Deterministic and stochastic phase-field modeling of anisotropic brittle fracture

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Anisotropic materials with cubic lattice such as silicon exhibit a four-fold symmetry in their elastic as well as fracture properties. Such materials are characterized by two weak directions that are equally favourable for crack propagation. In certain orientations of the material, the two weak directions are located symmetrically with respect to the loading. This leads to the distinctive phenomenon of the crack alternating between the two weak directions as it propagates, resulting in saw-tooth or zigzag crack patterns.

In this work, we investigate the variational fourth-order phase-field formulation of anisotropic brittle fracture to model crack zigzagging. We analytically derive and numerically test the fundamental behavioral aspects predicted by the two main available models [1, 2]. In this regard, the presence of second gradients of the phase-field variable in the weak form demands C_1 continuity of the shape functions for finite element discretization. To this end, we adopt isogeometric analysis [3] and discretize the displacement field, the phase-field and, as per the isoparametric concept, the geometry by quadratic B-Splines.

Furthermore, non-uniqueness of the phase-field solution associated to non-convexity of the governing energy functional [4] is observed to be quite pronounced in the anisotropic case (more than in the isotropic case). Hence, we transition from a deterministic to a stochastic model by introducing a material-related random field in the anisotropic phase-field energy functional. We introduce a small perturbation to the direction-dependent fracture toughness to trigger multiple crack-paths. We employ Monte Carlo, randomized quasi-Monte Carlo sampling and surrogate-based approaches to estimate statistical moments of the phase-field variable.

Numerical results establish the large variation in the responses of the two fourth-order phase-field models, both in their deterministic and stochastic versions. For either modeling choice, the stochastic model, which captures several possible zigzag crack paths, holds significant promise to enable meaningful predictions of anisotropic fracture with phase-field models.

References

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