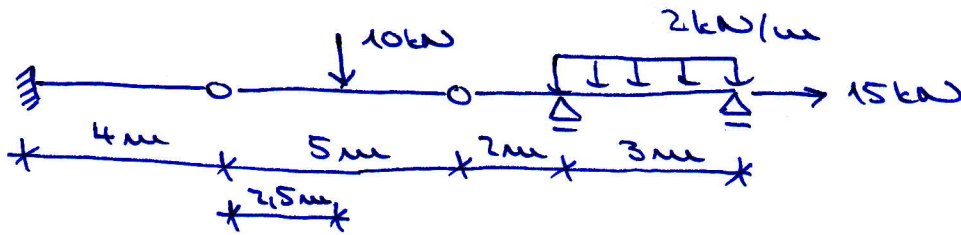


SMO1 - 7. cvičení

1.

ÚPOČET REAKCÍ SLOŽENÝCH SOUSTAV

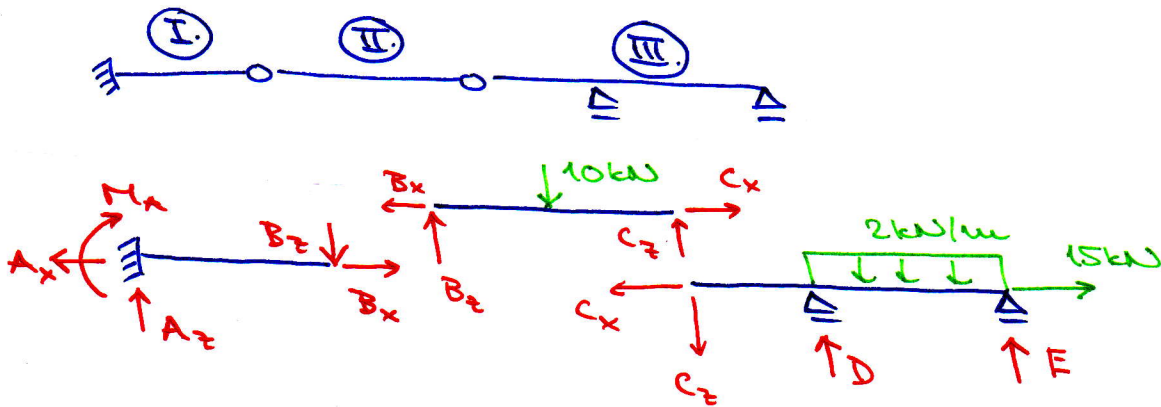


$$S = 3 \times 3 - 3 - 2 \times 2 - 2 \times 1 = 0$$

$$r_{ext} = 3 + 2 = 5 \geq 3$$

} staticky určitá konstrukce

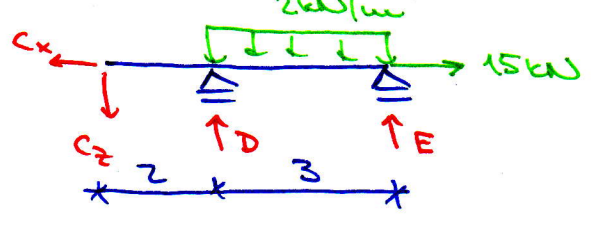
- Postup:
- konstrukci rozřešíme na jednotlivé desky a na každou zavedeme přibližně reakce za uvolněné vazby.
 - U konstrukce tohoto typu (tj. Gerberova nosníku) hledáme nesoucí a nesoucí desky, abychom našla místo, odkud máme začít počítat.



- Na desce I. máme zavedených 5 reakcí a z toho 3 nejsou horizontální.
 Na desce II. máme zavedené 4 reakce a z toho 2 nejsou horizontální.
 Na desce III. máme zavedené 4 reakce a z toho 3 nejsou horizontální.

→ Svislé reakce začneme počítat od desky II., která je nesoucí (podleprávné nejmenší reakce). Vodorovné reakce od desky III.

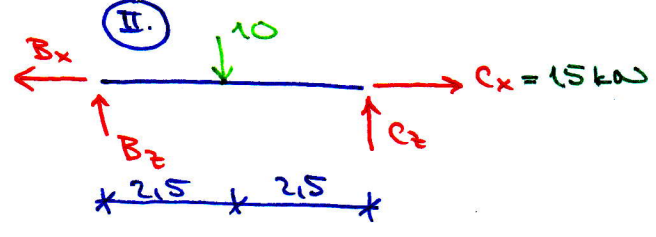
III.



→ : $-C_x + 15 = 0$, $C_x = 15 \text{ kN}$

že soustře' a ni momentové' podmínky už nic v tuto chvíli nepočítáme, proto se přesouváme na část II.

II.

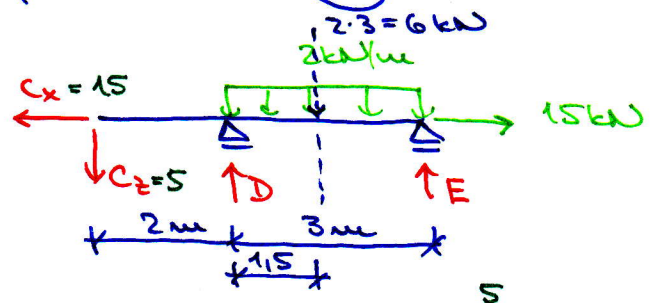



→ : $-B_x + C_x = 0$, $B_x = 15 \text{ kN}$

⊙ : $C_z \cdot 5 - 10 \cdot 2,5 = 0$, $C_z = 5 \text{ kN}$

↑ : $B_z + C_z - 10 = 0$, $B_z = 5 \text{ kN}$

zpět na část III.

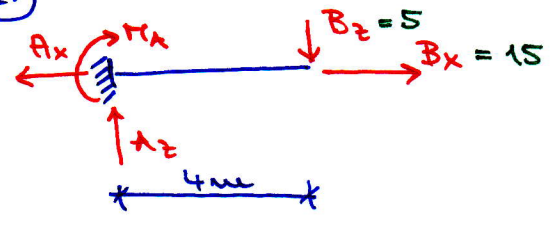


pozn.  spojitel' zatížen' nahrazují výslednicí působící u téžisti obrázce o velikosti plochy (tzn. zde obdelník).

⊙ : $E \cdot 3 - 6 \cdot 1,5 + C_z \cdot 2 = 0$, $E = -0,333 \text{ kN}$

↑ : $-C_z + D + E - 6 = 0$, $D = 11,333 \text{ kN}$

část I.

