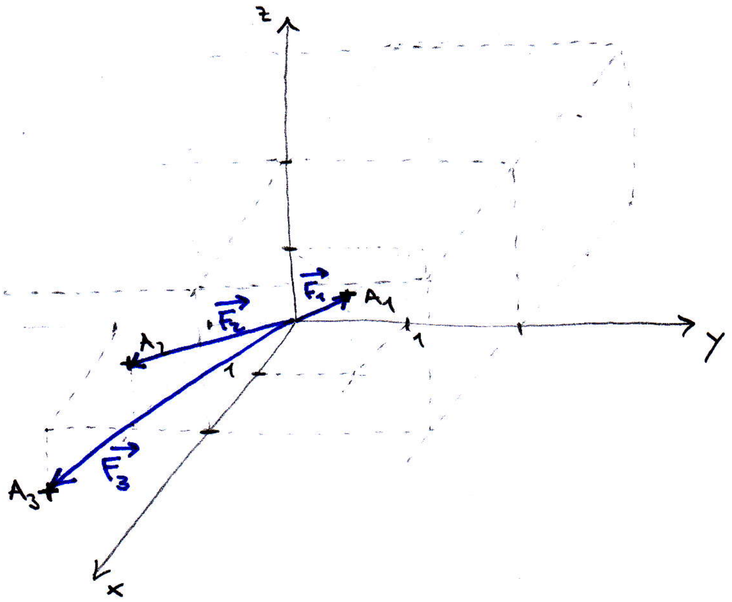
$P_1 = -\frac{7{,}686}{0{,}707} = -10{,}871\, kN$

kontrola správnosti řešení soustavy lin. rovnic:

$0{,}707 \cdot (-10{,}871) + 0{,}866 \cdot (-3{,}249) = -10{,}5$

$-10{,}499 \doteq -10{,}5$ ✓ ok

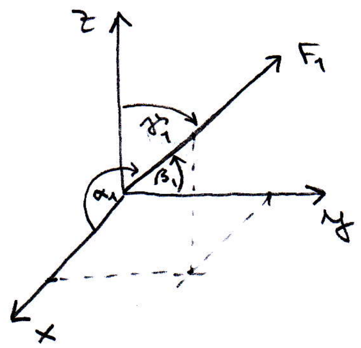
Př. **Naleznete výslednici zadaného svazku sil.**



$F_1 = 5\,kN$, $A_1\,[1,1,1]$
$F_2 = 10\,kN$, $A_2\,[-2,-1,1]$
$F_3 = 8\,kN$, $A_3\,[2,-2,-1]$

Nalezení směrových cosinů

1) pro F_1



$$|\vec{e_1}| = |OA_1| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

$$\cos\alpha_1 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cos\beta_1 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cos\gamma_1 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

kontrola: $\cos^2\alpha_1 + \cos^2\beta_1 + \cos^2\gamma_1 = 1$

$\frac{3}{9} + \frac{3}{9} + \frac{3}{9} = 1$ ✓ o.k.

2) pro F_2

$$|\vec{e_2}| = |OA_2| = \sqrt{(-2)^2+(-1)^2+1^2} = \sqrt{6}$$

$\cos\alpha_2 = \frac{-2}{\sqrt{6}} = -\frac{\sqrt{6}}{3}$, $\cos\beta_2 = \frac{-1}{\sqrt{6}} = -\frac{\sqrt{6}}{6}$, $\cos\gamma_2 = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$

kontrola: $\frac{4}{6} + \frac{1}{6} + \frac{1}{6} = 1$ ✓ o.k.

3) pro F_3

$$|\vec{e_3}| = |OA_3| = \sqrt{2^2+(-2)^2+(-1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

$\cos\alpha_3 = \frac{2}{3}$, $\cos\beta_3 = \frac{-2}{3}$, $\cos\gamma_3 = \frac{-1}{3}$

kontrola: $\frac{4}{9} + \frac{4}{9} + \frac{1}{9} = 1$ ✓ o.k.

