

# Phase-Field Fracture Modeling using Physics-Informed Deep Learning

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Scientific modeling and computation is increasingly leveraging advances in Deep learning [1]. The developments in deep learning allowing it to utilize the physical principles and not depend only on swathes of data, has contributed to wide scientific interest in it. A range of physics-informed deep learning approaches have been developed to learn the solution field of a partial differential equation governing a physical phenomenon including in mechanics. They hold promise to improve computational efficiency compared to the traditional approaches in modeling.

Phase-field fracture modeling [2, 3] recasts the problem of fracture as a variational problem which completely determines the fracture process including crack nucleation, propagation and bifurcation and obviates the need for ad-hoc conditions. In this approach, a phase field is introduced in the formulation which smears a crack. It is however a nonlocal model which includes a small length scale. Resolving this length scale in computation is expensive. Hence, uncertainty quantification, material parameter identification, design optimization, among others, using this approach become prohibitively expensive. We explore the application of physics-informed deep learning to parametric phase-field fracture modeling to overcome this challenge. We use Deep Ritz method (DRM) [4] in which the solution field is represented by a neural network (NN) and the training of the network proceeds by directly minimizing the variational energy of the system.

We first study crack nucleation in a 1-D homogeneous bar with prescribed end displacements ( $U_t$ ). We turn  $U_t$  as a parameter of the problem. Then  $U_t$  and coordinate  $x$  are both inputs to a NN and displacement ( $u$ ) and phase ( $\alpha$ ) fields are outputs. The complexity of the problem arises from the fact that for small  $U_t$ , the strain and phase fields are homogeneous. Above a threshold  $U_t$  however, solution bifurcates and the homogeneous solution becomes unstable. Phase field localizes in the stable solution lead-

ing to a jump in the energy- $U_t$  curve. An NN finds it difficult to approximate the resulting discontinuity. So, we utilize domain-decomposition [5] along parameter axis and use independent NNs to approximate the solution in each domain. Then the overall NN solution is able to represent the discontinuous solution and it agrees well with the finite element analysis (FEA) solution.

We also study crack propagation in a single edge notched plate under prescribed displacement which is a combination of tension and shear. The presence of multiple energy minima makes it challenging to obtain the correct solution when prescribed displacement is not purely tensile. The details of the models and the challenges in obtaining the correct solution will be discussed.

## References

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