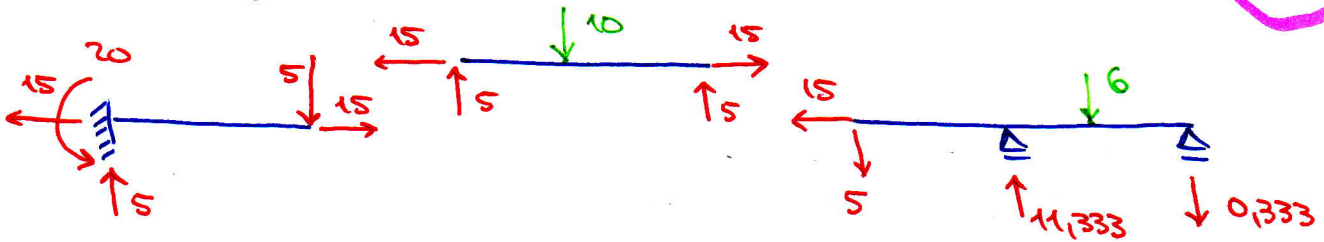


→ : $B_x - A_x = 0$, $A_x = 15 \text{ kN}$

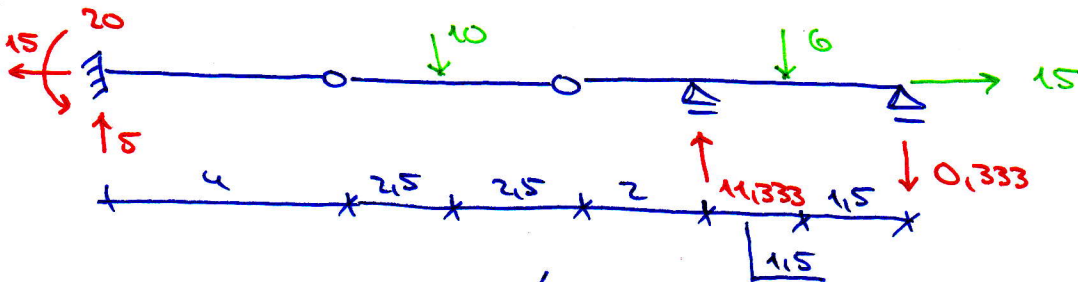
↑ : $A_z - B_z = 0$, $A_z = 5 \text{ kN}$

⊙ : $-M_A - B_z \cdot 4 = 0$, $M_A = -20 \text{ kNm}$

Skutčné pôsobení:



Podľa konštrukciu "složiť" zase dohromady, reakcie vo vnútorných väzboch sa mi vyrušia. Podmienky rovnováhy na celej konštrukcii využijú pro kontrolu správnosti výsledku.

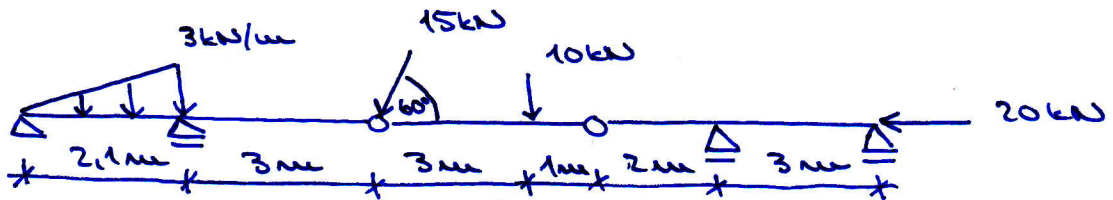


$\rightarrow : -15 + 15 = 0 \quad \checkmark$

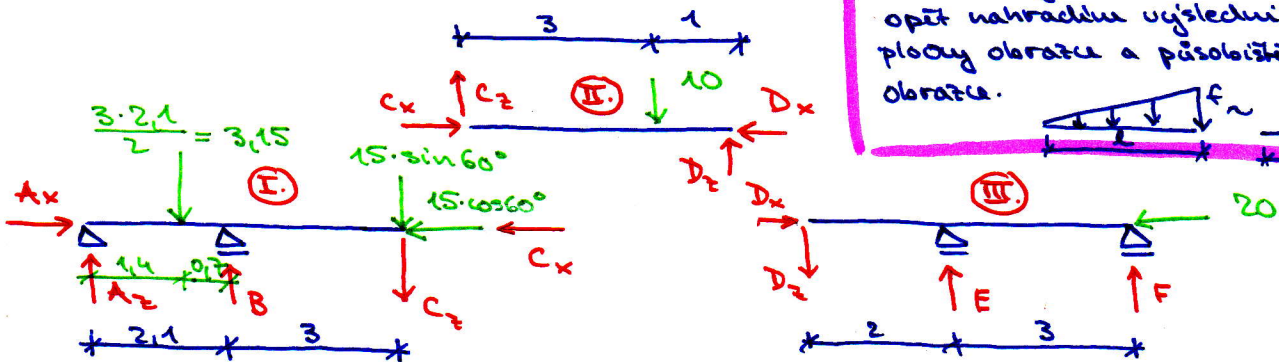
$\uparrow : 5 - 10 + 11,333 - 6 - 0,333 = 0 \quad \checkmark$

$\odot : 20 - 10 \cdot (4 + 2,5) + 11,333(4 + 5 + 2) - 6 \cdot (4 + 5 + 2 + 1,5) - 0,333 \cdot (4 + 5 + 2 + 3) = 0 \quad \checkmark$

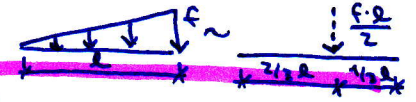
Pr.



$S = 3 \times 3 - 2 - 3 \times 1 - 2 \times 2 = 0$
 $r_{ext} = 2 + 3 \times 1 = 5 \geq 3$
 staticky určitá konštrukcia



Pozn. trojuholnikove zatíženie opäť nahradíme výslednicou o veľkosti plochy obrázka a pôsobí v ňom v ňom obrázka.



III.

$\rightarrow : D_x - 20 = 0 \quad , \quad D_x = 20 \text{ kN}$

IV.

$\curvearrowright : 10 \cdot 1 - C_z \cdot 4 = 0 \quad , \quad C_z = 2,5 \text{ kN}$

$\uparrow : C_z + D_z - 10 = 0 \quad , \quad D_z = 7,5 \text{ kN}$

$\rightarrow : C_x - D_x = 0 \quad , \quad C_x = 20 \text{ kN}$

V.

$\rightarrow : A_x - 15 \cdot \cos 60^\circ - C_x = 0 \quad , \quad A_x = +27,5 \text{ kN}$

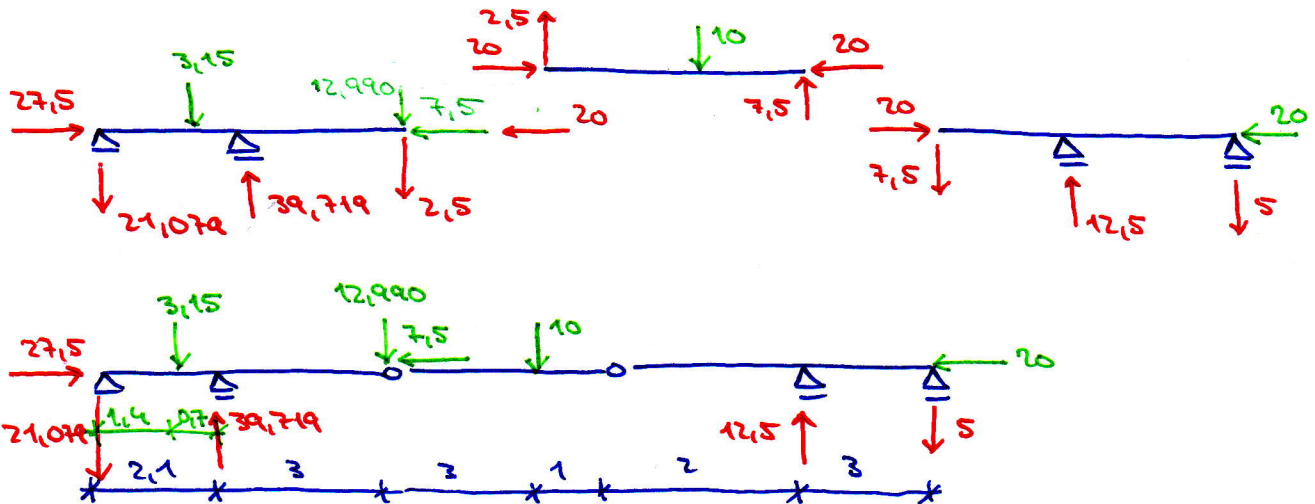
$\curvearrowright : B \cdot 2,1 - 3,15 \cdot 1,4 - C_z \cdot 5,1 - 15 \cdot \sin 60^\circ \cdot 5,1 = 0 \quad , \quad B = 39,719 \text{ kN}$

$\uparrow : A_z - 3,15 + B - C_z - 15 \cdot \sin 60^\circ = 0 \quad , \quad A_z = -21,079 \text{ kN}$

VI.