Výslednice F_R:

$$F_{Rx} = \sum F_{ix} = F_1 \cdot \cos\alpha_1 + F_2 \cdot \cos\alpha_2 + F_3 \cdot \cos\alpha_3$$

$$F_{Rx} = \sum F_{ix} = 5 \cdot \frac{\sqrt{3}}{3} + 10 \cdot \frac{\sqrt{6}}{3} + 8 \cdot \frac{2}{3} = \underline{\underline{16{,}385\,kN}}$$

$$F_{Ry} = \sum F_{iy} = \sum F_i \cdot \cos\beta_i = F_1 \cdot \cos\beta_1 + F_2 \cdot \cos\beta_2 + F_3 \cdot \cos\beta_3$$

$$F_{Ry} = 5 \cdot \frac{\sqrt{3}}{3} + 10 \cdot \frac{-\sqrt{6}}{6} + 8 \cdot \frac{-2}{3} = \underline{\underline{-6{,}529\,kN}}$$

$$F_{Rz} = \sum F_{iz} = \sum F_i \cdot \cos\gamma_i = F_1 \cdot \cos\gamma_1 + F_2 \cdot \cos\gamma_2 + F_3 \cdot \cos\gamma_3$$

$$F_{Rz} = 5 \cdot \frac{\sqrt{3}}{3} + 10 \cdot \frac{\sqrt{6}}{6} + 8 \cdot \frac{-1}{3} = \underline{\underline{4{,}303\,kN}}$$

a její směrové cosiny a směrové úhly:

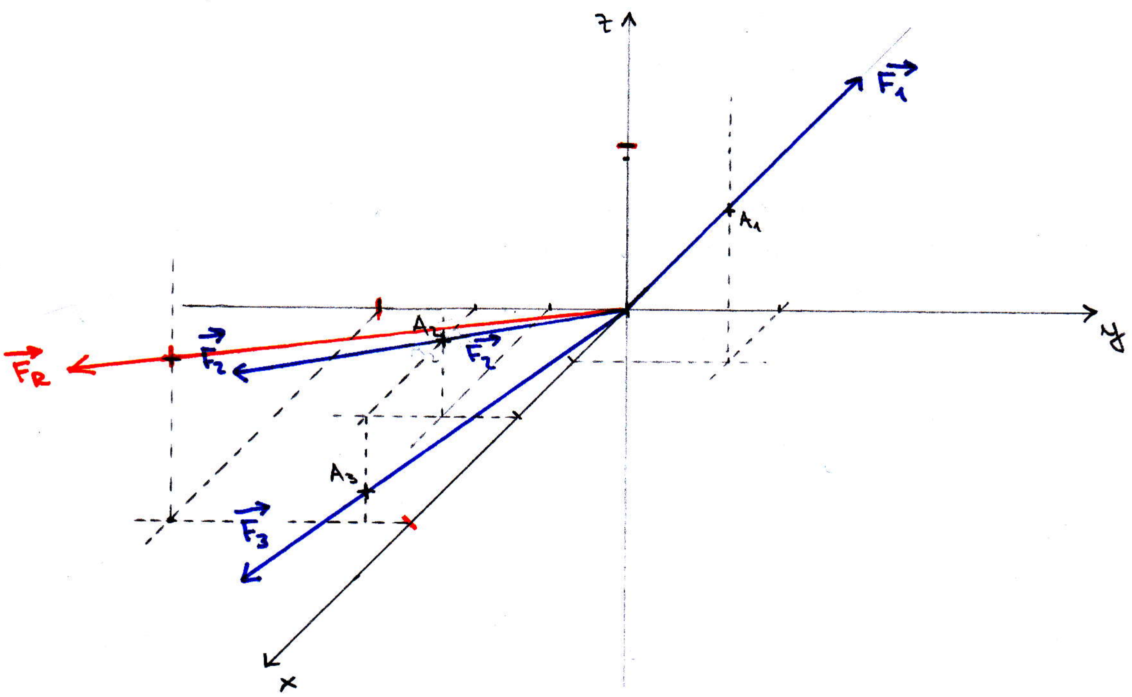
$$\cos\alpha_R = \frac{F_{Rx}}{F_R} = \frac{16{,}385}{18{,}155} = \underline{\underline{0{,}9025}} \rightarrow \alpha_R = \underline{\underline{25{,}511}}^\circ$$

