Stabilized formulation for phase-field fracture in nearly incompressible hyperelasticity

B. Li[1*], I. Ang[2], N. Bouklas[2]

[1] Department of Mechanical Engineering, Guangdong Technion - Israel Institute of Technology, 241 Daxue Rd., Shantou 515063, China, bin.l@gtiit.edu.cn

[2] Sibley School of Mechanical and Aerospace Engineering, Cornell University, 124 Hoy Rd., Ithaca 14853, USA

The phase-field fracture method has been well established within the context of linear elasticity. Ongoing work in soft materials extended the use of the phase-field model from linear to finite elasticity in a compressible setting where the Poisson's ratio, ν, ranged from 0.3 to 0.45. Staying within a compressible setting allows for pure displacement formulations to be used [1].

In this work we presents a stabilized formulation for phase-field fracture of hyperelastic materials near the limit of incompressibility. At this limit, traditional mixed displacement and pressure formulations must satisfy the inf-sup condition for solution stability. The mixed formulation coupled with the damage field can lead to an inhibition of crack opening as volumetric changes are severely penalized effectively creating a pressure-bubble. To overcome this bottleneck, we utilize a mixed formulation with a perturbed Lagrangian formulation which enforces the incompressibility constraint in the undamaged material and reduces the pressure effect in the damaged material [2]. A mesh-dependent stabilization technique [3] based on the residuals of the Euler-Lagrange equations multiplied with a differential operator acting on the weight space is utilized [4], allowing for linear interpolation of all field variables of the elastic subproblem.

This formulation was validated with three examples at finite deformations: a plane-stress pure-shear test, a two-dimensional geometry in plane-stress, and a three-dimensional notched sample. In the last example, we incorporate a hybrid formulation with an additive strain energy decomposition to account for different behaviors in tension and compression. The results show close agreement with analytical solutions for crack tip opening displacements and performs well at the limit of incompressibility. From an energy perspective the variational phase-field fracture model matches closely with theoretical critical initiation load (i.e., Griffith's criterion). The fact that the fully fractured configuration does not exhibit high residual pressures and stresses is a significant finding that has not been captured in prior results. This low-order formulation greatly increases computational efficiency for complex three-dimensional simulations, and has the potential to be extended towards dynamic fracture problems in soft materials at the limit of incompressibility.

References

[1] C., Miehe and L. M., Schänzel, Phase field modeling of fracture in rubbery polymers. Part I: Finite elasticity coupled with brittle failure, J Mech Phys Solids, 65 (2014), 93–113.

[2] B., Li and N., Bouklas, A variational phase-field model for brittle fracture in polydisperse elastomer networks, Int J Solids Struct, 182 (2020), 193–204.

[3] O., Klaas and A. Maniatty and M. S., Shephard, A stabilized mixed finite element method for finite elasticity.: Formulation for linear displacement and pressure interpolation, Comput Methods in Appl Mech Eng, 180 (1999), 65–79.

[4] I., Ang and N., Bouklas and B., Li, Stabilized formulation for phase-field fracture in nearly incompressible hyperelasticity, Int J Numer Methods Eng, 123 (2022), 4655–4673.