$\curvearrowright : D_z \cdot 2 + F \cdot 3 = 0 \quad , \quad F = -5 \text{ kN}$

$\uparrow : -D_z + E + F = 0 \quad , \quad E = 12,5 \text{ kN}$

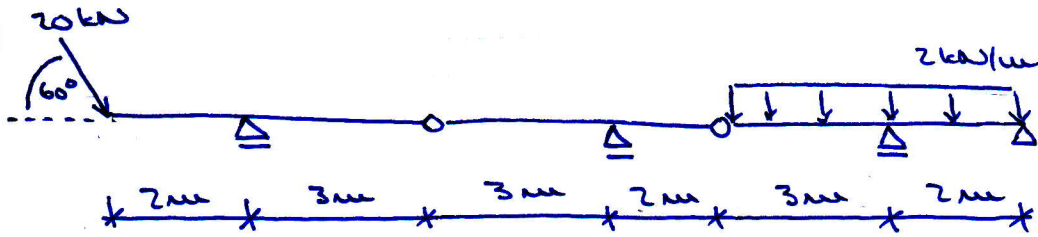


Kontrola:

$\uparrow : -21,079 - 3,15 + 39,719 - 12,990 - 10 + 12,5 - 5 = 0 \quad \checkmark$

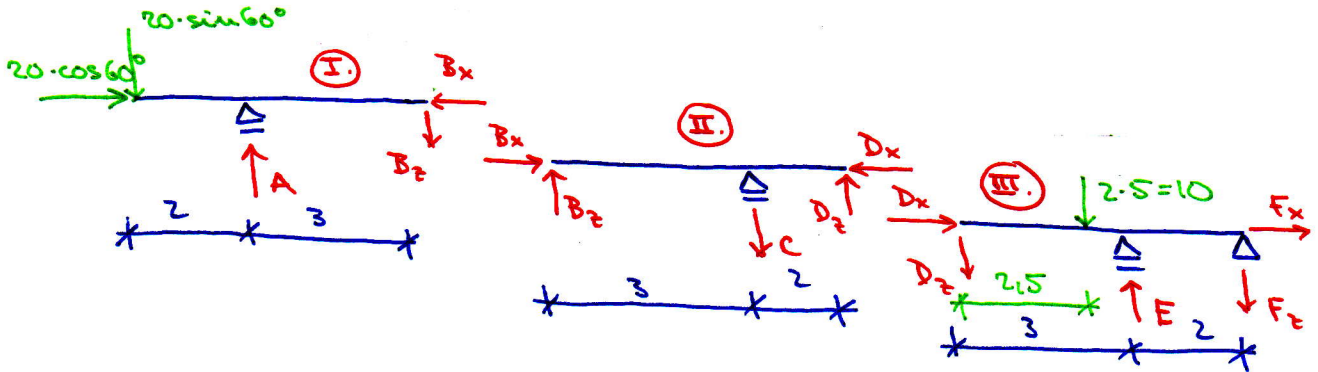
$\rightarrow : 27,5 - 7,5 - 20 = 0 \quad \checkmark$

Pf.



$S = 3 \times 3 - 3 \times 1 - 2 - 2 \times 2 = 0$
 $r_{ext} = 2 + 3 \times 1 = 5 \geq 3$

} staticky určitá konstrukce



I.

$\rightarrow : 20 \cdot \cos 60^\circ - B_x = 0 \quad , \quad B_x = 10 \text{ kN}$
 $\curvearrowright : -B_z \cdot 3 + 20 \cdot \sin 60^\circ \cdot 2 = 0 \quad , \quad B_z = 11,547 \text{ kN}$
 $\uparrow : -20 \cdot \sin 60^\circ + A - B_z = 0 \quad , \quad A = 28,868 \text{ kN}$
 (kontrola: $\curvearrowright : 20 \cdot \sin 60^\circ \cdot 5 - A \cdot 3 \stackrel{?}{=} 0$)

II.

$\rightarrow : B_x - D_x = 0 \quad , \quad D_x = 10 \text{ kN}$
 $\curvearrowright : -B_z \cdot 3 + D_z \cdot 2 = 0 \quad , \quad D_z = 17,321 \text{ kN}$
 $\uparrow : B_z - C + D_z = 0 \quad , \quad C = 28,868 \text{ kN}$

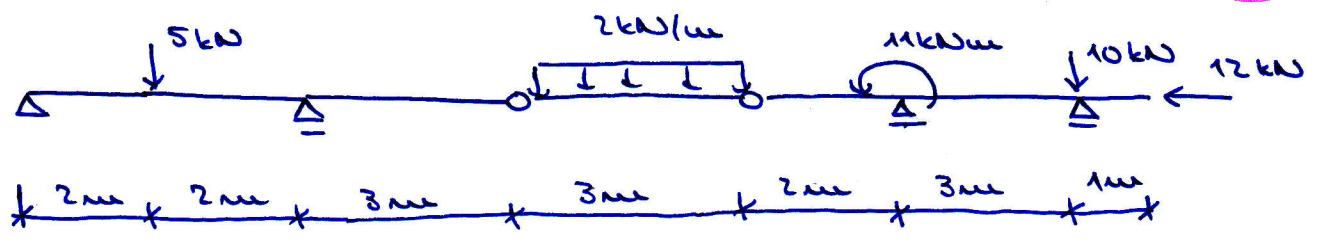
III.

$\rightarrow : D_x + F_x = 0 \quad , \quad F_x = -10 \text{ kN}$
 $\curvearrowright : D_z \cdot 3 + 10 \cdot 0,5 - F_z \cdot 2 = 0 \quad , \quad F_z = 28,482 \text{ kN}$
 $\uparrow : D_z \cdot 5 + 10 \cdot 2,5 - E - F_z = 0 \quad , \quad E = 55,803 \text{ kN}$
 (kontrola: $\uparrow : -D_z - 10 + E - F_z \stackrel{?}{=} 0 \checkmark$)

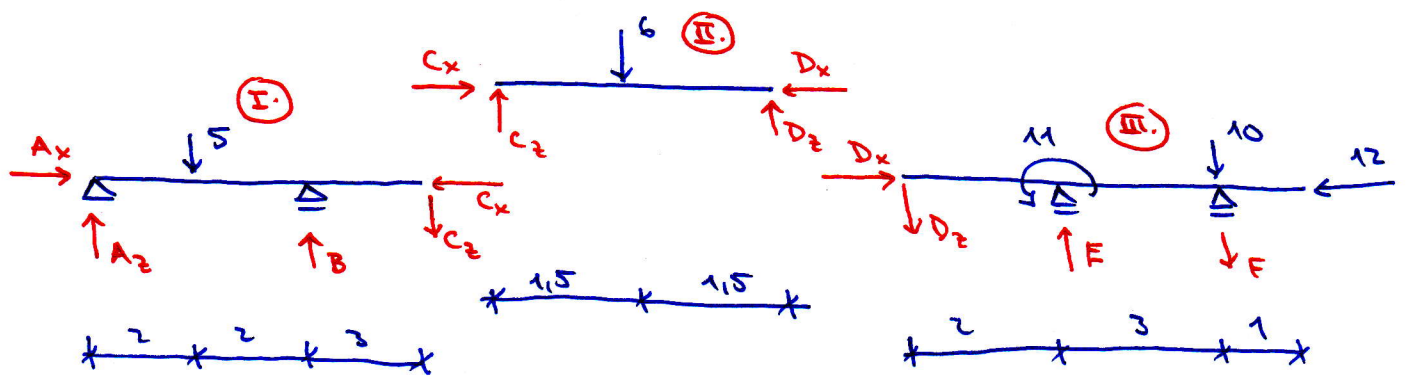
I. + II. + III.

