

## Adaptive and variable model order reduction for damage modelling using explicit time integration

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Finite Element (FE) Analysis is widely used in both academia and industry to analyse large structural models. Recent advances in material architectures and manufacturing techniques have resulted in geometrical features that, whilst improving structural performance, result in a length-scale that is orders of magnitude smaller than a typical structural scale. In FE modelling this requires a large number of degrees of freedom and time-steps, in case of dynamic analyses using explicit time integration. This is particularly challenging when modelling the damage behaviour where the computational cost is proportional to the geometrical and material non-linearities. To overcome this computational bottleneck, a reliable Reduced Order Modelling (ROM) method is required.

ROM methods function by substituting the high-dimensional models by an equivalent lower-dimensional model with reduced computational expense to achieve a similar level of numerical accuracy [1]. In many implementations this is achieved by adopting an offline-online approach. This pre-computed reduced-basis approach for online computations with ROM works well for linear problems, but they are not suitable for modelling non-linear responses, such as associated with damage propagation. This is because a local change in stiffness degradation in multi-axial loading requires a large parametric-space for the accurate calculation of reduced-basis functions.

The solution developed in this work is aimed at developing a ROM method in explicit time integration without offline training, while retaining the computational accuracy and achieving efficiency. Unlike conventional ROM methods, the snapshots are collected and the reduced basis is calculated in the online model with minimal inputs from the user. Although ROM reduces model dimension, the calculation of the internal force vector is still required. To overcome this, a hyper-reduction method called Energy Conserving Sampling and Weighting (ECSW) is used [2]. This

calculates the hyper-reduced internal force vector by analysing just a few elements with non-negative weights computed online. This serves as an approximation to global internal force vector.

As a result of hyper-reduction, some elements are skipped from the time increment and their history variables are not updated. Although methods such as Gappy Data reconstruction are available, they are often expensive when performed during multiple time increments. A computationally efficient reconstruction is introduced by using mixed-time integration in the current work.

The percentage of ROM in online calculation can be varied depending on the accuracy required. Furthermore, Full Order Modelling (FOM) is performed at regular intervals such that linear momentum balance is achieved between ROM and FOM modelling.

The developed framework is tested by analysing mode-I, mode-II and mixed-mode damage propagation. The results are examined by comparing numerical accuracy and computational efficiency when compared to FOM. This is further extended by analysing different mesh sizes to understand the reduction in the number of operations and overall computational cost.

### References

- [1] de Frías, G.J., Aquino, W., Pierson, K.H., Heinstein, M.W. and Spencer, B.W. (2014), A multiscale mass scaling approach for explicit time integration using proper orthogonal decomposition. *Int. J. Numer. Meth. Engng*, 97: 799-818.
- [2] Farhat, C., Avery, P., Chapman, T. and Cortial, J. (2014), Dimensional reduction of nonlinear finite element dynamic models with finite rotations and energy-based mesh sampling and weighting for computational efficiency. *Int. J. Numer. Meth. Engng*, 98: 625-662.