velikost výslednice

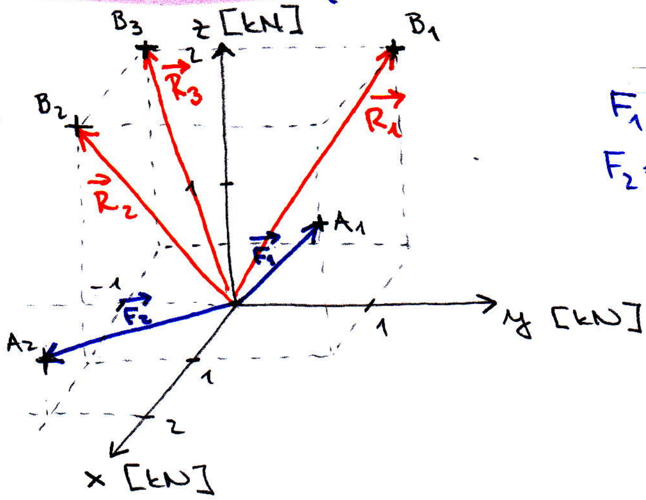
$$F_R = \sqrt{16{,}385^2 + (-6{,}529)^2 + 4{,}303^2} = \underline{\underline{18{,}155\,kN}}$$

$$\cos\beta_R = \frac{F_{Ry}}{F_R} = \frac{-6{,}529}{18{,}155} = \underline{\underline{-0{,}3596}} \rightarrow \beta_R = \underline{\underline{111{,}077}}^\circ$$

$$\cos\gamma_R = \frac{F_{Rz}}{F_R} = \frac{4{,}303}{18{,}155} = \underline{\underline{0{,}2370}} \rightarrow \gamma_R = \underline{\underline{76{,}29}}^\circ$$



Př: Uveďte svazek sil $\vec{F_i}$ svazkem sil $\vec{R_i}$ do rovnováhy.



$F_1 = 5\,kN,\ A_1\ [1,1,1]$

$F_2 = 7\,kN,\ A_2\ [2,-1,1]$

$B_1\ [-1,1,2]$

$B_2\ [1,-1,2]$

$B_3\ [-1,-1,2]$

Směrové cosiny:

$F_1 : |l_1| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\cos\alpha_1 = \frac{1}{\sqrt{3}},\quad \cos\beta_1 = \frac{1}{\sqrt{3}},\quad \cos\gamma_1 = \frac{1}{\sqrt{3}}$

$F_2 : |l_2| = \sqrt{2^2+(-1)^2+1^2} = \sqrt{6}$

$\cos\alpha_2 = \frac{2}{\sqrt{6}},\quad \cos\beta_2 = \frac{-1}{\sqrt{6}},\quad \cos\gamma_2 = \frac{1}{\sqrt{6}}$

$R_1 : |l_3| = \sqrt{(-1)^2+1^2+2^2} = \sqrt{6}$

$\cos\alpha_3 = \frac{-1}{\sqrt{6}},\quad \cos\beta_3 = \frac{1}{\sqrt{6}},\quad \cos\gamma_3 = \frac{2}{\sqrt{6}}$

$R_2 : |l_4| = \sqrt{1^2+(-1)^2+2^2} = \sqrt{6}$

$\cos\alpha_4 = \frac{1}{\sqrt{6}},\quad \cos\beta_4 = \frac{-1}{\sqrt{6}},\quad \cos\gamma_4 = \frac{2}{\sqrt{6}}$

$R_3 : |l_5| = \sqrt{(-1)^2+(-1)^2+2^2} = \sqrt{6}$

$\cos\alpha_5 = \frac{-1}{\sqrt{6}},\quad \cos\beta_5 = \frac{-1}{\sqrt{6}},\quad \cos\gamma_6 = \frac{2}{\sqrt{6}}$

Podmínky rovnováhy:

$\swarrow : \quad 5\cdot\frac{1}{\sqrt{3}} + 7\cdot\frac{2}{\sqrt{6}} + R_1\frac{-1}{\sqrt{6}} + R_2\frac{1}{\sqrt{6}} + R_3\frac{-1}{\sqrt{6}} = 0$

$-\frac{R_1}{\sqrt{6}} + \frac{R_2}{\sqrt{6}} - \frac{R_3}{\sqrt{6}} = -8{,}6022 \quad |\cdot\sqrt{6}$

$-R_1 + R_2 - R_3 = -21{,}071$

$\leftrightarrow : \quad 5\cdot\frac{1}{\sqrt{3}} + 7\cdot\frac{-1}{\sqrt{6}} + R_1\frac{1}{\sqrt{6}} + R_2\frac{-1}{\sqrt{6}} + R_3\frac{-1}{\sqrt{6}} = 0$

