A new computational procedure for singularity analysis of interface cracks with frictional contact in anisotropic bimaterials

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For the study of interface cracks in bimaterials, we must go back to 1959 when Williams [1] introduced an asymptotic analysis of elastic fields at the tip of an interface crack. This model is known as the open model of the interface crack and its solution leads to oscillatory displacement and stress fields around the crack tip. These oscillations in the displacement field imply that interpenetrations between materials near the interface crack tip may occur [2]. To solve this physical inconsistency Comninou [3] proposed the frictionless *contact model for interface* cracks in isotropic bimaterials. Later on, Comninou [4] generalized her model considering frictional contact between crack faces coming to the conclusion that friction makes the singularity weaker, i.e. $\lambda>0.5$.

In this work a generalization of the Comninou contact model of interface cracks to linear elastic anisotropic bimaterials considering the Coulomb isotropic friction law is developed, based on the Stroh formalism [5, 6] and using the concept of bimaterial matrix by Hwu [7]. The present semi-analytic procedure computes the singularity exponents λ that define the asymptotic stress and displacement fields at the interface crack tip. Unlike several previous approaches found in literature, this work does not impose a direction of sliding between both materials, considering the sliding angle ω as an unknown. Once the singularity exponents λ and corresponding sliding angles ω are computed, the displacement and stress fields at the crack tip associated with each singular mode can be obtained. After analyzing the displacement and stress fields in the frictional sliding interface, it is concluded that, as Comninou [4] pointed out for isotropic bimaterials, in the case of monoclinic materials with a symmetry plane $x_2 = 0$, that is, any stacking of layers in a composite laminate, friction also weakens the friction interface crack tip singularity.

One advantage of the developed methodology is that, despite using the Stroh formulation for mathematically non-degenerate anisotropic materials, the final expressions of the two nonlinear eigenequations depend on the bimaterial matrix, which can be expressed in terms of the real Barnett-Lothe [8] tensors, **H**, **L** and **S**. This allows a direct application of the present methodology to isotropic and transversely isotropic materials.

The weak singularity allows the use of the crack tip solutions computed by the present methodology to predict the growth of such interface cracks by the Coupled Criterion (CC) [9] of Finite Fracture Mechanics (FFM) as proposed by García and Leguillon [10] and Graciani and Mantič [11].

References

[1] M.L. Williams, The stresses around a fault or crack in dissimilar media, Bull. Seismol. Soc. Am. 49 (1959) 199-204.

[2] A.H. England, A crack between dissimilar media, J. Appl. Mech. 32 (1965) 400-402

[3] M. Comninou, The interface crack, J. Appl. Mech. 44 (1977) 631-636.

[4] M. Comninou, Interface crack with friction contact zone, J. Appl. Mech. 44 (1977) 780-781.

[5] A.N. Stroh, Dislocations and cracks in anisotropic elasticity, Philos. Mag. 3 (1958) 625-646.

[6] A.N. Stroh, Steady state problems in anisotropic elasticity, J. Math. Phys. 41 (1962) 77-103.

[7] C. Hwu, Anisotropic Plates, Springer, 2010.

[8] D.M. Barnett, J. Lothe, Synthesis of the sextic and the integral formalism for dislocations. Green's function and surface waves in anisotropic elastic solids. Phys Norv 7 (1973) 13–19.

[9] D. Leguillon, Strength or toughness? A criterion for crack onset at a notch. European Journal of Mechanics A/Solids 21 (2002) 61–72.

[10] I.G. García, D. Leguillon, Mixed-mode crack initiation at a v-notch in presence of an adhesive joint, Int. J. Solids Struct. 49 (2012) 2138-2149.

[11] E. Graciani, V. Mantič, Evaluation of the ERR in interface cracks with frictional contact and its application in the coupled criterion of finite fracture mechanics, Mech. Res. Commun. (to be submitted).