$\uparrow : -20 \cdot \sin 60^\circ + A - C + E - F - 10 \stackrel{?}{=} 0 \checkmark$
 $\rightarrow : 20 \cdot \cos 60^\circ + F_x \stackrel{?}{=} 0 \checkmark$

PF



$s = 3 \times 3 - 2 - 3 \times 1 - 2 \times 2 = 0$ } staticky určitá konstrukce
 $r_{ext} = 2 + 3 \times 1 = 5 \geq 3$



II. \circlearrowleft : $-6 \cdot 1,5 + D_z \cdot 3 = 0$, $D_z = 3 \text{ kN}$
 \uparrow : $C_z + D_z - 6 = 0$, $C_z = 3 \text{ kN}$

I. + II. + III. \rightarrow : $A_x - 12 = 0$, $A_x = 12 \text{ kN}$

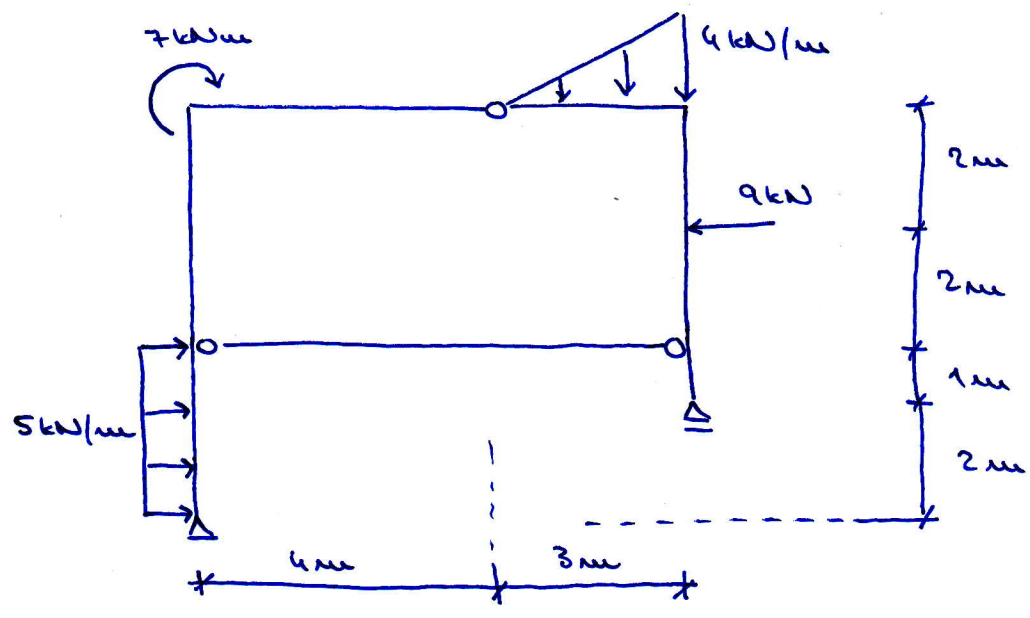
I. \rightarrow : $A_x - C_x = 0$, $C_x = 12 \text{ kN}$
 \circlearrowleft : $-A_z \cdot 4 + 5 \cdot 2 - C_z \cdot 3 = 0$, $A_z = 0,25 \text{ kN}$
 \uparrow : $A_z + B - C_z - 5 = 0$, $B = 7,75 \text{ kN}$

II. \rightarrow : $C_x - D_x = 0$, $D_x = 12 \text{ kN}$

III. \circlearrowleft : $D_z \cdot 5 + 11 - E \cdot 3 = 0$, $E = 8,667 \text{ kN}$
 \uparrow : $-D_z + E - F - 10 = 0$, $F = -4,333 \text{ kN}$

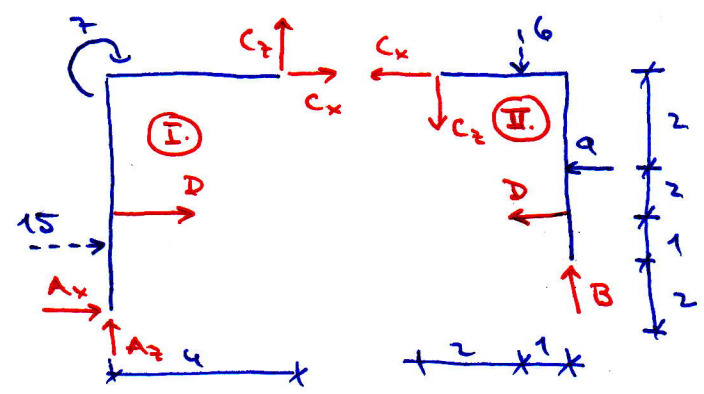
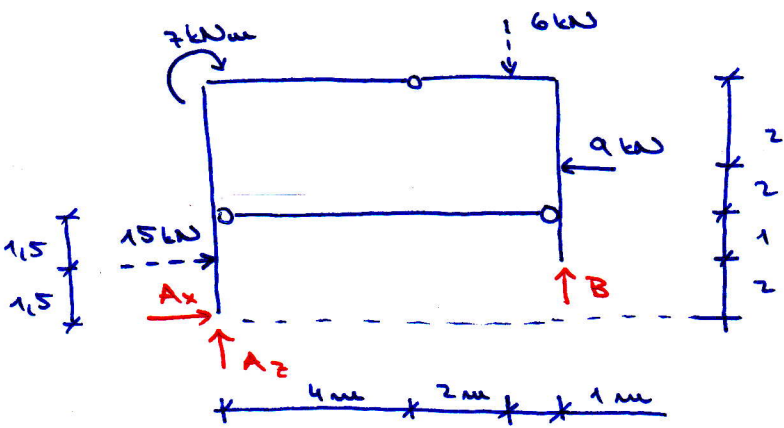
kontrola: I + II + III : \uparrow : $A_z - 5 + B - 6 + E - F - 10 \stackrel{?}{=} 0$ ✓

Pf.



$$S = 2 \times 3 - 2 - 1 - 1 - 2 = 0$$

$$r_{ext} = 2 + 1 = 3 \geq 3$$
 } stat. určitel konstrukce



I. + II.:

$$\rightarrow: A_x + 15 - 9 = 0 \quad \boxed{A_x = -6 \text{ kN}}$$

$$\curvearrow: -15 \cdot 1,5 - 7 + B \cdot 7 + 9 \cdot 5 - 6 \cdot 6 = 0 \quad \boxed{B = 2,929 \text{ kN}}$$

$$\uparrow: A_z + B - 6 = 0 \quad \boxed{A_z = 3,071 \text{ kN}}$$

II.:

$$\uparrow: -C_z - 6 + B = 0 \quad \boxed{C_z = -3,071 \text{ kN}}$$

$$\curvearrow: -6 \cdot 2 - 9 \cdot 2 - D \cdot 4 + B \cdot 3 = 0 \quad \boxed{D = -5,303 \text{ kN}}$$

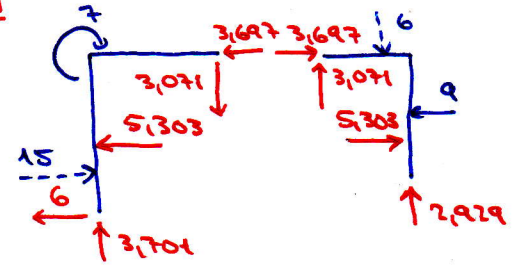
$$\rightarrow: -C_x - 9 - D = 0 \quad \boxed{C_x = -3,697 \text{ kN}}$$

kontrola: I.