$\frac{R_1}{\sqrt{6}} - \frac{R_2}{\sqrt{6}} - \frac{R_3}{\sqrt{6}} = -0{,}02901 \quad |\cdot\sqrt{6}$

$R_1 - R_2 - R_3 = -0{,}07106$

↕ : $5\cdot\frac{1}{\sqrt{3}} + 7\cdot\frac{1}{\sqrt{6}} + R_1\frac{2}{\sqrt{6}} + R_2\frac{2}{\sqrt{6}} + R_3\frac{2}{\sqrt{6}} = 0$

$$\frac{2}{\sqrt{6}}(R_1+R_2+R_3) = -5{,}7445 \quad \Big|\cdot\frac{\sqrt{6}}{2}$$

$$R_1+R_2+R_3 = -7{,}0355$$

$$\begin{pmatrix} -1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -21{,}071 \\ -0{,}07106 \\ -7{,}0355 \end{Bmatrix}$$

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & -21{,}071 \\ 1 & -1 & -1 & -0{,}07106 \\ 1 & 1 & 1 & -7{,}0355 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 1 & -1 & -21{,}071 \\ 0 & 0 & -2 & -21{,}142 \\ 0 & 2 & 0 & -28{,}107 \end{array}\right) \begin{array}{l} \\ (1)+(2) \\ (1)+(3) \end{array}$$

$$R_2 = \frac{-28{,}107}{2} = \underline{\underline{-14{,}054\ \text{kN}}}$$

$$R_3 = \frac{21{,}142}{2} = \underline{\underline{10{,}571\ \text{kN}}}$$

$$-R_1 - 14{,}053 - 10{,}571 = -21{,}071$$

$$R_1 = \underline{\underline{-3{,}553\ \text{kN}}}$$

Poznámka : Řešení soustavy rovnic

1. krok : eliminuji vše v 1. sloupci pod 1. řádkem

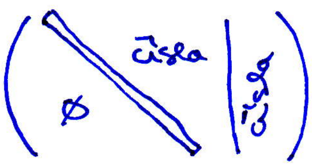
$$\left(\begin{array}{ccc|c} a_1 & a_2 & a_3 & f_1 \\ b_1 & b_2 & b_3 & f_2 \\ c_1 & c_2 & c_3 & f_3 \end{array}\right) \begin{array}{l} \cdot(-b_1) \\ \cdot(a_1) \end{array} \left(\begin{array}{ccc|c} a_1 & a_2 & a_3 & f_1 \\ -a_1\cdot b_1 + a_1 b_1 = 0 & -a_2 b_1 + a_1 b_2 & -a_3 b_1 + a_1 b_3 & -b_1 f_1 + a_1 f_2 \\ -a_1 c_1 + a_1 c_1 = 0 & -a_2 c_1 + a_1 c_2 & -a_3 c_1 + a_1 c_3 & -c_1 f_1 + a_1 f_3 \end{array}\right)$$

2. krok : stejným způsobem eliminuji vše ve 2. sloupci pod 2. řádkem

⋮

(n-2). krok : eliminuji vše v předposledním sloupci pod předp. řádkem

→ dostávám nuly pod diagonálou



zpětnou substitucí (dosazováním) získám postupně všechny potřebné hodnoty - začínám od posl. řádku a končím 1. řádkem.

Př. Nahraďte svazek sil $\vec{F_i}$ svazkem sil $\vec{P_i}$.

