Optimal scaling laws for ductile fracture derived from strain-gradient microplasticity

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Ductile fracture is the process whereby a material separates across a failure surface through mechanisms, such as void nucleation, growth and coalescence, that entail large amounts of plastic work. Such extensive plastic deformation notwithstanding, ductile fracture remains quintessentially a fracture process, in the sense that failure takes place by separation across a plane or surface and entails a well-characterized amount of energy *per unit area*, or *specific fracture energy*, to operate. Experimentally, ductile fracture is easily identified fractographically, as the crack surfaces exhibit a characteristic dimpling—the dimples being vestiges of voids—that is in contrast to the sharp specular cracks that result from brittle fracture. Furthermore, the measured specific fracture energies attendant to ductile fracture, e. g., from Charpy tests or from J-testing, are much larger than those of brittle solids.

We [1, 2, 3] carry out an optimal-scaling analysis of ductile fracture in metals. We specifically consider the deformation, ultimately leading to fracture, of a slab of finite thickness subject to monotonically-increasing normal opening displacements on its surfaces. We posit two competing constitutive properties, namely, *sublinear energy growth* and *strain-gradient hardening*. Sublinear growth (the energy of linear elasticity exhibits quadratic growth, by way of comparison) is a reflection of the work-hardening characteristics of conventional metallic specimens and gives rise to well-known geometric instabilities such as the necking of bars, sheet necking, strain localization and others. Strain-gradient hardening [4] has been extensively investigated and demonstrated by means of torsion tests in wires [5], nanoindentation [6], and by other means. It results in deviations from volume scaling, i. e., in *nonlocal behavior* and *size dependency*, in sufficiently small material samples. We show that ductile fracture indeed emerges as the net outcome of these two competing effects: whereas the sublinear growth of the local energy promotes localization of deformation to failure planes, strain-gradient plasticity stabilizes this process of localization in its advanced stages, thus resulting in a well-defined specific fracture energy. Specifically, we show that ductile fracture requires a well-defined energy per unit area that can be bounded above optimally by a void-sheet construction. This specific fracture energy bears a power-law relation to the prescribed opening displacement. This power-law relation may be regarded as an effective cohesive potential, thus indicating that ductile fracture is cohesive in nature. In particular, fracture processes involving distributed—possibly fractal—damage are ruled out by the analysis.

References

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