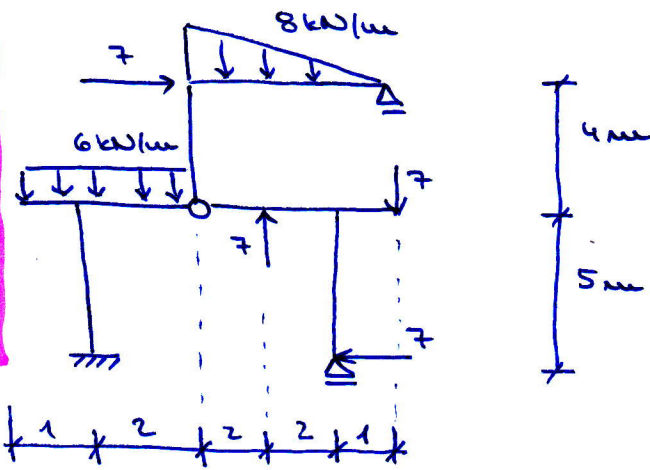
$$\rightarrow: C_x + D + A_x + 15 \stackrel{?}{=} 0 \checkmark$$

$$\uparrow: A_z + C_z \stackrel{?}{=} 0 \checkmark$$

$$\curvearrow: A_x \cdot 7 - A_z \cdot 4 + 15 \cdot 5,5 + D \cdot 4 - 7 \stackrel{?}{=} 0 \checkmark$$



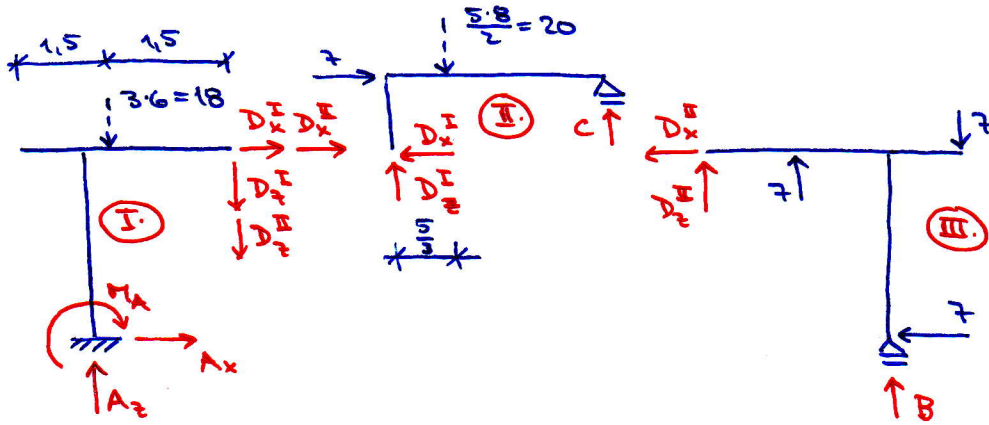
Př.



$$S = 3 \times 3 - 3 - 1 - 1 - 4 = 0$$

$$\Gamma_{ext} = 3 + 1 + 1 = 5 \geq 3 \checkmark$$

staticky určitá konstrukce



I. + II. + III.

$$\rightarrow : A_x + 7 - 7 = 0 \quad \boxed{A_x = 0}$$

III.

$$\rightarrow : -D_x^{II} - 7 = 0 \quad \boxed{D_x^{II} = -7 \text{ kN}}$$

$$\curvearrow : 7 \cdot 2 - 7 \cdot 5 - 7 \cdot 5 + B \cdot 4 = 0 \quad \boxed{B = 14 \text{ kN}}$$

$$\uparrow : B + 7 - 7 + D_z^{II} = 0 \quad \boxed{D_z^{II} = -14 \text{ kN}}$$

II.

$$\curvearrow : -7 \cdot 4 - 20 \cdot \frac{5}{3} + C \cdot 5 = 0 \quad \boxed{C = 12,267 \text{ kN}}$$

$$\uparrow : D_z^I - 20 + C = 0 \quad \boxed{D_z^I = 7,733 \text{ kN}}$$

$$\rightarrow : -D_x^I + 7 = 0 \quad \boxed{D_x^I = 7 \text{ kN}}$$

I.

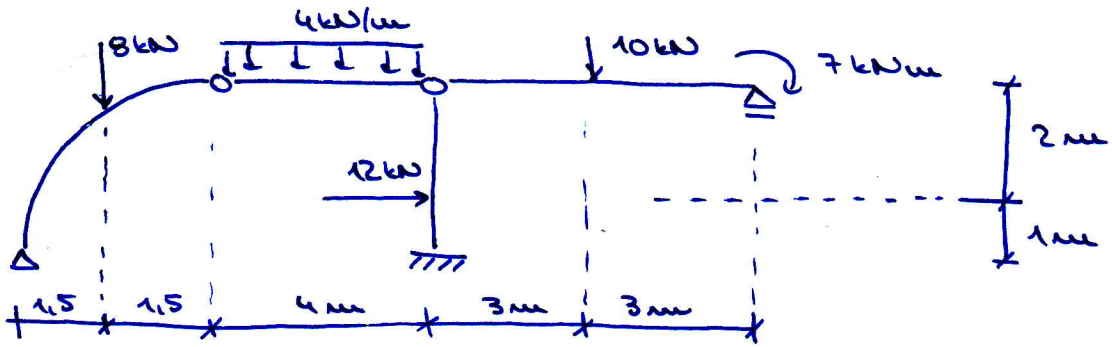
$$\uparrow : A_z - 18 - D_z^I - D_z^{II} = 0 \quad \boxed{A_z = 11,733 \text{ kN}}$$

$$\curvearrow : 18 \cdot \frac{3}{2} - A_z \cdot 2 + A_x \cdot 5 - M_A = 0 \quad \boxed{M_A = 3,534 \text{ kN}}$$

kontrola: $\rightarrow : A_x + D_x^I + D_x^{II} \stackrel{?}{=} 0 \checkmark$

kontrola: I + II + III.