$F_1 = 6\,kN$ $\quad B_1\,[-5,0,6]$

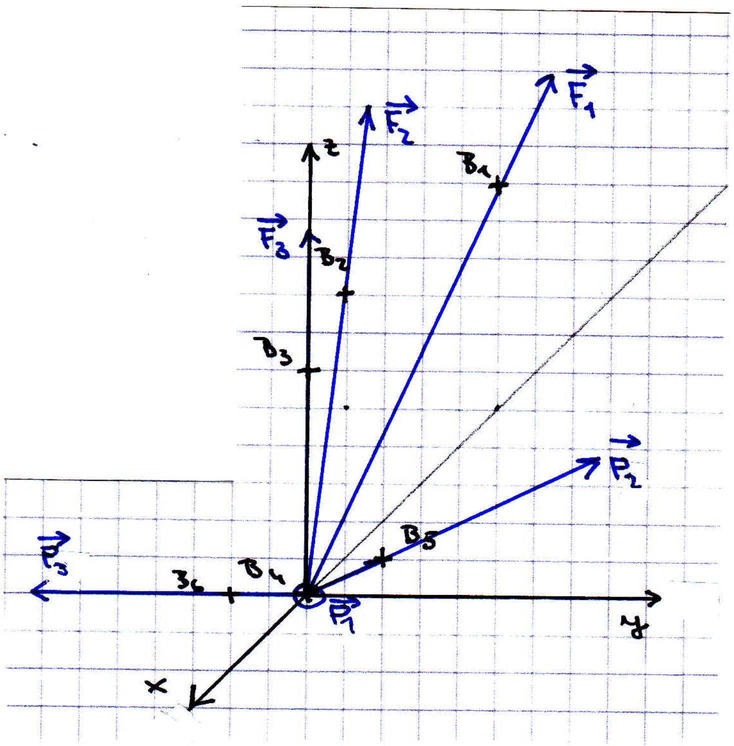
$F_2 = 8\,kN$ $\quad B_2\,[-5,-4,3]$

$F_3 = 12\,kN$ $\quad B_3\,[0,0,6]$

$P_1 = ?$ $\quad B_4\,[1,1,1]$

$P_2 = ?$ $\quad B_5\,[-1,1,0]$

$P_3 = ?$ $\quad B_6\,[1,-1,1]$



Směrové cosiny:

síla	$\lvert l_i \rvert$	$\cos\alpha_i$	$\cos\beta_i$	$\cos\gamma_i$
F_1	$\sqrt{61}$	$-\frac{5}{\sqrt{61}}$	0	$\frac{6}{\sqrt{61}}$
F_2	$\sqrt{50}$	$-\frac{5}{\sqrt{50}}$	$-\frac{4}{\sqrt{50}}$	$\frac{3}{\sqrt{50}}$
F_3	6	0	0	1
P_1	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
P_2	$\sqrt{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	0
P_3	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$

Podmínky ekvivalence: $\{\vec{F_i}\} = \{\vec{P_i}\}$

$$\swarrow\!\!\!\nearrow:\ 6\cdot\frac{-5}{\sqrt{61}} + 8\cdot\frac{-5}{\sqrt{50}} + 12\cdot 0 = P_1\cdot\frac{\sqrt{3}}{3} + P_2\,\frac{-\sqrt{2}}{2} + P_3\,\frac{\sqrt{3}}{3}$$

$$0{,}57735\,P_1 - 0{,}70711\,P_2 + 0{,}57735\,P_3 = -9{,}4980$$

$$\leftrightarrow:\ 6\cdot 0 + 8\cdot\frac{-4}{\sqrt{50}} + 12\cdot 0 = P_1\,\frac{\sqrt{3}}{3} + P_2\,\frac{\sqrt{2}}{2} + P_3\,\frac{-\sqrt{3}}{3}$$

$$+0{,}57735\,P_1 + 0{,}70711\,P_2 - 0{,}57735\,P_3 = -4{,}5255$$

$$\updownarrow:\ 6\cdot\frac{6}{\sqrt{61}} + 8\cdot\frac{3}{\sqrt{50}} + 12\cdot 1 = P_1\cdot\frac{\sqrt{3}}{3} + P_2\cdot 0 + P_3\cdot\frac{\sqrt{3}}{3}$$

$$0{,}57735\,P_1 + 0{,}57735\,P_3 = 20{,}003$$

$$\begin{array}{c} \left(\begin{array}{ccc|c} 0{,}57735 & -0{,}70711 & 0{,}57735 & -9{,}4980 \\ 0{,}57735 & +0{,}70711 & -0{,}57735 & -4{,}5255 \\ 0{,}57735 & 0 & 0{,}57735 & 20{,}003 \end{array}\right) \end{array} \begin{array}{l} \\ \cdot(-1) \\ \cdot(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 0{,}57735 & -0{,}70711 & 0{,}57735 & -9{,}4980 \\ -0{,}57735 & -0{,}70711 & +0{,}57735 & 4{,}5255 \\ -0{,}57735 & 0 & -0{,}57735 & -20{,}003 \end{array}\right) \begin{array}{l} \text{sečtu (1) a (2)} \\ \text{sečtu (1) a (3)} \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 0{,}57735 & -0{,}70711 & 0{,}57735 & -9{,}4980 \\ 0 & -1{,}41422 & 1{,}1547 & -4{,}9725 \\ 0 & -0{,}70711 & 0 & -29{,}501 \end{array}\right)$$

