Phase-field damage models for time-discontinuous crack evolution

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The modelling of damage behaviour has been an intensively researched topic for decades – both from the mechanical as well as from the mathematics point of view. Since the modelling of sharp cracks/interfaces and the resulting free boundary problem has been shown to be numerically very challenging, phase-field theories have become very popular, cf. [1].

Within this talk, the focus is on rate-independent damage models and their implementation. In this case, the resulting phase-field approximation is characterized by (incrementally defined) non-convex optimization problems. This non-convexity leads to two problems addressed here:
1. A discontinuous evolution of the crack in time (so-called brutal crack growth) might occur. Therefore two different internal time-scales have to be considered and modeled.
2. Different mathematical solution concepts are applicable to these kinds of rate-independent systems which are generally not equivalent, for instance the concepts of global energetic solutions and balanced viscosity solutions are here to be named. In combination with a time-discrete numerical approximation it is necessary to prove the convergence to one of the solutions concepts, see [2]. Furthermore these approximations have to be carefully evaluated with respect to the predicted physics.

One mathematically sound concept bridging the different, aforementioned time-scales is the time-discrete scheme proposed by Efendiev & Mielke, cf. [3]. Within this talk, the framework [3] is implemented into the Finite-Element-Method and carefully analysed from a physics point of view, see [4]. Particularly, the implementation and effect of the different norms and time-increments in the framework [4] are investigated by the means of numerical experiments.

References

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