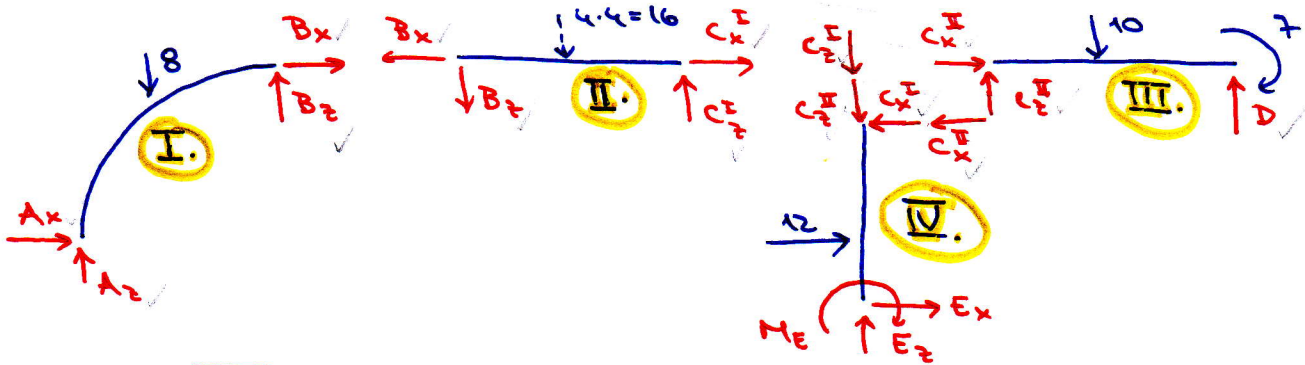
$$\uparrow : A_z - 18 - 20 + C + 7 - 7 + B \stackrel{?}{=} 0 \checkmark$$



$$s = 4 \times 3 - 2 - 3 - 1 - 2 - 4 = 0$$

staticky určitelá konstrukce

$$r_{ext} = 2 + 3 + 1 = 6 \geq 3$$



III. $\rightarrow: C_x^{\text{II}} = 0$

$\curvearrowleft: -10 \cdot 3 + D \cdot 6 - 7 = 0, D = +6,1667 \text{ kN}$

$\uparrow: C_z^{\text{II}} - 10 + D = 0, C_z^{\text{II}} = +3,8333 \text{ kN}$

II. $\curvearrowleft: -16 \cdot 2 + C_z^{\text{I}} \cdot 4 = 0, C_z^{\text{I}} = 8 \text{ kN}$

$\uparrow: -B_z - 16 + C_z^{\text{I}} = 0, B_z = -8 \text{ kN}$

I. $\curvearrowleft: -8 \cdot 1,5 + B_z \cdot 3 - B_x \cdot 3 = 0, B_x = -12 \text{ kN}$

$\uparrow: A_z - 8 + B_z = 0, A_z = 16 \text{ kN}$

$\rightarrow: A_x + B_x = 0, A_x = 12 \text{ kN}$

II. $\rightarrow: -B_x + C_x^{\text{I}} = 0, C_x^{\text{I}} = -12 \text{ kN}$

IV. $\rightarrow: E_x + 12 - C_x^{\text{II}} - C_x^{\text{III}} = 0, E_x = -24 \text{ kN}$

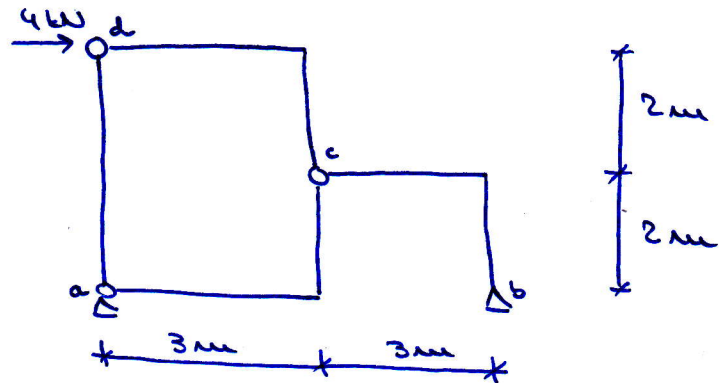
$\uparrow: E_z - C_z^{\text{II}} - C_z^{\text{III}} = 0, E_z = 11,8333 \text{ kN}$

$\curvearrowleft: 12 \cdot 2 + E_x \cdot 3 - M_E = 0, M_E = -48 \text{ kNm}$

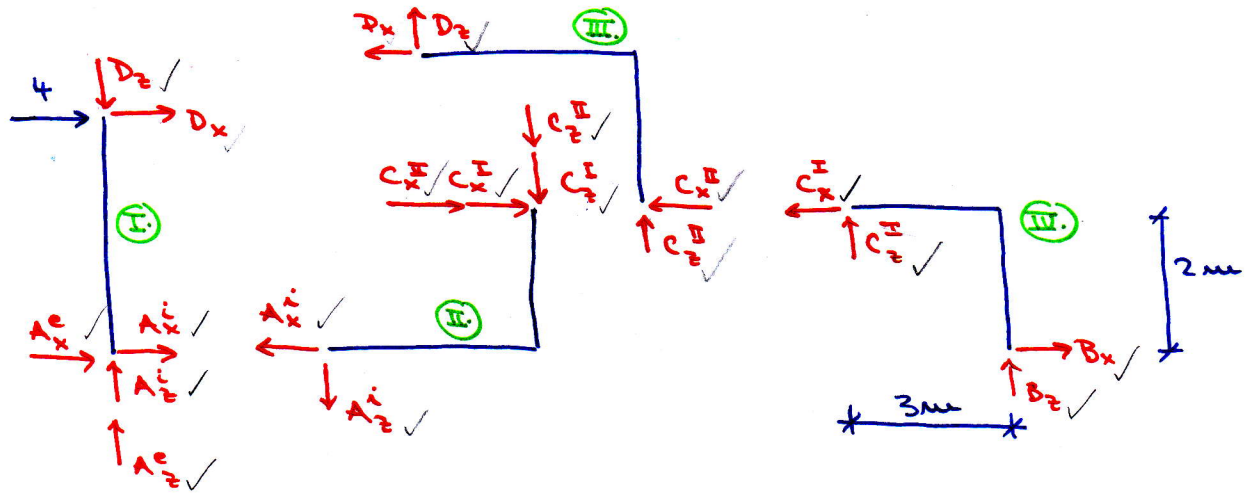
Kontrola: I+II+III+IV $\rightarrow: A_x + 12 + E_x \stackrel{?}{=} 0 \checkmark$

$\uparrow: A_z - 8 - 16 + E_z - 10 + D \stackrel{?}{=} 0 \checkmark$

PF



$s = 3 \times 4 - 2 \times 2 - 2 - 2 - 4 = 0$
 $\Delta p \downarrow \downarrow$
 $r_{ext} = 2 + 2 \geq 3$
 staticky urata' konstrukce



I. $\curvearrowright a : -4 \cdot 4 - D_x \cdot 4 = 0 \quad D_x = -4 \text{ kN}$

III. $\curvearrowright c : -D_z \cdot 3 + D_x \cdot 2 = 0 \quad D_z = -2,6667 \text{ kN}$

$\uparrow : D_z + C_z^I = 0 \quad C_z^I = 2,6667 \text{ kN}$

$\rightarrow : -D_x - C_x^I = 0 \quad C_x^I = 4 \text{ kN}$

USE $\curvearrowright a : B_z \cdot 6 - 4 \cdot 4 = 0 \quad B_z = 2,6667 \text{ kN}$

