Dynamic crack growth in viscoelastic materials with memory

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We study a model of crack growth in viscoelastic materials based on the following ideas (see [4]):

(a) the displacement solves viscoelastodynamics out of the crack, with traction-free boundary conditions on the crack;

(b) the dynamic energy-dissipation balance is satisfied: the sum of the kinetic energy and of the elastic energy at time t, plus the energy dissipated by viscosity and crack growth between time 0 and time t, is equal to the initial energy plus the total work done by external forces between time 0 and time t;

(c) a maximal dissipation condition is satisfied, which forces the crack to run as fast as possible, consistent with the energy-dissipation balance.

It is known (see [3]) that in the Kelvin-Voigt model of viscoelasticity conditions (a) and (b) prevent crack growth (viscoelastic paradox). To overcome this problem we study a different viscoelastic model, the Maxwell model (see [5]). It is governed by the following system, containing a memory term:

$$\ddot{u}(t) - \operatorname{div}\big((\mathbf{C}+\mathbf{V})Eu(t)\big) + \operatorname{div}\Big(\int_0^t \mathrm{e}^{\tau-t}\,\mathbf{V}Eu(\tau)\,\mathrm{d}\tau\Big) = \ell(t),$$

where $u(t)$, $Eu(t)$, and $\ddot{u}(t)$ are the displacement at time t, the symmetric part of its gradient, and its second derivative with respect to time, $\mathbf{C}$ and $\mathbf{V}$ are the elasticity and viscosity tensors, $\ell(t)$ is the external load at time t, and the system is satisfied out of the crack.

We prove an existence result in the case of planar elasticity with a free crack path, with suitable a priori assumptions on the regularity of the crack and of its time evolution. Also the maximal dissipation condition is satisfied only among suitably regular competitor cracks.

The proof is based on a careful analysis of the properties of the solution of the system with given initial and boundary conditions, in the case of a prescribed time dependent crack. In particular, to prove the result with a free crack path (see [1]) we use the results on existence, uniqueness, and continuous dependence on the cracks obtained in the case of prescribed time dependent cracks (see [2] and [6]).

References

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