

# An adaptive cycle-jump method to accelerate phase-field computations of fatigue

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Phase-field models of fatigue, e.g. [1, 2], proved to be a versatile approach capable of reproducing the characteristic experimental features of fatigue. However, their high computational cost makes the cycle-by-cycle analysis of components in the high cycle fatigue (HCF) regime, that is cycle counts of  $n > 10^4 - 10^5$ , unfeasible. One strategy to solve this issue is to employ a time acceleration scheme whose simplest form is the *cycle-jump* approach, where selected state variables are extrapolated over a certain number of cycles  $\Delta n$  based on an explicitly (cycle-by-cycle) computed evolution of the system.

To exploit the full potential of this procedure while keeping the accuracy of the solution, an adaptive cycle-jump algorithm is proposed for the model presented in [1] which degrades the fracture toughness of the material as a representative fatigue history variable  $\bar{\alpha}$  accumulates above a certain threshold value  $\bar{\alpha}_T$ . The core idea of the presented acceleration technique is to decide whether to jump cycles or not and how many cycles  $\Delta n$  to skip based on both the phase-field  $d$  and fatigue history variable evolution. This specific choice makes the adaptation of the proposed acceleration strategy to other phase-field fatigue models straightforward.

The computation is subdivided into four main stages: (i) early stage before fatigue effects are triggered, namely until  $\bar{\alpha} \simeq \bar{\alpha}_T$  for the first time, (ii) crack nucleation stage, i.e. between the end of the first stage and the instant in which the phase field reaches  $d \simeq 1$  for the first time, (iii) stable crack propagation (also known as Paris regime) and (iv) unstable crack propagation before failure. During the first stage, the system is essentially linear elastic, which can be exploited to compute how many cycles are needed to meet the condition  $\bar{\alpha} = \bar{\alpha}_T$  and thus directly jump to that point. Within the second stage, the system exhibits a stable evolution of the phase field variable  $d$  until right before the complete nucleation of a crack, i.e.  $d = 1$ . In this case, the determination of  $\Delta n$  is based on the rate of change of the  $L_2$ -norm of the

phase-field  $\|d\|_2$  with respect to the cycle count, obtained during a small number of explicitly resolved cycles. In particular,  $\Delta n$  is computed as the maximum number of cycles which can be skipped before the rate of  $\|d\|_2$  overcomes an appropriate threshold. After that, the system experiences a rapid and highly non-linear behavior lasting for a limited number of cycles, which are resolved explicitly. Once this phase is over, the stable crack propagation starts which, encompassing the vast majority of the fatigue cycles in the HCF regime, offers the highest potential for accelerating the simulation. In this third stage, the computation of the number of cycles to skip is again based on the rate of change of  $\|d\|_2$ , similarly to the second stage. The monitoring of the  $L_2$ -norm of the phase-field also allows to detect the onset of unstable crack propagation, during which the algorithm reverts to a cycle-by-cycle computation until final failure.

The proposed methodology is first investigated using different choices for its parameters and extrapolation strategies. Then, it is used to analyze the fatigue behavior of different virtual specimens. Finally, the results are compared to those obtained with a cycle-by-cycle and a fixed (i.e. non-adaptive) cycle-jump approach.

## References

- [1] Carrara, P., Ambati, M., Alessi, R. and De Lorenzis L., A framework to model the fatigue behavior of brittle materials based on a variational phase-field approach *Comp. Meth. App. Mech. Eng.* 361 (2020)
- [2] Alessi, R., Vidoli, S., and De Lorenzis, L., A phenomenological approach to fatigue with a variational phase-field model: The one-dimensional case *Eng. Frac. Mech.* 190(1) (2018) 53–73.