Griffith criterion for phase field fracture

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We provide a phase-field version of Griffith's criterion [4] and show that it is satisfied by the evolution obtained from the staggered scheme, both in continuum and in finite element settings [1].
We employ the second order Ambrosio-Tortorelli approximation of the Griffith's energy, $AT_2(u, v)$, that is defined as the sum of the phase-field elastic energy $\mathcal{E}(u, v)$ and of the dissipated energy $G_c\mathcal{L}(v)$, where $\mathcal{L}(v)$ is the phase-field approximation of the crack length. We introduce a notion of *phase-field energy release* $\mathcal{G}(v)$.
We consider a quasi-static evolution in a time interval $[0, T]$. The irreversibility of the crack is modeled by the monotonicity of the phase field variable v. This hypothesis does not directly imply the thermodynamic consistency of the dissipated energy, therefore we require that $\dot{\mathcal{L}}(v(t)) \geq 0$.
The Griffith's criterion in phase-field setting is written in terms of the following Karush-Kuhn-Tucker conditions: $\mathcal{G}(t, v(t)) \leq G_c$ and $(\mathcal{G}(t, v(t)) - G_c)\, \dot{\mathcal{L}}(v(t)) = 0$.
The evolutions are obtained by *staggered (alternate) minimization* for the energy AT_2, where we require monotonicity of the phase-field variable in time and not over each iteration as in [2]. The time interval $[0, T]$ is discretized and at each time step t_k the solutions u_k and v_k are defined as the result of the staggered minimization scheme, flanked by a stopping criterion and a constraint on the numer of iterations.
Passing to the limit as the time step approaches zero we find an evolution $t \mapsto (u(t), v(t))$ such that in the steady-state regime:

- $(u(t), v(t))$ is an equilibrium point for the energy $AT_2(\cdot, \cdot)$;
- $v(t)$ satisfies the Griffith's criterion and the thermodynamical consistency;
- an energy balance identity holds.

Obviously the evolution may depend on the discrete scheme and may present also unstable regimes, where propagation is catastrophic and Griffith criterion does not hold.
Analogously, if we discretize the domain with finite elements, the limit evolution as the time step approaches zero satisfies all the properties listed above.

References

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