$\uparrow : A_z^e + B_z = 0 \quad A_z^e = -2,6667 \text{ kN}$

IV. $\uparrow : B_z + C_z^I = 0 \quad C_z^I = -2,6667 \text{ kN}$

$\curvearrowright c : B_z \cdot 3 + B_x \cdot 2 = 0 \quad B_x = -4 \text{ kN}$

$\rightarrow : -C_x^I + B_x = 0 \quad C_x^I = -4 \text{ kN}$

II. $\uparrow : -C_z^I - C_z^II - A_z^i = 0 \quad A_z^i = 0 \text{ kN}$

$\rightarrow : -A_x^i + C_x^I + C_x^II = 0 \quad A_x^i = 0 \text{ kN}$

kontrola: $\curvearrowright a : -(C_z^I + C_z^II) \cdot 3 - (C_x^I + C_x^II) \cdot 2 \stackrel{?}{=} 0 \checkmark$

I. $\rightarrow : 4 + D_x + A_x^e + A_x^i = 0 \quad A_x^e = 0 \text{ kN}$

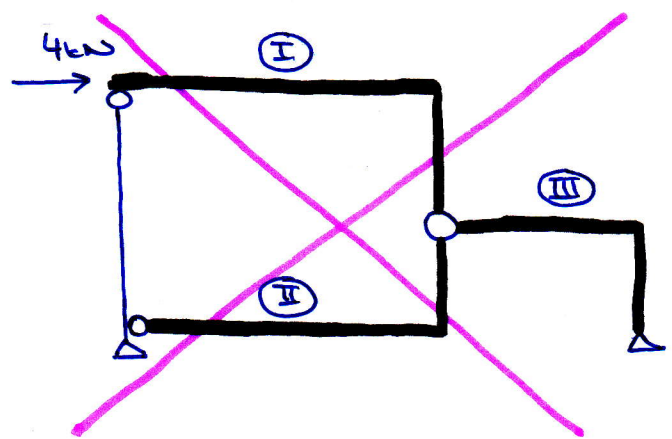
$\uparrow : A_z^e + A_z^i - D_z = 0 \quad A_z^e = -2,6667 \text{ kN}$

kontrola: USE $\rightarrow : A_x^e + B_x + 4 \stackrel{?}{=} 0 \checkmark$

$\curvearrowright a : B_z \cdot 6 + B_x \cdot 4 + A_x^e \cdot 4 \stackrel{?}{=} 0 \checkmark$

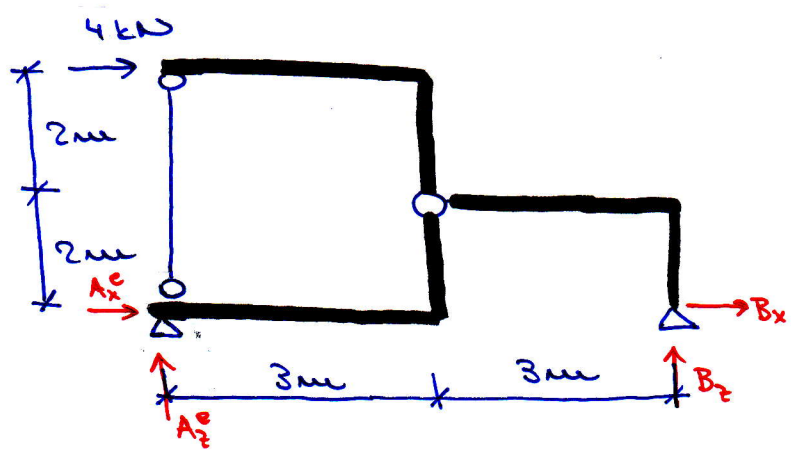
Je možné tuto úlohu formulovat tak, že deska I je nosná rovinná kývavým prutem?

VARIANTA B



Tato formulace není možná, protože z desky II budou vydaty 2 reakce $\leftarrow \parallel$, kývavý prut ale umí přenést jen jedno z nich.

VARIANTA C

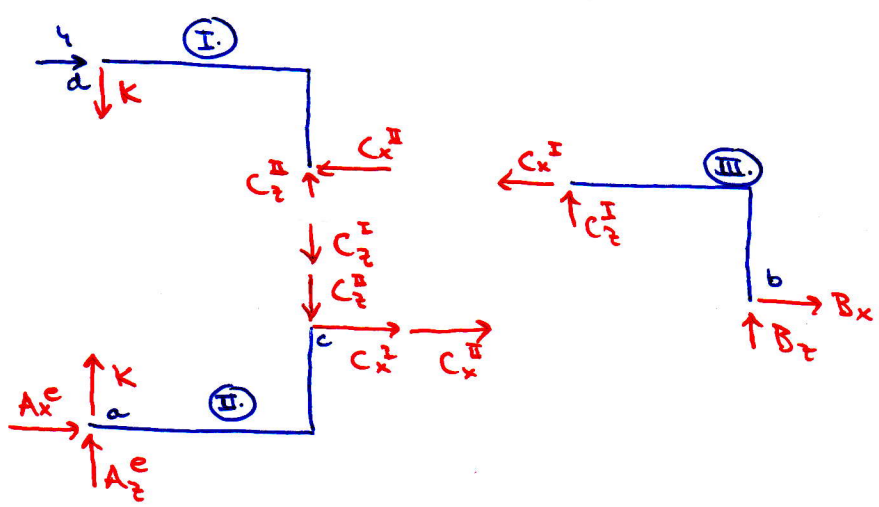


$$S = 3 \times 3 - 2 \times 2 - 4 - 1 = 0$$

$$\Delta \quad \phi \quad \circ \circ$$

$$r_{ext} = 4 \geq 3 \checkmark$$

st. určitá konstrukce



USE: $\curvearrowright a : -4 \cdot 4 + B_z \cdot 6 = 0 \quad \boxed{B_z = 2,6667 \text{ kN}}$

$\uparrow : A_z^e + B_z = 0 \quad \boxed{A_z^e = -2,6667 \text{ kN}}$

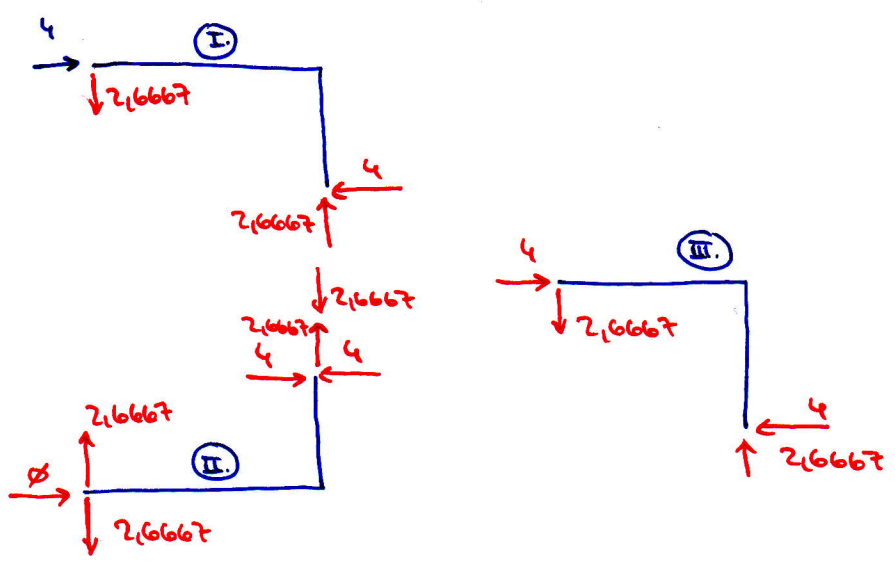
III.
 $\uparrow: C_z^I + B_z = 0 \quad , \quad C_z^I = -2,6667 \text{ kN}$
 $\circlearrowleft: -C_z^I \cdot 3 + C_x^I \cdot 2 = 0 \quad , \quad C_x^I = -4 \text{ kN}$
 $\rightarrow: -C_x^I + B_x = 0 \quad , \quad B_x = -4 \text{ kN}$