$$-0{,}70711\, P_2 = -29{,}501$$

$$\underline{\underline{P_2 = 41{,}721 \text{ kN}}}$$

$$-1{,}41422\, P_2 + 1{,}1547\, P_3 = -4{,}9725$$

$$1{,}1547\, P_3 = -4{,}9725 + 1{,}41422 \cdot 41{,}721$$

$$\underline{\underline{P_3 = 46{,}792 \text{ kN}}}$$

$$0{,}57735\, P_1 - 0{,}70711\, P_2 + 0{,}57735\, P_3 = -9{,}4980$$

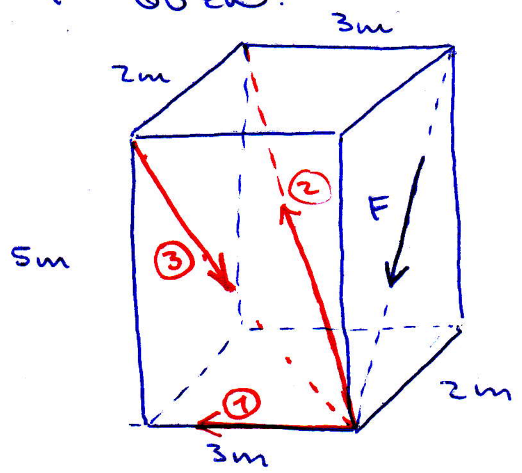
$$\underline{\underline{P_1 = -12{,}145 \text{ kN}}}$$

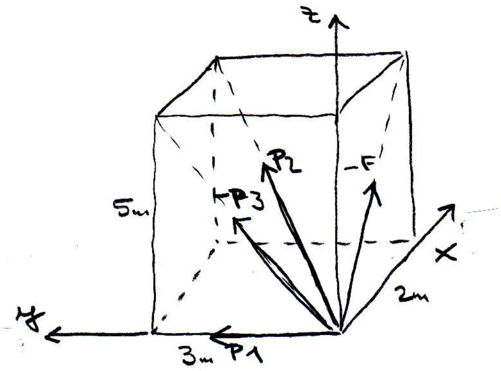
Kontrolu provedu dosazením hodnot do 2. a 3. řádku neupravené rovnice. Tímto ověřím správnost řešení soustavy, nikoliv sestavení rovnic!

Příklady na procvičení:

(Př.) Sílu F nahraďte silami na paprscích 1, 2 a 3.

$F = 60\,kN$.





síla	bod, kt. síla prochází			$\lvert e_i \rvert$	$\cos\alpha_i$	$\cos\beta_i$	$\cos\gamma_i$	F_i
F	2	0	5	$\sqrt{29}$	$\frac{2}{\sqrt{29}}$	0	$\frac{5}{\sqrt{29}}$	−60
1	0	3	0	3	0	1	0	P_1
2	2	3	5	$\sqrt{38}$	$\frac{2}{\sqrt{38}}$	$\frac{3}{\sqrt{38}}$	$\frac{5}{\sqrt{38}}$	P_2
3	0	3	5	$\sqrt{34}$	0	$\frac{3}{\sqrt{34}}$	$\frac{5}{\sqrt{34}}$	$-P_3$

Úloha ekvivalence: $\{\vec{F}\} = \{\vec{P_i}\}$

$$\swarrow\!\!\nearrow: \; -60\cdot\frac{2}{\sqrt{29}} = 0 + P_2\cdot\frac{2}{\sqrt{38}} + 0 \quad\longrightarrow\quad P_2 = \underline{\underline{-68{,}682\,kN}}$$

$$\leftrightarrow: \; -60\cdot 0 = P_1 + P_2\cdot\frac{3}{\sqrt{38}} - P_3\cdot\frac{3}{\sqrt{34}}$$

$$\updownarrow: \; -60\cdot\frac{5}{\sqrt{29}} = P_1\cdot 0 + P_2\cdot\frac{5}{\sqrt{38}} - P_3\cdot\frac{5}{\sqrt{34}}$$

$$P_3\frac{5}{\sqrt{34}} = -68{,}682\cdot\frac{5}{\sqrt{38}} + 60\cdot\frac{5}{\sqrt{29}}$$

$$P_3 = \underline{\underline{0{,}00016703\,kN}}$$

$$P_1 = -P_2\cdot\frac{3}{\sqrt{38}} + P_3\frac{3}{\sqrt{34}} = \underline{\underline{34{,}425\,kN}}$$

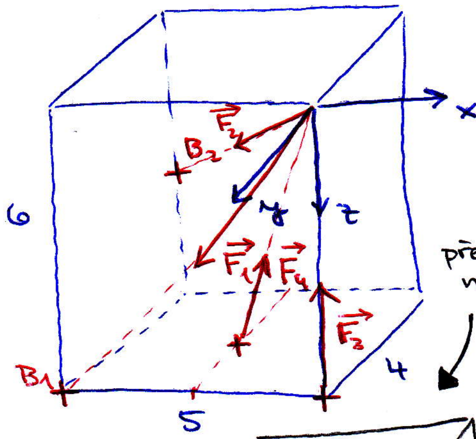
Př. Určete výslednici zadaného svazku sil.

$F_1 = 6\,kN$ | $B_1\,[-5;0;6]$

$F_2 = 8\,kN$ | $B_2\,[-5;-4;3]$

$F_3 = 12\,kN$ | $B_3\,[0;0;6]$

$F_4 = 5\,kN$ | $B_4\,[-2{,}5;-2;6]$



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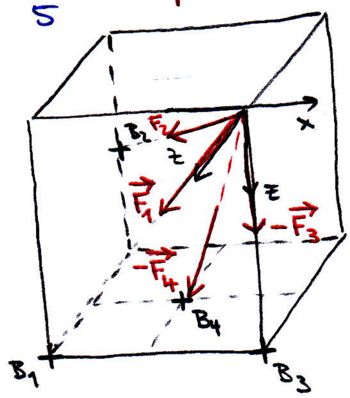

i	F_i	$\lvert e_i \rvert$	$\cos\alpha_i$	$\cos\beta_i$	$\cos\gamma_i$
1	6	$\sqrt{61}$	$-\frac{5}{\sqrt{61}}$	0	$\frac{6}{\sqrt{61}}$
2	8	$\sqrt{50}$	$-\frac{5}{\sqrt{50}}$	$-\frac{4}{\sqrt{50}}$	$\frac{3}{\sqrt{50}}$
3	−12	6	0	0	1
4	−5	$\sqrt{46{,}25}$	$\frac{-2{,}5}{\sqrt{46{,}25}}$	$\frac{-2}{\sqrt{46{,}25}}$	$\frac{6}{\sqrt{46{,}25}}$

$$F_{Rx} = 6\cdot\frac{-5}{\sqrt{61}} + 8\,\frac{-5}{\sqrt{50}} + 12\cdot 0 - 5\cdot\frac{-2{,}5}{\sqrt{46{,}25}} = -7{,}6599\,kN$$

$$F_{Ry} = 6\cdot 0 + 8\,\frac{-4}{\sqrt{50}} - 12\cdot 0 - 5\cdot\frac{-2}{\sqrt{46{,}25}} = -3{,}0551\,kN$$

$$F_{Rz} = 6\cdot\frac{6}{\sqrt{61}} + 8\,\frac{3}{\sqrt{50}} - 12\cdot 1 - 5\cdot\frac{6}{\sqrt{46{,}25}} = -8{,}4078\,kN$$

$$\|F_R\| = \sqrt{(-7{,}6599)^2 + (-3{,}0551)^2 + (-8{,}4078)^2} = 11{,}777\,kN$$

$$\cos\alpha_R = \frac{F_{Rx}}{\|F_R\|} = \frac{-7{,}6599}{11{,}777} = -0{,}65041 \rightarrow \alpha_R = 130{,}57°$$

$$\cos\beta_R = \frac{F_{Ry}}{\|F_R\|} = \frac{-3{,}0551}{11{,}777} = -0{,}25941 \longrightarrow \beta_R = 105{,}04°$$

$$\cos\gamma_R = \frac{F_{Rz}}{\|F_R\|} = \frac{-8{,}4078}{11{,}777} = -0{,}71392 \longrightarrow \gamma_R = 135{,}55°$$