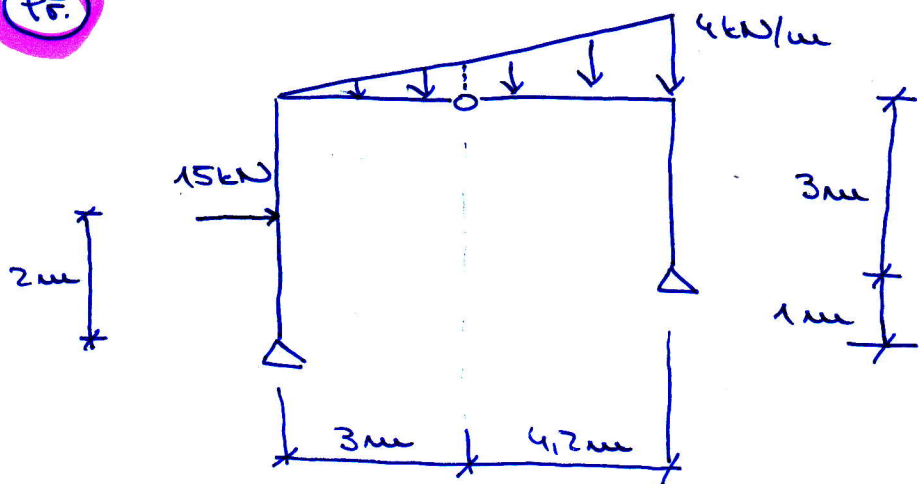
I.
 $\rightarrow: -C_x^H + 4 = 0 \quad , \quad C_x^H = 4 \text{ kN}$
 $\circlearrowleft: C_z^H \cdot 3 - C_x^H \cdot 2 = 0 \quad , \quad C_z^H = 2,6667 \text{ kN}$
 $\uparrow: -K + C_z^H = 0 \quad , \quad K = 2,6667 \text{ kN}$

II.
 $\rightarrow: A_x^P + C_x^I + C_x^H = 0 \quad , \quad A_x^P = 0 \text{ kN}$

kontrola: $\uparrow: A_z^P + K - C_z^I - C_z^H \stackrel{?}{=} 0$
 $-2,6667 + 2,6667 - (-2,6667) - 2,6667 = 0 \checkmark$

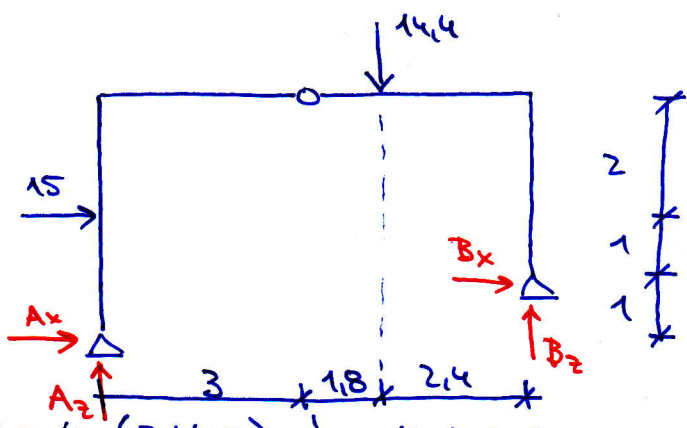
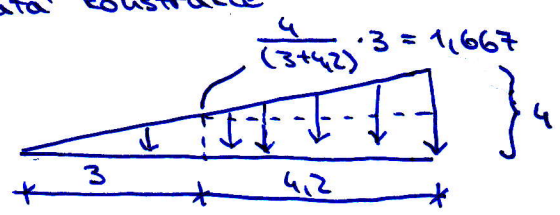
kontrola:
 USE: $\uparrow: A_z^P + B_z \stackrel{?}{=} 0$
 $-2,6667 + 2,6667 = 0 \checkmark$



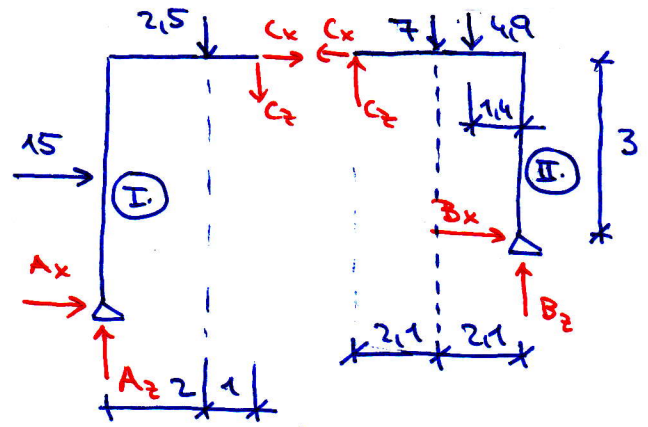


$s = 2 \times 3 - 2 - 2 - 2 = 0$
 $\Delta \quad \Delta \quad - \quad 0$
 $r_{ext} = 2 + 2 = 4 \geq 3 \checkmark$

statický uřáta' konstrukce



$F = 4 \cdot (3 + 4.2) \cdot \frac{1}{2} = 14.4 \text{ kN}$
 $l = (3 + 4.2) \cdot \frac{2}{3} = 4.8 \text{ m}$



$F_1 = 1.667 \cdot 3 \cdot \frac{1}{2} = 2.5 \text{ kN}$
 $F_2 = 1.667 \cdot 4.2 = 7.001 \text{ kN}$
 $F_3 = (4 - 1.667) \cdot 4.2 \cdot \frac{1}{2} = 4.899 \text{ kN}$

I. $\sum \mathcal{M}_a : -15 \cdot 2 - 2.5 \cdot 2 - C_z \cdot 3 - C_x \cdot 4 = 0$

II. $\sum \mathcal{M}_b : -C_z \cdot 4.2 + C_x \cdot 3 + 7 \cdot 2.1 + 4.9 \cdot 1.4 = 0$

$-4C_x - 3C_z = 35 \quad | \cdot 3$
 $3C_x - 4.2C_z = -21.56 \quad | \cdot 4$

 $-25.8C_z = 18.76$

$C_z = -0.727 \text{ kN}$

$C_x = (35 + 3 \cdot C_z) : (-4) = -8.205 \text{ kN}$

14.

$$\textcircled{\text{I.}}: \rightarrow : A_x + 15 + C_x = 0 \quad , \quad \boxed{A_x = -6,795 \text{ kN}}$$
$$\uparrow : A_z - 2,5 - C_z = 0 \quad , \quad \boxed{A_z = 1,773 \text{ kN}}$$

$$\textcircled{\text{II.}}: \rightarrow : B_x - C_x = 0 \quad , \quad \boxed{B_x = -8,205 \text{ kN}}$$
$$\uparrow : C_z - 7 - 4,9 + B_z = 0 \quad , \quad \boxed{B_z = 12,627 \text{ kN}}$$

$\textcircled{\text{I. + II.}}$ pro kontrolu

$$\uparrow : A_z + B_z - 14,4 \stackrel{?}{=} 0 \quad \checkmark$$
$$\rightarrow : A_x + B_x + 15 \stackrel{?}{=} 0 \quad \checkmark$$

UYSLEDNE REAKCE:

