

# Computational Modeling of Fracture and Failure of Materials and Structures

CFRAC 2013

Edited by:

M. Jirásek, O. Allix, N. Moës and J. Oliver



# **Computational Modeling of Fracture and Failure of Materials and Structures**



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Prague, Czech Republic, 5–7 June 2013**

**CFRAC 2013:  
The Third International Conference on  
Computational Modeling of Fracture and  
Failure of Materials and Structures**

**Proceedings**

**edited by**

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## Preface

The series of International Conferences on Computational Modeling of Fracture and Failure of Materials and Structures (CFRAC) started at Ecole Centrale de Nantes in June 2007 and continued at the Technical University of Catalonia in Barcelona in June 2011. On 5-7 June 2013, the third edition of CFRAC is held at the Czech Technical University in Prague. Its scientific program consists of 5 plenary lectures and 37 parallel sessions assembled into 13 minisymposia, in which 26 keynote lectures and 141 regular lectures are given. Each lecture is represented in this proceedings book by an abstract.

CFRAC belongs to the large family of ECCOMAS Thematic Conferences, organized with the support of the European Community on Computational Methods in Applied Sciences. It is also one of the Special Interest Conferences of IACM, the International Association for Computational Mechanics. The purpose of CFRAC is to bring together academic researchers and industrial partners involved in the development and application of numerical procedures for fracture and failure simulation of materials and structures.

The growing need for efficient and robust numerical methods dealing with fracture and failure can be attributed to the fact that designers of engineering structures must respond to ever increasing demands on performance in terms of safety, reliability, durability, low cost and low energy consumption. As new materials and new applications emerge, traditional design rules and conventional testing methods become insufficient or inapplicable, which strengthens the role of computational methods in the design process. Numerical tools can substantially accelerate the development of new products and their optimization. The ambition of CFRAC conferences is to serve as a platform for presentation and discussion of exciting advances in various branches of computational fracture and failure mechanics, and to promote collaboration among research groups in Europe and worldwide.

One of the characteristic features of the CFRAC series of conferences is the emphasis on a participant-friendly environment, encouraging the presentation of the most recent results. For instance, the final deadline for submission of contributions to CFRAC 2013 was just four months before the actual conference, and the authors had the opportunity to update the abstracts for two more months. At the conference, at most five sessions run in parallel, in lecture rooms located next to each other. The coffee break refreshments and the lunches are served in one spacious lobby, so that the participants can remain together and continue their discussions in a relaxed atmosphere. The organizers believe that such conditions facilitate the exchange of ideas and contribute to the success of the conference.

The present proceedings volume has been assembled from individual abstracts submitted by their authors. To ensure uniformity of style and to minimize the number of misprints, the abstracts have been checked and corrected by graduate students Václav Nežerka and Michael Somr, and the final unified pdf file with a table of contents and list of authors has been produced by Jan Zeman, associate professor at the Czech Technical University in Prague. We would also like to thank the minisymposia organizers and the plenary speakers and acknowledge their important contribution to the preparation of CFRAC 2013.

Finally, let us mention that the complete proceedings can be freely downloaded from the conference web site via <http://mech.fsv.cvut.cz/cfrac/proceedings.pdf>. We hope that the readers will find here stimulating ideas and enjoy the reading.

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- Jean-François Molinari
- Anna Pandolfi
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# **Plenary Lectures**





## Convergent Erosion Schemes for Three-Dimensional Fracture and Fragmentation

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Brittle fracture can be formulated as a *free-discontinuity problem*, i. e., a problem where the solution can have discontinuities and, in addition, the jump sets of the solution are *a priori* unknown. Within this variational framework, [1] have proposed an approximation scheme, which they term *eigenfracture*, based on the notion of *eigendeformation*. Eigendeformations are widely used in mechanics to describe deformation modes that cost no local energy. In the eigenfracture scheme, the energy functional depends on two fields: the displacement field  $u$  and an eigendeformation field  $\varepsilon^*$  that describes such cracks as may be present in the body. Specifically, eigendeformations allow the displacement field to develop jumps at no cost in local elastic energy. In addition, in the eigenfracture scheme the fracture energy is set to be proportional to the volume of the  $\varepsilon$ -neighborhood of the support of the eigendeformation field, suitably scaled by  $1/\varepsilon$ . The optimal crack set is obtained by minimizing the resulting energy functional with respect to both the displacement and the eigendeformation fields, subject to irreversibility constraints.

The eigenfracture scheme of Schmidt *et al.* [1] has been specialized to element erosion by Pandolfi and Ortiz [2], and to material-point erosion by Li *et al.* [3], and referred to the resulting schemes as *eigen-erosion*. Eigenerosion is derived from the general eigenfracture formulation by restricting the eigendeformations in a binary sense: they can be either zero, in which case the local behavior is elastic; or they can be equal to the local displacement gradient, in which case the corresponding material neighborhood is failed, or *eroded*. When combined with a finite-element approximation, this scheme gives rise to *element erosion*, i.e., the elements or material points can be either intact, in which case their behavior is elastic, or be completely failed — or eroded — and have no load bearing capacity. The implementation of the method, included the all-important  $\varepsilon$ -neighborhood construction, is exceed-

ingly simple and applies to general situations, possibly involving complex three-dimensional fracture patterns such as branching and fragmentation. The accuracy and convergence of the eigenerosion approach is comparable — at a much reduced implementation cost and complexity — to that of other numerical fracture schemes.

Element erosion has been extensively used to simulate fracture in a number of areas of application, including terminal ballistics. However, some of these methods fail to converge or converge to the wrong limit. By contrast, the eigenfracture scheme is known to properly converge to Griffith fracture in the limit of vanishingly small mesh sizes [1]. In particular, the local-neighborhood averaging of the energy which underlies the calculation of the effective energy-release has the effect of eliminating spurious mesh-dependencies. The accuracy and fidelity of the eigenerosion approach has been assessed through convergence studies for Mode I crack growth, both in two and three dimensions and for structured and random meshes, and validated against quasistatic and dynamic experimental data.

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## Issues on Ductile Failure Modelling: Stress State Dependence, Non-Locality and Damage to Fracture Transition

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### Ductile failure modelling

The recent investment on the research and improvement on ductile damage and fracture modelling may be explained by the necessity of exploiting and extending of traditional materials capacities, for different industrial applications, whilst maintaining safety rules and abiding to economic restraints. Another important factor has to do with the use of new materials with improved strength properties and the need of understanding their behaviour at severe stress/strain conditions. This is the case, in particular, in metal forming processes involved in the manufacture of a huge variety of structural parts in many industrial sectors, such as, for example, automotive, aeronautics or consumption goods in which materials are subject to significant changes in shape at the solid state that encompass large plastic strains.

Numerical modelling has become indispensable in the design, development and optimization of metal forming processes in the industry. The powerful commercial codes which are available nowadays allow for a reliable prediction of deformation, strain and stresses at critical points of industrial parts. One crucial issue is formability, i.e., the capability of produce a part without defects but within the material limits. The flow of the material, forced by contact with tools involve large plastic strains that, after a given threshold level, may trigger a ductile damage process occurring concomitantly with the plastic deformation due to the nucleation, growth and coalescence of micro-voids.

It is still common practice at the industrial level to use a posteriori fracture criteria, based on the computational evaluation of functions of some state variables that depend on the deformation story (e.g. [1]). These variables may be the total plastic work, the maximum plastic shear work or the equivalent

plastic strain. Other criteria may be based on the growth of defects which include geometric aspects (e.g. [2, 3]), growth mechanisms, dependent on principal stresses or hydrostatic pressure or material behaviour coupling (e.g. [4, 5]).

Present production of many components in the industry require complex deformation paths and these criteria very often fail to give the appropriate information. Situations in which the damage localizes away from the sites where the maximum equivalent plastic deformation is concentrated or where damage evolves differently for different compression or traction stress states, different triaxialities or diverse shear stress states are hardly handled by these criteria.

### Coupled models. Stress state dependence.

Models in which damage evolution is taken into account through the deformation process are for that purpose more reliable. Two major routes are often taken in building those models. One is based on Continuous Damage Mechanics (CDM) and the thermodynamics of irreversible processes. The most popular ones are based on Lemaitre's model which includes the evolution of internal damage, as well as non-linear isotropic and kinematic hardening in the description of the behavior of ductile materials [6].

Another route is grounded on micro mechanical considerations coupling damage and plasticity at the constitutive level [7, 8]. The Gurson-Tvergaard-Needleman (GTN) model, which is one of the most well-known extensions of Gurson's model, assumes both isotropic hardening and damage. This model includes strong coupling between plastic strain and damage variable, porosity or the void volume fraction, representing the presence of micro voids. More recently Xue [9] has proposed the introduction of

a shear mechanism to improve the model performance at low levels of stress triaxialities.

Many researchers have shown that the third invariant of the deviatoric stress tensor, through the so-called the Lode angle, is an essential parameter in the characterization of the effect of the stress state on material yielding and on ductile fracture (e.g. [10, 11]). In particular, Bai and Wierzbicki have suggested a three dimensional fracture loci on the space of equivalent strain, stress triaxiality and Lode angle.

An assessment on the performance of some of the referred models will be carried out here at different stress states.

### Non-local models

Nevertheless, most of those models are based on the assumption of the so-called local continuum in which the behaviour of the material is completely represented by a point-wise constitutive law, independently of the influence of surrounding material points. The material is assumed to be continuous at any scale and, therefore, size effects are inherently neglected. However, it is well known that the softening induced by the standard implementation of those models in finite element solutions may lead to mesh and orientation dependence. This fact is associated to the local change of the underlying type of differential equations representing the problem whenever a negative stiffness is locally included due to softening. As a consequence localization effects are not correctly dealt with by mesh refinement.

One of the solutions for this problem is the use non-local models (e.g. [12, 13, 14, 15, 16]). The non-local theory incorporates an intrinsic length, into the traditional continuum theory, trying to mimic those size effects at the constitutive level and, as a side effect, if conveniently formulated it alleviates or solves numerical problems associated with local models, either by means of gradient-enhanced or integral-type formulations.

The derivation of any non-local theory requires the choice of the variable or variables to be enhanced by non-locality. Typical choices are, amongst others, the regularization of variables related to kinematics (such as the strain tensor), regularisation of internal variables (e.g. scalar measurements of the amount of plastic strain or damage) or regularisation of ther-

modynamic forces power-conjugated with internal variables (for instance, the elastic energy release rate in damage models). In fact, the choice of the non-local variable depends on the kind of material to be modelled and on the nature of the problem to be solved. In the particular case of elasto-plastic damaging ductile solids, the internal degradation of the material, which in the CDM theory is usually treated by means of some damage measurement as an internal variable, is closely linked to the localisation phenomenon.

Different choices for non-local variables (damage, void volume fraction, hardening, elastic energy release rate, equivalent plastic strain) will be here numerically assessed for different combinations of stress triaxiality ratio and Lode angle.

### Damage to fracture transition

Continuum models successfully describe most the stages of material behaviour. Nevertheless, when it comes to the final stages of failure, these models are not able to represent the initiation and propagation of macro-cracks within a structure. To correctly address surface decohesion and avoid spurious damage growth, the use of a discontinuous approach becomes imperative. One of the most successful simulations of ductile fracture processes, in a finite element method framework, lay in strategies that involve relatively fine meshes and continuous remeshing. Models based on the smeared crack model, more common in brittle failure applications, in which the effects of a discontinuity are incorporated in the stress field and not at the displacement or strain field level, avoid the need for remeshing.

A successful alternative approach is to locate displacement or strain discontinuities intra-element. A large number of intra-element discontinuity models falls within the embedded discontinuities class [17]. These models are in general characterized by the introduction of new deformation modes in the standard finite element. These deformation modes are able to represent discontinuities with an arbitrary orientation both at the strain level (weak discontinuity) or at the displacement level (strong discontinuity) (e.g [18, 19]).

Another powerful technique to represent discontinuities intra-element is the eXtended Finite Element Method (XFEM) [20] in which the standard

displacement field approximation is enriched with functions able to capture the decohesion between two surfaces. Extra degrees of freedom are added to the nodes of the elements containing the discontinuity, allowing free propagation through the mesh. The XFEM possesses interesting characteristics to develop a successful simulation of ductile failure processes.

A simple strategy on how to apply it to ductile failure is briefly described here. When damage reaches a critical value a crack is included into the continuous and subsequently propagates following the damage pattern. To ensure thermodynamical consistency and to avoid a singularity in the continuum equations the transition is made by adding a cohesive law to the model built upon the underlying damage model.

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## Finite Element Simulations of Cutting Processes of Thin-Walled Structures

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Thin walled structures appear in many applications of engineering interest. The intentional or accidental cutting of this type of structures by means of a sharp object is a complex phenomenon, whose accurate description is of obvious interest in many instances. Early studies on the analysis of the mechanics of blade cutting were for example devoted to the analysis of ship grounding. The process of cutting involves several types of nonlinearities, such as large deformations, contact, crack propagation and, in the case of laminated shells, delamination, whose effective description requires state of the art computational technologies. In addition to this, a special difficulty in blade cutting is represented by the blade sharpness, whose accurate geometric resolution would require meshes with characteristic size of the order of the blade curvature radius.

The purpose of this work is to propose and discuss a computational finite element approach for the simulation of blade cutting of thin shells. As it is more convenient in the presence of highly nonlinear problems, the approach is developed in an explicit dynamics framework. Structures of this type are mostly analyzed using shell finite elements belonging to either one of two main categories: shell elements derived on the basis of the classical or degenerate shell concept, in conjunction with the assumption of plane stress state; solid-shell elements, directly derived from three-dimensional continuum elements, using displacement degrees of freedom only and allowing for the implementation of fully three-dimensional constitutive laws. Solid-shell elements are claimed to present several advantages over classical shell elements: more straightforward enforcement of boundary conditions, possibility to incorporate complex 3D material models, no need for complex update algorithms for finite rotations, easy usage in combination with 3D solid elements, possibility to obtain good accuracy in the through-the-thickness stress distribution in laminated composites. On the other hand, solid-shell elements exhibit several types of locking behavior which de-

mand for corrections of the element kinematics. In the present work, the element proposed in [1] has been implemented in an explicit dynamics framework. In this element, shear locking and curvature thickness locking are cured by the ANS (Assumed Natural Strain) method, while Poisson thickness locking is controlled by means of an EAS (Enhanced Assumed Strain) approach by enriching the strain field with one parameter. Reduced integration with hourglass stabilization is also adopted to reduce the computational cost and to avoid volumetric locking.

The possible use of solid-shell elements in an explicit context is attracting particular attention and requires a specific treatment. In fact, in this kind of elements, usually developed in an implicit framework, the computation of the enhanced strain parameters, which are to be condensed out at element level, requires the iterative solution of a nonlinear problem in each element and at each time step, which turns out to be computationally too expensive in explicit analyses, where a very small time step size is dictated by stability requirements. To reduce the computational cost, the element formulation has therefore been reconsidered by introducing an explicit update of the enhanced variables.

Another problem connected with the use of solid-shell elements follows from the fact that the thickness dimension is often significantly smaller than the in-plane dimensions, leading to a high ratio of transverse to in-plane normal stiffnesses, with a high finite element maximum eigenfrequency. Hence, very small time-steps are required to guarantee the analysis stability in explicit dynamics. To circumvent the problem, a new selective mass scaling technique has been proposed based on a linear transformation of the solid-shell element degrees of freedom. The transformation allows to separate middle plane degrees of freedom, which govern rigid body motions, so as to be able to selectively increase the masses associated to out-of-plane degrees of freedom only, preserving the diagonal

structure of the mass matrix (see e.g. [2] for a similar approach based on thickness scaling, and [3] for mass scaling). Important features of the proposed approach are that mass lumping is preserved after the transformation, allowing for a direct computation of nodal accelerations, and that only minimal modification to existing codes are required for its implementation. Furthermore, for solid-shell elements, where the thickness dimension is significantly smaller than the in-plane ones, the highest element eigenfrequency always turns out to be given by the square root of the eigenfrequency  $\omega^2$  corresponding to the thickness vibration mode. In the case of a regular parallelepiped, in [4] it has been shown that the critical time-step resulting from this eigenfrequency can be analytically computed. This allows to define a strategy for the optimal selection of the mass scaling parameter, so as to maximize the time-step without significant accuracy loss.

Special attention is required in considering the action of a sharp blade on a thin shell, since the blade can interfere with the transmission of cohesive forces between the crack flanks in the cohesive process zone. Standard cohesive interface elements are not suited for the simulation of this type of cutting, dominated by the blade sharpness, unless extremely fine meshes, with characteristic size comparable to the blade curvature radius, are used. To circumvent the problem, the use of a new type of “directional” cohesive interface element, first proposed in [5], for the explicit dynamics simulation of crack propagation in elastic shells, is further developed. According to this concept, when a fracture propagation criterion is activated at a node, the node is duplicated and a cohesive string element is introduced between the separating nodes. String elements are geometric entities which can detect contact against the blade. When this happens, the string transmits cohesive forces to the crack flanks in different directions. Specific issues connected with the use of reduced integration elastoplastic, thin solid-shell elements in connection with “directional” cohesive interface element, such as the definition of a suitable fracture activation criterion and the implementation of a computationally effective hourglass stabilization are addressed.

Applications of the proposed methodology to different types of cutting problems are shown in Figure 1. These examples, by means of comparisons with an-

alytical and experimental results, allow to define the range of applicability of the proposed methodology and to highlight its possible limitations.

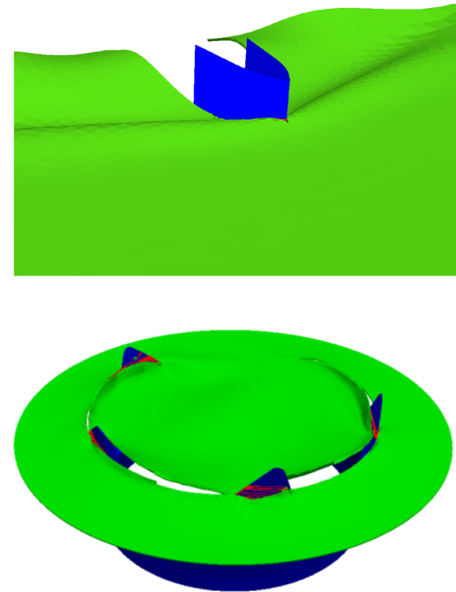


Figure 1: Application to blade cutting problems.

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## Dynamic Fracture: Discrete Versus Continuum Damage Modeling

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The modeling of catastrophic failure of materials and structures, including dynamic fragmentation, is a long standing scientific challenge with important societal impact. For example, the observation and prediction of fragment sizes have profound implications on the resistance of a material to ballistic impact, energy absorption capacity during crash, hydraulic fracturing, and clustering of galaxies. Upon severe loading, multiple micro-cracks initiate at seemingly random locations. High-speed cameras reveal that these cracks then propagate at high velocities. Their paths may be tortuous, single cracks may form complex branches, but eventually the cracks coalesce, resulting in the formation of fragments. Material failure is accompanied by a complex stress-wave communication network. At first glance, this catastrophic process appears chaotic and unpredictable. Yet, universal features, which can be explored through numerical calculations, emerge from the chaos.

In this presentation, we discuss two different classes of methods to investigate dynamic fracture and fragmentation: the cohesive element method (discrete approach) and a non-local integral-type continuum damage model (continuum approach).

We begin with the analysis of explosive fragmentation of simple structures, including brittle plates and hollow spheres, using cohesive elements [1, 2]. A comparison between our numerical results and analytic energy models [3] reveal an identical scaling law exponent for the dependence of the average fragment size on strain rate. However, our simulations, which include explicitly stress wave interactions, yield a higher number of fragments. The calculations give also access to statistics on fragment shapes and orientations. We show that thin membranes generate roughly structured orientations, whereas for larger membranes thicknesses, crack branching mechanisms bring random fragment orientations.

The robustness of our predictions regarding fragment shapes should be contrasted with the fact that

cohesive approaches suffer from mesh dependency. In the last part of the presentation, we explore the extension of classical static non-local continuum damage models to dynamical problems. This approach is used to smear the crack front over several mesh elements to achieve crack path mesh independence. We discuss the non trivial choice of material parameters, in particular for the non-local regularization, and present recent results on a benchmark problem involving dynamic crack-branching instabilities in a pre-cracked PMMA plate [4].

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## Fracture Scaling and Safety of Quasibrittle Structures: Atomistic Basis, Computational Challenges and New Advances

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The main objective of structural analysis is safety. This happens to be a particularly tricky problem for quasibrittle materials, i.e. heterogeneous materials with brittle constituents in which the inhomogeneity size, and thus the fracture process zone (FPZ), is not negligible compared to the structural dimension  $D$ .

The problem is that the type of strength distribution must be known up to the tail of failure probability  $10^{-6}$ , which is the maximum failure probability tolerable in engineering design. Such a small probability is beyond direct experimental verification by repeated tests (since at least  $10^8$  of identical structures would have to be tested). So the distribution type must be based on a physically justified theory, verified indirectly in other ways.

For ductile (or plastic) failure, the distribution must be Gaussian (normal) because the failure load is a weighted sum of contributions of random strength values of representative volume elements (RVE) of material along the failure surface, all of which fail simultaneously. For perfectly brittle failure, in which the structural failure is caused by failure of one negligibly small RVE, the structural strength distribution must be Weibullian [16].

So, in these extreme cases, the load of failure probability  $10^{-6}$  can be determined from the mean and standard deviation, which are easily measured or computationally simulated. Not, however, for quasibrittle structures, made of quasibrittle materials such as concrete, tough ceramics, fiber composites, rigid foams, many geomaterials, bio-materials, and all brittle materials on the micrometer scale [6, 2].

The analysis of interatomic bond breaks and multiscale transitions to the RVE has shown that the strength of one RVE must have a Gaussian distribution transiting to a power law in the tail of prob-

ability  $< 10^{-3}$ . For the typical case of Type 1 size effect, which occurs in structures with the so-called positive geometry (i.e., the geometry for which the energy release rate at constant load increases with the crack extension), the quasi-brittleness means that a larger structure fails if a single RVE fails, in other words, if it behaves as a chain of links, each representing one RVE. It was shown that as the structure's size  $D$  (or the number  $N$  of RVEs in the chain) increases, the failure load distribution gradually changes from Gaussian to Weibullian in such a way that a Weibull tail gradually grows into the Gaussian core.

In quasibrittle structures,  $N$  is not large enough (not  $> 10^5$ ) to make the distribution completely Weibullian. So one has a Gauss-Weibull graft, for which the mean and the coefficient of variation do not suffice to locate the tail probability of  $10^{-6}$ . Since, for the Weibull distribution, the ratio of the load of  $10^{-6}$  failure probability to the standard deviation is almost double the ratio for the Gaussian distribution, determining the Gauss-Weibull graft is crucially important. This means that the structure must be probabilistically modeled as a *finite* (rather than infinite) chain of RVEs [5]. Unlike ductile or perfectly brittle structures, the number of RVEs and their weighting becomes very important for safety assessments [13, 14, 15].

The present lecture reviews the nano-mechanical argument for fracture on the atomistic scale, the multiscale transition to the RVE level, and the computational challenges in calculating the failure probability tail. Some typical comparisons with test data for concrete and tough ceramics, documenting the applicability of the theory, are displayed (Fig. 1). Extension to the size effect on the lifetime distribution of quasibrittle structures subjected to static or

cyclic fatigue is also explained [9, 10].

Although the present theory can be calibrated from histograms of strength of many identical specimens of different sizes, it is argued that test data on the mean size effect of a broad enough range allow a much more effective and simpler calibration. Then, the recently developed boundary-layer non-local method for computing the failure probabilities and the size effect on the probability distribution are briefly explained and their application illustrated [11].

In the second part of the lecture, two new advances are briefly presented. One involves the computation of failure probability of residual strength under sudden overload after a sustained period under constant stress.

The other involves the probability distribution of failures after large stable crack growth, which exhibit the so-called Type 2 size effect. Since this is a venture into an unexplored realm, let us now explain it briefly.

These failures, whose statistics has apparently not yet been subjected to fundamental probabilistic modeling, are typical of reinforced concrete structures as well as some unreinforced ones, which typically have an initially negative geometry and fail once the crack growth switches the geometry to positive. They are known for a strong energetic (or deterministic) size effect roughly following Bažant's size effect law. The statistical part of size effect has been considered as negligible, mainly because the mechanics of fracture dictates the stable crack growth in structures of different sizes to follow nearly homologous paths despite the randomness of the material. Nevertheless, the coefficient of variation of strength is likely to depend on the structure size, and there might be some size effect on the mean as well.

Like in all studies of strength statistics, we assume that the tip of the dominant crack may lie, at the maximum load state, at various locations—particularly at the centers of square elements  $i = 1, 2, \dots, N_D$  in a square grid, as exemplified for shear failure of a reinforced concrete beam in Fig. 2. Each square element is assumed to represent an RVE of the material of characteristic size  $l_0$  (which implies an autocorrelated random strength field). Based on Bažant's size effect law as [6, 2] it can be shown

that

$$\sigma_{N_{i,\beta}} = \frac{B_{i,\beta}}{\sqrt{1 + D/D_{0i,\beta}}} f'_t \quad (1)$$

$$\text{where } D_{0i,\beta} = \frac{g'_{i,\beta}}{g_{i,\beta}} c_f \quad (2)$$

$$B_{i,\beta} = \sqrt{\frac{l_0}{g'_{i,\beta} c_f}}, \quad l_0 = \frac{E' G_f}{f_t'^2} \quad (3)$$

Here  $F$  = given load (maximum load),  $b$  = beam thickness,  $E$  = elastic modulus,  $G_f$  = fracture energy,  $l_0$  = Irwin's characteristic material length roughly equal to the length of the FPZ,  $c_f$  = material characteristic length for size effect ( $c_f/l_0 \approx 0.44$  for 3PB tests [7]);  $g_{i,\beta}$  = dimensionless energy release rate for crack extension from element  $i$  in the direction  $\beta$ , and  $g'_{i,\beta}$  = derivative with respect to the crack extension length in direction  $\beta$ ; and  $\sigma_{N_{i,\beta}}$  = nominal strength of structure of size  $D$  when the failure is due to crack extending in direction  $\beta$  from the  $i$ -th RVE with tensile strength  $f'_t$  (this means that we consider the crack growth to be governed by the cohesive crack model or crack band model, or some nonlocal damage model, in which  $f'_t$  is one basic material fracture characteristic, which is considered random). Before undertaking the failure analysis, the values of  $g_{i,\beta}$  and  $g'_{i,\beta}$  are evaluated by J-integrals and finite elements for different directions  $\beta$  of crack propagation. The direction  $\beta$  that gives the overall maximum energy release rate is determined and fixed (for the sake of simplicity, with same value for all tips  $i$ ). The energy release rate also depends on the shape of the crack (which itself is a result of Markov random process of crack growth) and on the location of its starting point, but these effects appear to be secondary and are here neglected, for the sake of simplicity.

What greatly simplifies the problem is the fact that the deterministic size effect law in Eq. (1), which is already well established, can be imposed as the input, rather than being solved as part of the analysis. It is through this law that the energetic aspects of fracture mechanics are conveniently introduced.

The probability that the structure of size  $D$  fails due to a crack extending in direction  $\beta$  from the  $i$ -th RVE of tensile strength  $f'_t$  may be written as

$$P_{f_{i,\beta}} = \text{Prob}(\sigma_{N_{i,\beta}} < \sigma_{ND}^L) \quad \text{where } \sigma_{ND}^L = F/bD \quad (4)$$

where  $\sigma_{ND}^L$  is the nominal stress of structure under given load  $F$ . Then, since:

$$P_{f_{i,\beta}} = \text{Prob} \left( f'_t \leq \frac{\sqrt{1 + D/D_{0i,\beta}}}{B_{i,\beta}} \sigma_{ND}^L \right) \quad (5)$$

The following basic statistical hypothesis now appears logical:

$$P_{f_{i,\beta}} = \Phi_{GW} \left( \frac{\sqrt{1 + D/D_{0i,\beta}}}{B_{i,\beta}} \sigma_{ND}^L \right) \quad (6)$$

where  $\Phi_{GW}$  = cumulative probability distribution function (cdf) characterizing the tensile strength of one RVE. Based on previous work [5, 9], it must have the form of a grafted Gauss-Weibull probability distribution, which was derived theoretically from nano-mechanics and multiscale transition of probability tail, and was extensively verified and calibrated by Type 1 tests of size effect and histograms.

In deterministic analysis, only one specific crack tip location corresponds to failure. But if the material is random, every location could correspond to failure, albeit with a very different probability  $P_{f_{i,\beta}}$ . For a random material, the structure of size  $D$  will survive under load  $F$  if none of the crack tips  $i$  leads to failure. So, according to the joint probability theorem:

$$1 - P_{f_D} = \prod_{i=1}^{N_D} \left[ 1 - \Phi_{GW} \left( \frac{\sqrt{1 + D/D_{0i,\beta}}}{B_{i,\beta}} \sigma_{ND}^L \right) \right] \quad (7)$$

where  $P_{f_D}$  = failure probability of the structure of size  $D$ ; and  $\Phi_{GW}$  = cumulative grafted Gauss-Weibull probability distribution of the strength of one RVE of the material, derived theoretically from nano-mechanics and multiscale transition of probability tail, and calibrated experimentally by Type 1 size effect tests.

The point to note is that, like in Type 1 strength analysis, the number of independently contributing material elements is greater in a large structure than it is in a small one; see Fig. 2. This intuitively explains the cause of the statistical size effect in Type 2 failures.

What allows us to base the failure probability analysis on the strength (rather than the critical energy release rate) of one RVE is that fracture of a quasibrittle material is properly analyzed in terms of

the cohesive crack model or crack band model, for which the material strength is one essential material property characterizing the distributed damage in the FPZ. The lecture presents and reviews the results of this analysis for typical reinforced concrete beams of various sizes, including the statistical effect on the mean nominal strength and on the coefficient of variation of nominal strength.

*Final comment:* The results of computer analysis must be subjected to various safety factors, particularly the overload factors and the understrength factors dictated by the design code. At present, these factors are mostly empirical and highly uncertain, mainly because they do not properly reflect the tails of probability distributions and the size effect. Thus, in fact, *the design code provisions accounting for uncertainty are more uncertain than anything else in design*. Therefore, to benefit from accurate computer analysis of structures, these tails and the size effect must be realistically incorporated into the system of safety factors. Until that happens, a highly accurate computer simulation of the strength of concrete structures has only little practical value.

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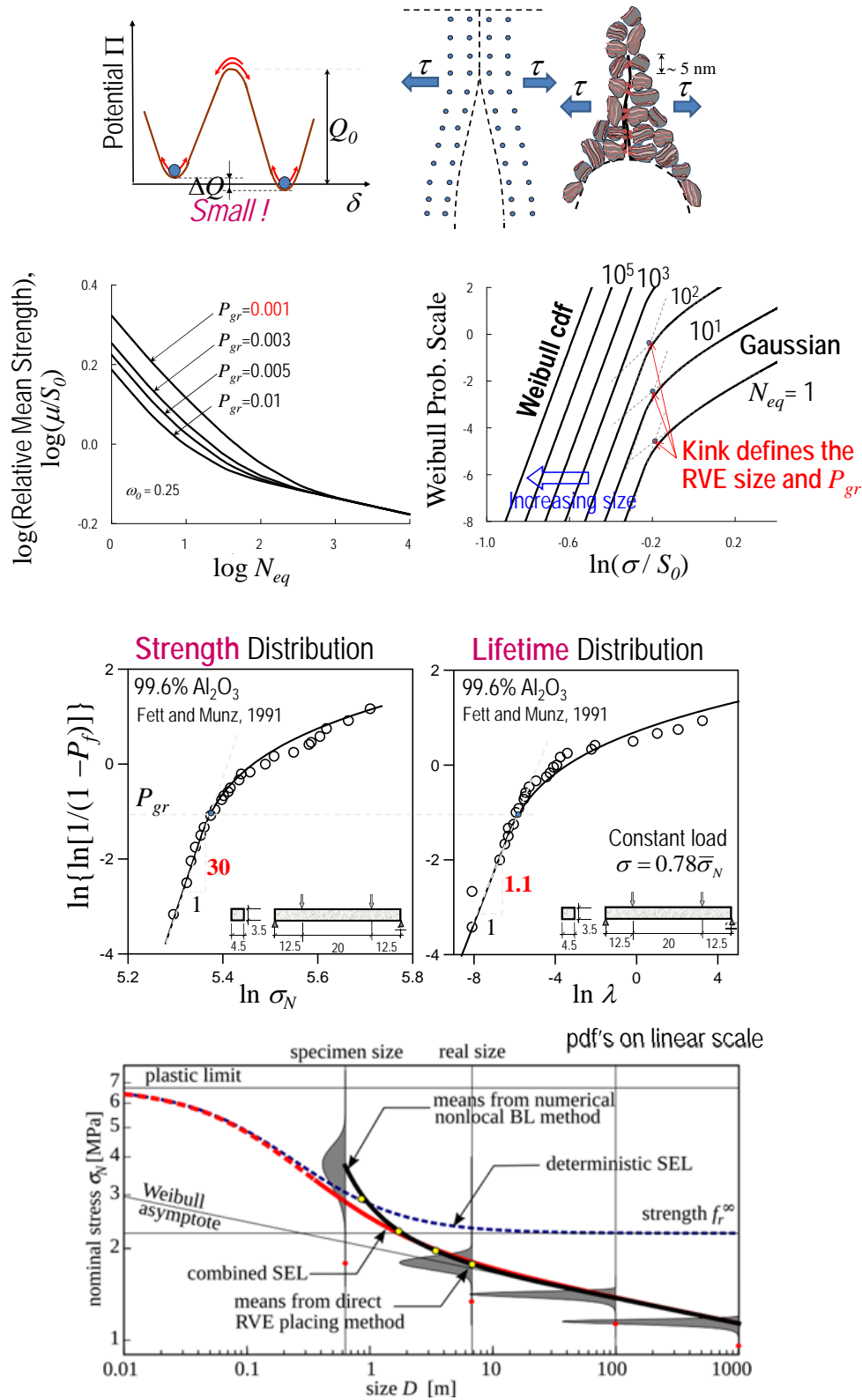


Figure 1: Illustrations of basic concepts of probability of quasibrittle failure. First row: Small change of activation energy barrier due to failure of an interatomic bond in an atomic lattice of nano-scale element; Second row: calibration of the Gauss-Weibull grafted distribution parameters using size effect data; Third row: optimum fits of strength and lifetime histograms of 99.99% Al<sub>2</sub>O<sub>3</sub>; Last row: Type 1 size effect on the mean strength and on the strength distribution evolving from mostly Gaussian toward Weibullian, calculated for collapsed Malpas Dam.



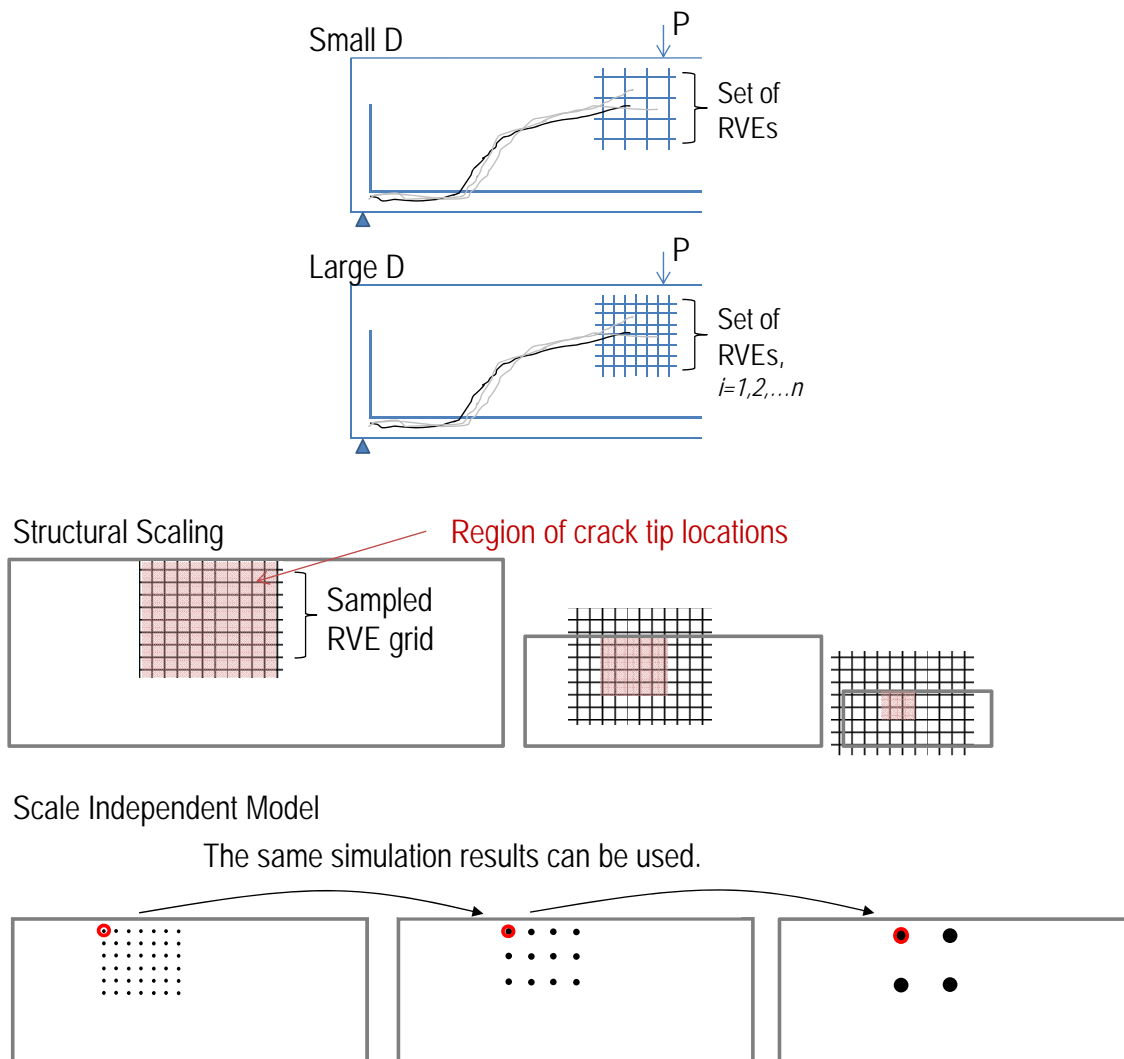


Figure 2: Top two rows: Schematic of half of a 4-point bending shear failure illustrating the approach used to determine the probability of quasibrittle failure of Type 2. A number of gray geometrically scaled crack paths from size effect tests are averaged to obtain an average crack path shown in black. The crack tip of the averaged crack path is placed in each RVE within the zone of possible locations to compute geometric fracture parameters. Bottom row: Schematic of the scaling of the set of RVEs employed to utilize a single set of strain energy release rate calculations to determine the size effect on the CoV of the structural strength. In a scaled down structure (on the right), there are fewer RVEs that influence the maximum load since their spacing is governed by the ratio of  $l_0/D$ .



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## Modeling of Damage in Unidirectional SiC/SiC Composites and Multi-Scale Experimental Validation

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Because of their favorable mechanical properties at high temperatures and under irradiation, SiC/SiC composites are prospective candidates for functional uses in future nuclear reactors.

Elaborated from woven tows constituted by 500 Hi-Nicalon type S fibers and a SiC matrix deposited by a Chemical Vapor Infiltration process (CVI), such materials present a complex and multi-scale microstructure. Moreover, they exhibit a nonlinear behavior due to the accumulated damages occurring between and inside the tows, such as through matrix cracking, fiber/matrix debonding as well as fiber breaking. So, a multi-scale approach is under development to build a predictive model of their complex and damageable mechanical behavior taking into account their heterogeneous microstructure [1].

In this context, this paper focuses on the study of damage process at the scale of the tow under a uniaxial tensile loading. Thus, a micromechanical approach has been developed to get a better understanding of interactions between the various microscopic damage mechanisms as well as their impact on the macroscopic behavior of the unidirectional composite. Beyond the building of a 1D probabilistic model of damage evolution, the key point of this approach lies in its identification and validation at both local and macroscopic scales.

Therefore, in addition to classical tensile tests on a macroscopic device, damage mechanisms within minicomposites (1D composites containing a single bundle of fibers) have been experimentally characterized using two specific in-situ tensile tests. Statistical data about the cracking as a function of the applied load – such as initiation, opening and spacing of matrix cracks – have been obtained from scanning electron microscope (SEM) observations. In order to fully characterize damage, these observations were complemented by an X-ray computed microtomography investigation

to observe the propagation of matrix cracks [2]. In addition, radiographs observation provided statistical data on fiber breaking.

Then, a numerical probabilistic model, including both matrix cracking and fiber breaking, is proposed on the basis of existing modeling tools separately addressing these mechanisms. It is based on matrix and fiber failure probability laws and a stress redistribution assumption in the vicinity of matrix cracks or fiber breaks [3, 4]. The identification of interfacial parameters was conducted to fit the experimental characterization, and shows that conventional assumptions of 1D probabilistic models can adequately describe matrix cracking at both macro- and microscopic scales. However, it is necessary to enrich them to get a proper prediction of ultimate failure and fiber break density.

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## Thermomechanical Couplings in Metallic Polycrystals: Full-Field Measurements and Thermoplastic Simulations

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Polycrystalline metallic materials are made of an aggregate of grains more or less well oriented, with respect to the loading axis, for plastic gliding. Under mechanical loading, this leads to a heterogeneous deformation at the microstructure scale. This local plasticity triggers a heterogeneous thermal dissipation caused by mechanical irreversibility. Some original experimental works enabling the simultaneous determination of thermal and strain fields, in the same area, at this scale have already been realized in house on a A316L stainless steel [1,2]. Two complementary ways have been followed: some numerical treatments in order to access to experimental dissipations and the development of a consistent constitutive model. Both aspects are presented in this communication and a dialogue between microstructural texture coming from EBSD analysis, local deformation mechanism and thermal localization phenomenon is introduced.

The treatment of the full-field measurements is done with respects to the polycrystalline texture of the material. Under different assumptions, strain and thermal fields are obtained grain to grain [3]. The local disorientation, the different grain sizes and the crystallographic orientations are the main aspects, which have an influence on the strain localization process at the grain scale. The analysis of the local temperature evolutions is a key important feature of this coupled analysis as it enables, for example, the determination of the critical resolved shear stress from grain to grain. The values are in good accordance with the CRSS determined on monocrystals and coming from literature.

The numerical implementation in a FE code of a fully coupled crystalline plasticity constitutive model has been realized and is the other main part of this work [4,5]. It enables to compare the local kinematic and thermal distributions during monotonic tests and to study the heterogeneity of the stored energy at grain scale. Some energy balances are conducted at the global scale but also

grain to grain. It exhibits some large heterogeneity at this scale with an evolution with the loading. These analyses of thermomechanical couplings at the grain scale could lead to the definition of new thermodynamically based strain localization criteria.

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## Identification of a Cohesive Zone Model at the Micron Scale

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Since the pioneering work by Barenblatt [1] and Dugdale [2], cohesive zone models have been widely used. One of the reasons for this success is their ability to model non-linear fracture processes that is suitable for numerical simulations: mesh dependency is avoided as long as appropriate the size of cohesive elements is considered, computational cost is preserved as the non-linear processes are concentrated along lines in 2D, planes in 3D.... However, the length scale involved in cohesive fracture is usually small compared to the length of a crack or a structure. Therefore, the observation and/or identification of non-linear processes inside the cohesive zone is a challenging topic. We propose a methodology based on the use of digital images and X-FEM. Digital Image Correlation (DIC) is a powerful method that allows to compute (in its 2D version) the displacement field on the surface of solid between two states. Its is based on the resolution of the passive advection of the grey level functions of a reference image  $f$  and a deformed one  $g$ :

$$f(z) = g(z + \bar{u}(z)). \quad (1)$$

In a first analysis, the displacement is searched for as a decomposition over Williams' series [3]. As shown in [4], it allows for extracting not only stress intensity factors but also the position of the crack tip. In the case when a cohesive zone is developing, based on the analysis of far-field local non-linearity are ignored. The equivalent elastic crack is thus determined as the crack within a perfectly elastic material which produces the same far-fields. The evolution of the equivalent elastic crack position is obtained from this analysis. During the analyzed experiment, the advance of this tip corresponds to the development of crazing within a cohesive zone. In a second step, X-FEM simulations with a cohesive model are performed using the displacement obtained by DIC as boundary conditions. The obtained displacement field is projected onto the Williams' series so that the equivalent elastic crack from

the X-FEM simulation is determined. The parameters of the cohesive law are then adjusted so that the gap between the crack tip position obtained the experimental analysis and the numerical simulation is minimized. A test on a PMMA sample is performed and analyzed with the proposed methodology. The set identified parameters agrees well with values found in the literature. Further, the proposed methodology allows to access to the mechanical state inside the cohesive law whereas its length is far smaller than the usual resolution of DIC. The methodology is exemplified in a fracture test on PMMA but the framework allows for investigation of the cohesive zone between in other polymers in which no cohesive zone are available at the moment and cohesive zone representing the interface between assembled bulk part.

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## Fracture of a $\gamma$ -TiAl Polycrystal: Model Versus Experiment

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Titanium Aluminides have excellent mechanical properties at high temperature, which makes them attractive for aeronautical applications such as turbine blades. However, the material suffers from poor ductility at room temperature and is known to be extremely sensitive on the underlying microstructure. Consequently, in order to understand the influence of the different microstructural parameters on the fracture toughness, this work presents a micromechanical study of cleavage and grain boundary fracture in a small  $\gamma$ -TiAl polycrystal.

In earlier work [1, 2, 3], a methodology was developed in order to identify the parameters of a constitutive law for these materials by confronting Crystal Plasticity Finite Element Method (CPFEM) calculations with experimental strain fields at the sample surface. The latter were measured during an *in situ* four-point bending test in a scanning electron microscope. The microscopic strain fields were measured using digital image correlation techniques between the undeformed and the deformed images of a microgrid which was deposited at the surface beforehand. The strain fields were also correlated with the grain orientations and morphologies using EBSD. In the method special attention is paid to the way in which the boundary conditions are applied in the CPFEM calculations, and also to the representation of the unknown subsurface microstructure.

The objective of the current work is to present the first steps to extend this method to crack propagation. In  $\gamma$ -TiAl cracks initiate and propagate along grain boundaries or along cleavage planes inside the grains. Both mechanisms are modeled with cohesive zone elements (CZE). A first set of approximative parameters for both mechanisms is adopted, based on literature values and on comparing experiments from the literature with bicrystal models. One well-known difficulty with using CZE is that the crack path is not known beforehand, and that CZE cannot be inserted between all volume elements of the Finite Element mesh, because the final

crack path would become strongly mesh-dependent. Much effort has therefore gone into the development of numerical tools (remeshing and transfert of internal variables) which allow insertion of cleavage crack planes and the associated CZE on the fly, using a cleavage stress threshold calculated in a region close to the crack front. On the experimental side, an *in situ* four-point bending test has been carried out on a small  $\gamma$ -TiAl sample. A special sample holder made out of Al was designed in order to prevent catastrophic crack propagation, which would not leave sufficient time for image acquisition. As before, the grain morphologies and orientations at the surface were determined using EBSD.

A first comparison between the experiment and the corresponding CPFEM simulations is presented, and the (dis)agreements between them are discussed. This demonstrates that the basic ingredients are now in place in order to initiate the full inverse optimisation procedure.

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## A Digital Image Correlation Controlled Multiaxial Machine to Perform Mixed Mode Crack Propagation Tests

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The tests performed by Nooru-Mohamed (NM) in 1992 [1] on double-notched concrete specimens are interesting for several reasons. First, crack propagation occurs under mixed-mode condition. Second, during a test the ratio  $K_I/K_{II}$  may evolve and the resulting final crack geometry may be complex. Third, numerous tests were performed with various loading paths (proportional or not, displacement or force controlled), offering a potential rich benchmark for numerical models.

The goal of this study is to perform “new” NM tests, i.e. to keep the test principle and add new experimental techniques, like full field measurements which are a priceless technique for experimental/numerical confrontation.

The machine is a electromechanical hexapod (6 Degrees Of Freedom) with sufficient force capacity and displacement resolution to perform a NM test [3]. Beyond the obvious application of mixed-mode propagation loadings, the interest of such a machine are the extra DOF it offers. They may be used to control unwanted loadings, e.g. out-of-plane ones, unlike a usual biaxial machine. One may force the crack tilt (or mode III for example) to zero, or on the contrary impose one.

These type of tests can be obtained with a displacement control (*stricto sensu* the crack propagation is not controlled) or a Stress Intensity Factor (SIF) control (propagation is controlled). In this last case, the use of an outer loop with full-field measurement techniques like Digital Image Correlation (DIC) is evident. It may be direct, giving an SIF by using relevant shape functions [2], or indirect, introducing a Linear Elastic Fracture Mechanics calculation with a mesh updated by the crack tip position obtained by DIC. For a displacement control, an outer loop is also needed because of the non-negligible

compliance of the machine. A technique based on DIC has been developed as a preparation for more complex SIF setup since several difficulties are similar: high number of DOF, 3D motions, computation time, DIC algorithm, links between components.

Besides the general approach of this study, the presentation will emphasize the displacement-controlled setup as a first step toward SIF controlled one.

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## Quantification of Three-Dimensional Surface Deformation Using Global Digital Image Correlation

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A change in surface height profile is characteristic for many mechanical phenomena, e.g. a) surface relief during cycling fatigue loading, b) grain boundary sliding during many creep experiments, c) surface height changes during loading of a metal, due to crystallographic orientation differences between grains and dislocations escaping the free surface, or d) in contact and wear problems.

In these phenomena, pronounced local variations in the surface height appear. Quantifying this surface height evolution, both in-plane and out-of-plane, can provide valuable information on the underlying mechanisms. This quantification is typically performed by studying average surface height values or the statistical height-height correlation [1]. While these techniques give insight into the amplitude of the height change and how the (average) height evolves, they do not provide actual information on local material deformation.

This local information is particularly important in the case of deformation-induced surface roughening of a polymer-coated steel used, for example, for food and beverage packaging. During production, the material is subjected to large deformations at high temperatures and strain rates. After production, the material must not exhibit any damage, even after a relatively long shelf-life period, as this triggers corrosion and compromises the quality of the content [1].

A new methodology to obtain the three-dimensional surface deformations was developed. The method uses Global Digital Image Correlation to extract the surface deformations from evolving surface height profiles and is independent of the measurement technique used to obtain these profiles.

For validation of the method, the deformation-induced surface roughening of a polymer-coated steel during a uniaxial tensile test was studied. During the experiment, the surface height profile was

captured *in-situ* using confocal microscopy. This case study was chosen because of its complexity, i.e. a) the initial surface roughness of the steel is smooth in the rolling direction of the steel, creating a low contrast (height) profile in this direction, b) the steel shows Lüder-bands in a uniaxial tensile test, which is accompanied by a sudden change in the surface profile, and c) the surface profile continuously and progressively changes due to the deformation-induced roughening.

The displacement fields, extracted from the experiment, reveal the full-field kinematics accompanying the roughening mechanism. Local deviations from the (average) global displacements are the result of the formation, growth, and stretching of hills and valleys on the surface.

This new way of characterizing an evolving surface height profile provides quantitative information for various surface deformation phenomena, and hence out-performs conventional methods using average height values. This information can be used, for example, in numerical simulations of roughening-induced interfacial de-adhesion, directly affecting the local interface integrity.

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## A Three-Dimensional Analysis of Fatigue Crack Paths in Thin Sheets

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Fatigue crack growth normal to the tensile axis becomes unstable in thin metallic sheets, above a material and environment-dependent loading amplitude, even though, in many cases, small scale yielding conditions still prevail.

Shear lips development has been investigated mainly in aluminium alloys by Schijve and co-workers [1-2] or Zuidema et al. [3-4], while very few studies were devoted to steel [5].

Fatigue crack growth in thin sheets of 7075 T651 aluminium alloy and S355 steel were characterized in 3D, using crack front markings and topographic reconstructions of fracture surfaces. Tests performed in air or in salt water produced different crack paths for similar mechanical conditions, shear lips being reduced by corrosive environment, in the aluminium alloy as well as in the steel.

Tunnelling was progressively reduced and cancelled as slanted crack growth developed, even though  $\Delta K_I$  was reduced locally by crack twisting. This indicates a significant contribution of shear modes to the crack driving force, even though mode I striations are present in slanted zones.

Elastic three-dimensional X-FEM computations were performed with real crack geometries to analyse the observed crack growth kinetics, based on  $\Delta K_I$ ,  $\Delta K_{II}$  and  $\Delta K_{III}$ . The crack growth rates correlated much better to

$$\Delta K_{eq} = \sqrt{\Delta K_I^2 + \Delta K_{II}^2 + \frac{\Delta K_{III}^2}{(1-\nu)}}$$

than to  $\Delta K_I$ .

Elastic-plastic finite element simulations and the local application of a fatigue criterion with an

amplitude-dependent critical plane were found to capture qualitatively the transition in fracture mode and its inhibition by side grooves.

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## Numerical Modeling to Analyse Optical Fiber Measurements Along a Steel-Concrete Interface

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The durability of nuclear reactor confinement vessels is a crucial issue in civil engineering. Inside such a large structure, vertical and orthoradial prestressing cables apply compression to the reinforced concrete in both directions. Thus, the steel-concrete interface is loaded in shear due to the material stiffness difference while the concrete is in compression. Furthermore, this load level may vary in time due to material creep and ageing or decennial pressure tests. Such a stress state must be accounted for in numerical simulations and it must be previously analysed from an experimental point of view.

Many researches have already been performed on steel-concrete bond in civil engineering structures, leading to numerous numerical models (such as Dominguez [1] or Richard [2]). However, in most cases (such as Eligehausen [3] and Pijaudier-Cabot and La Borderie [4]) the interface was loaded by means of tension in rebars (through pull-out tests) with or without confinement. Besides, experimental results are often global and do not provide local information along the interface. Therefore, a specific setup has been developed to analyse locally the interface behaviour with rebar under compression.

A 16 mm diameter steel rebar is set up along the axis of a 16x32 cm concrete cylinder. A push-in load is applied on the top of the steel bar while the concrete cylinder is supported at the bottom. Pre and post-peak behaviours using various interface lengths are studied with load - unload cycles remaining in the compressive domain.

Distributed optical fiber sensors [5] are embedded in the concrete for measuring at different load steps the vertical strain parallel to the steel bar. This minimally intrusive instrumentation, supported by Ifsttar, provides a convoluted measurement of the local strain. Thus, the post-processing leads to transfer function issues. Therefore, we propose to use an experiment-model dialog in which the

numerical parameters are calibrated on the convoluted strain signals to assess the local strain quantities. Very useful information on the local behaviour of the interface are thus obtained regarding the three possible failure modes of the specimen: (steel bar yielding, concrete splitting or interface sliding).

The methodology is firstly validated when the steel bar yielding rupture occurs, while the concrete and the interface remain in their own elastic domain. It is then applied when the interface sliding occurs and compared to numerical results using the interface model developed by Richard [2]. Results are bringing a new and direct lightening on the local mechanisms acting inside the steel-concrete bond.

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## **Crack Characteristics in Complex D-Regions Designed Using Strut-and-Tie Models in Reinforced Concrete Structures**

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Strut-and-tie model (STM) is recognized as the most rational and simplest method for designing D-Regions and recommended in several codes of practice, such as those in appendix A of ACI318-08. Since these code rules were primarily derived to be taken as being sufficiently general and conservative only for capacity assessment (ultimate limit states), members designed using these STM provisions may not necessary exhibit satisfactory performance under service load levels [1] e.g. acceptable crack width and propagation.

In this research, to understand cracking characteristics and propagations in D-regions designed using STM, an experimental [2] and computational [3] program was conducted. A total of 17 test specimens of four different types of complex D-Regions were designed based on the strut-and-tie approach and the associated code provisions in Appendix A of ACI318-05. The shapes of STM used in those specimens were selected from the different multiple truss shapes. The numerical approach so called “Topology Optimization” was also adopted to guide the selection of STM shapes. All specimens were fabricated, cast, and instrumented with both tradition and advanced measurement system e.g. Krypton K600 Dynamic Measurement Machine (DMM) for accurately measuring 3D coordinates of Light Emitting Diodes (LEDs) mounted to the surface of the test structure [1,2]. The use of high-resolution photographs taken during the experimental programs was also adopted for recording crack markings. All specimens were then loaded test to failure using displacement controls.

The experimental program results in the following significant findings: 1) the cracking load, crack development, pattern, and propagation on the surface of specimens according to all loading steps was reported to be significantly influenced by STM shapes including orientation and amount of main

reinforcement; 2) the distributed reinforcement required in ACI318-05 for design of D-Regions was also reported to significantly dominate the failure mode relevant to unstable crack propagation; 3) the shapes of STM based on Topology Optimization are also reported to show the most superior structural performance under service loads in all specimens.

Based on the findings, to eliminate the deficiency and impracticability of the current STM, a new design and analysis procedure for complex D-Regions is thus proposed and presented in this research by integrating topology optimization for STM shape selection and a new nonlinear Finite Element Analysis (FEA) with the existing STM code provisions which results in the effective structural performance design of D-Regions validated experimentally.

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## Application of a Vanishing Viscosity Procedure to a Fiber-Matrix Debonding Problem

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Quasistatic inelastic processes on interfaces of solid bodies usually referred to as interface fracture or damage and also as debonding or delamination [1, 2, 3] are examined. The so-called energetic solution [1, 2, 3, 4] tends to produce in some applications to delamination problems an unrealistically too early delamination [5, 6]. However, when a certain small viscosity is assumed for approximately elastic bodies, stress-driven rupture may occur, which seems to be more natural and also in agreement with usual engineering applications. Damage along the interfaces can be taken into account by introducing an interface damage variable. In the present work, Kelvin-Voigt rheology [7, 8, 9], which is the simplest rheology commonly used in engineering practice, is assumed to approximate well the behaviour of solid bodies studied. We study one of quite complex failure mechanisms in fiber reinforced composites referred to as matrix cracking or inter-fibre failure under transverse uniaxial tension. This failure mechanism usually initiates by fibre-matrix debonds. Although, there is a large amount of analytical and numerical studies of this problem available in the literature, there is still a controversy regarding the initiation of this failure mechanism. In particular, considering a single fibre specimen, two main debond geometries are predicted by numerical studies either with one or two symmetrically situated debonds. The present work tries to clarify which of these two debond geometries is preferable. The present numerical implementation is accomplished using the collocation boundary element method for the spatial discretization.

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## Isogeometric Analysis of Mode-I Delamination in Composites

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Applications of fiber-reinforced composites in the aircraft, automobile and civil engineering industries are tremendously increasing. Safe and efficient use of these materials requires proper account for the failure mechanisms of the composite structural components, which presents challenges due to their highly heterogeneous nature. Neglecting the damage mechanisms within the bulk materials (matrix and fibers), this work focuses on interface degradation phenomena, in particular on debonding of two adjacent laminae within a laminate (delamination).

In the framework of finite element methods, delamination phenomena are most often treated using interface elements. However, these methods suffer from oscillations at large stress gradients that are only alleviated by impractically fine meshes. Non-conforming discretization frameworks, more suitable for large deformation settings, suffer from the lack of smoothness of the discretized contact surfaces in standard (Lagrange) finite element interpolations, which causes deficiencies in both accuracy and robustness. A framework where better geometrical accuracy is combined with higher (and tailorable) inter-element continuity is provided by isogeometric analysis. In particular, the isogeometric basis functions used in this work are Non-Uniform Rational B-Splines (NURBS).

Motivated by the impressive results of NURBS-based interpolations for the solution of contact problems [1, 2, 3], and considering that contact and delamination phenomena can be treated in a unified manner by properly extending unilateral contact formulations, this research develops a computational modeling framework to describe the interface damage mechanisms for laminated composite materials exploiting the advantages of the NURBS-based isogeometric setting.

The model for the interface between adjacent laminae incorporates frictionless contact in compression, and a mode-I cohesive zone model featuring a non-linear relation between interface normal open-

ing and traction under tension. In the numerical setting, the interaction between non-conforming discretizations for the adjacent laminae is treated with a master-slave approach by enforcing the contact or cohesive-zone model constraints at every surface integration point on the slave side.

The methodology is explored in a three-dimensional finite deformation setting for mode-I delamination cases. A few different cohesive zone models available in the literature are implemented. The numerical results show that mode-I cohesive zone formulations in conjunction with NURBS-based discretizations allow for a more accurate and robust treatment of delamination phenomena in comparison to standard linear and higher-order Lagrange discretizations.

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## Towards a Coupled Thermo-Mechanical Numerical Model of TBC Delamination

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Thermal barrier coatings (TBC) are widely used to prevent components from high temperature attack and extend their durability. Nevertheless, in service TBC systems are subjected to severe thermo-mechanical loading and they may break by spallation of large areas of the protective ceramic layer. Due to coefficient of thermal expansion mismatch between the layers the TBC failure often initiates near the interface between the brittle oxide layer and the ductile substrate. Microcracks initiate from the growing oxide, expand along the interface which subsequently leads to the spallation of the TBC layer. This failure mechanism shall be discussed and modeled in this work by means of coupled thermo-mechanical cohesive zone model. The object of study is a multilayered structural system loaded by an applied heat flux obtained from an aero-thermal computation of the turbine. The cracks expanding along the interface induce a temperature jump and modify the heat flow conducted across the interface. The aim of this work is to develop a physically motivated, computationally efficient and complete thermo-mechanical cohesive zone model including its finite element formulation. The thermal and mechanical problems will be solved in a coupled way to simultaneously consider the changes in load transfer due to crack propagation and the heat flux variations as a result of the mechanical damage of the interface.

The mixed finite interface element for cohesive zone models [1] is implemented in the finite element code Z-set to mesh the crack path located between the TBC and the substrate. Its variational formulation is based on an augmented Lagrangian, the degrees of freedom of which are the displacement jumps and the density of cohesive forces. The model accounts for the well-known fact that the fracture toughness of the interface is not a constant but a function of the mode mixity (or phase angle). Within the framework of this model, the co-

hesive energy and hence the cohesive strength are not chosen to be a constant, but rather functions of the mode mixity parameter. Continuum Based shell elements including Multi-Point Constraints to ensure kinematic continuity between the cohesive elements and the shell's reference surface are used to mesh the TBC.

The description of thermal transport includes a thermal cohesive zone model inspired by [1] in which the degrees of freedom are the temperature jumps and the heat flux across the interface. This thermal cohesive zone can describe the breakdown of the interface conductance with increase in material separation. The cohesive zone conductance, based on [2], takes into account the current state of interfacial failure, the presence of gas trapped between the crack surfaces, the radiative heat transfer across the crack and a contact conductance between the crack faces. The thermal gradient through the thickness of the TBC also plays an important role in modeling the TBC delamination. A Continuum Based thermal shell element with a linear approximation of the temperature in the thickness is introduced to mesh the TBC in the thermal problem. A coupled numerical framework for modeling the TBC failure is presented and applied to a representative test case. The thermo-mechanical coupling is performed with the partitioned coupling algorithms of Z-set.

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## Influence of Contact in Fragmentation Phenomena

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Fragmentation consists in a rapid and catastrophic failure of a material. When subjected to extreme loading, internal local defects cause nucleation and extension of cracks. These cracks propagate and coalesce forming fragments, which can move and impact each other.

Dealing with this problem is very challenging for several reasons: it is fast transient, many complex mechanisms lie behind it, and it is characterized by a high unpredictability. Some aspects, like fragment size distribution, begin to be well understood (see [1, 2]) while others, such as fragment residual velocity distribution, still represent an open question. Studying this phenomenon experimentally is difficult, so numerical methods represent a useful tool for this purpose. Numerical methods that are usually utilized are divided in two categories: particle and continuum methods. In the first case materials are represented by a certain amount of discrete particles. This approach permits to handle discontinuities and big deformations more easily, but at the same time usually leads to highly complex computations and can potentially cause non-physical behaviors (only at atomic scale materials can effectively be represented by particles). In the second case materials are represented as continuum objects in order to easily obtain a physical behavior together with high efficiency. However this makes it hard to deal with discontinuities (cracks) and big deformations.

No matter which numerical method is used, the main issue in simulating a dynamic fragmentation is properly modeling contact. The aim of this work is to analyze its importance and effects on such simulations with a particular emphasis on residual velocities. For this purpose a very simple case is considered: a brittle quasi 1D bar under uniform traction modeled through the Finite Element Method with cohesive elements (see [3]). In this problem it is possible to study also contact because fragments impact each other. In fact when cracking oc-

curs, stress waves propagate inside the material and consequently fragments repeatedly shrink and expand. First a reference case without energy dissipation and contact is analyzed. Then these two levels of complexity are sequentially added. The following data was examined:

- number of fragments;
- dissipated energy;
- residual velocities.

It emerged that contact plays a fundamental role when dealing with materials characterized by low fracture energy dissipation, even in the context of tensile loading. Indeed, while both number of fragments and dissipated energy increase, residual velocities change significantly. A power law relation was observed in the difference between final residual velocities and initial ones.

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## Identification of Cohesive Zone Models from Thermomechanical Imaging Techniques

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Cohesive-zone models (CZMs), which were first introduced by Dugdale [1] and Barenblatt [2], are widely used in numerical simulations to describe the fracture processes in various materials and structures. However, the identification of the CZM constitutive equations still remains a difficult problem. In this work, we focus on overall elastoplastic damageable behaviors of ductile materials. The CZMs are associated with a purely damageable softening surface behavior embedded between purely elastoplastic bulk elements with non-negative hardening.

With the recent developments in imaging techniques, local thermo-mechanical measurement fields can now be reached. This paper aims at proposing an experimental approach combining Digital Image Correlation (DIC) and InfraRed Thermography (IRT), to characterize the shape and the parameters of a CZM by analyzing the kinematic fields and checking the thermo-mechanical consistency of the identified model with respect to the calorimetric measurements.

An experimental setup, which includes a mechanical testing machine, a CCD camera and an InfraRed (IR) camera, is used. The kinematic measurements are extracted from the CCD images using DIC and the thermal measurements are obtained from the IR images using IRT. The local thermo-mechanical fields can be derived from these experimental measurements under certain assumptions.

The normal component of the cohesive response is identified during tensile loading on standard specimens. From the material experimental response and from the damage level obtained by volume variation measurements, an isochoric elastoplastic response associated to the bulk behavior is firstly characterized. Then, the surface response associated to the CZM is deduced as the difference between the material response and the bulk response [3].

For such a ductile material, the data identified for the bulk can be directly associated with the volume behavior and the related material parameters to be introduced in a Finite Element Model. However, the identified surface behavior cannot be directly used in CZM-based simulation since a characteristic length is necessary to transform the experimental stress-strain surface response into a traction-separation law. The characteristic length is identified by a mixed numerical-experimental approach [4]. This dimension represents the physical scale at which the CZM was identified.

The thermo-mechanical consistency of the identified model can be ensured using energy balances. This thermo-mechanical coherence is stronger than the simple consistency of the mechanical responses. We underline on various examples that simple rheological models taking into account elasticity, plasticity and damage effects can ensure the mechanical consistency but the thermo-mechanical consistency of the identified CZM is only reached when the elastoplastic bulk behavior dissipate the appropriate amount of energy.

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## A Cohesive Zone Model Coupled with In-plane Stretch of an Interface

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Interfaces play an important role in mechanical and thermal responses of a body simply because they can possess different properties than that of the bulk. They are also an inevitable part of components including two or more different materials that undergo cyclic loading therefore fatigue crack growth is a concern. As a result, interfaces combined with cohesive zone model have been used to study fracture, especially if the location of the crack is known before hand. The numerical modelling of a solid with finite deformation, using the finite element method, including mechanical interfaces, can be performed to investigate these effects.

To numerically model a geometrically non-coherent interface, a decohesion element with mixed-mode capability and zero thickness, based on a normal-shear decomposition of displacement discontinuity vector is used, exploiting two different traction-separation laws: bi-linear and exponential [1]. The constitutive equation used, relates the traction vector to the displacement jump and the normal vector to the mid-plane for loading and unloading procedures [1, 2].

The mid-plane is the average of coordinates of nodes on the two surfaces of the interface since it has been shown that this choice of mid-plane provides a physically sound frame work [4]. To numerically implement a geometrically non-coherent interface the degrees of freedom of the interface nodes are doubled.

The cyclic loading is applied to study the irreversibility of constitutive equations by keeping track of the maximum value of the displacement jump across the interface. Furthermore, the effects of the normal or shear components of the displacement vector can be pronounced by assigning different weights to them.

In addition to classical strategies, interfacial elements are allowed to have an in-plane stretch resistance by assuming a hyperelastic interface Helmholtz energy  $\bar{\Psi}$ . Let  $\bar{\mathbf{F}}$  denote the interface

deformation gradient of the mid-plane, a surface between two faces of interface. Motivated by the surface/interface elasticity theory [3], the interface Helmholtz energy is a function of the interface deformation gradient,  $\bar{\Psi}(\bar{\mathbf{F}})$ .

The nonlinear governing equation are given. They are solved then, using the finite element method. The results are illustrated through a series of three-dimensional numerical examples for different interfacial and material parameters exploiting the afore-said traction-separation laws.

In particular, a comparison has been made between the implementation of the classical (cohesive) and novel (cohesive and elastic) interfaces. In addition to the purely mechanical interface implementation, thermomechanical coupling of the interface is performed and the results are discussed.

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## A Variational Formulation of Mixed-Mode Decohesion Processes

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The cohesive zone concept has become the most common approach to the computational simulations of delamination. This has been enabled by realistic constitutive modeling of interfaces under general mixed-mode conditions, along with the development of robust numerical tools [1]. The aim of this contribution is to reconcile these issues in a variational format, utilizing recent advances in mathematical theory of rate-independent evolution [2] and its application to delamination problems [3].

To this purpose, the system under consideration is described by a stored energy functional, quantifying the reversible changes, and a dissipation rate functional, related to irreversible interfacial processes. Both functionals are constructed to incorporate a wide class of interfacial constitutive models available in existing literature, e.g. [1]. Among others, this implies that the dissipation must be state-dependent to account for the mode mixity effects.

Evolution of the system is then provided by the energetic solution [2], defined in terms of energy stability and energy equality, ensuring that no energy is lost during the process. Moreover, using a (semi)-implicit time discretization, the energetic solution can be naturally approximated by an incremental energy minimization procedure.

Restricting ourselves to the time-discrete framework, we will comment on the existence of the incremental solution and demonstrate that it satisfies the stability condition and two-sided energy inequalities, closely related to the energy equality. This will be complemented with convergence analysis for discretization with standard conforming finite elements.

The numerical treatment of the problem is based on two ingredients. The first one is the incremental energy minimization algorithm due to Bourdin et al. [4] that allows us to split the incremental

minimization problem to two strictly convex sub-problems that can be efficiently resolved using specialized solvers. The second one represents a time stepping strategy based on the two-sided energy estimates [5], ensuring that the energy remains conserved during the inelastic process.

Basic features of the formulation will be demonstrated using energy landscapes of typical interfacial models, performance of the time-stepping procedure for a simple discrete system, and by the finite element analysis of the mixed-mode flexure delamination test.

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## A Multiscale Hydro-Thermo-Mechanical Cohesive Zone Model in Concrete

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Concrete as a significant construction material, is a very complex material, which is comprised of aggregates, cement paste and interfacial transition zone between aggregates and cement paste at the mesoscale [1]. The interfacial zone is inferior due to more pores and calcium hydroxide, therefore, many experiments have already observed that more cracks concentrate in the interfacial zone. In addition, the work on the thermal conduction and the water diffusion through the crack zone in the concrete should be investigated.

In this contribution, aggregates with a random distribution embedded in the homogenized cement paste represents the mesoscale of the concrete, which is generated through the take-place method, not taking into account the interfacial zone yet [1]. The mesoscale representation is discretized with the tetrahedral element, where the mesh can match geometrical boundaries between aggregates and matrix. The zero-thickness interface elements are produced through doubling the nodes on contact meshes, where the mechanical behavior of the interfacial transition zone can be predicted. Cohesive Zone Method (CZM) is used to describe the crack phenomenon in the interfacial transition zone, which is based on nonlinear traction-separation relations between the normal and tangential components of the interface tractions and relative displacements [2]. The mechanical deterioration of the cement paste is represented by the Mazars damage model, which can capture behaviors of tensile and compressive tests individually.

The behavior of the heat flux through the open crack can be described through the relation between the thermal flux and temperature jump of two separate faces [3, 4], and one thermal damage variable with the function of the interface opening can capture the resistance or the reduction of thermal conduction through the interface.

On the contrary, the diffusion coefficient of the wa-

ter will be increased with the enlargement of the crack opening according to the experiment. The obtained relation between the crack opening and the diffusivity of the water is incorporated into the equation between the diffusion flux and the relative humidity jump of two separate faces, thus enabling to reflect the evolving diffusion flux through the open crack with the dependence on the crack opening.

The staggered method is used to combine the well established cohesive zone method with the heat flux-opening relation and diffusion flux-opening through the interface, allowing solving each field individually. In the end, a 3D multiscale hydro-thermo-mechanical work is present to capture the influences of crack opening on the thermal conduction and the water diffusion in the concrete.

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## **A Multiscale Cohesive Zone Model for Fibre-Reinforced Polymer Sheets Bonded to Concrete**

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Fiber-reinforced polymer (FRP) sheets bonded to the surface of reinforced concrete members are a relatively recent technique for strengthening of civil engineering structures. Failure of the FRP-concrete bond controls the capacity of the strengthened member and thus has been extensively investigated.

Several numerical mesomechanical analyses of concrete have been developed to deduce its macroscopic constitutive behaviour [1-3]. Using this approach the material mesostructure is described explicitly and the influence of size, shape, grading and distribution of the coarse aggregates on the mechanical response can be assessed. On the other hand, fracture in concrete has also been extensively interpreted with macroscopic cohesive zone (CZ) laws. However, there have been limited attempts to derive the shape and magnitude of these laws based on a detailed consideration of the mesostructure.

Considering the case of FRP sheets bonded to concrete substrates, interfacial failure may occur by formation of a crack prevalently within the concrete, a few millimeters from the bond line, or within the adhesive. Several phenomenological CZ laws are available for the FRP-concrete interface in macroscopic mode II. However, these models are unable to interpret the competition between different cohesive crack paths based on the mesomechanical details of the substrate. Moreover, they cannot deal with mixed-mode loading. Very limited investigations have focused on such interfaces in mixed-mode loading cases which very often result from geometry, loading and/or deformation-induced effects. They have pointed out a complex influence of the mode mixity on the mesostructural details of the failure mechanism and on the consequent strength of the interface.

In this work, a mesomechanical analysis of the interface between FRP sheets and concrete is presented, leading to a multiscale CZ model for the same interface. The mesostructure of concrete is modeled using aggregates of polygonal shapes with

size distribution based on the Fuller curve. These aggregates are generated numerically with a Monte Carlo random sampling procedure combined with a take-and-place positioning method. The cement matrix is modeled with an isotropic damage law available in the literature [2], where the damage variable is evaluated from an appropriate combination of tension and compression damage. The behavior of the aggregates is assumed linearly elastic. In the present formulation, aggregates and matrix are taken as perfectly bonded but future developments will include consideration of an interfacial transition zone.

A micromechanical test setup is developed where the interfacial behavior between FRP and concrete is investigated and leads to the obtainment of a macroscopic CZ law via a suitable micro-macro transition. Upon the examination of the representativity of the numerical sample, the role of the aggregate shape, size and distribution on the macroscopic law is examined. Single-mode as well as mixed-mode effects are investigated, and appropriate comparisons are made with available test results.

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## Crack Growth Analysis under Low-Cycle Fatigue at High Temperature Using a Cohesive Zone Model

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Cohesive zone models (CZM) have been used successfully in a series of crack growth analysis problems under monotonic [1-3] and dynamic loading [4, 5] conditions. Recently, attempts have been made to use CZMs to study fatigue crack growth problems under small-scale yielding conditions [6-8], where linear elastic fracture mechanics (LEFM) is relevant.

This study focuses on crack growth analysis under low-cycle fatigue loading. An original cohesive zone model was proposed by Besson and Bugat [9] to account for monotonic creep fracture. This former model was adapted to deal with fatigue crack growth on the basis of bulk plastic energy dissipation. Non-local method was used to overcome element size dependence problem. The adapted model was tested on fatigue crack growth experimental results obtained for a Cobalt-base superalloy Ha188 at 900°C under large scale yielding condition. The initiation and propagation of cracks were satisfactorily described with this model for different strain levels.

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## Modelling of Fatigue Crack Growth with a Cohesive Zone Model Approach Including a Local Endurance Limit

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This presentation addresses the modelling of interfacial damage behaviour under monotonic and cyclic loading conditions. A cyclic cohesive zone model (CCZM) is proposed wherein a single evolution equation for the scalar damage variable  $D$  allows the description of both monotonic and cyclic loading cases. The traction-separation relation characterising the separation law in the monotonic case is based on the well-known exponential approach of XU and NEEDLEMAN (1993). It forms an upper bound for the tractions attained under cyclic loading conditions (damage locus). In contrast to comparable CCZM, e.g. [1, 2], a lower bound of damage evolution is introduced by a local damage-dependent endurance limit (endurance locus). In combination with a non-linear approach for the unloading behaviour, a smooth transition from reversible to dissipative processes is guaranteed, see [3].

The capabilities of the CCZM to simulate interfacial fatigue crack growth is investigated with a 2D finite element boundary layer model under plane strain and small scale yielding conditions loaded by the mode I cyclic stress intensity factor  $\Delta K_I$ . Complete fatigue crack growth rate (FCGR) curves (FCGR vs. cyclic load) between a threshold value  $\Delta K_{th}$  and static failure are generated. Parameter studies are carried out to investigate the influence of the model parameters with respect to static failure load, threshold value and PARIS region. As expected, the first corresponds to an energy release rate above the fracture energy density,  $\Gamma_0$ , i.e. the area under the monotonic envelope. Concerning the lower bound of FCG, the threshold value,  $\Delta K_{th}$ , is found to be dependent on the endurance properties in terms of the stress type initial endurance limit and the shape of the endurance locus. Of course, below  $\Delta K_{th}$  no crack propagation occurs. Nevertheless, damage initiates here and evolves but newly formed cracks arrest due to the low load level. The lower bound of this region is determined by the

initial endurance limit. A relation between the parameters of the PARIS equation describing the region of constant slope in the log-log FCGR curve and the CCZM parameters is of particular interest. As assumed in [2], the PARIS parameters correlate with the parameters governing the damage evolution equation. Thereby, the variations of the PARIS exponent,  $n_P$ , and  $\Delta K_{th}$  are coupled. Apart of this, slight dependencies of  $n_P$  with respect to the non-linearity of the unloading paths and the shape of the monotonic envelope at constant values of both  $\Gamma_0$  and  $\Delta K_{th}$  are observed.

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**Minisymposium CIE:**

**Cracking Induced by Environmental Processes**

*Organized by Ignacio Carol, Günter Hofstetter and  
Kaspar Willam*





## A Thermo-Hygro-Mechanical Model of Concrete Deterioration due to Alkali-Silica Reaction

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One of the most important causes of deterioration of concrete structures is due to the alkali-silica reaction (ASR), which is the most common form of alkali-aggregate reaction (AAR). ASR in concrete, originally observed by Stanton in 1940 [1], is a chemical reaction that occurs between reactive forms of silica in the aggregates and alkali and hydroxyl ions in pore solution. This produces an amorphous gel, which expands in presence of water and creates an increasing internal pressure resulting into a drastic reduction of the mechanical properties. Even though the ASR process is complicated and consists of several stages, a simplified description, consisting in 2 stages, was proposed by Dent Glasser and Kataoka [2]: the first step is characterized by the hydroxyl ions attack of the siloxane groups on the surface of the aggregates forming silanol groups that further react hydroxyl ions forming the amorphous alkali silica gel; the second step is the expansion of the formed gel, by absorption of free water. Several factors may influence the ASR process such as the mineralogy and the size of the aggregate, the alkali content in pore solution, the temperature and the relative humidity. As regards to the numerical modeling of this phenomenon, during the last decades and thanks to the upgraded computational resources, many different and more complex models have been developed. Recalling some of the most important ones: the mesoscopic model based on fracture mechanics approach presented by Bazant and Meyer [3] together with the one for the kinetic of the reaction and the diffusion processes by Bazant and Steffens [4]; the thermo-chemo-mechanical model of the material swelling, by Ulm et al. [5], in the framework of chemoelasticity; the coupled chemo-hygro-mechanical model, within the framework of porous media, by Bangert et al. [6]; the chemo-thermo-damage model proposed by Comi et al. [7] and the chemo-hygro-thermo-mechanical model by Pesavento et al. [8] at variable hygro-thermal conditions. In this work the ASR evolution and the resulting concrete degradation have been implemen-

ted in the coupled thermo-hygro-mechanical finite element code developed at Padua University, called NEWCON3D [9], which treats concrete as a multi-phase system. The enhanced code will find application into the study of the degradation mechanism due to alkali-aggregate reaction in spent nuclear fuel casks subjected to accelerated aging.

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## Multiscale Material Model for ASR-Affected Concrete Structures

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The sustainability and serviceability of concrete structures has become highly relevant. Adequate modelling is important when engineers have to deal with long term deterioration process. One of these phenomena is the Alkali-Silica Reaction (ASR), that is the chemical process between the alkali available in the cement and the silica in the aggregates [1].

Its product is a hydrophilic gel which swells and causes damage, possibly influencing the integrity and capacity of the structure. The expansion process is strongly linked to the stress state of the structure [2, 3] and it indeed results in the degradation of the mechanical properties [4].

The multiscale nature of the problem does not yield to a straightforward modelling approach, because a small difference at micro scale can result in substantial discrepancy at macro scale. Noting that the micro-mechanical nature of the phenomenon cannot be excluded from a macro-mechanical model, the authors present a new material model making use of Homogenization theory.

The concrete is modelled, at micro level, as a heterogeneous material: aggregates and cracks filled by the gel are considered as embedded inclusions in the cement paste, that is, the matrix. It is assumed that the gel formation starts at the rim between aggregates and cement paste and that subsequently the gel flows in the matrix causing cracks and damage. The aggregate and the surrounding gel are modelled by spherical inclusions; the cracks as ellipsoidal inclusions. The effective properties of the medium are evaluated by the Mori-Tanaka homogenization scheme [5].

In the first place the model should be calibrated on the basis of experimental laboratory tests. These tests should provide the evolution of the expansion and the degradation of global mechanical properties in time for a sample in free expansion condi-

tions. In this way, the gel mass production can be calibrated and it becomes the internal driver in the model for modelling the ASR process in a structural analysis. The model allows studying the coupling phenomenon between the internal ASR loading and the external mechanical loading.

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## Damage of Concrete Caused by Corrosion of Reinforcement: 3D Coupled FE Model

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Reinforced concrete structures exposed to aggressive environmental conditions, such as structures close to the sea or highway bridges and garages exposed to de-icing salts, often exhibit damage due to corrosion [1]. Damage is usually manifested in the form of cracking and spalling of concrete cover caused by expansion of corrosion products around reinforcement. The reparation of corroded structure is related with relatively high direct and indirect costs. Therefore, it is of great importance to have a model, which is able to realistically predict influence of corrosion on safety and durability of RC structures.

In the present contribution recently developed coupled 3D chemo-hygro-thermo-mechanical model for concrete is discussed. The model takes into account the interaction between non-mechanical processes (distribution of heat, humidity, oxygen, chloride and rust) and mechanical properties of concrete (damage). The mechanical part of the model is based on the microplane model.

To predict the increase of volume of the corrosion products it is necessary to calculate the corrosion current density in the corrosion unit. Furthermore, the transport of corrosion products into the pores and through cracks needs to be taken into account in order to consider decrease of the pressure due to the expansion of corrosion products around the reinforcement cross-section. Generally, calculation of corrosion current density depends upon following physical, electrochemical and mechanical processes: (i) Transport of capillary water, oxygen and chloride through the concrete cover; (ii) Immobilization of chloride in the concrete; (iii) Transport of OH<sup>-</sup> ions through electrolyte in concrete pores; (iv) Cathodic and anodic polarization, (v) Transport of corrosion products into pores and cracks and (vi) Damage of concrete due to mechanical and non-mechanical actions [2].

The expansion of corrosion products is modeled using 1D contact corrosion elements on the

reinforcement-concrete contact surface. Once the reinforcement is depassivated (beginning of corrosion), corrosion rate is calculated and corrosion contact elements are automatically activated generating radial compressive pressure, which can cause cracking of concrete cover.

To demonstrate the application of the model, 3D FE analysis is performed on the beam-end specimen assuming aggressive environmental conditions (splash zone). The numerical results are compared with the experimental data. In the first part of the study the corrosion induced damage is predicted for different levels of corrosion. The influence of the geometry (concrete cover and bar diameter), position and size of anode and cathode and the influence of the distribution of rust through corrosion induced cracks on current density and damage of concrete cover is studied. Subsequently, the influence of the corrosion induced damage on pull-out capacity of reinforcement bar is investigated.

The results can be summarized as: (i) Concrete cover, bar diameter and position and size of anode and cathode significantly influence corrosion rate and corrosion induced damage; (ii) Pull-out capacity of reinforcement is reduced due to the corrosion induced damage of concrete; (iii) Transport of reinforcement through cracks reduces damage of concrete cover; (iv) Corrosion induced damage increases corrosion rate and (v) Good agreement with experimental data is obtained.

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## Cracking of Concrete Under Environmental Processes Using Zero-Thickness Interface Elements

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A number of environmental process such as drying, temperature changes or chemical attacks of various sorts, may affect concrete behavior and may cause material cracking. In general, the immediate cause of cracking is the differential volume changes between different concrete components, or between different parts of the same component at different locations within the specimen or structural member. Volume changes may be due to chemical or physical processes, for instance, losing or increasing the free water content in the pores of cement due to drying, or certain chemical reactions taking place between external agents and components of the aggregates of cement paste, that result in products that occupy different volume than the ingredients.

Through the last two decades, the group of Mechanics of Materials at the School of Civil Engineering in Barcelona (ETSECCPB), has been developing a meso-mechanical representation of concrete, under the effects of mechanical as well as environmental actions, with special emphasis on cracking. The underlying geometric description of concrete consists of the explicit representation of the larger aggregates, which are immersed in a homogeneous matrix representing the average homogenized behavior of mortar. From the mechanical viewpoint, those two continuum phases are assumed to behave elastically or visco-elastically, and the possibility of non-linear behavior and cracking is introduced via zero-thickness interface elements that are inserted in between continuum elements, along all aggregate-matrix and some of the matrix-matrix lines (2D) or edges (3D). For most analysis, the aggregate geometry is generated using Voronoi tessellation, followed by some contraction algorithm for the polygons/polyhedra, plus some fill-up of the spaces left between them. But some other algorithms have also been used, such as numerical generation of circles/spheres, or mapping particle geometries from real material images. In the first developments, devoted to purely mechanical actions, the model was applied to a variety of 2D

and 3D loading scenarios with very satisfactory results [1], including crack initiation, development, branching, merging, and spontaneous localization of microcracks into macrocracks.

In more recent years, the model has been extended to environmental phenomena coupled with cracking. The main peculiarity of the approach is that zero-thickness interface elements are also considered in the diffusion formulation, with a localized diffusivity that is directly related to the mechanical opening of the crack. Initial applications in 2D were drying shrinkage [2] and external sulfate attack [3]. The model was also applied to high-temperature problems from a purely mechanical viewpoint (different expansion curves for each concrete component) [4]. Currently, the model is being extended to the 3D version of the high-temp. mechanical analysis, to incorporating the high-temp moisture flow, and to the modeling of alkali-silica reactions (ASR), the latter involving at least a second diffusion process to couple with the mechanical analysis. Examples of all these applications are presented.

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## Investigation of Rehabilitating a Deteriorated RC bridge

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Adding a concrete overlay to an existing concrete structure is one of the most widely used methods for repair, retrofitting and rehabilitation. Most of the experimental research contributions regarding this structural strengthening technique found in the literature are restricted to lab test programs ([1], [2]). Thus, in the present contribution the strengthening of a reinforced concrete bridge by a concrete overlay will be presented. The plate girder bridge to be investigated, which shows four webs and three spans in longitudinal direction, was built in 1970 as part of the Austrian road network. In order to maintain the load bearing capacity of the bridge with deteriorated material properties and in order to improve the structural capacity of the bridge due to increasing traffic loads a 90 mm thick reinforced concrete overlay was added.

In course of the experimental investigation the evolution of both, the depth dependent moisture distribution and the deformations measured in longitudinal and lateral direction of the concrete bridge were monitored. The in-situ measurements were started before, continued during rehabilitation work and are still going on.

Since the restraint of shrinkage deformations is one of the main factors influencing the serviceability and durability of bonded concrete overlays the in-situ measurements of the overlay and the adjacent substrate were accompanied by a comprehensive laboratory testing program concerning the evolution of shrinkage strains. The latter was based on the experimental program described in [3]. Similar to [3] water desorption isotherms, ultimate drying shrinkage strains, moisture distribution profiles as function of time and autogenous and drying shrinkage strains as function of time as well as the temperature evolution during the hydration process were determined.

Comparison of the lab results with the in-situ measurements allows an assessment of environmental influences on the bonded overlay behaviour and allows a better understanding of the impact of surface preparation and finally of the resulting restraint effects at the interface.

Furthermore, advices regarding conclusions from experimental results determined on composite lab tests to real bridge decks strengthened by an overlay can be formulated.

In order to get a deeper insight into the effects of drying shrinkage of concrete overlays parallel to the experimental investigations numerical simulations of the laboratory tests were carried out. To this end a physically based model of drying shrinkage relying on a multi-phase concrete formulation, in which concrete is considered as a porous material, [4] was used. The determination of the respective material parameters for drying shrinkage from measurement data was a main focus in the framework of the research work. The computed response compared to available experimental data demonstrated the capabilities of the numerical model.

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## Universal Meshes for the Simulation of Hydraulic Fractures

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We describe our approach to simulating hydraulic fractures based on the use of Universal Meshes. This problem is challenging to simulate because of the non-linear coupling between the fluid pressure and the crack opening, and also because of the presence of two moving boundaries, the crack tip and the fluid front. Often in simulating hydraulic fracture, the rock is modeled as a homogeneous, isotropic, infinite elastic medium. This has the advantage of bypassing the 2D elastostatics equations for the rock and instead solving a 1D integral relation between the fluid pressure and crack opening. While this approach has been highly successful for this simplified case, there would be great difficulty in extending it to more general problems, for example when it is desired to model the effects of poroelasticity or the intersection of hydraulic fractures with pre-existing natural fractures. Finite-element-based approaches are attractive in the simulation of hydraulic fractures because of their ability to easily handle inhomogeneities in the material and more general geometries.

Constructing finite-element-based approximations for hydraulic fracture problems faces a crucial obstacle though: a suitable mesh is needed over the faces of a possibly-curved-crack to solve for the pressure distribution in the fracturing fluid. Since the crack itself is part of the solution, it is not possible to a priori know where the crack will be and hence where to construct such mesh. Standard solutions for crack propagation, such as cutting elements as in the extended finite element method, lead to very irregular meshes over the crack surfaces not suitable for computation. Such meshes can lead to accuracy and conditioning problems.

To this end, we have introduced the idea of a Universal Mesh. A Universal Mesh is one that can be used to mesh a class of geometries by slightly perturbing some nodes in the mesh, and hence the name universal [1, 2, 3]. In this way, as the crack

evolves, the Universal Mesh is always deformed so as to exactly mesh the crack surface. The advantages of such an approach are: (a) the crack faces are exactly meshed with a conforming mesh at all times, and the quality of the surface mesh is guaranteed to be good, (b) apart from duplicating degrees of freedom when the crack grows, the connectivity of the mesh and the sparsity of the associated stiffness matrix remains unaltered; this has the positive effect of enabling efficient iteration over the crack geometry, needed for the satisfaction of Griffith's criterion at the crack tip.

We have devised one algorithm to simulate plane-strain, straight [4] and curvilinear hydraulic fractures that takes advantage of a Universal Mesh. The algorithm is capable of handling the non-linear coupling between the pressure and crack opening profile, and to separately track the evolution of the fluid front and the crack tip. For straight fractures, we validate the algorithm by exactly reproducing some asymptotic exact solutions.

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## Numerical Studies of Remanufacture of Cracked Impeller

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When the impeller was used for a period of time including the whole life cycle, wear and crack can be found. The remaining values of such used impeller can be recovered by remanufacture [1]. The remanufacture quality can rely on the component replacement and reprocessing on the product surface [2].

The use of the replacement and reprocessing technologies can change the mechanical properties of the surface material. Finite element method was used to study the effect of the mechanical property changes of the surface material on the contact status in the impeller-axle system. When laser melting coating is used on the surfaces of the impeller and the axle, the calculated contact pressure and the corresponding contact displacement are very similar. This means that the change of the mechanical properties caused by laser melting coating hardly affects the contact status in remanufacture.

In the impeller-axle system, the remanufacture of the blade is much more important due to the wear at the root of the blade. The new blade can be joined to the used impeller by welding technologies in remanufacture. Three welding technologies are compared in current work: friction stir welding (FSW), tungsten inert gas welding (TIG), and laser welding (LW). Results show that the longitudinal residual stress in LW is the highest and the one in FSW is the lowest.

The impeller to be re-manufactured usually consists of cracks. The cracks must be studied for the determination of the possibility for remanufacture of such an impeller with cracks. Virtual crack closure technique (VCCT) and the superposition approach are employed for the investigation on the influence of the residual stress on the crack propagation. Benchmark is tested for validation with comparison to Ref. [3].

When the residual stress is considered, the total stress intensity factor is increased by 187.5% at the position of 11 mm from the weld center. The effect

of the residual stress on the crack propagation rate becomes more obvious in lower nominal stress ratio. When the nominal stress ratio is 0.1, the crack propagation rate for FSW is the lower at the welding line than the other two joining technologies. When the crack grows to the shoulder border, the crack propagation rate for FSW becomes higher. When the nominal stress ratio is increased to 0.8, the crack propagation rates of the three welding technologies become very similar. The crack propagation rate is not only dependent on the residual stress distributions but also on the nominal stress ratio.

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## Meso-Level Modeling of Concrete under Nuclear Radiation

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Concrete under severe environmental conditions is a topic of wide interest for various applications in engineering. In particular, when dealing with concrete shielding in nuclear facilities, a thermo-hydro-mechanical modeling of concrete behavior is mandatory to investigate the load bearing capacity under thermal effects and the durability of the structure.

Our work deals with concrete conceived for a next generation nuclear facility, designed to produce high energy radioactive ion beams (RIB), at the Legnaro National Laboratories (LNL), National Institute of Nuclear Physics (INFN) location in Padua, Italy. The point with this structure is that it is subjected to the prolonged exposure of high energy neutron fluxes, which can become dangerous for the mechanical properties of the material above critical quantities [1,2].

The numerical approach consists of a 3D Finite Element phenomenological model for concrete [3], based on the findings by Bažant and co-authors for mass diffusion and heat convection-conduction equations in terms of relative humidity [4-6].

The model is developed within the theoretical framework of porous media ultimately applied to visco-elastic materials and it allowed us to make considerations on damage due to radiation under the normal operative condition and accident. The most relevant results are offered by a meso-scale modeling of concrete, in which the multiphase system is distinguished in: aggregates, cement paste and interfacial transition zone (ITZ). The interfacial transition zone between aggregates and cement paste is found to be the weakest region, therefore it becomes the first possible location for fracture triggering, moreover the role of aggregates is shown to influence the deposition of energy generated by radiation exposure.

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**Minisymposium DFQ:**

**Damage and Fracture of Quasibrittle Materials**

*Organized by Ignacio Carol and  
Gilles Pijaudier-Cabot*

*in honor of Professor Zdeněk P. Bažant  
on the occasion of his 75th birthday*



## Correlations Involved in the Failure of Quasi-Brittle Materials: Analysis with a Meso-Scale Model

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The degradation of quasi-brittle materials encompasses micro-crack propagation, interaction and coalescence in order to form a macro-crack. These phenomena are located progressively within the so-called Fracture Process Zone (FPZ), i.e. in a small region of the solid ahead of the tip of a macro-crack. The shape and growth of the FPZ, and its interaction with boundaries lead to typical phenomena such as size effects, boundary effects and shielding effects.

Meso-scale models encompass the meso-structure details and therefore capture the failure process at the scale of the heterogeneities with details that continuum based model cannot provide. Typical meso-scale models such as the one used in this contribution consider aggregate embedded in a mortar matrix and introduce also a special response of the interfacial transition zone between mortar and the aggregate. With the help of the 2D lattice model used in this paper, we were able already to provide predictions of the experimental mechanical responses obtained from three-point bending tests on notched and un-notched concrete beams of different sizes. Both the maximum loads and the softening regime were recovered and the energy dissipation within the FPZ was analysed [1]. The aim of this contribution is to provide a further insight in the description of failure with the help of statistical analyses of local damage.

In the numerical models, the histograms of relative distance between locations at which damage grows within a given loading step have been built first. These histograms reveal the correlation between individual failure events. They can also be compared to the histograms of the relative distances between the locations of successive acoustic events in experiments. These numerical and experimental histograms have been obtained in the case of 3 point bending tests on notched and un-notched beams. They are similar, especially those in which the distance between damage

locations is projected onto a horizontal axis, perpendicular to the direction of propagation of the crack.

The interpretation of bending test is biased by the correlation introduced by the stress/strain gradient within the structure. In order to facilitate the analysis, further interpretations have been carried out on tensile geometries for which results on histograms between successive failure events already exist [2]. Numerical results are considered only. The material heterogeneities are described either explicitly or with the help of auto-correlated random fields capturing the material heterogeneities at a scale lower than the meso-scale. Ripley functions [3] are used to characterise the correlation patterns in the course of failure. As expected, it is observed that the failure patterns deviates from random fields in the course of localisation of damage and from auto-correlated random fields resulting from the initial disorder in the material too. Patterns are then compared to those resulting from auto-correlated random fields in order to extract a correlation length involved in the localization of damage during failure.

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## Crackling Noise in a Discrete Element Model of Heterogeneous Materials

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We present a discrete element model of brittle materials with highly heterogeneous micro-structure and investigate the uniaxial compression of cylindrical specimens. In order to capture the heterogeneous micro-structure of materials the sample is generated by sedimenting randomly sized spherical particles inside a cylindrical container with a log-normal size distribution. The cohesive interaction of particles is represented by beam elements which can break when they get overstressed. The breaking rule takes into account the stretching and shear of particle contacts. When particles not connected by beam come into contact their interaction is described by the Hertz contact law. The time evolution of the system is generated by molecular dynamics simulations in three dimensions [1]. The modelling approach is similar to the one presented in Refs. [2, 3].

Computer simulations revealed that under strain controlled uniaxial loading of the system first micro-cracks nucleate in an uncorrelated way all over the sample. As loading proceeds localization occurs, i.e. the damage concentrates into a narrow band which has an angle of 30-45 degrees with the load direction. Inside the damage band the material is not completely crushed, it is composed of fragments which are embedded into fine powder of single particles. The mass distribution of fragments proved to be a power law with an exponent 2.1.

In order to analyze the temporal fluctuations of the breaking process we introduce a correlation time of local breaking events of particle-particle contacts: if two beam breakings follow each other within the correlation time they are considered to belong to the same correlated breaking avalanche. Similar burst definition based on temporal correlation of consecutive events has also been used to investigate crackling noise emerging during three-point bending of heterogeneous brittle materials [4, 5]. Our calculations showed that during the fracture process local breaking events form correlated trails which are analogous to acoustic bursts of experiments. Bursts

are characterized by their size, i.e. the total crack surface generated by the burst, by the released elastic energy, and by the duration. Interesting information is encoded also in the waiting time between consecutive bursts. Our analysis showed that characteristic quantities of bursts have power law probability distributions over a broad range. We found that the energy and duration of bursts have power law dependence on the crack surface created by bursts. An analytic expression was derived for the relation of burst exponents which was confirmed by numerical calculations. As the system approaches macroscopic failure we pointed out that consecutive bursts get correlated: the average waiting time to the next event proved to be an increasing function of the burst size, furthermore, the formation of the damage band is marked by the decrease of the average distance of consecutive bursts.

The simulation results are in reasonable agreement with the experimental findings on the fracture process of some quasi-brittle materials.

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## Damage Enhanced Creep in the Fiber Bundle Model

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We present a theoretical study of the creep rupture process of heterogeneous materials based on a fiber bundle model which provides a direct connection between the microscopic damage mechanism and the macroscopic time evolution. In the model, material elements fail either due to immediate breaking or undergo a damage accumulating ageing process [1, 2]. For the damage rate a power law dependence on the local stress is assumed. In the limit of equal load sharing after local failure events we show analytically that the model reproduces the Basquin law of creep lifetime and the time-to-failure power law behaviour of strain rate [1, 2, 3].

Based on computer simulations of localized load sharing we carried out a detailed investigation of the interplay of quenched structural disorder and of the inhomogeneous stress field during the creep rupture process. The sensitivity of the system to the details of the stress field is controlled by the exponent of the law of damage accumulation. Assuming strongly localized load redistribution around failed fibers, we showed that the system has two phases depending on the amount of strength disorder and on the damage accumulation exponent  $\gamma$ : For high disorder and low values of  $\gamma$  simultaneously growing cracks are spread homogeneously over the entire system. However, for low disorder and high  $\gamma$  values the damage gets localized to highly stressed regions giving rise to a single growing crack. The jerky time evolution of the bundle is characterized by the size distribution of bursts and by the distribution of the waiting times elapsed in between. Both distributions proved to have a power law behavior with a load-dependent cutoff [4].

The most important outcome of the simulations is that the exponent of the burst size distribution proved to be independent of the details of the damage process such as the  $\gamma$  exponent, however, it depends on the external load which controls the efficiency of triggering. For the waiting time distributions the opposite behavior is observed, i.e. the load only affects the cutoff of the distribution, while the

exponent  $z$  decreases from 2 to 1.4 as  $\gamma$  is raised from 1. We analyzed the micro-structure of damage in the last stable configuration of the system just before catastrophic collapse. Simulations showed that under the dominance of disorder the crack structure is analogous to percolation lattices, i.e. power law distribution is obtained with a varying exponential cutoff. However, when a single crack propagates, the cluster size distribution becomes steeper [4].

We show that the time evolution of the rupture process can be described as a non-homogeneous Poissonian process, where the rate of bursts obeys the (inverse) Omori law. The Omori parameters exhibit a high degree of universality. Studying the temporal and spatial evolution of single bursts we show that the average pulse shape of bursts encodes information about the range of load redistribution, namely, for long range load sharing pulses are symmetric, however, for localized interaction right-handed asymmetry is obtained. For localized load sharing burst are found to be compact geometrical object having a fractal frontier with a fractal dimension 1.25. We argue that burst frontiers fall in the universality class of loop erased random walks.

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## A Computational Approach to the Statistical and Size-Dependent Failure of Cellular Ceramics

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Bio-inspired cellular ceramics appear to be attractive solutions to support the need for new stronger and lightweight materials in strategic applications as diverse as tissue engineering, energy storage or catalysis. The microstructure of these cellular ceramics can be controlled by varying the process parameters, yielding materials with very different functional properties. The challenge of developing such materials can only be met through a sound understanding of the relationships between materials architecture and their mechanical response.

Cellular ceramics exhibit a quasi-brittle mechanical behavior with a statistical and size-dependent failure. A previous work on cellular ceramics made by robotcasting [1] has shown that predicting the ceramic scaffold strength from the rod scale using a classical Weibull approach leads to very conservative results. The successive breakings of scaffold walls and the resulting stress redistribution need to be considered to accurately predict the final failure of the structure. Here, we propose a computational approach to quantitatively understand the mechanics of cellular ceramics at multiple length scales, and to predict the size-dependent and statistical stiffness and strength of such structures.

For a typical cellular ceramic scaffold, we consider periodic representative volume elements (RVE) of increasing size, i.e. containing from one to several hundreds of unit cells. The mechanical response of these RVE is computed using the Finite Element method for several loading paths. The statistical distribution of the cell wall strengths is introduced through the Weibull framework. Extensive computations eventually allow to obtain the strength distribution for each RVE size and the distribution of the scaffold stiffness evolution. The latter can possibly be used to derive a volume-dependent continuum mechanics model [2]. The statistical charac-

teristics of the strength size effect are discussed. A scaling law is proposed and compared to the classical power law size effect. Finally, characteristic fracture lengths are identified and correlated with the scaffold architecture and material properties.

We expect our approach will provide a valuable insight into the mechanics of cellular ceramics and should help materials scientists to build more reliable materials.

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## Simulation of Micromechanical Fragmentation and Removal Processes during Wire Sawing

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Today, industrial wire sawing is the most efficient and economic technology to cut brittle materials into thin wafers. An important field is the production of silicon wafers for photo-voltaic and micro-electronics applications. For the cutting process a web of wires moving with a velocity of about 15 m/s is fed into the material ingot, while a fluid suspension (slurry) with abrasive particles ( $\sim 15 \mu\text{m}$ ) is added. The material is fragmented and removed in the sawing kerf by micro-mechanical fracture processes that occur by interaction forces between rolling abrasive particles and the silicon surface. The sharp edges of the particles are pressed into the surface and lead to indentation fracture and chipping off small amounts of material. This entire process is denoted by "rolling indentation" [1].

In order to study this micro-mechanical process, the model of a moving and interacting particle system in a viscous shear flow was developed in [2]. The numerical realization is based on the 2D discrete element method, which has been extended by sharp edged polygonal particles and by drag and lift forces acting from the fluid flow on the particles. Concepts of indentation fracture mechanics are included to account for the elementary fragmentation processes, see [3]. Due to high local stresses under the indenting particle, median and radial cracks are formed. During unloading lateral cracks appear beneath the plastic zone and, if they reach the surface, the material is chipped away.

The numerical simulations are capable of reproducing fundamental aspects of the sawing process. Qualitatively, the idea of "three body abrasion" is in principle verified to happen as basic phenomenon in the complex particle system, too. Moreover, this view could be generalized to the multiple interaction of abrasive particles in the sawing kerf with the silicon surface and among each other, driven by the slurry and the moving wire.

The model allows to study the influence of essential

process parameters like wire speed, lapping pressure, particle shape, particle size distribution a. o. on the material removal process. A parametric study towards the influence of wire speed and lapping pressure on the amount of material removal is also presented. The results obtained by rather extensive and expensive simulations confirm the phenomenological law of Preston for the removal rate stating that it is proportional to wire speed and lapping pressure. Moreover, the simulations enable correlations between the Preston coefficient and micro-mechanical process parameters. The obtained results agree qualitatively quite well with experimental observations. The generalization to three-dimensional DEM-simulations is subject of ongoing research [4].

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## Fictitious Elastic Stiffness Parameters of Zero-Thickness Interface Elements to Recover Accurate Nodal Stresses

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Zero-thickness interface elements are often (but not only) used in the discrete simulation of cracks in brittle and quasi-brittle materials. Two major approaches can be identified. In one case, interfaces are inserted amongst the mesh lines only after certain cracking conditions have been reached. The interfaces behave as non-linear elements that dissipate energy while they open [1]. In the other approach, interfaces are inserted a priori along the mesh lines, to include as many crack path scenarios as possible [2]. To keep the interface closed while the cracking conditions are not yet satisfied, a fictitious elastic behaviour is imposed through penalty stiffness parameters. These are elastic stiffness parameters set as high as possible to guarantee that no penetration or separation take place while the cracking nucleation conditions are not satisfied, but not so high to avoid ill-posed problems.

When using the above mentioned second approach, the elastic solution in the standard continuous elements is not affected by the presence of the fictitious elastic interface elements. However, this paper shows that the elastic static variables (stresses and forces) at the nodes of the interfaces are affected by errors that are often higher than 10%, hence not negligible. The error seems to be affected by the distribution in the space of the interface elements concurrent to a node, but not by their length.

A solution is proposed here that is based on the satisfaction of the kinematic conditions at the node to which several interfaces converge [3] and the assumption of the existence of a unique stress tensor at the node. Concurrent interface weights are introduced in the stiffness matrix governing the elastic behaviour of the interface. Several examples are presented showing the goodness of the solution and

the accurate recovered values of the nodal interface stresses.

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## An Interface Damage Model Depending on the Body Degradation

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The present work deals with the structural response of quasibrittle and elastic domains in adhesion, that can be significantly influenced by the process of decohesion.

In order to model the deterioration of the interface between the materials, both volume and interface damaging behaviors and their interactions are taken into account [1].

A coupled interface-body nonlocal damage model is proposed. A nonlocal damage and plasticity model is developed for the quasi-brittle material.

The model is able to reproduce the main mechanisms of cohesive materials under cyclic loadings, i.e. degradation of the mechanical properties, development of irreversible strains and recovery of stiffness due to crack closure. The internal stiffness degradation is measured by the introduction of two scalar damage variables: a tensile and compressive damage. Moreover, it is assumed that the damage occurring in compression directly induces damage in tension.

In cohesive heterogeneous materials subjected to loading conditions, the initially smooth distribution of strain changes into a highly localized one. Typically, the strain increments are concentrated in narrow zones while the major part of the structure experiences unloading. Thus, when high strain gradient occurs, the standard ‘local’ models fail to describe localized failure patterns in an objective way. Nonlocal approaches are needed in order to overcome the problem due to the strain localization [2]. The nonlocal formulation is not affected by mesh sensitivity. In particular, an integral-type of nonlocal model based on the weighted spatial averaging of a strain-like quantity is adopted. In particular, the growth of the compressive damage is governed by the nonlocal value of the accumulation of the plastic strain, while the evolution of the tensile damage state mainly depends on the nonlocal measure of the equivalent elastic strain linked to the positive principal elastic strains. The plastic flow is controlled by the

effective stress through the introduction of a suitable yield function.

For the interface, a model, which accounts for the mode I, mode II and mixed mode of damage and for the unilateral contact and friction effects, is developed.

In the adhesion zone there is interaction between damage at a point and damage in its neighborhood. Thus, there is interaction between the interface and the quasi-brittle material behavior. This interaction is modeled introducing two different coupling laws in which the damage evolution at the adhesion zone is governed not only by the local difference of displacements but also by the damage occurring in the neighbor quasi-brittle materials [3,4]. In particular, in the first approach the degradation state of interface is equal to the highest value between the interface damage and the body damage evaluated at the bond surface [3]. The second approach is based on micromechanical considerations [4].

The presented methodology is implemented in a numerical code. Finally, numerical applications are performed in order to assess and compare the performances of the proposed coupled models and of the developed procedure.

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## Constitutive Model for Timber Fracture under Tension and Shear

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Sensitive restoration of historical timber structures or efficient design of modern ones often requires a detailed nonlinear analysis by the finite element method (FEM). To this end, the FEM code must be equipped with appropriate constitutive model.

In this contribution we present a two-dimensional constitutive model for timber fracturing under tensile and shear loads [1]. The model captures the following phenomena: (1) elastic and inelastic behavior in small deformation range, (2) material orthotropy, both in linear and non-linear range, (3) cracking across and along fibers, (4) behavior under unloading/reloading. The model can be applied both to hardwood and softwood by selecting appropriate material characteristics. We idealize timber as a homogeneous quasi-brittle material. Prior to cracking, the material is characterized by elastic orthotropic stress-strain relationship. An orthotropic fracture criterion, which can distinguish between crack forming across or along fibers is proposed. Post-cracking response is treated by means of the fixed smeared crack model.

As the primary fracture criterion we use the generalized Rankine-type condition proposed by Lourenco [2]. This condition is defined by three parameters: direct tensile strengths in two directions (parallel and perpendicular to fibers) and pure shear strength. Furthermore, we assume that if the maximum principal stress diverts from the fibers direction by an angle less than a certain limit value (which is considered as a material parameter), a crack forms across fibers in the direction perpendicular to the principal stress (crack type CT1). If the angle exceeds the threshold, then a crack forms along fibers regardless of the principal stress direction (type CT2). This implies that at crack initiation, the surfaces of CT1 are exposed to normal traction only, but in the case of CT2 both normal and shear tractions may be nonzero at this instant. It is assumed that only one crack (either CT1 or CT2) can occur at a given material point and its direction remains fixed throughout the loading history.

Response of a crack is governed by the traction-separation law, which relates the crack-bridging tractions to the relative displacements of the crack surfaces. This relationship is defined in the local coordinates  $n$ - $m$ , perpendicular and parallel to the crack. The same form of the traction-separation law is assumed for both crack types CT1 and CT2, albeit with different parameters. In the crack-normal direction, the traction is assumed to decrease with crack opening displacement according to exponential function [3]. In the tangential direction, the traction-separation law is proposed using an arctangential function, which fulfills the following assumptions: (1) at constant crack opening displacement  $\delta_n$  the shear traction  $t_m$  increases with increasing crack sliding displacement  $\delta_m$ , but never exceeds the shear strength  $f_s$ , (2) at constant slip  $\delta_m$ ,  $t_m$  decreases with increasing opening  $\delta_n$ , (3) the function of  $t_m$  can take any value from the interval  $(-f_s, f_s)$  at the crack initiation state. The latter property ensures that the proposed formulation ensures a smooth transition from the stress state on the failure surface to the state governed by the traction-separation law even for crack type CT2. Additionally, the traction-separation law is limited in both  $n$  and  $m$  directions by critical crack opening/slip.

The model performance is demonstrated for various load cases and the results are consistent with expected timber behavior.

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## Fracture Properties of Cement Composites Reinforced by Carbon Nanotubes

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The main objective of this work was to determine the influence of carbon nanotubes (CNTs) reinforcement to the fracture energy and tensile strength of the cement paste. The work was divided into experimental and computational parts. The effect of the CNTs amount, length and clustering was studied. The clustering of CNTs in volume was found as the governing factor of CNTs reinforcement. The results were already published in [1].

In the experimental part, samples with various CNTs content were produced and the fracture energy was measured. The CNTs were added to the cement paste in the form of so-called cement hybrid material (CHM). The CHM is an ordinary Portland cement with CNTs directly synthesized on the surface of the grains. The CHM was produced by L. Nasibulina's group from Aalto University, Finland [2] with the average CNTs length 2.5 μm.

The CHM can be simple intermixed with regular cement and water. CHM avoids a flocculation of separately added CNTs. No special treatment as demanding sonification of the cement paste with CNTs is needed for the CNTs dispersion. At least three samples of each batch were evaluated.

The micromechanical simulations aimed at reproducing the measured fracture energies. The task was solved using the finite element method in OOFEM package [3]. The mesh was created from a 100×100 μm 2D microstructure generated by CEMHYD3D model. The various chemical phases were simplified into three parts: unreacted cement particles, hydration products and voids. Each pixel of the 2D microstructure corresponds to one quadrilateral finite element with four nodes. CNTs were incorporated to the mesh as trusses. Isotropic damage material model was assigned to all four components. A simple cohesive crack model with linear strain softening and Mazars' measure of strain was used.

The material parameters of the hydration products were fitted from the experimental results on the

plain paste. The final values yielded: fracture energy  $G_f=12.9$  N/m, tensile strength  $f_t=5.57$  MPa and Young's modulus  $E=21.7$  GPa. The mechanical properties of CNTs and cement grains were estimated from recent results [4]; for CNTs  $G_f=200$  N/m,  $f_t=3000$  MPa and  $E=231$  GPa, for cement grains  $G_f=118.5$  N/m,  $f_t=1800$  MPa and  $E=135$  GPa.

The initial volume of the cement grains in the fresh paste is about 50% for w/c 0.35. The CNTs could not enter this space even if the cement is hydrated. This leads to the CNTs cluster formation. The micromechanical simulations proven the measured data,  $G_f$  increased from 16.5 N/m (plain paste) to 19.7 N/m (CNTs in clusters) and  $f_t$  from 5.5 MPa to 6.3 MPa. On the other hand the theoretical uniform distribution of 2.5 μm long CNTs led to the improvement of the fracture energy from 16.5 N/m to 38.8 N/m and of the tensile strength from 5.5 MPa to 6.7 MPa. It has been also shown that there is no significant effect of CNTs length to the tensile strength and fracture energy until the CNTs length does not exceed the average cement particle size (about 20 μm).

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## Fracture Properties of Recycled Aggregate Concrete

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Demolished Portland cement concrete can be crushed into small pieces, grouped into different sizes and reutilized as aggregates (called recycled aggregates) in a new construction. The concrete made of either fully or partly recycled aggregate is called recycled aggregate concrete (RAC). The major difference between RAC and regular concrete made of natural aggregate is that there is a layer of residual cement paste on the surface of recycled aggregate, which results in poor mechanical properties of RAC. As a result, RAC has only been used sporadically as a structural material. The residual cement paste has high water absorption capacity, high porosity, and weaker bond of interfacial transition zone between the residual cement paste and new cement mortar. In order to use RAC as a structural material, various combinations of mixing approaches, surface pretreatment methods with pretreatment materials were developed and optimized to improve the compressive strength of RAC [1]. The present paper focuses on fracture properties of RAC made of 100% recycled coarse aggregate and prepared by an optimized mixing and pretreatment method. The present paper includes experimental study and theoretical analysis.

In the experimental study, RAC notched beams of four different sizes with similar geometry were prepared. Load-displacement curves of the beams were measured. The fracture properties of RAC such as the fracture process zone, fracture energy, and R-curve were determined based on the size effect method developed by Z.P. Bazant in 1984 [2,3,4]. Comparisons were then made between the mechanical and fracture properties of the RAC and those of ordinary concrete.

In the analytical study, a multi-scale analytical approach was developed for evaluating fracture process zone,  $c_f$  of RAC.  $c_f$  is characterized by fineness of aggregates as well as the interface bond, which depends mainly on the surface pretreatment method and material for RAC. The model combines micromechanics models and cement chemistry to simulate the multiple layered

internal structure of recycled aggregate particles (natural aggregate as the core, residual cement paste, surface pretreatment layer, and new cement paste as outer shells) and to estimate the fracture process zone of RAC. When the fracture process zone is estimated by the model, the fracture energy of RAC can be determined by testing of concrete beam of one size instead of multiple sizes.

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## High-Order Microplane Theory for Elasticity and Softening of Quasi-Brittle Materials

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For quasi-brittle materials, failure is often caused by nonlinear phenomena, such as fracture, damage localization, and frictional shearing, occurring at weak locations in the internal material structure. These weak locations coincide, for example, with interfaces among particles in particulate materials, weak matrix layers in composites with hard inclusions, compliant interfaces between stiff material grains, and they are typically characterized by a specific geometry and well determined orientations.

Due to this peculiarity, classical tensorial constitutive equations typically fail to provide a satisfactory representation of the mechanical behavior of these materials especially if failure mechanisms are associated with strain-softening. A better approach in these cases is the so-called Microplane Model, first introduced by Bažant and coworkers [1] to simulate concrete. The microplane model is based on the following assumptions: (1) material weak locations are idealized through the concept of “microplane” which represents a plane with a given orientation at a point of the 3D continuum; (2) on each microplane the material behavior is described in terms of stress vector and strain vector; (3) microplane strains are kinematically constrained to the macroscopic strain tensor; and (4) microplane stresses are related to the macroscopic stress tensor through the principle of virtual work.

Since its introduction in the early eighties, the microplane model for concrete evolved through several progressively improved versions [2]. Microplane models were also successfully developed for other quasi-brittle materials such as rock, sand, clay, rigid foam, shape memory alloys, fiber reinforced concrete, and composite laminates.

Despite the undoubtedly success of all the aforementioned microplane formulations, their main shortcoming is that they lack an explicit representation of material internal characteristic length.

Models equipped with an intrinsic material char-

acteristic length are the ones based on discrete approaches (lattice and particle models) in which solids are discretized “a priori” according to the features of their internal material structures. Very successful discrete formulations have been proposed for geo-materials and concrete materials [3].

In this study, a new high-order microplane model is derived directly from a discrete particle formulation, which gives rise to an enhanced kinematic constraint where the microplane strains are obtained through projection of both strain and strain gradient tensors. In addition, high-order stresses, energy conjugate of the strain gradient are computed naturally from the microplane stresses by means of the principle of virtual work.

The new formulation has the following characteristics: (1) it is conceptually simple; (2) it can be implemented within the framework of the classical finite element method and used to retrofit available finite element computer programs; and (3) it suppresses mesh sensitivity in the softening regime.

In this presentation, several examples in both the elastic and inelastic regime are presented to demonstrate the effectiveness of the new theory.

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## Circumventing Bifurcations

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Researchers generally apply increments of load or displacement and then iterate hoping to find equilibrium. The increment size is finite and controlled. This means that within a single step multiple points may crack and start softening simultaneously. Numerically, this leads to negative tangent stiffness and ill-conditioning due to alternative equilibrium states or bifurcations. Physically, multiple cracks initiate but only a few survive while others unload. This loading/unloading competition process gives local jumps and hampers convergence. Advanced path-following techniques and the use of imperfections only partly remedy the problem. The issue is relevant for concrete and masonry structures where the scale is large, the cracks jump in suddenly and the reinforcement has a crack spreading tendency.

Load scaling presents an alternative. A series of linear analyses is applied and the global load is scaled such that only one single point changes its status at a time. The critical point is traced by comparing stress to current strength in a saw-tooth softening diagram. Local damage increments drive the global solution. The method applies to lattices [1], continua and interfaces. It was termed event-by-event, sequentially linear, saw-tooth or non-iterative [2] while similarities exist with implicit/explicit methods [3].

This contribution presents three recent advancements with the model.

First, the automatic transfer of bifurcation points to limit points is re-evaluated for asymmetric fracture in symmetric notched tensile specimens and for dispersed fracture in RC bars. The role of load scaling and positive secant stiffness is explained.

Second, the model is extended to non-proportional loading. Suppose the structure carries an existing initial load  $F_A$  and the task is to sequentially add a new load  $F_B$ . The aim is to keep load multiplier  $\lambda_A$  for load A at 1 while searching for the critical load multiplier  $\lambda_B$  for load B by comparing for all points

their total principal tensile stress ( $1\sigma_A + \lambda_B\sigma_B$ ) with the current saw-tooth strength. This leads to sets of quadratic equations in  $\lambda_B$  and a unique solution not always exists. Different points may lead to conflicting constraints for  $\lambda_B$ . Physically, the new cracking for load B leads to redistribution of the initial load A in the system, which in turn may violate the stress criterion for other points and inner cascade cracking may result. Solutions are to moderate the constitutive requirements [1], to add numerical damping or to use a double-load multiplier method. The latter method if required temporarily scales the last present total load ( $F_A + \lambda_{B,previous}F_B$ ) on the system via  $\lambda_{temp}$  allowing for redistribution cracking until all points recover, i.e. all stresses are below their current saw-tooth strength. Thereafter,  $\lambda_{temp}=1$  and the scaling for just load B via  $\lambda_B$  proceeds. Examples include Nooru-Mohamed's test, a masonry façade and a pre-stressed beam.

Third, it is shown that stepwise softening is a natural way to rotate a smeared crack. A tension-shear model problem is included.

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## Simulation of Crack Propagation Through the Static Application of Peridynamics

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Peridynamics is an integral theory of continuum mechanics that can easily describe crack propagation. Cracks in a solid represent a discontinuity in the material, and the classical theory of solid mechanics, based on partial differential equations, presents many difficulties in these circumstances. Peridynamics can overcome these problems, since it does not require spatial derivatives to be evaluated [1].

The most relevant advantage introduced by this theory in the study of crack propagation is the lack of necessity to define a-priori the crack path or at least the cracked zone, as it happens in the finite element method (interface elements [2] or x-fem capabilities [3]). In fact, with peridynamics, the crack is free to appear in every part of a structure, following only physical and geometrical constraints.

Bond-based peridynamics' main assumption is that the body is composed of material points which are linked to any other point within a finite distance named horizon. This connection is called bond.

This represents the biggest difference between this theory and Finite Elements, where the direct interaction between two nodes exists only if they belong to the same element (i.e. they are in contact).

A force, named pairwise force function, acts along the bond direction and does not produce any moment.

The original peridynamics formulation regarded dynamic problems and was adapted to the static case by introducing a substantial amount of numerical damping [4]. The novel aspect of the present work is the implicit static implementation of the theory applied to the simulation of crack propagations.

Through this theory, the crack propagation in a structure can be simulated by adopting appropriate constitutive laws of the material: nonlinear (suitable to model a progressive bond damaging phenomena), brittle or quasi-brittle relationships.

In this paper, an appropriate constitutive law has been chosen to allow the bonds to break and the crack to propagate. Therefore, the problem becomes nonlinear as a whole and an iterative procedure based on the Newton-Raphson method is used to solve it numerically.

In fact, when some bonds overcome the linear elastic limit and become damaged, the stiffness of the entire structure changes and needs to be updated. Iterations are accomplished until the equilibrium between external and internal forces is verified.

In order to simulate the propagation of a crack in a structure, a quasi-static procedure has been implemented, consisting in the incremental application of external controlled displacements, within which a convergence criterion checks the compliance of the equilibrium.

The results obtained with the developed procedure are presented for different structure configurations, boundary and load conditions.

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## Application of a Coupled Continuous-Discontinuous Approach to Concrete Fracture

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Modelling of quasi-brittle materials like concrete is a demanding task due to the presence of fracture that is a fundamental phenomenon. At the beginning of loading, a region with several micro-cracks is formed. Later these micro-cracks create a macro-crack. There exist two basic approaches to simulate cracks in solid bodies. The first idea is based on a continuum description. The material can be described using e.g. elasto-plastic, damage mechanics or coupled elasto-plastic-damage constitutive laws. These formulations include softening, so they have to be equipped with a characteristic length of microstructure to preserve the well-posedness of the boundary value problem. It can be done by means of e.g. non-local or gradient theories. Alternatively, a crack can be regarded as a discrete macro-crack with a displacement jump (by omitting a micro-crack phase). The oldest solutions used interface elements defined along element edges. The modern ones allow for considering cracks in the interior of finite elements using embedded discontinuities or XFEM (eXtended Finite Element Method) [1] based on a concept of the partition of unity. Continuum constitutive laws are more realistic in describing strain localization phenomena, but they cannot properly capture the crack formation and crack propagation. Discontinuous models, on the other hand, can handle macro-cracks, but they are not able to simulate localized zones. A combination of continuous and discontinuous approaches enables to capture all stages of a fracture process. Such coupling can be done in several ways, e.g. the latest approaches combine XFEM with implicit gradient [2] or integral-nonlocal [3] isotropic damage models.

The paper presents results of our FE simulations of a fracture process in concrete elements using a coupled continuum-discontinuous description. An isotropic damage model with non-local softening was defined as a continuous approach [4]. To describe discontinuous jumps in the displacement field, XFEM was assumed [4]. Cohesive laws on a traction-separation surface in a damage format were formulated. Several benchmarks were tested starting from uniaxial cases to mixed-mode

conditions. The problem of choosing a transition point between continuous and discontinuous displacements was numerically analysed and FE results were compared with experimental results of measured displacements on the surface of notched concrete beams under 3-point bending using the digital correlation image (DIC) technique [5]. In experiments, a transition point was marked on the load-deflection diagram. The influence of the so-called shading effect of newly created crack segments on non-local calculations was investigated.

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## Simple, Cheap and Efficient Explicit Error Estimator for Adaptive Fracture and Damage Analysis of Quasibrittle Materials

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In [1], the explicit residual-based (Babuška-Miller type [2]) error estimator, which is free of an *unknown* multiplicative constant, has been currently derived for the standard FEM in 2D isotropic linear elasticity using piecewise linear elements:

$$\|\mathbf{u} - \mathbf{u}^h\|_{\Omega} \leq UB := \frac{\tilde{c}_P \tilde{c}_K}{\sqrt{2\mu + \tilde{c}_{SE}\lambda}} \left( \sum_T \eta_T^2 \right)^{\frac{1}{2}} \quad (1)$$

where

$$\begin{aligned} \eta_T &:= h_T \|\mathbf{f} + \operatorname{div} \boldsymbol{\sigma}(\mathbf{u}^h)\|_{\mathbf{L}^2(T)} \\ &+ \frac{h_T}{|T|^{\frac{1}{2}}} \sum_{\ell=1}^3 |E_{\ell}|^{\frac{1}{2}} \|[\boldsymbol{\sigma}(\mathbf{u}^h) \cdot \mathbf{n}]_{E_{\ell}}\|_{\mathbf{L}^2(E_{\ell})} \end{aligned} \quad (2)$$

and the constants  $\tilde{c}_P$ ,  $\tilde{c}_K$  and  $\tilde{c}_{ES}$  read explicitly as

$$\tilde{c}_P := \frac{4(\sqrt{17} - 1)^{\frac{1}{2}}}{(\sqrt{17} + 7)(\sqrt{17} + 3)^{\frac{1}{2}}} \quad (3)$$

$$\tilde{c}_K := 2 \left( \frac{\pi}{3\pi + 2} \right)^{\frac{1}{2}}, \quad \tilde{c}_{SE} := \frac{2\pi + 4}{3\pi + 2}. \quad (4)$$

The pronounced advantages of the estimator is, in the first place, that it is computationally simple and cheap. The upper bound  $UB$  on the discretization error is guaranteed and, under the shape regularity assumption for an underlying mesh, it is also accurate – in [1], for a number of singular and regular benchmark linear problems the effectivity indices  $\theta := \frac{UB}{\|\mathbf{u} - \mathbf{u}^h\|_{\Omega}}$  were found to be of order 1.3–1.6, which is treated as practically (very) acceptable. Finally, the derivation ideas for the above  $\tilde{c}_P$ ,  $\tilde{c}_K$  and  $\tilde{c}_{ES}$  are applicable to the 3D FE setting, that is, the extension of (1)–(4) to the three-dimensional linear formulations is seemingly straightforward.

In this talk, we consider two types of practical engineering problems, crack propagation and crack initiation in quasi-brittle materials (so far,

in 2D and a single-scale setting), for which accurate non-adaptive solutions are seemingly out of reach (unless the advanced FE techniques like e.g. GFEM/XFEM are implemented) or can be very costly to obtain when using the higher-order elements. Instead, we employ the most simple  $\mathbb{P}_1$ -triangular approximations driven by error-controlled adaptivity due to (1)–(4) to gain very accurate, yet affordable results for those problems.

In this context, modeling of cracks *initiation* (here, in a single-edge notched concrete beam undergoing four-point unsymmetric loading) using an isotropic gradient-enhanced damage formulation [3] is of our most interest. Estimator (1) is used to generate meshes which adaptively follow evolving non-local equivalent strain localizations and damage patterns, significantly improving the overall accuracy of the quantities of interest. Also, a subsequent accurate transition from continuous damaged state to actual discontinuity (fracture) is also possible in our case.

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## On the Calibration of Nonlocal Models for Tensile Fracture in Concrete

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Tensile fracture in concrete is characterised by a quasi-brittle response in the form of a gradual reduction of stress with increasing displacements. This softening response is caused by the development of a fracture process zone in the heterogeneous concrete, involving mechanisms such as branching and bridging of micro-cracks. The fracture process zone evolves from distributed and disconnected micro-cracks to a strongly localised tortuous macroscopic crack, whereby the tortuosity is governed by the presence of aggregates, which represent the dominant heterogeneity of the material. For the analysis of concrete structures, it is important to include this quasi-brittle softening response, since it strongly influences the load capacity of structures and is the source of a particular size effect on nominal strength, which neither follows the strength theory nor linear elastic fracture mechanics [1].

For many structures, the heterogeneities in concrete are too small to be modelled explicitly and too large to allow for the use of linear elastic fracture mechanics. Therefore, concrete is often assumed to be homogeneous on the scale studied and the fracture process zone caused by interactions on the lower scale are modelled by localised but regular strain profiles using nonlocal models [2]. In these models the stress at a point is evaluated by the weighted spatial averaging of history variables in the vicinity of this point. Nonlocal approaches do not rely on the assumption that strains localise in a mesh dependent zone. Therefore, they are capable of describing the simultaneous occurrence of distributed and localised failure mesh-independently. Additionally, fracture patterns obtained with nonlocal models are less sensitive to the alignment of the finite element mesh than those obtained by other models.

The spatial averaging in nonlocal models requires the input of a length parameter, which should be related to the width of the fracture process zone. Calibration approaches to determine this parameter have been proposed in literature [3, 4]. However,

more research is required to provide simple procedures which would allow for the determination of this length parameter.

Here, a recent approach based on the meso-scale analysis of the fracture process of a unit cell subjected to tension [5] is further developed to determine this parameter. The tortuosity of the final localised crack is linked to the width of the fracture process zone and the length parameter used in the nonlocal model. The influence of aggregate size and volume fraction on the nonlocal length will be investigated with this method. Furthermore, suggestions for experimental techniques to determine the length parameter will be proposed.

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## **A New Method to Assess the Most Likely Tensile Strength Field in Quasi Brittle Materials Complying with the Probabilistic Scale Effect**

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Numerical prediction of cracking in quasi brittle large structures homogeneously loaded has to face in tensile strength dependence on loaded volume. This phenomenon is related to the probability to find a defect, which increases with the solicited volume. The tensile strength reduction with the specimen size phenomenon is also known as “Weibull scale effect” and often modeled thanks to the “weakest link theory”. In concrete structures for example, if the tensile strength mean value is obtained performing classical split tests, the scale effect leads, for a metric element, to a reduction of 50% of the tensile strength, the non-negligible character of this size effect was also confirmed and studied by Bazant for beams more longer than 2 m [1] who proposed a “non-local Weibull” formulation to consider both the energetic and the probabilistic size effect. In fact, and as explain below, his formulation was more a “classical Weibull formulation” applied to a “non-local model” than a non-local Weibull method. The probabilistic scale effect is due to the material random properties at the meso-scale. Many authors have tried to consider it thanks to meso-scopic approaches, or through a random sampling of tensile strength field. However these last techniques, even if they explain more or less the scale effect origin and facilitate the localization, needs numerous Monte Carlo simulations to assess the mean behavior of the structure. As resorting to Monte Carlo simulations is particularly time consuming for nonlinear finite element analysis of large structures, a new method was proposed by A. Sellier [2] and implemented in Castem with the aid of A. Millard in the framework of the French research program Mefisto. This method reconciles probabilistic scale effect and determinist

simulations, and then avoids resorting to random simulations. Most of the method’s assumptions are classical: the material tensile strength complies with a Weibull distribution, the behavior is brittle in tension and must be modeled thanks to a classical regularization procedure; the weakest link theory is applicable. But a few non-classical assumptions have to be formulated to achieve the formulation: specifically a “probabilistic influence function” is introduced in the weakest link theory to consider that the influence of a volume loaded too far from a considered point has a neglected probabilistic influence on this point, mainly due to the redistribution capability of the 3D structure which is not considered in the classical weakest link theory. At last, the proposed method must be used in parallel to a regularization technic, not in substitution, since its role is not to regularize the localization problem, but to determine the most likely tensile strength to be used in the damage model.

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## On the Determination of Softening Curves for Cementitious Materials by Inverse Analysis of Fracture Tests

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The measurement of complete load-displacement curves in deformation controlled fracture tests and subsequent inverse analyses of these tests are a widely used procedure of determining fracture properties to be adopted for numerical simulations of cracking in concrete structures. According to this approach, also referred to as work-of-fracture method, the external mechanical work necessary to completely split the respective specimen is divided by the nominal (plane) ligament area and, in this way, an apparent fracture energy is obtained. If the specimen boundaries are sufficiently remote from the fracture process zone, the latter is propagating in a stationary way and the fracture energy defined as the flux of energy into this zone is a material property [1]. In laboratory tests of normal-scale concrete specimens, however, this condition can usually not be met. When the crack tip comes close to the specimen boundary, the size of the fracture process zone will be reduced and only a portion of the fracture energy is activated, i.e., the locally required energy for crack propagation is getting smaller [2]. As a consequence, the apparent fracture energy of concrete determined on the basis of the work of fracture in bending or wedge splitting tests becomes smaller with decreasing ligament length. Due to the aforementioned boundary effect, the local softening curve will also show variations along the crack path. The intention of the authors was to consider this boundary effect in inverse analyses of fracture tests in order to obtain ligament length independent fracture properties of the respective material [3].

Wedge splitting tests with different specimen dimensions were carried out. For the inverse analyses of these tests, the cohesive crack model with multi-linear softening curves was adopted and the experiments were numerically simulated by using 2D Finite Element models. As optimization method an evolutionary algorithm has been used [3, 4]. Such algorithms replicate the process of biological evolution of a population by consecutive genetic operations and selection. When compared to a deterministic gradient descent optimization,

the major advantage of the applied evolutionary algorithm is the improved objectivity of the results. It may be shown that in the case of fracture tests objective inverse analysis results require very good fits of the numerical load-displacement curves to the experimental ones [4]. The corresponding error function has multiple local minima and evolutionary algorithms are suitable means for solving such non-convex optimization problems.

If the boundary effect on the local fracture properties is taken into account in the inverse analyses of the wedge splitting tests, the variation of the softening curve along the crack path may be studied. It was found that the tail of the softening curve is shortened and lowered when the crack tip approaches the specimen boundary whereas tensile strength and initial slope of the softening curve appear to be not affected [3]. This observation is in agreement with the generally accepted theory that the so-called initial fracture energy is size-independent whereas the tail of the softening curve may exhibit a boundary effect [1].

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## Dynamic Fracture of Quasi Brittle Materials: Failure Mode and Crack Branching

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The experimental and theoretical evidence show that loading rate significantly influences the resistance and failure mode of concrete structures [1]. In case of linear elastic material there is no rate sensitivity. However, in materials that exhibit damage and fracture phenomena, such as concrete, there is significant influence of loading rate on material and structural response. This implies that rate sensitivity is closely related to damage of the material, i.e. more is the damage the stronger will be the influence of loading rate on the structural response. The confirmation for this statement can be found in the fact that concrete like materials exhibit the highest rate sensitivity for tensile load.

It is assumed that the response of concrete structures depends on time dependent loading through three different effects [2]: (1) through the rate dependency of the growing microcracks (influence of inertia at the micro level), (2) through the viscous behavior of the bulk material between the cracks (viscosity due to the water content) and (3) through the influence of structural inertia forces (macro scale), which can significantly change the state of stresses and strains of the material. At macro scale the first two effects can be accounted for by the constitutive law and the third effect should automatically come out from dynamic analysis where the constitutive law interacts with structural inertia [2, 3].

Depending on the material type and the loading rate, the first, second or third effect may dominate. For quasi-brittle materials, such as concrete, which exhibit cracking and damage phenomena, the first two effects are important for relatively low and medium strain rates. However, for higher strain rates (impact) the last effect dominates. Moreover, inertia cause the change of the failure mode and it is responsible for crack branching phenomena.

The present contribution addresses two important phenomena, rate dependent failure mode and crack branching. With increasing loading rate failure mode changes, i.e. there is a transition from mode I to localised shear failure mode and crack branching

phenomena takes place. Both phenomena are studied experimentally and numerically.

The experiments are performed on the compact tensile specimen (CTS) dimensions 200×200×25 mm made of normal strength concrete. The specimen is loaded by controlling displacement ranging from 0.01 to 5 m/sec. Beside CTS similar experiments are carried out also on L-shaped concrete specimen dimensions 500×500×200 mm. The main objective of the experimental research was: (i) Study the influence of the loading rate on the failure mode, crack branching phenomena and speed of crack propagation and (ii) To confirm recent numerical results obtain for the same specimens using 3D FE analysis that was based on the rate sensitive microplane model [2, 3].

The results of experimental investigations can be summarised as follows: (i) With increasing loading rate there is a change of failure mode from mode-I to mixed mode; (ii) Maximal crack speed measured in experiments is approximately 500 m/sec; (iii) Once the crack reaches maximal velocity there is crack branching and (iv) The experimental results nicely confirmed recent numerical results obtained by the 3D finite element studies [2, 3].

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## 3D Global / Local Analysis of Cracking of Reinforced Concrete

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The assessment of crack features — pattern, spacing, opening — is a requirement for the design of reinforced concrete structures. Those assessments cannot rely on formula or experiment only, the first one being too simplistic and the second one too heavy. Therefore, numerical tools are developed to tackle the problem.

On one hand, non-linear finite element analyses based on continuum damage mechanics [1,2] are efficient to obtain the global behavior of large reinforced concrete structures. RicRag [3] is a finite element model coupling elasticity, isotropic damage and internal sliding. It can describe the local mechanisms related to concrete such as the asymmetry between the tensile behavior and the compressive behavior, the inelastic strains and the unilateral effect. Moreover, it is robust and can handle large-scale computation. However, this model does not model the discontinuities and thus makes the study of the cracks complex.

On the other hand, discrete element models [4,5] are inherently capable of representing discontinuities such as cracks. DEAP [6] is a lattice model for which the main physical mechanisms of quasi-brittle materials rupture are recovered, such as spatial correlation, crack tortuosity or scale effects. Unfortunately, the mesh density needed for such modeling is prohibitive to treat the case of industrial structures.

A global/local analysis has been proposed [7] to take advantage of each model at their adequate scale. This non-intrusive technique allows the use of finite element models at a structural scale and a decoupled local analysis of some interesting areas, i.e. around cracks, for which a discrete element model is used.

The presentation will focus on the extension of the global/local method for 3D analysis of reinforced concrete beams. The treatment of reinforcement and interface will be addressed.

The numerical strategy is confronted to two different experiments to show its capabilities. The experimental crack pattern and crack openings are obtained on the surface with a digital image correlation analysis. The first experiment is a laboratory three points bending test performed on a concrete beam with light reinforcement. The second experiment is an industrial four points bending test on a massive concrete beam with heavy reinforcement.

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**Minisymposium DuF:**

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## Micro-Mechanical Numerical Analyses on the Effect of Stress State on Ductile Damage and Failure

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The presentation deals with the effect of stress state on the damage behavior of ductile metals. Experimental results obtained from different tension and shear tests with carefully designed specimens allow identification of basic macroscopic parameters. To be able to get more insight in the complex damage and failure mechanisms series of three-dimensional numerical analyses on the micro-scale of void containing unit cells have been performed. The calculations cover a wide range of stress triaxialities and Lode parameters. They allow development of equations of damage criteria and damage strains as well as identification of corresponding material parameters.

The continuum damage model [1] has been generalized to take into account the effect of stress state on damage criteria and on evolution equations of damage strains corresponding to various physical mechanisms at different scales. It is based on the introduction of damaged and corresponding undamaged configurations. Different elastic potential functions are introduced to be able to take into account the effect of damage on the elastic material properties. In addition, plastic behavior is modeled by a Drucker-Prager type yield condition and a non-associated flow rule formulated in the effective stress space (undamaged configurations). In a similar way, a damage criterion is proposed characterizing the onset of damage which is formulated in the damaged configurations. Different branches of this criterion are considered corresponding to various damage mechanisms depending on stress triaxiality and Lode parameter [2, 3]. Furthermore, a non-associated damage rule describes the evolution of damage strain rates caused by the growth and coalescence of voids as well as the simultaneous formation and growth of micro-shear-cracks.

Experiments with smooth and notched tension and shear specimens have been performed and the test results are used to identify elastic-plastic material

parameters as well as the equivalent stress at onset of damage. However, it is not possible to identify all parameters appearing in the continuum model based on these experiments [4]. To be able to get more insight in the complex damage mechanisms under different types of loading conditions, additional series of three-dimensional micro-mechanical numerical analyses of void containing unit cells have been performed. These finite element calculations on the micro-scale cover a wide range of stress triaxialities and Lode parameters in the macroscopic tension, shear and compression domains. The numerical results are used to show general trends and to understand the physical mechanisms of ductile damage under complex loading paths as well as to propose equations for the stress-state-dependent damage criteria, to develop evolution equations for the macroscopic damage strains, and to identify parameters of the continuum damage model.

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## Micromechanical Analysis of Size Effects in Ductile Failure

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Ductile fracture is an important failure mechanism in metallic materials and there is a huge number of studies about this topic (for an overview see e.g. [1]). Ductile failure is caused by nucleation, growth and coalescence of microscopic voids. Usually, these voids nucleate at inclusions or second-phase particles and start to grow due to the plastic deformation of the surrounding matrix material. The onset of internal necking between the voids corresponds to the start of acceleration of void growth leading to final failure by void coalescence.

In structural metals there are several types of defects with different sizes and mean spacings such as carbides, precipitates and non-metallic inclusions. Under mechanical loading voids nucleate first at the larger particles and start to grow. These large voids represent the primary porosity. Upon further loading smaller secondary voids nucleate at the smaller particles so that there are now two populations of voids on different size scales. The final coalescence process between the larger voids can take place by two mechanisms: Either there is coalescence by a flat rupture mode or by development of a micro shear band resulting in the so-called void-sheet formation. Which type of coalescence mechanism gets active depends on the applied type of loading and the microstructure of the material.

Regarding the influence of an existing or nucleating population of secondary voids, Faleskog and Shih [2] performed micromechanical simulations under different strain biaxialities with discretely resolved secondary voids in order to study both mechanisms. Furthermore, unit cell analyses, e.g. Kuna and Sun [3], were performed, where the secondary porosity was introduced by employing the GTN-model for the matrix material containing the secondary voids. These investigations showed, that the effect of secondary voids is significant if the secondary voids are present right from the start and that it is negligible if the secondary population nucleates late.

In this contribution the influence of a population of

secondary voids on the flat ductile rupture mode is investigated numerically by 3D cell model calculations. The present study focuses on materials with a large primary void volume fraction like nodular cast iron. The non-local GTN-model of implicit gradient-enriched type developed by Linse et al. [4] is used to model the behavior of the porous matrix material surrounding the primary voids. Due to the internal length scale incorporated in the non-local GTN-model the size or the spacing, resp., of the secondary voids can be taken into account in the simulations. Hence, the influence of different ratios between the size of primary to secondary voids on void growth and coalescence is examined. Furthermore, the effect of different primary void arrangements is studied for certain combinations of stress triaxiality and Lode parameter. Finally, the numerically obtained failure strains are compared to experimental values for materials with different microstructures.

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## A Gurson-Type Criterion for Plastically Anisotropic Material Containing General Ellipsoidal Voids

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Since the pioneering work of Gurson [1], who proposed a macroscopic criterion for porous isotropic plastic solids containing spherical voids, several works have been extended this ductile fracture model, taking account of the void shape and the material anisotropy. For example, Monchiet *et al.* [2] proposed a criterion for a Hill material containing spheroidal voids, and very recently, Madou and Leblond [3, 4] proposed a criterion for isotropic material containing general ellipsoidal voids.

The aim of this work is to extend both criteria of Madou and Leblond [3, 4] and Monchiet *et al.* [2], to Hill materials containing general ellipsoidal voids.

To this end, we perform a limit analysis of some ellipsoidal cell made of a Hill material and containing a confocal ellipsoidal void. The velocity fields used are exactly the same than in the isotropic case, since "isotropic" fields have been shown by Monchiet *et al.* [2], in the spheroidal case, to provide accurate results. Then, simplifications of the macroscopic plastic dissipation are proposed, based on asymptotic studies, and lead to a yet non fully determined approximate yield function. This yield function, very similar to that of Madou and Leblond [3, 4], consists of a quadratic form of the stress tensor plus a hyperbolic cosine of a linear form of the stress tensor. Full determination of the criterion parameters is conducted in distinct ways for hydrostatic and deviatoric loadings.

For hydrostatic loadings, finite elements simulations are performed in a number of significant cases in order to refine the limit analysis. Some parameters of the criterion are then fitted to match the results of these numerical limit analyses. We take advantage of the hypothesis of isotropic velocity fields to perform these numerical studies on isotropic materials, the anisotropy intervening only in the exploitation of results.

For deviatoric loadings, limit analysis is dropped

and parameters are determined using a rigorous bound applicable to non linear composites, specialized to porous materials, because this bound is well known to provide accurate results for this kind of loadings. Consequently, parameters pertaining to deviatoric loadings are adjusted in order to enforce coincidence of the approximate yield function and the bound.

The criterion is then assessed through numerical limit analysis in a number of representative cases, performed using finite elements on representative cells of various geometries, with anisotropic material. Both the yield locus and the flow rule are investigated, and the results obtained are quite good.

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## Finite Element Modeling of Void Nucleation, Growth and Coalescence for Large Plastic Strain and Complex Loading Paths

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Ductile damage prediction is still an important issue in the mechanical industry. Ductile failure criteria as well as coupled damage models were enhanced in the last decade by accounting for the influence of stress triaxiality ratio and Lode angle. For the Gurson-Tvergaard-Needleman (GTN) model, these improvements were essentially made for the growth and coalescence stages based on unit-cell models in which the behavior of a single ellipsoidal void is studied under various loading conditions [1]. In the present work, the stage of nucleation is also addressed in addition to void growth and coalescence for large plastic strain and different particle/void morphologies.

- Microstructure representation

Heterogeneous microstructures with particles and voids in a metallic matrix are considered. This heterogeneous microstructure is meshed using level-set functions - to define interfaces between matrix, inclusions and voids - and anisotropic mesh adaptation. This anisotropic mesh adaptation is particularly well suited for macroscopic [2] and microscopic [3] ductile damage problems.

- Mechanical failure criteria

Nucleation starts either by particle failure or by debonding at particle-matrix interfaces. For debonding nucleation (respectively particle failure), the failure criterion is based on interfacial normal and tangential stresses (respectively maximum principal stress in particles). The void growth stage is based on plastic strain around void during the deformation stage. Finally coalescence is activated when a critical plastic strain value is reached between two growing voids. It has to be noticed that this coalescence criterion can be easily transformed in a criterion based on the distance between neighbouring voids.

- Numerical failure techniques

Once the failure criterion is reached, two different techniques can be used to model failure:

- Germ-based method: a void germ, represented by a new level-set function, is created at the location for which the failure criterion was reached. The interface between this void and the matrix (or particle) is well described thanks to the anisotropic mesh adaptation.

- Failure plane method: an instantaneous failure plane, modelled by a level-set function, is created once the critical failure criterion is reached. For good accuracy, particular attention has to be paid to the mesh refinement in the area in which this failure plane is created.

The two methods are compared and their advantages and drawbacks are presented in terms of numerical accuracy and physical representation of ductile failure mechanisms.

- Applications

Several 2D and 3D applications are presented. The influence of failure parameters and loading conditions are studied for simple microstructures and for real microstructures coming from scanning electron microscopy (SEM) images.

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## Understanding Apparent Ductility of Martensite in Dual Phase Steels

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Dual Phase (DP) steels are a class of Advanced High Strength Steels, combining a good combination of high strength and ductility. Despite the fact that the DP steels have already been on the market for a while, their damage initiation and propagation behaviour is not yet fully understood, as these seem to differ from the classical void growth mechanism. As a result, in the industrial practice, DP steels sometimes show failure at strain levels which should be save according to the conventional failure criteria.

The excellent properties of DP steels are the result of their microstructure, consisting of a ductile ferritic phase and a high strength martensitic phase. Despite the natural expectation that the decohesion of the phases would be responsible for damage (at least its initiation), in more recent experimental observations, the severe plastic straining of ferrite near to the martensite islands and martensite cracking have been observed [1]. In particular, the high strains and the apparent ductile behaviour of martensite islands [2] seems to contradict the generally observed brittle behaviour of martensite.

In this work, an attempt is made to systematically study the possible mechanisms of apparent martensite ductility, as well as their potential contribution to the overall properties of DP steels.

To this aim, the internal hierarchical substructure of a martensite island is considered, known to consist of packets, blocks, laths and variants, having particular crystallographic features and related to a parent austenite grain orientation by a so-called orientation relationship [3]. In addition, the role of thin layers of retained austenite, possibly present either between the martensite laths, or at prior austenite boundaries, is studied.

A microstructural unit cell model is created, incorporating several martensite laths (of the same crystallographic variant) with a thin layer of austenite between them. The crystallographic orientations of the laths and the austenite are

assigned according to the Kurdjumov-Sachs (K-S) orientation relationship. Both phases are modelled by a classical crystal plasticity model, with FCC and BCC slip systems for the austenite and martensite, respectively.

The results of the simulations show that as long as there are enough carriers for plasticity present in the thin austenite layer, shearing along the lath habit plane yields high deformations. Under such conditions, the austenite phase acts like a greasy plane on which stiffer martensite laths can slide. This mechanism is dominated by the orientation relationship between the phases, which is such that the lath habit planes are approximately parallel to the slip planes of the parent austenite. In the absence of the crystallographically correct orientation relationship the shearing mechanism is not present to the same extend.

Therefore, it has been concluded that this shear mechanism due to the presence of the interlath retained austenite can indeed be a plausible reason explaining the observed apparent ductility of the lath martensite.

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## A Crack Insertion Strategy Based on a Gradient-Type Criterion and Local Remeshing

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The failure process of ductile materials, such as metals, can be divided into two main phases. The first stage involves extensive plastic strains together with the nucleation and growth of voids in localized areas. During this phase, a continuous model may be used to describe the failure of the underlying microstructure in an average sense by means of a damage variable. This continuous description is acceptable up to the onset of fracture. At this point, voids coalesce to form macroscopic cracks, and a continuous model cannot properly describe the kinematics associated with a crack opening. A modern approach consists in introducing a discontinuous description to model crack propagation, e.g. using a remeshing strategy [1]. A crack initiation and propagation strategy using a new orientation criterion and global remeshing has recently been proposed by the authors [2, 3] for both 2D and 3D cases. In this contribution, we present the latest developments devised to make this strategy more robust and more efficient, as well as some new test cases.

The proposed orientation criterion relies on the Projected Gradient of a Smoothed Field (PGSF) to determine the position of the next crack increment. Until now, the considered field has been the damage field. However, the fact that damage is a bounded variable (between 0 and 1), would make it difficult to study the sign of the gradient in the areas where damage had almost reached its maximum value. To overcome this difficulty, the authors now propose to use any unbounded field which is representative of the material degradation.

One of the main advantages of the PGSF method is its cost, since it relies only on a scalar field, as opposed to a second order tensor. Besides, this orientation criterion is particularly interesting if a crack initiates completely inside the structure (i.e. far from the boundary). This strong feature will be illustrated by some numerical examples, where criteria based on weighted averages might fail.

Once the position of the considered crack increment has been determined, a mesh intersection algorithm is used to insert the crack surface in the structure's mesh [4]. The fields are then transferred from the old mesh to the new mesh so that computation can be resumed. The main drawback with this remeshing strategy is that the field transfers introduce numerical errors which may result in different mechanical states before and after remeshing. In order to reduce these errors, a local remeshing approach is preferred. The only areas which undergo remeshing are those where the mesh is too coarse – according to an error estimator – and those where a crack increment will be inserted. A least square method is used to transfer the fields in the areas where the mesh has changed. Everywhere else the fields can be transferred exactly. Numerical examples illustrating the advantages of this approach will be presented.

Finally, other challenging issues regarding the transition between damage and fracture will be addressed (e.g. crack branching or merging), with new numerical examples on different structures.

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## Simulating Ductile Crack Growth Using the Cohesive Band Model

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Fracture lies at the heart of many failure phenomena of man-made and natural structures. For quasi-brittle and ductile fracture, where the length of the fracture process zone is not small compared to a typical structural size, cohesive surface models have proven particularly successful. One type of cohesive model can be characterized by a fracture initiation criterion, and after initiation, by a cohesive opening that is governed by the work of separation. The fracture process zone is lumped into a plane. Its opening is governed by the shape of the decohesion curve, which sets the relation between the normal and the shear tractions across the cohesive surfaces and the relative displacements between these surfaces. Fracture is then a natural outcome of the loading process.

A conventional zero thickness cohesive surface formulation involves a relation between tractions and displacement jumps across a surface. Stress components that do not affect the tractions are not accounted for in the cohesive constitutive relation and neither are deformation components that only involve displacements and gradients parallel to the surface. This limits the modelling capability in a variety of circumstances, including ductile failure of metals [1] where stress triaxiality plays an important role. One approach to overcome this limitation is to insert the normal stress from a neighbouring integration point in the continuum into the cohesive surface relation, e.g. [2]. Another approach is to introduce a band of finite thickness upon the occurrence of the loss of ellipticity at a material point [3]. Recently, the cohesive band method has been proposed as a promising alternative [4]. Here, the cohesive zone is represented by a band with a specified, constant thickness. The various strain terms of the in-band response are derived by exploiting the partition-of-unity property of finite element shape functions. This in-band response is taken to augment the cohesive surface relation so that, as in the

usual cohesive surface formulation, the band constitutive relation is independent of the volumetric material relation. In addition, the band thickness appears to be a numerical parameter, rather than a material parameter, and the formulation is such that as the band thickness goes to zero, a conventional cohesive surface formulation is recovered. Finally, the cohesive band approach is fully discrete, with continuity of the discontinuity gap at element boundaries.

In this paper, we carry out analyses of ductile crack growth using the cohesive band method. The analyses use a rate dependent Gurson type relation in the band and a rate dependent Mises solid outside the band. The performance of this model is compared with simulations performed by Siegmund and Brocks [2] and with simulations using the finite band model performed by Huespe et al. [3].

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## Modeling of Damage to Crack Transition Using a Coupled Discontinuous Galerkin / Cohesive Extrinsic Law Framework

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One current challenge related to computational fracture mechanics is the modelization of ductile fracture and in particular the damage to crack transition. In this paper we propose to achieve this goal by combining

1. A non-local damage model
2. The hybrid discontinuous Galerkin (DG) / extrinsic cohesive law (ECL) formulation

As classical damage models for finite element formulations lose the solution uniqueness and face the strain localization problem when strain softening of materials is involved, the damage model is herein formulated in a so-called implicit non-local approach, following the developments in [1]. In this formulation, a new non-local variable, the non-local accumulated plastic strain eg, representative of an internal variable and its derivatives, results from the resolution of a new boundary value problem. Besides the advantage of using  $C^0$  elements, although the elements have now one additional degree of freedom per node, this approach also possesses the feature of being fully non-local.

The hybrid DG/ECL method was recently proposed [2] to circumvent the drawbacks of the cohesive element methods. Indeed, with this DG/ECL method, prior to fracture, the flux and stabilization terms arising from the DG formulation at interelement boundaries are enforced via interface elements in a way that guarantees consistency and stability, contrarily to traditional extrinsic cohesive zone methods. Upon the onset of fracture, the traction-separation law (TSL) governing the fracture process becomes operative without the need to modify the mesh topology as the the cohesive elements required to integrate the TSL are already present. This DG/ECL method has been shown to be an efficient numerical framework that can easily be imple-

ment in parallel with excellent scalability properties to model fragmentation, dynamic crack propagation in brittle and small-scale yielding materials, for 3D problems and for thin structures [2, 3].

In this work, the DG/ECL method is extended to account for the damage process, as described in the non-local setting. One main advantage of the DG/ECL formulation is the existence of interface elements in which the damage model can be solved, the hydrostatic pressure can be resolved, and through which discontinuities can easily be introduced with a physically-based criterion.

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## Error Analysis for Submodels with Cracks: Extension to Elastic-Plastic Material

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The integrity of nuclear shipping casks has to be guaranteed under severe test conditions, regulated by the guidelines given by the IAEA [1] and BAM [2]. Among others this includes impact conditions, such as a drop from 1 m onto a steel punch. Within this context applying methods of fracture mechanics is inevitable if the cask is made of cast iron.

To analyze the safety margin against brittle failure, finite element simulations are carried out. Since the model of the cask can be very large submodeling technique is used to analyze specific regions, e. g. stress concentrations more in detail. To do so, a submodel of the region of interest is generated with a higher mesh density. Its boundary is driven by the displacement of the coarse global model. A disadvantage of the technique is, that the boundaries have to be far enough away from the stress concentration. Furthermore, the stiffness of the region should not be changed in the submodel.

To use submodeling technique for fracture mechanics analysis the following procedure is followed: The drop test simulation is based on a global model without defects. From the results positions are derived at which a crack initiation from a defect would be most likely. At each of these positions a submodel with a crack is analyzed. The size of the crack is defined based on the limits of detection of non-destructive material testing methods.

However, the insertion of the crack reduces the stiffness and the stress of the submodel region. Therefore, the stress intensity factor of the crack in the submodel is too low. The bigger the submodel the lower its stress intensity factor deviates from the reference value found without using the submodel technique.

This deviation can be determined only from one submodel with the method developed in [3] calculating the difference of the traction vector between the submodel with opened and with closed crack. Its scalar product with the weight function results

in the contribution of the submodel boundary to the deviation. Integrated over the boundary it yields the deviation of the submodel. The shape of the submodel can be optimized by expanding sections of the submodel boundary with a high contribution. So, the required submodel size for a given maximum deviation can be deduced in a few steps.

However, the method described above is limited to linear-elastic material behavior, since weight functions are used. But the cast iron material is exceeded by high plastic deformations during the drop test. Therefore, an extension of the method to elastic-plastic material laws is intended. The plastification due to the insertion of the crack in the submodel is big. The objective is to replace this plastic deformation by volume forces and apply them to a linear-elastic analogous submodel. The deviation of its stress intensity factor can be calculated by the method described above. Finally, this deviation has to be converted to the deviation of the J-integral of the real elastic-plastic submodel.

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## Crack Driving Forces and Fatigue Crack Propagation in Elastic-Plastic Materials

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Extending the tools of linear elastic fracture mechanics to elastic-plastic materials is not a straightforward task, in particular with respect to the appropriate definition, and consequent computation, of the crack driving force associated with the advancement of a preexisting singular macroscopic crack. It is also important to establish new propagation laws by using measures in standard laboratory experiments applicable for describing crack growth in arbitrary structures. For crack growth in elastic-plastic materials under large-scale or general yielding conditions, the common approach is to use criteria based on the crack tip opening, the Rice J-integral, the energy dissipation rate, the theta-approach or material forces. However, the use of each of these criteria is somewhat problematic. The theta approach is used widely in dissipative context [1]; however, there seems to be a lack of consensus as to the appropriate basic formulation. Moreover, in quasi-static crack propagation of an elastic-perfectly plastic material under monotonic increasing load, the theta-method or, more generally, any Griffith-like energy balance supplies vanishing value of the energy release rate; this result is commonly accepted in literature and known as the Rice paradox. In addition Bui [2] show with a free boundary problem that a damage interface in an elastic-perfectly plastic body must have a non-zero curvature radius. Rice also argued that for an elastic-plastic material with work-hardening, a Griffith-like energy balance could predict a nonzero value of the surface energy; this conclusion was based on the assumption that an elasticplastic material with a high degree of hardening can be treated as an elastic one, for which finite energy release rate is found. The goal of this paper is to clarify these issues. Our approach is to first use thermodynamic arguments to derive an explicit and simple crack driving force that can be computed by domain differentiation (theta method). To do that, we make a strong

separation between the two sources of dissipation of a cracked elasticplastic body: the propagation of the crack tip and the plastic deformation in the body. Then, we explicitly identify the numerical difficulties that are associated with this quantity. To do that, we review the classical aspects of numerical validation for the G-theta method that are domain independence, residual and mesh convergence and other aspects specific to plasticity like convergence in unloading, history dependence, locality in time. Moreover, we make a comparison between this quantity and the others used with theta method in dissipative materials. The different quantities are found equivalent when computed using the theta method. Our computations show the path-invariance of the theta method integral, mesh convergence of the resulting crack driving force, and qualitative agreement with experimental ratio effect, overload effect and underload effect. Finally, a simple framework is adopted to understand the interaction between our quantity and fatigue crack propagation. Since fatigue crack growth is often modelled by plasticity-modified Paris law, we use a cyclic loading [3] and verify if the reproduction of classical effects like overload and block load effects are taken into account by the proposed quantity.

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## A Two Length Scale Non-Local Model to Describe Ductile Rupture at Low Stress Triaxiality

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Ductile rupture of metals is often described as a three stage process involving void nucleation, void growth and void coalescence. Void nucleation sites are primary inclusions such as MnS or CaS in steels or particles containing Fe and Si in aluminum alloys [2]. This rupture scenario prevails at high stress triaxiality ratios. However at low stress triaxiality, void nucleation on a second inclusion population is often observed. These inclusions are usually much smaller than the primary nucleation sites mentioned above. They consist in iron carbides in steels [3] or in fine strengthening dispersoids in aluminum alloys [1].

Constitutive models for ductile rupture have been developed since the 70's. These models explicitly take into account the different damage stages including both nucleation mechanisms but also microstructural features such as void shape change, void rotation, coalescence by internal necking... (see reviews in [2, 4]). Their use in finite element codes leads to severe numerical problems including strain and damage localization, dependency of the simulation results (energy dissipation, crack path) on mesh size and mesh type. Solving these problems is essential in order to fully benefit from advanced descriptions of damage processes to predict failure of structures.

In this work, the chosen solution to avoid mesh dependency is to use so called non-local models. In order to be able to represent failure by void growth and coalescence and failure by secondary void nucleation, a two length scale model is proposed. Using two characteristic length scales is needed as the mean spacings between primary voids and between secondary nucleation sites strongly differ. The proposed solution combines the formulation developed in [5] to avoid localization of volume variation (corresponding to void growth and coalescence) and the formulation proposed in [6, 7] to represent fracture driven by softening plasticity (corresponding

to secondary nucleation). A special finite element is developed which uses a four field formulation: displacements, assumed pressure, assumed volume variation, and non-local plastic strain. This element is used together with a set of constitutive equations based on the GTN model [8] modified to deal with the non-local treatment of the plastic strain. The model is applied in the case of axisymmetric and plane strain notched specimens.

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## Ductility of an X100 Steel under Different Triaxiality and Lode Angle Histories: Axisymmetric and C(T) Fracture Specimens

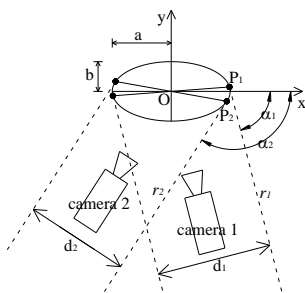
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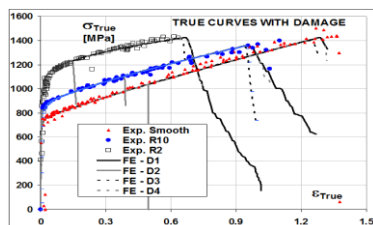
Failure of an X100 steel for piping is studied in order to determine how the triaxiality parameter  $TF$  and the Lode angle parameter  $X$  affect the failure of such a highly anisotropic steel.

The anisotropy poses a significant preliminary problem for determining a reliable hardening characterization, as the progressive flattening of the cross section becoming more and more elliptical makes it impossible to determine the true stress-true strain curve with the usual methods.

A method developed by the author is shown to satisfactorily overcome this issue; it is based on two video acquisition of the evolving tension specimen shape, from different viewpoints.

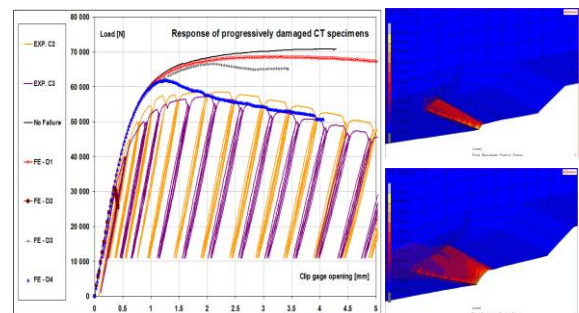
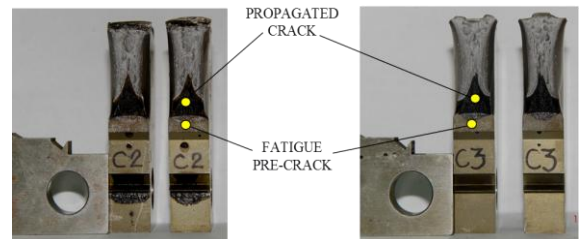
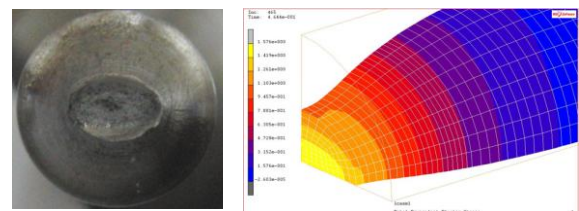


Ductile damage evaluation and progressive failure by elements removal is implemented in FE analyses where the dependence of damage and failure on  $TF$  is modeled through the Bao-Wierzbicki model, while the effect of  $X$  is modeled by different functions whose responses are compared each other.



The evolving crack size and the load-notch opening of C(T) specimens from FE is checked against the compliance-determined experimental data, so obtaining information about the interaction between triaxiality, Lode angle, failure and stress-strain response.

Finally, the responses of the FE analyses where different effects of  $X$  are implemented, also allow to identify some characteristics the damage-Lode angle trends should have for effectively reproducing the experiments.



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## Lie-Group Interpolation and Variational Recovery for Internal Variables

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The transfer of field data from one mesh to another is a need that arises frequently within the context of mesh adaption for fracture and failure in the finite element method [1, 2, 4, 5, 6, 7, 8]. Fields that are available at the nodes may be directly mapped by using the corresponding interpolation functions. The situation is more complicated, however, in simulations that carry state information in internal variables, as these are normally available only at integration points.

We propose a variational procedure for the recovery of internal variables, in effect extending them from integration points to the entire domain [3]. The objective is to perform the recovery with minimum error and at the same time guarantee that the internal variables remain in their admissible spaces.

The minimization of the error is achieved by a three-field finite element formulation. The fields in the formulation are the deformation mapping, the *target* or mapped internal variables and a Lagrange multiplier that enforces the equality between the *source* and target internal variables. This formulation leads to an  $L_2$  projection that minimizes the distance between the source and target internal variables as measured in the  $L_2$  norm of the internal variable space.

To ensure that the target internal variables remain in their original space, their interpolation is performed by recourse to Lie groups, which allows for direct polynomial interpolation of the corresponding Lie algebras by means of the logarithmic map. Once the Lie algebras are interpolated, the mapped variables are recovered by the exponential map, thus guaranteeing that they remain in the appropriate space.

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

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**Minisymposium DyF:**

**Dynamic Fracture**

*Organized by Alain Combescure and Joško Ožbolt*





## Modelling Dynamic Fracture of Shells Filled with Fluid

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The paper is devoted to the presentation of simulation of dynamic fracture of shells filled with fluid. The proposed simulations are also carefully compared with experimental results.

The first part of the presentation shall be devoted to the presentation of the damage fracture transition in case of transient loadings. The implementation of these concepts in an SPH context shall then be introduced. The definition of a crack in an SPH structural modeling shall be presented. The extension of SPH method to shell formulation shall then be presented as well as the fracturing SPH shell strategy.

The finite element SPH coupled model for fluid structure interaction shall then be presented and explained. For instance one shall explain how the unilateral contacts are treated.

The method shall then be compared to a series of experiments. These comparison shall show the interest the potentials and the limits of such an approach. For instance one will show that the method is able to explain why the flow shows regularly spaced knots. The method also predicts the case of failure of the shell as well as the directions and length of cracks propagated by the fluid flow. The flow through the cracks is also predicted. On the other hand the long, rm pressure field in the fluid does not compare well with the experiments. This is due to the poor representation of the pressure field in the fluid model chosen into these simulations in case of low speed flows

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## **A Micromechanical Damage Model for Ductile Solids Incorporating Micro-Inertial Effects and Application to Dynamic Crack Extension**

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Dynamic fracture is an important issue in a number of industrial applications. For example, understanding dynamic crack propagation and arrest is mandatory for the evaluation of the integrity of aircraft structures, pipelines and nuclear pressure vessels.

The fracture of ductile materials is often the result of the nucleation, growth and coalescence of microscopic voids. In dynamic fracture, microvoids sustain an extremely rapid expansion which generates strong acceleration of particles in the vicinity of microvoids. Several recent works indicated that accounting for micro-inertia (related to void growth) is of primary importance in the modelling of spall fracture [1-2]. Studies on the role of micro-inertia in dynamic ductile crack growth are very seldom. Glennie [3] proposed a simple analytical model for void growth in the vicinity of a blunted crack tip in which the effect of microscale inertia is taken into account. Even if the effect of damage due to void growth on the stress state in the vicinity of the crack tip is neglected in the modelling, the work of Glennie [3] suggested that the inertial resistance to void expansion limits the speed at which cracks can propagate.

In the present contribution, a constitutive damage model for porous plastic materials taking microscale inertia into account is proposed. Micro-inertia effects have been incorporated using a dynamic version of Hill-Mandel's homogenization approach, proposed by Molinari and Mercier [4], and are a direct consequence of local material accelerations around the growing voids. Material rate dependence and adiabatic heating are also taken into account in the analysis. The accuracy of the proposed constitutive framework was assessed through comparisons with dynamic finite element cell computations [5].

The developed continuum damage model has been implemented in the finite element code ABAQUS / Explicit. The fracture of an axisymmetric notched

bar and of a double edge cracked specimen were investigated. In both cases, the influence of microscale inertia was found to be significant. Because micro-inertia prevents damage to develop too rapidly, a regularizing effect is observed which reduces the mesh sensitivity of the simulations. Micro-inertia was also found to lead to lower crack speed and higher fracture toughness, compared to situation where this contribution is neglected [6].

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## **Pre-Stress Effect on the Ballistic Behaviour of Ceramic Armor: Numerical Approach**

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A new method for simulation of pre-stressed ceramic targets under high velocity impact, using explicit software, AUTODYN<sup>®</sup>, is proposed. This method includes six steps: 1) target components are generated separately and the ceramic size is set primarily larger than confinement size, and damping parameters are set for subsequent static process; 2) an inward velocity is applied at the ceramic periphery, thereby compressing the ceramic to a size smaller than the confinement; 3) all the target components are put together in their correct positions; 4) the velocity on the ceramic periphery is removed, thereby increasing ceramic size until it contacts the inner surface of the confinement and a pre-stress is developed; 5) static damping parameter is gradually reduced to zero for subsequent impact simulation; 6) initial projectile impact velocity is set and impact simulation is performed.

Rigorous choices of damping and viscosity parameters, namely, static damping, hourglass damping and smooth particle hydrodynamics (SPH) linear viscosity, are critical in obtaining stable pressure contour before impact and finally good simulation results. It is shown that the simulation results accord well with available experimental data (radial pre-stress) performed by Lundberg et al. [1]. Comparison of simulation results with experimental data [1], is available in literature, however, either the impact simulation results of the pre-stressed target were not agreed well with experimental data of the pre-stressed target [2] or the simulation results of the targets without pre-stress were compared with experimental data of the pre-stressed target [3].

Simulations with two other pre-stress statuses, namely axial and hydrostatic, with small and large pre-stress values, as well as without pre-stress status are performed for the sake of comparison.

The effect of pre-stress on the ballistic behavior of ceramic targets is captured through simulations. Based on different ceramic/confinement geometric ratios, used to obtain the three pre-stress statuses,

the pressure contour formed in the ceramic is different. Two important concepts are discussed:

1. Interface defeat on the impact surface which is highly affected by the pressure contour at the impact side
2. Rate of penetration through the ceramic armor which is highly affected by the pressure contour within the ceramic armor.

Optimized pressure contour is sought through varying ceramic/confinement geometric ratios for the purpose of obtaining best ballistic behavior in terms of longest interface defeat time and lowest rate of penetration. Successful simulations of pre-stressed ceramic targets can be exploited for further optimization and design purposes of ceramic armor materials.

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## Dimensional Crossover in Impact Fragmentation

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Fragmentation, i.e. the breakup of materials into a large number of pieces due to an energetic load has a high technological importance and presents interesting challenges for basic research, as well [1]. Recently, Kadono et al. investigated the impact fragmentation of a thin glass plate by means of high speed imaging techniques [2]. As the main outcome of the experiments, no correlation was found between the mass and velocity of fragments, while a strong correlation was revealed between the velocity of fragments and their position inside the original body. However, in three dimensional fragmentation experiments of Nakamura et al. [3] a strong mass-velocity correlation was found for large fragments which decays as a power law with a universal exponent  $1/3$  [3].

We study the impact fragmentation of two- and three-dimensional brittle solids with a heterogeneous microstructure. We developed a discrete element model of heterogeneous materials where the disordered microstructure is represented by a packing of spherical particles with a random size distribution. Cohesion is introduced such that a Delaunay tetrahedrization is performed in 3D with the particle positions and we connect the particles by elastic beams along the edges of tetraheders. Crack formation is captured by the breaking of beams which can be caused by stretching and bending. Particles not connected by beam elements interact via Hertz contacts. The time evolution of the system is followed by molecular dynamics simulations solving the equation of motion of particles for the translational and rotational degrees of freedom.

Computer simulations were performed to understand the breakup process caused by an energetic impact varying the shape of the sample from quasi two-dimensional plate-like to three dimensional. Our calculations revealed that depending on the energy of impact the breakup process can have two different outcomes: at low impact energy the sample gets damaged, however, to achieve fragmentation the imparted energy has to surpass a critical

value. Based on large scale computer simulations we show that the position of fragments inside the original body with respect to the impact site determines their mass and velocity in the final state. A novel relation of the mass and velocity of fragments is revealed: In the damage phase fragment mass and velocity are strongly correlated. In the fragmented regime for small fragments the velocity proved to be independent of the mass, however, in the limit of large fragments a strong mass-velocity correlation occurs. The correlation function decays as a power law with a universal exponent in an excellent agreement with recent experimental findings.

Simulations revealed that the mass distribution of fragments has a power law distribution with an exponent which increases with increasing impact energy. We demonstrate that the apparent increase of the exponent is due to the competition of two fragmentation mechanisms which both have universal mass distributions. The mixing ratio depends on the impact energy and on the geometry of the sample.

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## Modelling Concrete under High Loading Rates and Impact

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The behavior of concrete structures is strongly influenced by the loading rate [1]. Compared to quasi-static loading concrete loaded by impact loading acts in a different way. First, there is a strain-rate influence on strength, stiffness, and ductility, and, second, there are inertia forces activated which influence the resistance and failure mode of concrete structure. The results of the experimental measurements show that after reaching some critical strain rate concrete resistance progressively increases with increase of strain rate. The reason for the phenomena is still under discussion. It is not clear whether this is due to the material strength or there some other reasons.

Recently, an explicit finite element code has been developed for the simulation of high velocity impact and fragmentation events. In the code the rate sensitive microplane material model for concrete [2] is used. The model is extended to account for large deformations. Failed elements on the way are removed with the technique based on element deletion. Large deformation frictional contact has been treated by forward incremental Lagrange multiplier method. In order to investigate the role of rate sensitive constitutive law and inertia forces, three different problems are studied using the mentioned finite element code: (i) Drop hammer test on notched three-point bending beam with impact velocity up to approximately 2 m/sec, (ii) The Hopkinson bar test (tension) with loading rates up to 300 s<sup>-1</sup> and (iii) High velocity mushroom impact experiment with impact velocity varied from 100 to 2000 m/sec. The results of the numerical simulations are evaluated and compared with available experimental results from the literature.

Based on the evaluation of numerical and experimental results (macro scale) it is shown that the true material strength is approximately a linear function of strain rate (log scale) and that it can be modeled by the rate sensitive constitutive law. Moreover, it is demonstrated that the progressive

increase of resistance (apparent strength) can be attributed to inertia of the softening zone and not to the material strength. The size of the fracture process zone and the size of the specimen significantly influence apparent strength (inertia contribution). Brittle materials with very small fracture process zone (e.g. glass) exhibit almost no rate sensitivity, i.e. apparent and true strengths are almost the same and there is no progressive increase of resistance with increasing strain rate. Furthermore, it is shown that the concrete fracture energy, similar to the true strength, is a linear function of strain rate (log scale). For relatively low and medium loading rates the structural response is controlled by rate sensitivity, i.e. apparent and true strength are almost the same. However, at high and very high loading rates inertia forces dominate although the influence of the rate sensitivity cannot be neglected.

With increasing loading rate failure mode changes, i.e. there is a transition from mode I to localized shear failure mode. Moreover, with increase of loading rate crack branching phenomena takes place. For these phenomena are responsible inertia forces [3]. The effects as well as the progressive increase of the structural resistance should come automatically from the dynamic finite element analysis and it should not be a part of the rate sensitive constitutive law.

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# A Novel Strain Rate Model for Concrete and its Influence upon Crack Energy

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The strain rate effect, i.e. strength increase of materials under high strain rates, is especially pronounced for the tensile strength of concrete. It is generally characterized by a relation for the dynamic strength increase factor (DIF), i.e. uniaxial strength related to a quasistatic reference value, depending on the strain rate. This relation may be approximated by a bilinear course in a double-logarithmic scale. The first branch up to a strain rate value of  $\approx 1 \text{ s}^{-1}$  is relatively flat up to a DIF-value of 2 for the uniaxial tensile strength. A second branch connects for higher strain rates which is much steeper. DIF-values of roughly 10 are experimentally determined for strain rates of  $\approx 10^2 \text{ s}^{-1}$  [1], although a larger scatter is documented due to the high demand on the experimental setup. Two different physical mechanisms may be considered for the two branches according to the current state of knowledge:

1. More or less physically bound water is moved within the different capillary systems of the mortar's micro structure during deformations. The resistance of this relative movement against actions increases for more rapid deformations. This phenomenon is connected to the flat branch of the double-logarithmic DIF-relation.
2. The first phenomenon also occurs for higher strain rates but is strongly superposed by retarded damage effects. Damage results from micro crack formation. Formation of cracks cannot occur arbitrarily fast. This is easily demonstrated on the macroscopic scale but presumably also valid for microscopic scales. Thus, damage is retarded in case of high speed actions compared to quasistatic actions.

These phenomena are described with a macroscopic constitutive law. This bases upon an strain based isotropic damage law [2], whereby damage itself is not only made dependent upon the actual strain

state, but also on damage acceleration and velocity. This approach leads to a retardation of damage and covers the second phenomenon. To cover the first phenomenon damaged elasticity is embedded in a viscoelastic frame of a Maxwell type [3]. The whole approach is fully triaxial. It uses two material parameters for damage retardation and two further parameters for viscosity in addition to those material parameters for the quasistatic base. Dynamic strength increase factors are treated as a special case. The four material parameters for the strain rate effect may be chosen such that experimentally determined uniaxial DIF-relations are reproduced.

This constitutive law is implemented into nonlinear Finite-Element-Methods. The paper discusses its application to the numerical simulation of spallation experiments [4]. Such experiments follow the Hopkinson bar setup whereby the cylindrical concrete specimen spalls into discrete pieces flying away. This setup allows for the estimation of the dynamic crack energy. Experimental results are discussed regarding corresponding numerical calculations. The latter allow for a detailed simulation of stresses, strains and energies. This is utilized with special regard of the concrete's crack energy.

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## Comminution in Concrete at Extremely High Strain Rates

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Deviations from quasistatic strain rates affect the concrete behavior in two main ways: (1) Increased strain rates increase the strength and brittleness while keeping the fracture energy mostly constant, (2) reduced strain rates cause creep of the intact material [1]. Although both of these phenomena must be addressed in the constitutive modeling of concrete to be able to take into account arbitrary strain rate effects, in this study we restrict our attention to the increased strain rates. A number of models for rate sensitive constitutive behavior of concrete has already been developed (e.g. [1]). Such models can routinely predict the increased strength and brittleness of concrete at higher strain rates (between  $1 \cdot 10^{-3}$  to  $0.2 \text{ s}^{-1}$ ) than quasistatic strain rates (on the order of  $1 \cdot 10^{-4} \text{ s}^{-1}$ ).

For higher strain rates (on the order of  $100 \text{ s}^{-1}$ ), finite element analyses show that the inertial forces reduce the stress intensity at the crack tips of large cracks and cause new cracks to nucleate and propagate. Consequently, enormous energy is dissipated by cracks distributed all over the specimen. This leads to very large apparent strength values at high strain rates. However, this is clearly not a constitutive behavior. At such rates, crack growth is still governed by activation energy theory [1].

Even higher strain rates (on the order of  $1 \cdot 10^4 \text{ s}^{-1}$ ) take place in the penetration of concrete walls by projectiles. Various microplane models have been used in such simulations [1, 2]. Microplane model M4 was successfully employed in the prediction of crater shapes in contrast to various plasticity based models at WES (U.S. Army Waterways Experiment Station) [1]. However, the prediction of exit velocities of the projectiles proved to be more challenging. Only recently a new microplane model based on the model M4 has been developed and calibrated to fit the exit velocities of these projectiles by scaling the microplane strain-dependent yield limits along both the strain and stress axes drastically for

such high strain rates [2]. However, this means that the fracture energy also increased, and even more drastically, which is contrary to many publications in the literature which show fracture energy to be mostly independent of strain rate.

In this study we propose that the dissipation by comminution of concrete must be the dominant dissipative mechanism at strain rates on the order of  $1 \cdot 10^4 \text{ s}^{-1}$ . A new rate sensitive constitutive model based on the new microplane model M7 [3], called M7R, is presented. The model M7R can simulate concrete behavior not only at high strain rates (up to  $1 \cdot 10^3 \text{ s}^{-1}$ ) but also at very high strain rates (on the order of  $1 \cdot 10^4 \text{ s}^{-1}$ ). To this end, the model M7R takes into account the comminution of concrete by the penetrating projectile using a strain rate dependent material viscosity. The model M7R has been used in the fitting and prediction of exit velocities of the projectiles as well as the simulation of the formation of the entry and exit craters in the perforation of concrete walls by projectiles.

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## Dynamic Comminution of Quasibrittle Solids at High-Rate Shear under Impact and Analogy with Turbulence

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In spite of the recent advances in the constitutive modeling of concrete, the finite element models severely overestimate the depth of penetration of projectiles into the walls of hardened structures and, in the case of perforation, the exit velocity [1, 2]. Inclusion of viscoelastic rate effect and the effect of crack growth rate does not suffice by far for obtaining correct predictions. The predictions are way off the mark even when the finite element code uses a constitutive model such as the new microplane model M7 [3], which provides very good fits of virtually the complete range of the experimental data from diverse types of uniaxial, biaxial and triaxial tests and, in particular, can give static energy dissipation due to shear under very high confinement. As it appears, the main problem is that the constitutive model is not capturing the effects of material comminution into very fine particles. Such comminution at very high strain rates can dissipate a large portion of the kinetic energy of the missile. The spatial derivative of the density of energy dissipated by comminution gives a compressive stress resisting penetration, and needs to be added to the nodal forces obtained from the static constitutive model in a finite element programs.

The authors present a new constitutive model for dynamic comminution inspired by analogy with turbulence. In high velocity turbulent flow, the energy dissipation rate is maximized by the formation of micro-vortices (eddies) which dissipate energy by viscous shear stress. Similarly, it is assumed that the energy dissipation at fast deformation of a confined solid gets maximized by release of kinetic energy of high shear strain rate of forming particles, whose shape in the plane of maximum shear rate is considered to be regular hexagons. The free energy density consisting of the sum of this energy and the fracture energy of the interface between the forming particle is minimized. This yields a relation between the

particle size, the shear strain rate, the fracture energy and the mass density.

It is concluded that the particle size is inversely proportional to the  $2/3$ -power of the shear strain rate and that the dynamic comminution creates an apparent material viscosity varying as the  $(-1/3)$ -power of the shear strain rate. Introduction of this viscosity to a finite element program based on the microplane model M7 leads to a good match of missile penetration into massive concrete walls, and in the case of penetration, to a good match of the exit velocities. At the same time, the use of the microplane model is essential for capturing the nonlinear triaxial effects in progressive degradation of concrete. A brief explanation of the latest version of this model, called M7, along with the modifications compared to the previous microplane models, is given. The presentation concludes with various comparisons with test results.

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## An Analysis of the Unstable Propagation in a Heterogeneous Peeling Test

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This paper is a contribution to the study of dynamic fracture in the framework of Griffith's theory [1, 3]. We analyze the propagation of a crack in a heterogeneous material (with variable toughness), focusing on the role of the kinetic energy. In particular, we wonder whether a (possibly modified) quasistatic model is a good approximation of the phenomenon, avoiding delicate and time-consuming dynamic computations.

Indeed, even under quasistatic loading, the material's answer is a priori dynamic when the crack reaches a zone where the toughness suddenly decreases [2]. In this case, since the energy release rate becomes greater than the toughness, the dynamic evolution presents a fast propagation, whose speed has the order of the speed of sound.

We consider here the simplified model of the debonding of a one-dimensional inextensible film, subjected to a monotonic loading with vanishing speed, under the hypothesis that the toughness is piecewise constant and takes only two possible values. Three energies interact: the potential energy, the surface energy (associated to debonding) and the kinetic energy. All the difficulty is to find how and when the potential is transformed into surface energy, and which is the role of the kinetic energy during that process.

Three cases are studied. We first focus on the case of toughness with one decreasing discontinuity. In this case we show that the dynamic solutions converge, as the loading speed tends to zero, towards the quasistatic evolution, upon assuming a criterion of energy conservation to govern the jumps. It is also possible to determine the length of the crack's jump.

We then consider the case of a defect (two discontinuities) of the toughness. If the size of the defect does not exceed a threshold, the dynamic solutions

do not converge to the quasistatic one, which overestimate the jump's length. Nevertheless, a good approximation can be obtained by a quasistatic model if the toughness is suitably modified: we give the expression of the alternative toughness.

Finally, we consider the case of many defects whose distribution could be periodic or random. In the quasistatic model (under the assumption of energy conservation), the solutions are staircase functions in the space/time plane, corresponding to phases of arrest and of jumps (fast propagations). In particular, fixed a level of load, the length of debonding can be higher than the one corresponding to a material with a constant low toughness. The dynamic solutions depends strongly on the density of the lowest toughness. As the density of the lowest toughness decreases from one to 1/2, the dynamic solutions change from a staircase function (consequently equivalent to the quasistatic solution) to a line whose slope is however different from the one corresponding to the mean toughness.

This behaviour reflects the influence of the kinetic energy on the dynamics. Indeed, we see here the competition of two opposite phenomena: the creation of kinetic energy as the toughness decreases and the relaxation observed in the case of a single defect.

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## Dynamic Response of Concrete – LS-Dyna Concrete Material Models Review

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Concrete has been widely used in the construction of security-sensitive structures, such as bridges, tunnels, government buildings, industrial and military facilities. These structures should be designed against extreme dynamical loads, as close-in explosions and impact. Therefore, understanding the concrete material behaviour under these loading conditions is crucial.

Concrete is a brittle material and its resistance and damage (softening) behaviour are strongly influenced by the loading rate [1,2]. The physical mechanisms which describe the concrete dynamic response are qualitatively reasonably well understood, but an important question is how to capture the rate dependency in numerical models. The dynamic response should therefore be divided into (i) material and (ii) structural response.

The integrated response of concrete up to complete failure is represented in the macroscopic load-deformation relation. The observed response of concrete at macro level is determined by damage initiation and damage accumulation mechanisms at meso and micro scale level. The failure process is governed by (i) the stress condition, (ii) the mechanisms governing micro-crack nucleation, propagation and obscuration of critical flaws, (iii) the ability to absorb energy in the fracture zone and (iv) the energy flow from the surrounding material into the fracture zone. In dynamics all four conditions vary in time and depend on the loading rate. The challenge is how to separate material response given in the constitutive model and the structural response captured in the numerical model in detailed and global computational analyses.

Recent studies (e.g. [3]) show that in detailed analyses with discrete and microplane models, the micro and meso-structure response during the failure process can be captured. So, the structural inertia forces, which are activated in the undamaged part of the material, should be captured by the dynamic analysis and should be excluded from the applied constitutive model.

The questions we address in this paper are (i) how the dynamic material and structural response are covered in advanced commercial codes like LS-Dyna and (ii) the validity of the dynamic analysis.

Three different dynamic concrete models are evaluated in impulsive, uniaxial compression and tension tests. The results are compared to experimental data. Special attention is given to the damage development, width of the failure zone and spatial and temporal energy dissipation.

At last, the K&C concrete model is studied in particular considering different Dynamic Increase Factor (DIF) and Equations Of State (EOS) formulations (suggested by [5,6]) in order to evaluate their effect on the results.

The study provides a survey of the dynamic response mechanisms of concrete, how these should be covered in numerical modelling and to what extent these can be captured in advanced commercial codes.

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## Finite Element Modeling of the Drop Weight Impact Test of PMMA Specimens

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The drop weight impact test consists in a falling weight that impacts the specimen. This impact striker, also known as a tup, falls through a vertical guide tube that directs it to the center of a flat circular specimen (see ASTM D3763 for details). A main advantage of using drop weight impact test over other standard tests, like Charpy and Izod tests, is its ability to reproduce conditions under which real life component would be subject to impact loading. In this way, it is possible to induce different failure modes on the specimen by changing the test parameters. In particular, experimental tests performed as part of this work have shown that the failure mode of polymethyl metacrylate (PMMA) specimens changes from punching to flexural with the increment of the tup diameter,  $d$ , with respect to the specimen diameter,  $D$ .

It is presented in this work a finite element modeling of the drop weight in order to better understand the change in the specimen failure mode with the tip diameter.

The finite element analyses were conducted using three-dimensional models of the specimens, which were discretized using linear tetrahedral elements (C3D4) in combination with cohesive elements (COH3D6) to account for material damage and crack propagation. Cohesive models were placed everywhere in the model in order to allow the cracking spontaneous onset. The bilinear traction versus displacement law with scalar damage proposed by Camacho and Ortiz [1] was adopted for the cohesive elements. Following Pandolfi and Guduru [2], mixed-mode fracture was introduced into the fracture criterion by means of the coefficient  $\beta = K_{IIc}/K_{Ic} = K_{IIIc}/K_{Ic}$ . Models were solved using the software Abaqus/Explicit [3].

Elastic and fracture material properties were measured by means of tensile and impact fracture tests. Tensile tests were conducted on PMMA samples according to ASTM D638, while the dynamic fracture tests were performed according to ASTM D5045. Moreover, finite element models

were carried out for both, the tensile and the impact fracture tests. These models served to devise strategies to compensate the effects of the spurious addition of mass and reduction in stiffness that result from the extensive inclusion of cohesive elements into the models. These compensations showed to be a key factor when dealing with dynamic crack propagation models.

The drop weight impact test of the PMMA circular specimens considered the four  $d/D$  ratios that result from using tup diameters  $d=12.7$  mm and 20 mm, and specimen diameters  $D=40$  mm and 76 mm. A number of finite element models were conducted for each case in order to study the variation of the results with the discretization and the randomness on the results. Very good agreement was achieved between the numerical predictions and the experimental results in terms of the force versus time plots and crack patterns. The models are able to predict the prevalence of punching failure for the lowest  $d/D$  values and a combination between punching and flexural failures as the  $d/D$  increases. Setting the adequate value of  $\beta$  of the PMMA was found to be the key point to achieve accurate predictions of the experimental results.

The finite element results allowed monitoring the sequence in the nucleation and propagation of the radial and circumferential cracks during the tests. These phenomena were correlated to the load peaks and other characteristic features of the force versus time plots.

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## Fracture in Reinforced Concrete Bridge Columns under Seismic Loading

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The energy dissipation of seismic response in reinforced concrete columns is very much related to the damage occurring in the columns' plastic hinges. Accurate damage cannot be predicted by typical damage indices. This is due to the fact that the damage intensity cannot be generalized for a structure under seismic loads, since they are dynamic and could have different loading rates, which produce very different damage states. Many researchers in Earthquake Engineering field assume that the peak ground accelerations PGA alone is not a sufficient intensity measure to determine the ultimate response [1], since different loading rates would result different responses, even with records of similar PGA.

The seismic design criterion SDC that is used by many building codes employs the demand/capacity ductility principle for a performance-based design, which assumes a low damage state in the member, allowing for a better serviceability of the structure after the seismic event. The damage in the plastic hinge, according to SDC is assumed to be limited to spalling of the column cover and yielding of the longitudinal reinforcement bars. However, the crack growth in the column core can lead to excessive strength degradation in the hysteresis behavior of the structure, which could lead to a total collapse.

In this paper, two approaches have been followed to predict the damage in a R/C bridge column under seismic loading. The first is a stress-based damage method, which is based on computing the concrete stress-strain constitutive elasto-plastic relation for selected points on the cover and core of the column section. The nonlinear dynamic analysis of this approach is performed by using the Fibre Element Method, for the column when subjected to a strong ground motion record. As a fibre model, the damage is considered due to flexural failure, and is measured in relation to the established stress-strain model for each fibre in the critical section.

The second approach is a small-scale analysis, by using the combined DE/FE Discrete

Element/Finite Element method, with Mohr-Coulomb and Rankine cut-off tension failure criterion, supplemented with a fracture model, which is developed by the Elfen-Explicit application. In this analysis the crack growth is determined in the zones of softening strains at the column's core and cover.

In this RC column, designed according to the SDC of the California Department of Transportations (Caltrans 2001), a large area of the column core zone at the plastic hinge is damaged when subjected to Lexington Dam, Loma Prieta, 1989 [2], and the column loses its integrity in the plastic zone, which affects the bar/concrete bond substantially. This could lead to total collapse especially if the reinforcement bars are quite exposed and start to buckle.

The problem was also analyzed by using Rankine failure criterion supplemented with fracture, which is more conservative when compared to the previous model since it is accounted for tension failure mode only. Despite its excessive computation time, the combined DE/FE model provides reliable information about the local damage state of the RC column core, which enhances the understanding of the seismic performance of the structural member.

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## Experimental Observation of Dynamic Ductile Damage Development under Various Triaxiality Conditions

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Fracture in ductile materials finds its origin in microscopic mechanisms: the nucleation of voids that growth and coalesce in order to form a crack. A large part of literature deals with the development of models predicting each stage of this complex phenomenon. The most popular of these models, proposed by Gurson [1], aims at describing the damage development with respect to the plastic behavior of a porous ductile material. The Gurson model has been extended by Perrin to describe damage evolution in ductile viscoplastic porous materials [2]. The so-called Gurson-Perrin model (GPM) allows representing damage development with respect to the stress triaxiality and strain-rate conditions.

In the GPM, free parameters have to be identified from suitable experiments. To characterize correctly the GPM, an experimental program must:

- Generate high strain-rates (more than  $10^5\text{s}^{-1}$ ) in order to achieve a dynamic evolution of damage.
- Allow a change on the triaxiality level.

Nevertheless, classical experimental procedures (tensile tests on Hopkinson bars and plate impact experiments) cannot satisfy both conditions. Tensile tests on Hopkinson bars indeed allow a variation of damage evolution with respect to the triaxiality level but the strain-rate is not high enough (less than  $10^3\text{s}^{-1}$ ). Reversely, in plate impact experiments, a damage evolution occurs under dynamic conditions but the influence of damage development with respect to the triaxiality level cannot be characterized.

We propose a new experimental design able to test and validate the GPM under various dynamic conditions and for different triaxiality levels. This new experimental concept has a hybrid configuration, which combines the two classical experimental principles: plate impact experiments and tensile test on notched bars. The principle of the experimental project will be detailed. This experiment is realized on a powder gun. A Cu plate impactor strikes a Cu target, which is then

dynamically set in motion. This target is a notched cylinder with a full back. A tensile state is generated in the notched part of the cylinder where damage develops. The experiment ends with the local failure of the notched part. This process allows a dynamic damage evolution. Moreover, a variation of the notch radius enables a variation in the triaxiality level. Three notch radii have been tested. Two observations with numerical cameras allow following the global behavior of the experiment and the shape of the notch, which is characteristic of the local damage development. The first images will be exposed. Several PDV measurements have been performed around the target in order to quantify its deformation. A first analysis of this experimental process will be shown.

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## Fractional Viscoplasticity Accounting for Isotropic Damage

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In the paper recent developments of *fractional viscoplasticity* are considered. Fractional viscoplasticity is a generalisation of Perzyna's type viscoplasticity (so simultaneously classical plasticity) using differential operators of arbitrary order. This concept was first proposed in Author's original papers [8, 7].

It should be emphasised that the introduction of the fractional calculus in viscoplasticity leads in fact to definition of new family of constitutive models. As shown in [7] such concept results in reduction of material parameters or even material functions in a particular model, what is of crucial importance in phenomenology [5].

One should notice that since 1695 when the fractional calculus was 'born' it finds many successful applications in e.g. Fluid Flow, Rheology, Dynamical Processes in Self-Similar and Porous Structures, Diffusive Transport Akin to Diffusion, Electrical Networks, Probability and Statistics, Control Theory of Dynamical Systems, Viscoelasticity, Electrochemistry of Corrosion, Chemical Physics, Optics and others ([1, 6, 3, 2] and cited therein).

In papers [8, 7] the general definition of fractional viscoplasticity was presented without introduction of damage. Because finally this new concept will be applied for modelling of extreme dynamics, under large deformations and temperatures reaching melting point it is necessary to include damage (finally anisotropic damage [4]).

The formulation presented assumes model description in Euclidean space under small deformation assumptions. This first step is necessary for further generalisation of the model to manifolds - necessary for proper description of finite deformations. It should be pointed out that fractional viscoplasticity introduces in natural way non-locality due to definition of differential operator of arbitrary order on an interval (we utilise Caputo's type operator in the formulation) - in contrast to classical one defined in a point.

Illustrative examples will be presented.

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## Elastodynamic Antiplane Analysis of Cracked Graded Piezoelectric Layer via DDT

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Piezoelectric materials have been applied for some advanced applications which necessitate the analysis of their behavior in different situations. Due to the material behavior of piezoelectric materials, they are usually susceptible to crack. Therefore these materials have been the subject of many researches in the field of fracture mechanics. Different loading types can be recognized in various applications of piezoelectric materials including static, time-harmonic and transient [1], [2]. Additionally, loading may be mechanical, electrical and thermal. The material behavior for piezoelectric materials may be homogeneous or nonhomogeneous. The most common assumption for material gradation is exponentially dependence to the location [3]. A general case is investigated by Mousavi et al. for a graded layer under static loading [4].

The crack boundary conditions may be assumed partially permeable or any of permeable or impermeable. The experimental results depicts that the electric loading has dominant effect on the fracture behavior of the piezoelectric materials which is coincident with the impermeable condition [5].

On the other hand, the type and arrangement of the defects may vary and change the geometry of the problem. Although many useful published articles are devoted to specific crack geometries in the layer, but the capability to characterize the behavior of the cracked domain in the presence of multi cracks under dynamic loading is still a challenge.

Distributed Dislocation Technique (DDT) provides the capability to tackle various configurations of cracks. The necessary step is to analyze the domain in the presence of a dislocation. Then utilizing the dislocation solution, superposition effect and the Buckner's principal, it is possible to form any configuration of the cracks and achieve the domain behavior and field intensity factors. DDT has been utilized for static and recently for elastodynamic

analysis of cracked domain under mechanical loading [6].

In the present article, the dislocation solution is achieved for a layer under antiplane elastodynamic condition. The electrically impermeable crack face condition is used as the dislocation condition. Then the distributed dislocation technique is utilized for the antiplane time-harmonic analysis of a functionally graded piezoelectric layer and the stress and electric displacement intensity factors are determined for the cracked layers. The configuration and arrangement of cracks in the layer are arbitrary.

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**Minisymposium EFE:**

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## Three-Dimensional Crack Propagation in Ductile Media Using the XFEM

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This work presents a numerical method to simulate crack propagation in plastic media in three-dimensional space using a damage model to examine crack propagation as well as the direction of growth. Most damage models naturally only induce material softening but no discrete separation of material yielding extensive straining in case of complete failure. Therefore, a combination of damage and discrete failure is appropriate to model ductile failure, presented in this work.

A widely spread method to model damage is the application of non-local or gradient enhanced damage models introduced by [1]. In contrast to these models, local models suffer from mesh sensitivity and are therefore not applied here. The basic idea of non-local or gradient enhanced models is to take into account the microstructure in average. This averaging can be reformulated into a HELMHOLTZ-type equation leading to additional nodal unknowns, but also to the desired mesh insensitivity. Therefore, non-local procedures are methods of our choice.

Combining non-local damage models with discrete fracture in a standard FE approach under the assumption of elasto-plastic material behavior was introduced by [4] for two-dimensional problems: A new crack segment is inserted once a certain damage threshold value is exceeded. This naturally leads to computationally expensive remeshing for each propagation step.

To avoid remeshing for propagating cracks, the extended finite element method (XFEM) introduced by [2] offers great accuracy and flexibility. Incorporating the level set method to describe crack surfaces as well as enrichment functions depending on level set fields to describe the mechanical behavior of cracks yields the today's XFEM. Thus, advancing crack fronts do not change the FE mesh, but only require a local update of the crack surface, presented

in this work. Compared to standard FE approaches, computationally expensive remeshing strategies as well as size differences of finite elements in the vicinity of the crack front and the rest of the domain are avoided.

As the XFEM traditionally was designed for linear elastic fracture mechanics (LEFM) and accordingly for singular stress fields at the crack, elasto-plastic problems require special attention. Stresses are bounded due to the introduction of a yield surface leading to different enrichment functions at the crack front introduced by [3] compared to the XFEM applied to LEFM problems.

The goal of this work is to combine elasto-plasticity, non-local damage mechanics and discrete failure in single computable model using the XFEM. Here, the focus is set to present a robust computational framework for this type of problems. This work offers promising results in terms of modeling ductile failure computationally efficient, without loss of computational stability.

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## 3D XFEM Modeling of Composite Failure Combining Discrete and Diffuse Damage

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For a consistent lightweight design the consideration of the nonlinear macroscopic material behavior of composites, which is amongst others driven by damage effects and the strain-rate dependent material behavior of typical polymeric matrices, is required. To this end, the authors developed a numerical modeling approach that combines the extended finite element method (XFEM) with suitable constitutive relations for the individual constituents.

The XFEM is a well recognized technique for the modeling of heterogeneous material structures. It allows for the modeling of both, weak and strong, discontinuities independent of the underlying FE mesh. In combination with a cohesive zone law it can be utilized to model discrete damage phenomena. In the present contribution the method is applied to model the interface failure between fibre and matrix material on the microscale of composites. Further failure mechanisms, such as the degradation of the matrix material, are incorporated by a continuum damage approach. In addition to these approaches, which account for the material structure and its failure mechanisms, viscoelastic and viscoplastic material models have been developed for the inelastic behavior of the polymeric matrix [3]. The structure of the overall approach allows for the description of the effective material behavior on both the micro and the meso scale.

The modeling of composites often requires the description of complex material structures. To this end, the XFEM is commonly linked to a level-set function, which is utilized to locate the discontinuity within the model domain. Since a closed analytical expression of such function is only available for special cases, e. g. for a cylinder or a sphere, the description of complex geometries can become challenging. Therefore, the authors developed a so called local level-set representation, where the discontinuity is localized elementwise [1]. The nodal level-set values are calculated corresponding to an elementwise parametric function which is again defined by a set of discrete points of the actual

interface. Such points can be easily obtained from a CT scan or a micrograph. The overall method has been extended to model curved, three dimensional structures. The usage of higher order shape functions reduces the discretization error and interelement discontinuities [2]. However, curved discontinuities require consistent integration techniques. To this end, different integration techniques have been examined regarding their error and convergence rate.

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## Modeling Crack Propagation in Heterogeneous Materials Using a Computational Homogenization Method

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The modeling of crack propagation in heterogeneous materials using a computational homogenization method is presented.

In the computational homogenization method, the macroscopic properties of each material point are obtained by solving a boundary value problem for a representative volume element (RVE). When localization occurs in the material, the assumption of homogeneity of stress and strain over the local-scale breaks down and one cannot use standard homogenization scheme.

An objective continuous-discontinuous homogenization scheme [1] is developed based on a failure zone averaging method [2] which can be used to homogenize the traction-separation law for a macrocrack from the heterogeneous local-scale. The macrocrack is modeled as a strong discontinuity using XFEM and a gradient-enhanced damage model is used to model diffuse damage at the local-scale.

The continuous-discontinuous scheme is extended to dynamic problems in which the macroscopic wave length is significantly larger than the local-scale length. Under this condition, the local-scale problem can be solved as a quasi-static problem.

In the continuous-discontinuous scheme, the loss of hyperbolicity criterion [3] is employed for crack initiation and propagation. Based on this criterion localization initiates when the momentum equation loses hyperbolicity and the vector that minimizes the hyperbolicity indicator is normal to the direction of the crack (localization). In the multi-scale analysis, this criterion can be used to detect localization in the RVE. At each time step, the acoustic tensor can be calculated from the homogenized tangent modulus. Initiation and direction of the crack can then be determined using the hyperbolicity criterion. The advantage of this criterion is that both initiation and direction of the crack can be obtained

from a local-scale analysis.

Furthermore, rate effects are taken into account in the model by relating the material properties of the RVE to the rate of the macroscopic crack opening. Using the present model, numerical examples are presented for crack propagation in heterogeneous media and the influence of the loading rate on cracking is studied.

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## Reinforced Concrete Modelling Using Enriched Finite Elements

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Reinforced concrete has become one of the most employed materials in civil engineering works. Its economy, efficiency, and stiffness, make this material an excellent choice for the construction of a wide variety of structures. Unfortunately, due to the complexity of its damage evolution to collapse, its mechanical behaviour is not fully understood. One of the first attempts to simulate this behaviour is presented in the work by Ngo and Scordelis, [1]. Currently, different researchers continue studying it through experimental tests and numerical simulations. The nonlinear behavior of this composite material, under different conditions, may lead to inelastic response and, eventually, take the structure to the collapse, *e.g.*, [2].

In this work, the variational formulation of the embedded discontinuity model, developed by Retama in his PhD thesis [3], and its approximation by means of the finite element method, to simulate the damage process in structural elements of reinforced concrete is presented. The effect of the reinforcement steel is introduced through truss elements considered to be embedded in the concrete finite element. In the modelling of the concrete behaviour, two dimensional finite element are used; together with a cohesive damage model and the embedded discontinuity approximation, [3]. In this work a perfect bond for the interface concrete-steel bars is considered.

To validate the correctness of this formulation, it is implemented in the finite element program FEAP, [4], and numerical examples are presented; and the results derived are compared with those of existing experimental tests.

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## Simulation of Steel Fibre Reinforced Concrete Behaviour Using Discrete Crack Approach

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The recent wide spread use of steel Fibre Reinforced Concrete (FRC) is mainly due to its numerous advantages compared to Normal Concrete (NC), namely by the possibility of changing the quasi-brittle behaviour of plain concrete structures to a behaviour of enhanced ductility, as a direct result of the addition of steel fibres. This work aims at developing a finite element formulation to specifically address the simulation of the behaviour of FRC members up to failure.

For this purpose, an embedded discrete strong discontinuity approach [1, 2] is adopted and steel fibres are explicitly introduced in the finite element mesh. In other words, the steel fibres are considered as discrete elements, embedded in concrete matrix, which increase the stiffness of the bulk finite elements. In this approach, new nodes (tips of fibre-elements) are introduced in the finite element mesh without increasing the global number of degrees of freedom. This is due to the fact that displacements at the new nodes are obtained from the displacements measured at the usual degrees of freedom. In this work the fibres are assumed to be perfectly bonded to the bulk.

This kind of approach provides a simple and efficient way to assess the influence of the steel fibres in the development of the crack pattern and the structural behaviour of the concrete elements. The proposed approach is validated using both numerical and experimental results. Finally, results are discussed and some conclusions are drawn.

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## Subscale Enrichment of Discontinuity for XFEM Crack Tip Element

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In the present contribution, we propose a new XFEM based enrichment of the displacement field to allow for cracks that end or kink within an element. The basic concept relies on the fact that the crack tip element is treated on a subscale, where, in addition to the macroscopic continuous ( $\varphi_c$ ) and discontinuous ( $\varphi_d$ ) displacement fields, a discontinuous fluctuation field  $\varphi_d^f$  with Dirichlet boundary conditions is introduced to allow for proper representation of the discontinuous kinematics. Hence, the finite element approximation of the total deformation map is obtained as:

$$\varphi^h = \underbrace{\sum_{i \in I} N^i \varphi_c^i + \sum_{j \in J} \tilde{H}_S \psi^j \varphi_d^j}_{\varphi_M^h} + \underbrace{\sum_{k \in K} \tilde{H}_S \psi^k \varphi_d^{f,k}}_{\varphi_f^h} \quad (1)$$

where  $\tilde{H}_S$  is the shifted discontinuity function and where, referring to Figure 1,  $I$  is the set of edge (master) nodes of the crack tip element,  $J \subset I$  is the set of edge nodes enriched with discontinuous degrees of freedom (black squares) and  $K$  is the set of internal nodes enriched with fluctuation (discontinuous) degrees of freedom (white circles). In fact, in this non-standard interpolation of the discontinuity field, additional internal nodes are utilised for the interpolation. In Eq. (2), we therefore introduced  $N^i$  and  $\psi^i$  to distinguish between shape functions that have support over the entire crack tip element and only partially over a subset of the subscale elements respectively.

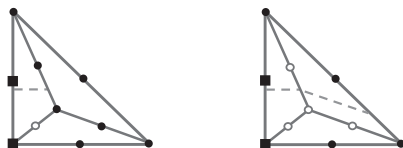


Figure 1: 6-noded shell element with quadratic interpolation for the discontinuity field  $\varphi_d$  enhanced with internal degrees of freedom for  $\varphi_d^f$ .

Adopting an explicit time integration scheme and utilising that the fluctuation field vanishes along the

crack tip element edges, the fluctuation field can be implicitly solved for in terms of the macroscopic field. By considering the discretised form of the momentum balance for the subscale problem

$$\begin{bmatrix} \mathbf{M}_{MM} & \mathbf{M}_{Mf} \\ \mathbf{M}_{fM} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}}_M^h \\ \hat{\mathbf{a}}_f^h \end{bmatrix} = \begin{bmatrix} \mathbf{f}_M^{ext} - \mathbf{f}_M^{int} \\ -\mathbf{f}_f^{int} \end{bmatrix} \quad (2)$$

it is clear that the discretised macroscopic form of the momentum balance for the crack tip element reduces to

$$\mathbf{M}_M \hat{\mathbf{a}}_M^h = \mathbf{f}_M^{ext} - \bar{\mathbf{f}}_M^{int} \quad (3)$$

with

$$\mathbf{M}_M = \mathbf{M}_{MM} - \mathbf{M}_{Mf} \mathbf{M}_{ff}^{-1} \mathbf{M}_{fM} \quad (4)$$

$$\bar{\mathbf{f}}_M^{int} = \mathbf{f}_M^{int} - \mathbf{M}_{Mf} \mathbf{M}_{ff}^{-1} \mathbf{f}_f^{int} \quad (5)$$

The current methodology is applied to ductile crack propagation in shells loaded at high strain rates, extending previous developments in [1] to allow for not only crack segments through the entire shell elements (edge to edge).

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## Mixed Finite Elements with Prescribed Primary and Secondary Variables

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In this paper, three mixed finite element formulations for the solution of elastic and nonlinear problems are approximated, implemented and validated. The mixed approximations are formulated using two fields: displacement-stress, corresponding to the Hellinger-Reissner formulation, the displacement-strain formulation and the three field formulation using displacements, strains and stresses also referred as Veubeke-Hu-Washizu. For these formulations each of the independent fields considered, displacement, strain and stress, is approximated with linear interpolation functions. The finite element implementations of these formulations have additional strain and stress fields depending of displacements, which emerge in natural way from the finite element approximation [1]. It is shown that the proposed approximations do not present numerical problems as others do which need the stabilization of the resulting system of linear algebraic equations to avoid them.

In the approximations proposed, not only primary variables, displacements, are prescribed on the boundary, but also secondary variables as strain and stress are constrained where they are intrinsically known, although these are not conventional restriction variables. The formulation of finite elements with additional restriction variables lead to improved approximations of the displacement, strain and stress fields, compared with what standard displacement finite elements give.

In the formulation of nonlinear problems involving damage a continuous damage model equipped with two internal variables at each integration point is used. The finite element implementation of the displacement-stress formulation presents numerical problems when the material is completely damaged, thus the continuous damage model was only implanted in the displacement-strain and the three fields formulations as these do not present numeri-

cal problems.

Numerical results of elastic problems are compared against analytic solutions, showing that the mixed approach provides better approximation on the strain and stress fields than a standard formulation. In addition, problems showing the locking problem as those involving incompressible solids are solved and the problem overcome with the two and three field formulations, having better results with the last one. To show the effectiveness of the mixed formulations with the continuous damage model, some numerical results of the representing numerical examples are presented.

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## Cohesive-Frictional Crack Model Applied to Dam-Foundation Joint

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The most realistic method used today for the numerical simulation of concrete fracture is the cohesive crack model, introduced by Barenblatt (1962) and Dugdale (1960) for elasto-plastic materials and by Hillerborg et al. (1976) for quasi-brittle materials. When this model is used in a large scale problem the process zone is completely developed and very often the friction operates when the crack faces are opened. In this case phenomena occurring at various scales interact each other causing instabilities of various kind.

As an example, following the benchmark problem proposed by the International Commission on large Dams [3], the propagation of a crack along the interface between a gravity dam and the foundation rock is analysed.

In order to understand the physical meaning of this instabilities and to stabilize the equilibrium iterations executed according to the Newton-Raphson method, Barpi and Valente [2] show that the knowledge of the asymptotic expansion of the displacement and stress fields at the fictitious crack tip is a useful tool.

In this paper an asymptotic expansion similar to that proposed by Karihaloo and Xiao [1] is applied. In this way many commonly-used softening laws, e.g. rectangular, linear, bilinear and exponential, can be taken into account.

The hydromechanical coupling between subpressures induced by water penetration into the crack and displacement discontinuity is evaluated following the work of Reich et al [4]

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## On the Use of Discontinuous Damage Models for Mixed-Mode Fracture

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The onset of micro-cracking in quasi-brittle materials commonly occurs perpendicularly to the maximum principal stress direction. Nevertheless, crack evolution may not necessarily occur in mode-I, due to e.g. aggregate interlocking, which develops between crack faces of the discontinuity. Consequently, the ability to simulate mixed-mode fracture in these materials is of utmost importance.

In the present work innovative damage-based material models for strong discontinuities in quasi-brittle materials are introduced. In the proposed models all failure modes are considered, namely mode-I fracture, pure mode-II fracture, mixed-mode fracture and also mode-II under compressive tractions.

The introduced material models are developed in a strong discontinuity framework, such as the discrete interface approach, the embedded strong discontinuity approach [1] and XFEM. In order to overcome the common and recurrent convergence problems associated with the iterative numerical schemes, when dealing with the softening behavior of quasi-brittle materials, the proposed models can make use of a non-iterative numerical scheme, first introduced by Graça-e-Costa et al. in [2], which was partially inspired by the sequentially linear approach by Rots et al, [3]. The use of these non-iterative schemes is made possible with the definition of both total and incremental approaches within the proposed material models.

The first material model makes use of kinematic internal variables and is presented both in: i) an isotropic setting, being a generalization of the mixed-mode model presented by Alfaiate et al. in [4], and in ii) a non-isotropic setting, in which crack closure under compressive stresses can be properly taken into account. In the second material model, the definition of a limit surface in the traction space is required, based on which the

damage evolution law is defined. This model is also presented in an isotropic and non-isotropic variant, in which, in the latter case, different evolution rates are adopted in mode-I fracture and mode-II fracture.

It is worth noting that the proposed models may also capture a characteristic feature of quasi-brittle materials, namely the dilatancy effect. In order to assess and validate the results of the proposed models, several relevant numerical examples were studied.

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## Enhanced Finite Element Modeling of Earthen Structural Materials with Weak Interfaces

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Many materials can fracture either along existing weak interfaces or through the bulk solid material, along with more diffuse failure modes. Layered composites, reinforced concrete, and welded joints are examples. Often such materials fail through some combination of mechanisms.

Masonry materials are a classical example of this type of material. In our research, we are particularly interested in studying earthen structural materials such as rammed earth and stabilized mud block [1]. While the former only has weak interfaces between compacted layers, the latter is an earthen brick that is often joined with standard or soil-cement mortar. Unlike standard masonry construction, the blocks may be weaker and more ductile than the mortar, admitting a wide range of failure modes [2].

To analyze these materials, we have developed an enhanced finite element that allows for failure both on predefined interfaces and through the bulk element at a critical orientation defined during the simulation.

The continuum deformation is defined by a 3-invariant cap plasticity model with isotropic and kinematic hardening. Initially developed for geomaterials, the model is currently under modification with a smooth tensile cap to better capture the behavior in that regime.

Bulk localization is determined by the now classical loss of ellipticity conditions, both for continuous and discontinuous bifurcation for this model [3]. Motion on existing weak interfaces is governed by a traction yield surface. This surface uses an elliptical relationship to account for the coupled shear and normal traction in tension, and a cohesive-frictional model in compression along the surface. A slip-weakening traction-displacement [4] relationship describes the loss of cohesive force with motion along the interface.

Bulk softening is governed by a similar constitutive model, though with different parameters, as it is typically more ductile for these materials. In the

case that both bulk and existing interfaces localize at once, the more critical interface governs. Propagating fractures are allowed to follow the bulk material or the interface, depending on which is most critical.

The models are used to examine the behavior of earthen structures under shear loading, both in-plane shearing and out-of-plane bending of walls. Critical importance is placed in examining the effects of openings, and the design of the brick laying to improve the performance of these structures.

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## Three-Dimensional Modelling of Embedded Coated Spherical Inclusions Through a Regularized XFEM Approach

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We propose a simple and reliable three-dimensional finite element approach for modelling particle reinforced composites where spherical and cylindrical coated inclusions are embedded within a matrix. Particle reinforced composites are employed for instance, in the automotive industry and in electronic products. Mechanical properties strongly depend on interfacial bonding quality. Hence the determination of the maximum radial stress at the particle surface as a function of the applied load and the adhesion parameters is of great interest.

Well established theoretical approaches such as the Eshelby approach for the "dilute" inclusion problem, and (generalized) self-consistent schemes for interacting particles are available [1]. Analytical solutions cannot however be obtained for any general geometry and material behavior. Finite element models appear therefore more feasible. In this context, the coating can be modeled by means of cohesive interface elements either placed along the finite element boundaries or embedded within the finite elements. The latter alternative has the advantage that the geometry of the interfaces is independent of the mesh. Based on the Partition of Unity Property of the finite element shape functions [2], the eXtended Finite Element Model (XFEM) is a broad spectrum technique for dealing with cohesive embedded interfaces. In the last years, the Authors have developed a variant, called Regularized XFEM approach. A two-dimensional application to the delamination problem of a FRP strip glued to a concrete block has shown an excellent comparison with experimental results [3]. Three-dimensional implementation is discussed in [4].

In the present work, imperfect interfaces are studied where discontinuous displacements fields across the interface and continuous traction vector occur. As usual in the XFEM approach, the surface of separation is implicitly defined via a level set function. The main steps are:

1. The assumption of the displacement field as the sum of a constant part and a disturbance part deriving from the coated inclusion.
2. The use of an "equivalent eigenstrain" concept.
3. The assumption of an extended Hill-Mandel work-equivalence principle.

The numerical solution is compared with the analytical results obtained by assuming the Hashin spring-like model [1]. We prove that the proposed approach correctly reproduces the analytical solution. The effect of the coating on the stress concentration is shown to strongly depend on the ratio between the elastic modulus of the matrix and the coating and on the Poisson's ratio of the coating.

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## Generalized Finite Element Method and the Splitting Method as a Framework for Multiple Site Damage

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In this work, fracture mechanics problems of multiple site damage in two-dimensional domains are treated by combining the Splitting Method [1, 2] and a partition of unity approach as the Generalized Finite Element Method (GFEM) [3, 4].

When applying the splitting method, the given problem is split into one global sub-problem consisting of the uncracked global domain which includes all the prescribed boundary conditions. The solution of this problem provides stress distributions which are interpolated over the previously known crack lines by using a polynomial basis. In the sequence of the procedure, a set of two auxiliary local and global problems is defined aiming to account for the stress concentration effects and also the interactions among the cracks.

The final solution for the given problem must be such that the stress distributions resulting from a linear combination of the first global problem and the set of auxiliary local and global problems are null on each crack faces.

On the other hand the main feature of the partition of unity methods such as the generalized finite element method (GFEM) is their ability for exploring a priori knowledge about the solution of a problem in the form of enrichment functions.

The Generalized Finite Element Method is hereby applied to the analysis of the local sub-problem generated by the splitting method and consisting of an isolated crack submitted to a certain number of loading cases applied on its faces. Accurate estimates of the stress intensity factors are provided by the GFEM, especially when customized enrichment functions are used in the local analysis. In order to assess the efficiency of the numerical framework, some examples varying from a single to multiple site crack problems are considered.

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## A Constrained Large Time Increment Method for Modelling Quasi-Brittle Failure

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The numerical modelling of the post-peak behaviour of quasi-brittle solids plays an important role in assessing the residual strength and failure mode of a structure. The mechanical response of these structures often exhibits snap-back behaviour (i.e. a decrease of both stress and strain under softening), necessitating the use of a robust algorithm capable of tracing this highly non-linear response. In this contribution, a non-incremental LATIN-based solution procedure capable of calculating the exact and complete loading behaviour, including snap-backs, is developed.

Unlike conventional incremental-iterative algorithms (e.g. the Newton Raphson scheme), the LATIN algorithm calculates the whole time domain in one single increment [1]. At variance with existing LATIN algorithms, the presented solution scheme can trace snap-back behaviour without the need for switching to conventional step-by-step procedures [2].

Although many advances have been made in incremental-iterative algorithms, they often require problem-specific adaptations such as the *a priori* selection of the most critical degrees of freedom [3]. However, in many large-scale engineering problems, these critical degrees of freedom are not known in advance. In the presented solution procedure, these degrees of freedom are automatically selected, rendering the algorithm to be more general.

Special attention will be given to implementational aspects and the choice of the algorithmic variables. Finally, the performance and robustness of the proposed algorithm is demonstrated by means of several numerical examples, including a GFEM-based mesoscopic masonry model involving many nonlinearities [4].

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## Three-Dimensional Modelling of the Concrete-CFRP Bond Behaviour

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The major problems found with external bond reinforcement are the local failure modes. In the last few years, several experimental and analytical studies have been carried out, which contributed to the understanding and quantification of the phenomenon related to the bond behaviour between concrete and fibre reinforced polymers (FRP). However, several issues still need to be clarified to allow an accurate quantification of the adherence between the concrete and the reinforcing material. To quantify the bond between FRP and concrete, several studies have been developed, in general, by means of two-dimensional tools. In general, the failure occurs by detachment of a thin concrete layer adjacent to the interface. Therefore, several authors presented a relation between the concrete properties and the cohesion. In general, for the most common concrete used in buildings structures, the cohesion is about 5 – 7 MPa. Assuming failure by concrete, the dissipated energy per unit area of cracked surface is defined as fracture energy of concrete. Taking into account 2D analyses, a dispersion of the mode-II fracture energy values greater than the one previously observed for cohesion has been found. In this case, from published works, for the most common concrete used in buildings structures, the ratio between maximum and minimum values of  $G_F^{II}$  is almost four. According to several authors, the parameters describing the bond behaviour are dependent on geometrical characteristics. One way of considering such influence is by means of a 3D analysis. Use of 3D models to analyse the bond behaviour between FRP and concrete is considered in few studies. Linear elastic 3D finite element analyses were carried out by [1] to investigate the stress state in a single shear test, in which perfect bond was assumed both between the adhesive and the FRP and between the adhesive and the concrete. Bond-slip laws were implemented in shear models [2, 3, 4].

In this study, based on previous works, a 3D analysis is presented. Pure shear models are

defined, using the finite element method, in order to describe the bond behaviour between concrete and sheets of carbon FRP externally glued. Modelling the behaviour of these specimens involves a non-linear phenomenon, namely: shear mode fracture interface behaviour between the concrete and the CFRP. In both models, the bond between the FRP and the concrete is modelled using a discrete crack approach. Interface elements with zero initial thickness are adopted. The shear and peeling stresses developed at these elements are dependent on the relative displacement between the strengthening material and the concrete surface, according to a local softening constitutive relationship. Both Poisson's ratio and the orthotropic behaviour of the CFRP are considered. The relation between adherent and substrate widths is considered in a parametric study. A discussion on the adoption of these parameters is made and some values are proposed.

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## A Stabilization Technique for the Extended Finite Element Simulation of Arbitrary Crack Geometries in 3D

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The mesh independent simulation of fracture processes in three dimensions using the extended finite element method (XFEM) often leads to badly conditioned equation systems due to the almost linear dependence of standard and enriched degrees of freedom. Especially for arbitrary crack geometries and crack propagation simulations it frequently happens that a crack barely intersects an element which leads to this undesired effect. In such cases different techniques can be applied. The geometry of the crack can be changed slightly by pushing it onto the node that is cut off the rest of the element by the crack. Alternatively the crack can be pushed away from the affected node or the node can be pushed onto the crack or away from the crack [1]. All these techniques lead to a slight modification of the crack geometry which is undesirable.

To avoid a change of the geometry and possibly the mesh, stabilization techniques can be applied. In [2] recently a global stabilization technique for the generalized finite element method (GFEM) is presented for one dimensional problems. This method promises to overcome such problems. Stabilization techniques on the element level have been applied in the context of standard and underintegrated finite elements for a long time already. Most of these methods are based on the works of Flanagan and Belytschko [3].

In this contribution we present a stabilization technique for the regularization of almost singular element stiffness matrices that occur within the extended finite element method. The method is based on an eigenvalue decomposition of the element stiffness matrix and a corresponding modification of the matrix and the right hand side for those eigenmodes with small or even zero eigenvalues. Physically or numerically reasonable zero eigenmodes like rigid body translations or rotations remain untouched. This technique has the advantage that neither the mesh nor the crack geometry is modified.

Additionally it is very efficient. The stabilization leads to a significantly improved convergence behaviour of iterative equation solvers used for large scale three dimensional problems, while the modification of the stiffness matrix and the righthand side does not affect the accuracy of the result significantly and the additional numerical effort on the element level is low. This is demonstrated with multiple examples of two and three dimensional fracture problems with arbitrary crack geometries.

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**Minisymposium FCM:**

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## The Influence of Friction and Plasticity on Mode II Delamination Fracture Toughness

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A central assumption in delamination analysis is that the fracture energy is a material characteristic which can serve as input. Whether the virtual crack closure technique is used or cohesive elements, it is assumed that the fracture toughness is constant, at least for given fracture mode. However, evidence exists that the fracture toughness in a pure mode test is not always constant but rather depends on size. Experimental measurements by Wisnom et al. [1, 2] on specimens with cut central plies under simple tension (a mode II setup) have displayed a significant size effect in the fracture toughness. To this date, the cause for this effect has not been fully understood and the conclusion that the fracture toughness might be influenced by anything other than the loading mode is mostly ignored in computational practice.

In the discussion of the measurements, Wisnom et al. [1, 2] have mentioned matrix plasticity and the presence of compressive stress on the interface as possible causes for the size effect. In the present work, these hypotheses are tested numerically with plasticity and friction models in combination with cohesive elements.

Nonlinear finite element simulations are performed with the cohesive law by Turon et al. [3] in combination with friction on the interface as proposed by Alfano and Sacco [4] and a damage/plasticity law for shear nonlinearity of the plies following Van der Meer et al. [5]. The range of geometries described by Wisnom et al. [1, 2] is analyzed. A parametric study is performed to investigate the effect of different input parameters on the response. It is shown that the correct size effect trend is captured. However, there is a strong sensitivity to parameters that are not very well defined such as the shear strength of the composite, the friction coefficient for to the cracked interface and the friction-free mode II fracture toughness. Notably, there is a nonlinear interaction between shear nonlinearity and friction in

the sense that their joint contribution is significantly larger than the sum of their isolated contributions.

Furthermore, unstable crack growth that has been observed in the tests on thicker specimens [1] is reproduced in the simulations. It is pointed out that this unstable crack growth disqualifies the data reduction technique that is used to compute the fracture toughness from the global response.

It is concluded that combined plasticity and friction is a likely cause for the measured size effect. As demonstrated, these effects can be simulated on the mesoscale. Nevertheless, it is acknowledged that the understanding of micromechanical processes that constitute distributed shear damage, mode II delamination, and friction is still rudimentary.

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## Intermittent Crack Growth along the Interface in a Woven Composite

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The crack growth in nonhomogeneous materials is a tricky problem where generally classical fracture mechanics tools do not apply. This is particularly the case for delamination of woven composites [1]. The interphase between two plies is rich in resin whereas the adjacent layers are rich in fibers forming either the warp or the weft and hence poor in resin (Fig. 1).

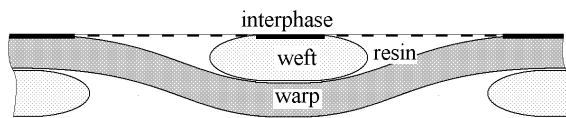


Figure 1.

The delamination crack runs successively within the resin (dashed line) and along the yarns (solid line). It is the origin of the intermittent growth. In a first step the simplified frame shown in Fig. 1 is simplified again (Fig. 2).

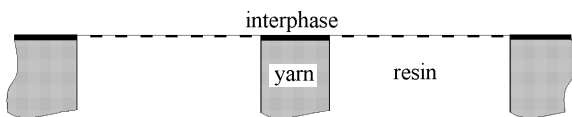


Figure 2.

Depending on its location, the delamination crack tip undergoes different singularities (Fig. 3). In the resin away from the yarn, it is the classical crack tip singularity ( $\lambda=0.5$ ,  $\lambda$  is the displacement singularity exponent). When the crack impinges on the yarn, the singularity is weak ( $\lambda>0.5$ ,  $\lambda=0.734$  if  $E_Y/E_R=50$ , where  $E_R$  and  $E_Y$  denote respectively resin and yarn Young's moduli). When the crack tip is located along the interface between the yarn and the interphase, it is an interface crack tip singularity ( $\lambda=0.5\pm i\varepsilon$ , the exponent is complex,  $\varepsilon=0.089$  if  $E_Y/E_R=50$ ). Finally when the yarn is totally disconnected from the interphase the singularity is strong ( $\text{Re}(\lambda)<0.5$ ,  $\lambda=0.354\pm i0.038$  if  $E_Y/E_R=50$ ). In cases 1 and 3 Griffith's criterion can be used to describe the crack growth, but not in cases 2 and 4 and weak and strong singularities have a major impact on the crack growth model.

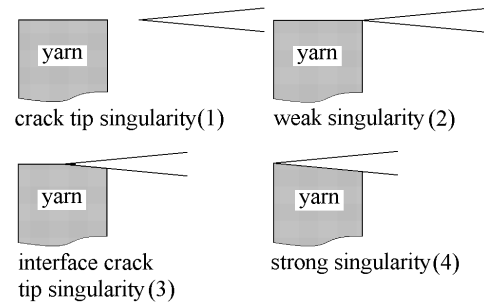


Figure 3.

The situation is similar to that of a crack in a laminated structure that grows perpendicularly to the layers [2]. In position 1 (Fig. 3) the crack slows down and is trapped ahead of the stiff area due to the weak singularity. If the load increases, the crack jumps in position 3, then accelerates (strong singularity) and goes beyond the stiff area to grow and be trapped again in the resin.

The coupled criterion [3] is used to describe the different stages. Mechanisms of slow-down, jump and acceleration depend on the elastic contrast between the resin and the yarns but also on the contrast in toughness and tensile strength between the resin and the interface yarn/resin.

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## Modeling of Delamination Migration in DCB test on Multidirectional Composite Laminates

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The different failure mechanisms occurring in fibres reinforced polymer matrix composites usually interact with each other, making the overall failure of a composite structure a complex process, difficult to predict. The interaction between delamination and matrix cracks in laminated composites has been investigated here, using the case of a Double Cantilever Beam (DCB) test on multidirectional laminates. A Cohesive Zone based approach was employed in the finite element analyses [1] and validating experiments were carried out. Delamination and matrix cracks cause a redistribution of stresses inside the plies, eventually leading to fibres breakage in the primary load-carrying plies and, therefore, to the final failure of a structure. The interaction between delamination and matrix cracks causes delamination migration through the thickness of laminates. Delamination finds its way to the weakest interface in a laminate, through matrix cracks, finally causing the complete failure of a structure [2]. The process is well known in industrial practice as an important cause of failure in laminated composite structures. Nevertheless, it is usually neglected in finite element analyses, because of the difficulty in modelling the interaction between the different damage mechanisms when using the currently available finite element tools [3].

In this work delamination migration through the thickness at  $\pm\theta$  interfaces was modelled using a full scale three-dimensional DCB test model on an aerospace grade carbon/epoxy composite. Hexahedral continuum elements were employed to represent a 40 ply laminate. Cohesive interface elements were employed to represent both matrix cracks and delamination in four groups of  $\pm\theta$  plies in the centre of the lay-up. Matrix cracks were modelled by bands of cohesive elements, parallel to the fibres orientation and equally spaced along the ply length. This allowed for the matrix cracks to initiate in any of the possible locations, according to the initiation criteria defined. Delamination was accounted for by 9 inter-ply layers of cohesive

elements tied to the surrounding elements. The models consisted of 540898 nodes, with 124668 cohesive elements. Numerical analyses were performed using the Abaqus/Explicit solver, requiring a solution time of 46 hours using 8 CPUs.

The fracture surfaces obtained from experimental DCB tests showed the typical “crack jump” behaviour. The main features of the damage mechanisms observed were captured by the proposed model. Crack jumps initiated at the free edges of the sample and damage propagated to different interfaces through cracks within the plies. The model can predict onset and propagation of both delamination and matrix cracks. Above all, the interaction between intersecting cracks on different planes was correctly managed, also for a number of different crack initiation locations. Although a complete agreement with the experimental results has not yet been achieved, this approach represents a step forward in the prediction of delamination migration in full scale composite structures, since it is able to manage the simultaneous presence and interaction of multiple cracks and delaminations.

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## Integral Identities for a Semi-Infinite Interfacial Crack in 2D and 3D Elasticity

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The problem of a semi-infinite crack at the interface between two dissimilar elastic half-spaces, loaded by a general asymmetrical system of forces distributed along the crack faces is investigated. The proposed original formulation is based on integral transforms and two fundamental notions of linear elasticity: the Betti reciprocal theorem and the weight function approach. The Betti identity has been extensively used in the perturbation analysis of two and three-dimensional cracks [5]. In linear fracture mechanics, the concept of weight function, defined as singular non-trivial solutions of the homogeneous boundary value problem for a solid with a crack, was introduced by Bueckner [1].

Recently, symmetric and skew-symmetric weight function matrices have been derived for 2D and 3D interfacial cracks in isotropic materials [5, 6] and for 2D interfacial cracks in anisotropic elastic bodies [2, 3]. Using these matrices together with the fundamental reciprocal identity (Betti formula), the elastic fracture problem is formulated in terms of singular integral equations [4] relating the applied loading and the resulting crack opening. The obtained integral identities are then used for studying some illustrative examples of plane interfacial cracks loaded by symmetric and skew-symmetric distribution of forces acting on the faces.

The singular integral formulation for 2D and 3D semi-infinite interfacial crack problems in both isotropic and anisotropic elastic materials is derived in explicit form by means of integral transforms. This approach seems to be unknown in the literature and avoids the use of Green's function and the tedious limiting process involved in the general procedure [7].

The derived compact formulation can be used to solve many problems in linear elastic fracture mechanics (for example various classic crack problems

in homogeneous and heterogeneous anisotropic media, as piezoceramics or composite materials). This formulation is also fundamental in many multifield theories, where the elastic problem is coupled with other concurrent physical phenomena.

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## A Rate-Dependent Cohesive-Zone Model Capturing Stick-Slip Crack Propagation

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Rate dependent crack initiation and propagation has been the subject of extensive experimental, theoretical, analytical and numerical studies. This is because in many problems of great engineering interest the dependence of fracture processes on the loading rate cannot be ignored and often plays a key role, particularly when it leads to unstable crack growth or stick-slip crack propagation, the latter being a sequence of transitions from slow and stable crack growth to very fast and unstable crack propagation and vice versa. For debonding in adhesive joints and delamination in composites, this may result in great difficulty in the characterisation of fracture toughness.

Theoretically, the problem can be studied in the framework of Griffith theory of fracture, by observing that in the rate-dependent case the fracture energy  $G_c$ , intended as the total energy dissipated per unit of new formed crack area, is a function of crack speed  $\dot{a}$ , i.e.  $G_c = \gamma(\dot{a})$ , whereby crack speed instabilities may occur if  $\gamma$  is decreasing in part of its domain, see for example [1, 2]. Within this theoretical framework, models to study stick-slip crack propagation are of a rather phenomenological nature, whereby  $\gamma$  is determined experimentally, which is particularly challenging in its decreasing branches [2].

Cohesive-zone models (CZMs) represent a widely used alternative method to analyse crack growth. If they are developed within a damage-mechanics formulation a damage variable  $D$  ranging between 0 and 1 can be introduced with the usual meaning. The natural extension of the above described phenomenological approach to model rate-dependent crack growth is to assume a rate-dependent evolution law for  $D$  in such a way that the entire power dissipated  $\Pi$  is a non-linear function of  $\dot{D}$ . With this approach, crack-speed jumps and possibly stick-slip behaviour may occur if  $\Pi$  is non-convex in  $\dot{D}$  [3].

An alternative approach consists of introducing other internal variables  $\alpha_i, i = 1, \dots, n$  within the CZM to capture different dissipation mechanisms, so that the entire dissipated power is a function not

only of  $\dot{D}$  but also of  $\alpha_i$  [4]. The advantage of this approach is that the internal variables  $\alpha_i$  and their evolution laws can provide a richer description of the actual dissipation mechanisms which occur at a micro-mechanical scale. This can lead to a model which is based more on first principles and less on phenomenological assumptions.

In this paper we will focus our attention on rate-dependent CMZs developed within the framework of thermodynamics with internal variables using the latter of the above described approaches. In particular, we will present a rate-dependent model in which (i) a rate-independent evolution law is assumed for the damage variable  $D$  and (ii) additional internal variables are associated with visco-plastic dissipation. The model results in an overall specific dissipation which is a decreasing function of the applied rate of displacement jump on the interface. In a structural problem this implies that the total dissipation is a decreasing function of the crack speed. Therefore, one would expect that crack velocity jumps and/or stick-slip crack propagation can occur. This is confirmed by the numerical results so far obtained, including those for two DCB specimens, designed to characterise (i) debonding in an adhesive joint of two aluminium plates and (ii) delamination in a unidirectional carbon-fibre reinforced poly-ether-ether-ketone (PEEK) matrix.

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## Damage Evolution Predictions in Large Laminated Composite Structures

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The paper aims at the prediction of the mechanical performance of large structures made from laminated composites including nonlinear material behavior of the plies.

In academia and research, the study of material nonlinearities of various kinds is widespread. Progressive damage, failure, plasticity, and delamination are considered for anisotropic constituents of composites. In combination with numerical solution schemes, e.g. the Finite Element Method (FEM), structural analyses can be performed. Also, periodic unit cells can be utilized for computational material characterization. However, the size of the investigated structures is limited because of the computational effort in solving such nonlinear problems.

In industrial practice, nowadays, the common approach for studying large composites structures is to perform linear elastic FEM computations in combination with first ply failure (FPF) analyses. These, of course, neglect any nonlinearities and do not tell what happens when exceeding the FPF limit.

To extend the predictive capabilities, a methodology is proposed which combines the two former approaches. Dissipated energies which are associated with the individual nonlinear mechanisms will be used as “damage evolution measures”. The approach will be presented for a textile laminate, based on unit cell considerations of a single ply.

The nonlinear behavior of the ply is considered first by means of a unit cell [1]. It consists of different nonlinear constituents, i.e. anisotropic tows of fiber reinforced matrix and unreinforced matrix material. The unit cell will be employed to predict the nonlinear behavior of the ply under plane stress assumption (as being typical for thin walled shell structures). Radial stress paths are simulated with a sufficiently fine resolution to cover all (plane) stress space and the responses are predicted. Of particular interest is the evolution of the dissipated energies of the individual mechanisms [2]. The results are

stored in a database and can be used in two ways. First, “damage evolution envelops” in stress space can be drawn corresponding to dissipation energy levels, e.g. for the onset of the individual mechanisms and for various energy levels. This way, an overview is given over “strength reserve”, sensitivity to overloads, directional sensitivity, damage evolution gradient, etc. Second, the data can be utilized in an automated way, for which the predicted stress from the FEM analysis at a location of the large structure is used as input.

Since the assumption of radial stress paths may not hold in general, additionally, strain controlled simulations are performed based on the linear elastic state of the structure. The evaluation scheme is the same as for the stress loading scenarios. Beyond the elastic limit, however, the response under the strain controlled loading may deviate from the stress controlled one. The two loading types give some upper and lower estimates on the possible behavior.

The utilization of these ideas for other composite materials with (complex) nonlinear behavior seems to be feasible and fairly straightforward.

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## Analytical and Numerical Modeling of the Impact Behavior of Fibre-Metal Laminates

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GLARE (GLAss-REinforced) Fibre-Metal Laminates (FMLs) present outstanding damage tolerance properties and meet the stringent requirements of aerostructures like the Airbus A380 fuselage panels and the Boeing B777 cargo floor panels [1].

In order to improve the impact resistance of this material, testing is mandatory. Yet, scatter in obtained data increases the number of tests to observe significant trends, which is time consuming and costly. Numerical modeling has become increasingly important in the development of advanced materials for impact performance. Good agreement between experiments and numerical results could be achieved [2]. However, the numerous numerical studies on FMLs focus their attention to the region of local damage, at the expenses of knowledge on the contribution of global plate deformation. Besides, the influence of the metal layers on the impact response and energy absorption remains unclear [3].

This situation calls for using another method to optimise FMLs for impact performance. To complement the scarcity of analytical studies, an analytical model was developed to assess the impact behaviour of FMLs [4, 5]. Constitutive equations address among other parameters:

1. Generic flexural deformation profile
2. Strain rate effect on material constituents
3. Contact area increase during perforation
4. Stress analysis with residual curing stresses
5. Contribution of failure modes
6. Impact location
7. Impact response

This recently developed model adapts the plate theory to an energy-balance model. The interaction between the plate variables - i.e. ply-angle orientation, aluminium thickness, plate dimensions, global flexure, failure modes - and the impact conditions - i.e.

impacting mass, impact force, and impact duration - can be explicitly determined. This approach reveals that the composite layers modify the flexure of the aluminium layers so that the global deformation is the most energy-absorbing failure mechanism. At the same time, these aluminium layers prevent projectile penetration and limit delamination.

In this paper, the developed analytical impact model for FMLs will be presented. Its capabilities and limitations will be discussed with respect to numerical models. Analytical methods capture the fundamental impact behaviour of FMLs while numerical studies indicate the impact damage extent. For this reason, the coordinately use of both methods is necessary to develop high-energy absorbing FML concepts before performing impact testing.

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## Study of Impact Damage Response of Fibre-reinforced Polymer Composites

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A three-dimensional meso-scale finite element model has been developed and implemented in user material subroutines in ABAQUS to predict the evolution of interlaminar and intralaminar damage response of unidirectional fibre-reinforced composite panels undergoing impact loads. Performance of the main user-defined material model subroutines under static loads was examined in [1]. Drop-weight impact tests representative of low-velocity high-energy impact loads were carried out according to [2] at the University of Limerick and are examined in this work. The combined model developed for this study predicts the evolution of fibre damage, shear-dominated and transverse matrix damage, and delamination. Impact energy levels are varied from 10J to 40J to evaluate the level of damage threshold and damaged area within the laminate. Nonlinear dynamic response of the laminate from implicit and explicit solvers has also been discussed. Incorporating an elastic-plastic definition of the composite material into the combined model, the effect of plasticity on the impact response has also been studied. The capability of the model to capture the transient behaviour of the laminate has been enhanced by implementation of strain rate dependence of the composite material.

The results of the combined model, in a very good agreement with the experimental data, provide an accurate characterisation of the damaged area through the thickness of the laminate. It has been found that the damage threshold is successfully predicted by the model. It is conclusively shown that delamination must be accurately modelled in order to correctly predict the impact damage response. Comparisons are provided between findings from this work and those from recent literatures [3, 4] on the same issue.

The project has been carried out at the University of Limerick as one of the main computational core of the European-wide MAAXIMUS project with the focus on fast simulation-based development of

composite aircraft structures, funded by the European Commission through FP7 work grant number 213371 and led by AIRBUS.

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# Numerical Simulation of Damage in Laminated Composite Structures under Lateral Impact

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Carbon-fibre reinforced laminated composites are becoming the material of choice for design of the main load carrying components in advanced structures such as aircrafts. Due to the critical role that these components play in the structural integrity of the aircraft, understanding and predicting the behaviour of composite materials under extreme loading conditions is crucial.

Unlike conventional construction materials such as metals with established theories for predicting their behaviour under ultimate loads, there is no comprehensive model for laminated composites that can reliably predict their behaviour under various loading scenarios.

The majority of the currently available models, deal with laminated composites at the ply level. These models are complicated and costly to run in terms of computational time and therefore cannot be practically used for simulation of large-scale structures.

The UBC COMposite DAMAGE Model (CODAM) introduced by Williams et al. [1] is a macro-scale model that takes the sub-laminate as its main building block. The objective of the sub-laminate approach is to provide a physics-based and computationally efficient model that is capable of predicting the effective and overall behaviour of the laminate in large scale structures.

The second generation of CODAM, called CODAM2, has been developed recently at UBC. This model employs a non-local averaging scheme that inhibits the spurious localization problem and results in a more realistic prediction of damage patterns [2]. CODAM2 and its non-local averaging formulation are implemented in the commercial explicit finite element code, LS-DYNA, as a built-in material model (MAT\_CODAM2 or MAT\_219).

A series of lateral impact tests on carbon fibre-reinforced laminated composite plates (CFRP) were performed at The University of British Columbia to study the response of such materials to

various impact parameters including the mass and velocity of the impactors [3]. In this study, the response of CFRP plates under low mass and high velocity impact tests are simulated.

The Finite Element Model is then employed to simulate the response of the impacted plate. The predictions of the numerical model such as force and deflection time histories, damage patterns and dissipated energies are compared with the experimental observations and measurements (e.g. Figure 1).

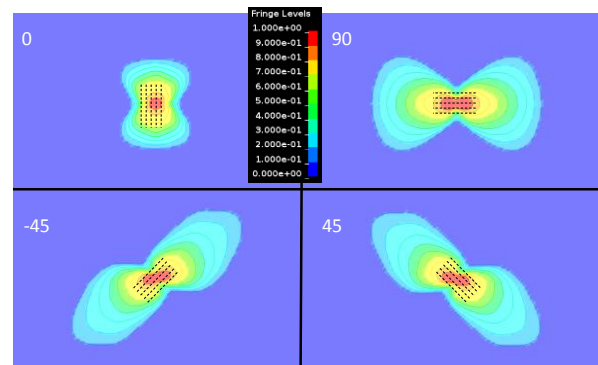


Figure 1: Predicted matrix damage pattern in various layers of the composite laminate.

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## A Plastic Smeared Crack Model for Polymer-Matrix Composites

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A new fully three-dimensional smeared crack model with uncoupled plasticity that is able to predict the onset and propagation of ply failure mechanisms in polymer composites reinforced by unidirectional fibers is proposed [1]-[2]. The failure criteria are used to predict not only the onset of the failure mechanisms but also the orientation of the fracture plane, which depends on the applied stress state and on the microstructure of the material.

The information about the orientation of the crack plane is used in a smeared crack model for transverse cracking that imposes a linear softening relation between the traction acting on the fracture planes and the crack opening displacements. The longitudinal failure mechanisms are represented using bi-linear softening relations and the corresponding material properties are obtained from the analysis of crack-resistance curves.

The model is validated using off-axis compression tests performed in unidirectional specimens as well as using tensile and compressive tests in multidirectional laminates containing central cracks and open-holes. A good correlation between experimental observations and numerical predictions is obtained.

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## Computational Micromechanical Model of Ply Failure: Matrix Cracking, Delamination and Crack Density

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The primary aim in this study is to capture the failure mechanisms in the transverse direction of the ply. The secondary one is to relate the ply stiffness with the crack density. We propose to use the computational micromodel, developed by Canal L.P. Bibliography entry, to simulate the crack propagation of transverse cracks, the evolution of the delamination promoted by matrix cracking and the crack density by means of the embedded cell approach. The model can also be used to study the relation between the ply stiffness with the crack density. According to Maimí et al. Bibliography entry, this relation has to be defined solving the Poisson's equation for the cracked ply embedded in the laminate. However, in the proposed approach, it is determined by using finite element simulation of the detailed microstructure.

The modeling approach is based on a representative volume element (RVE) of the composites microstructure using periodic boundary conditions to simulate an infinite laminate, which has demonstrated to be capable to account for nonlinear deformation and damage mechanisms Bibliography entry. The RVE represents a composite laminate with the following stacking sequence: [0/90n/0]. The model assumes homogeneous material properties and linear elastic mechanical behavior for the outer plies. The model also requires a suitable constitutive model to predict the non-linear response of the embedded ply. Therefore, it is assumed a continuum plastic-damage model for the bulk material. For the debonding mechanism of the interface between the epoxy matrix and the E-glass fiber, a cohesive crack model is considered in terms of a bilinear traction-separation law. Finally, the fibers are considered completely elastic.

As for the fiber distribution, a random absorption algorithm (RSA) is used to generate the coordinates of the particle centers using a uniform distribution Bibliography entry. Using this algorithm, different fiber distributions can be tested to analyze the effect of it on the failure process.

The model predictions are compared with experimental data.

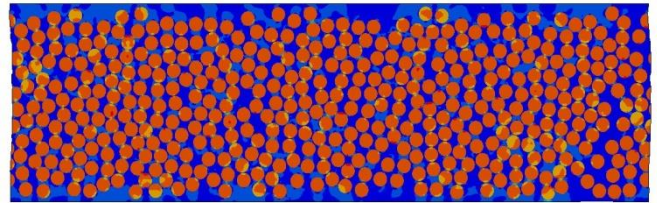


Fig 1: Von Mises stress distribution and deformed configuration in the embedded ply

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## Multiscale Modeling of Kinking in CFRPs: Validation and Competition Between Damage Mechanisms

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The intensive use of Carbon Fibers Reinforced Plastics in aeronautics or automotive applications implies to master the prediction of any behaviour up to final failure. The so-called virtual testing approach supports this goal and relies on the use of robust models for key physical mechanisms. Among them, the kinking phenomenon in compression along the fibers direction is a major one since it is involved either in static or dynamic complex test cases, which can lead to many fragments through the whole volume and consequently strong non-linear behaviour and important dissipated energies. While the physics of kinking is relatively well known at the scale of the fiber, its modeling at a meso-scale and its interaction with delamination is still challenging.

Previous works have focus on quasi-static loadings. A micro-modeling of a representative unit cell incorporating a carbon fiber in an epoxy matrix has been developed and is able to represent major degradation mechanisms associated to kinking [1, 2]. It is based on Fleck & Budiansky's kinking theory. This micro model has been used to extract the most important quantities (strength, dissipated energy, kink band size) and the associated scattering mainly due to straightness defects of fibers. From that point, a model at an upper scale [3] has been improved to account for compressive loadings [4]. For that, an approximate potential form has been proposed and the associated state and evolution laws identified based on an energy equivalence principle. The kinkband size play the role of localization limiter.

In this paper, two main activities are presented. First, a validation of the proposed single fiber micro-model is presented. It relies on the use of a more complete model with multiple fibers, associated defects and cracks or fiber/matrix debond-

ings. This allows to validate of the kinematic proposed for the single fiber micro-model as well as the choice of the representative defect among the existing wide spectrum of defects. Second, simulations are used to understand the competition between the kinking (represented using the previously built homogenized model) and other degradation mechanisms such as transverse cracking at the laminate scale. For that, different geometries of coupons and loading conditions have been carried out, leading to different degradation scenarios. Comparisons with experimental observations [5] have been done.

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## Global-Local Numerical Characterization of Damage Tolerance of a Composite Runout Specimen Subjected to Tensile Loading Conditions

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Structural applications of carbon fiber reinforced composite (CFRC) materials are in continuous expansion in different industrial areas. As a consequence of their superior stiffness/weight and strength/weight ratios in comparison to other engineering materials, the incorporation of composites has especially attracted the attention in the aerospace and in the aeronautical sectors. In modern aerostructures, composites are also used in the conception of the primary structure where stiffened panels constitute one of the principal typologies.

Current design constraints require the elimination of a certain number of stringers along the structural definition of the component, these locations being usually denominated as runout regions. The abrupt change in the geometrical configuration around these areas makes the load carrying mechanisms of the component resulting significantly modified. Hence, the loading in the stiffener must be absorbed by the skin to which it is attached to. It leads, consequently, at these locations, to complex three-dimensional stress states that include high stress concentrations. Based on the previous arguments, it is clearly noticeable that runout regions can be identified as crucial locations at which different damage mechanisms and failure processes can potentially occur in real components. Therefore, the development of reliable methodologies of analysis to provide a further insight into the phenomenology of damage in these components can yield to significant optimizations in their structural conceptions.

The analysis of runout areas has been carried out from experimental and numerical points of view. In particular, damage tolerance of different coupon-size configurations under uniaxial tensile and compressive actions were thoroughly investigated in [1] (see also references therein). Observations performed in the vast majority of these experiments evidenced the skin-stringer debonding process as

the predominant failure mechanism. Focused on the computational damage analysis, the Virtual Crack Closure Technique (VCCT) was successfully employed to predict the crack growth characteristics of the specimens tested [1].

The present contribution covers the FE simulation of the damage process of a composite runout specimen under uniaxial tensile loading action at component level. Alternatively to the investigations mentioned above, the numerical characterization of the damage in the component is carried out through the use of the Cohesive Zone Model approach [2]. Due to the high level of discretization that this approach generally requires in order to obtain accurate predictions, the insertion of interface cohesive elements along the entire stiffener-skin junction can result computationally unaffordable. One possible choice to overcome these limitations can be to use FE global-local techniques. In this sense, this work complements the investigation previously accomplished [3], and is part of a large experimental and numerical program aiming to characterize the damage tolerance of different runout concepts under identical supporting and loading conditions at structural component level.

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## **Computational Models for the Description of Multi-Particle Interactions in Random Structures, Meso- and Macrofailure of Unidirectional Fibre-Reinforced Composites**

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The structural stochasticity of unidirectional composites is caused by randomness of the shape, mutual disposition, and orientation of fibers and dispersion of their diameters. The principles of constructing algorithms for the computer generation of random-structure unidirectional fibre-reinforced composites are formulated, the maximal void fractions of inclusions are defined, and the effects of self-organization and self-regularization are detected and explained. A special investigation into structures with randomly distributed fibre diameters allowed us to establish that the dispersion of fibre diameters was a factor predetermining the occurrence of localization and the lack of periodic terms in random fields. Analytical expressions for the conditional and unconditional multipoint correlation functions (i.e. CF) of different orders for random-structure random stress and strain fields are obtained. Analytical expressions for derivatives of the conditional and unconditional CF of second and third order at the points corresponding to zero values of arguments are obtained. Applying a correlation analysis to the unidirectional fibre-reinforced random-structure composites created, new criteria for determining the characteristic size of the representative volume element, with account of the multi-particle interaction in the system of reinforcement aggregates, is proposed.

Computational experiments in transverse shear and tension, uniform tension in the reinforcement direction, and anti-plane shear showed that the effective elastic moduli of the composites did not depend on the symmetry and asymmetry of the distribution laws. But the asymmetry significantly affected the fractional structure of the materials and the character of multi-particle interaction at distances from half to two averaged fibre diameters. These length scales predetermined the character of strain and stress heterogeneity in undamaged composites and significantly affected the damage evolution scenario at the initial stage of quasistatic loading.

A nonlinear two-level structurally phenomenological model of damaged unidirectional fibre-reinforced composites was presented. The model allowed us to describe the inclination and coarsening of defects in the matrix as a multistage process and determine the instant of macrofailure as a result of the loss of stability of damage evolution. An iteration procedure is presented for an automatic selection of a quasi-static loading step for a composite with elastic-brittle structural elements in numerically solving boundary-value problems by using FEM. The procedure suggests that a minimum possible number of matrix domains change their deformation properties owing to the partial loss or recovery of the bearing strength if the type of stress-strain state changes on the structural level. For various schemes of combined triaxial proportional macrodeformation and anti-plane shear of composites, the main regularities in the evolution of damaged matrix domains were determined. The phenomenon of 'quantum' character of damage evolution under hydrostatic compression, which did not depend on the type of statistical distribution law of fibre diameters was detected and investigated. A qualitative correlation between the macrofailure character in computational experiments and the results of mechanical tests in the anti-plane shear of glass-epoxy plastics was shown to exist.

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**Minisymposium IAF:**

**Isogeometric Analysis of Failure in Solids and Structures**

*Organized by Clemens V. Verhoosel and  
René de Borst*



## Micromechanical Failure Modeling of Trabecular Bone Using Isogeometric Analysis

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Advanced computational methods are now well accepted in biomedical engineering and have found their way into clinical practice. An example is the determination of bone fracture risk, where numerical simulations play an important role in the calculation of reduced bone strength due to e.g. osteoporosis. Numerical simulations are required to adequately represent the microarchitecture of the bone in order to accurately identify bone strength.

In the last few decades microscale finite element techniques have been proposed that use a voxel conversion technique to represent the bone microstructure with brick elements. Although such analyses can well predict bone stiffness in some clinical research studies, the disadvantages associated with such analyses prevent wider application. Most importantly, due to their inability to accurately represent microscopic stresses, only ad-hoc strength indicators can be used.

Isogeometric analysis (IGA) [1, 2] is proposed to overcome the difficulties associated with traditional finite element analyses for microscale bone specimens. Given a voxel mesh obtain from a micro-CT scan, the developed numerical method is capable of representing bone structures of arbitrary topological complexity. As a first step in this method an implicit boundary representation is created by the construction of a volumetric B-spline level set function. In the second step, an immersed boundary method [3] is employed to construct an analysis-suitable spline space over the geometry of interest. Hierarchical refinement [4] is used to locally refine the spline space to ensure sufficient analysis accuracy (in e.g. the stresses).

The smooth parametrization of the computational domain and the corresponding smooth discretization of the displacement field allow for an accurate determination of stresses. This opens the doors to performing computationally reliable failure analyses. In this contribution we study the applicability

of gradient damage models – for which IGA has already been demonstrated to be an efficient discretization technique [5] – to mimic the evolution of damage in a trabecular bone specimen. Using standard numerical homogenization methods we derive direct failure indicators from these damage analyses.

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## T-Spline-Based Isogeometric Cohesive Zone Modeling of Interface Debonding

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Cohesive zone (CZ) models based on non-linear relationships between tractions and opening displacements are widely adopted within finite element frameworks to analyze fracture of materials and interfaces. Herein we focus specifically on debonding at bimaterial interfaces, or more in general on problems where the path of the debonding crack is known *a priori*.

A drawback of CZ models is that, unless a sufficiently fine mesh is provided around the crack front, the computed load-deflection response is non-smooth and may exhibit artificial snap-throughs and snap-backs [1]. This is due to the inability of coarse meshes to capture correctly the strain field in the process zone around the crack front during its propagation.

In contrast to refinement of the entire domain, local refinement of the process zone is a computationally more efficient alternative. To this end, different surface enrichment strategies have been developed in the literature using different types of enrichment functions for CZ interface elements [1,2], as well as for contact elements [3]. These techniques, however, only affect the interacting surfaces and leave the bulk behaviour of the solid unaltered. Moreover, they typically do not increase the degree of continuity of the parameterization at the inter-element boundaries which is also responsible for unphysical stress oscillations at the interface.

The isogeometric analysis framework [4] has already demonstrated to guarantee substantial advantages in the computational treatment of unilateral contact [5]. Differently from non-uniform rational B-splines (NURBS) built on rectangular grids in the parameter space, T-splines allow local refinement due to the introduction of T-junctions and extraordinary points. This approach is particularly suitable for CZ models, due to the high resolution required by these models in the process zone. Furthermore, in the isogeometric setting the discretized crack surfaces feature higher

order inter-element continuity with respect to classical finite elements.

In this contribution, debonding problems at known interfaces are treated with CZ modeling within the T-spline-based isogeometric framework. The interface is discretized with zero-thickness contact elements which account for both contact and debonding within a unified framework, using a Gauss-point-to-surface formulation. The continuum is discretized with cubic T-splines, as well as with arbitrary order Lagrange polynomial elements for comparison purposes. Results for mode-I double cantilever beam (DCB) and mode-II end-notched flexure beam (ENF) tests with varying resolutions of the process zone are presented and compared in terms of load-deflection relationship. The ability of T-spline interpolations for fracture modeling is discussed.

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## An Isogeometric Solid-Like Shell Element for Modelling Delamination

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Isogeometric analysis (IGA) has recently received much attention in the computational mechanics community. The basic idea is to use splines, which are the functions commonly used in computer-aided design (CAD) to describe the geometry, as the basis function for the analysis rather than the traditional Lagrange polynomial functions [1].

A main advantage of isogeometric analysis is that the functions used for the representation of the geometry are employed directly for the analysis, thereby by-passing the need for a sometimes elaborate meshing process. This important feature allows for a design-through-analysis procedure which yields a significant reduction of the time needed for preparation of the analysis model [1]. Indeed, the exact parametrization of the geometry can have benefits for the numerical simulation of shell structures, which can be very sensitive to imperfections in the geometry. Moreover, the higher-order continuity of the shape functions used in isogeometric analysis allows for a straightforward implementation of shell theories which require  $C^1$  continuity such as Kirchhoff-Love models [2, 3]. A Reissner-Mindlin shell formulation has been developed by Benson *et al.* [4] using NURBS basis functions.

A further benefit of basis functions that possess a higher degree of continuity is that the computation of stresses is vastly improved. In shell analysis this can be particularly important when materially non-linear phenomena such as damage, or delamination, which can occur in laminated spatial structures, are included in the analysis. In the latter case the computation of an accurate three-dimensional stress field becomes mandatory, and solid-like shell elements become an obvious choice. The latter class of shell elements is characterized by the absence of rotational degrees of freedom, which is convenient when stacking them, yet possess shell kinematics, and are rather insensitive to shear locking and membrane locking.

In this work, we develop a solid-like shell element that is based on the isogeometric concept. It therefore combines the advantage of an accurate geometric description of the shell reference-surface and the advantages in terms of meshing of isogeometric analysis with the three-dimensional stress representation of conventional solid-like shell elements. The formulation adopts NURBS (or T-spline) basis functions for the discretization of the shell reference-surface, while in the thickness direction a higher order B-spline basis function is used. A very useful property of the B-splines is their ability to model strong discontinuity by knot insertion [5]. Using this property, we model delamination in composite laminates. The proposed isogeometric solid-like shell element and its application for delamination modelling will be demonstrated by several examples.

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## A Phase-Field Model for Cohesive Fracture using Isogeometric Analysis

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The objective of this contribution is the development of a numerical procedure using a diffusive model that is able to characterise crack propagation and branching in the context of cohesive and quasi-static fracture. The performance of the obtained formulation is tested using a standard FEM formulation and isogeometric analysis.

The diffusive model, based on [1], introduces a crack phase-field to describe the evolution of the fracture, inherently removing the need for a discrete crack. A thermodynamically consistent framework was developed for the smeared description of the crack topology. The resulting model, derived from balances of storage and dissipation of energy, couples the kinematic and phase-field variables.

This model, initially derived for brittle fracture, is extended to include cohesive-zone processes. The introduction of an auxiliary field that approximates the displacement jumps across the formed crack was necessary in order to evaluate the cohesive forces.

The performance of the proposed approach is tested on 2D numerical examples. The coupled problem is solved in a monolithic approach first using standard finite elements, and second using isogeometric elements in a similar fashion to [2] and [3].

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**Minisymposium IAV:**  
**Industrial Applications and Validation of  
Fracture Models**

*Organized by Jaime C. Gálvez and  
Stéphane Guinard*



## Seismic Analysis of RC Structures Using Damage Model and Simplified Modelling

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When treated using nonlinear finite elements techniques, structural dynamic analysis generates, for RC structures, complex and large numerical problems. Damage is a major aspect of the behavior of concrete and the discretization technique is an important keypoint to control the size of the problem. Both aspects are considered in this paper and applications on seismic situations are proposed.

Based on previous work [1, 2], the  $\mu$  damage model is a new model set up around the use of a multi-surface threshold to activate the different damage effects linked to cyclic loading, including unilateral effects. Assumptions are formulated to simplify the writing, while allowing a correct description of the main non-linear effects. In that way the model

- couples elasticity and damage; damage is assumed isotropic (scalar variable);
- is described in a complete explicit form;
- does not include any permanent strain.

This last choice is the result of a long experience on the simulation of nonlinear behavior of RC structures, which leads to the conclusion that permanent strains come mainly from plasticity of steel, therefore the part coming from concrete can be neglected. The general 3D constitutive relationship is:  $\sigma = (1 - D_\mu)\Lambda_0 : \varepsilon$ , where  $\Lambda_0$  is related to the initial mechanical characteristics and  $\sigma$ ,  $\varepsilon$ , are the stress and strain tensor respectively. Driving the stiffness evolution, the damage variable  $D_\mu = F_\mu(Y_\mu, A, B)$  depends both, on the historical path of loading and, on the actual state of the load. Consequently (and it is an original concept),  $D_\mu$  changes during the closing-opening of cracks,  $A$  and  $B$  are used to fit the good trend of the curves of evolution and  $Y_\mu$  is the pilot variable for  $D_\mu$ .

Two internal variables are chosen,  $Y_{\mu t}$  and  $Y_{\mu c}$  which are the maximum values of two equivalent strains,  $\varepsilon_{\mu t}$  and  $\varepsilon_{\mu c}$  respectively:

-  $\varepsilon_{\mu t}$  is the equivalent strain in tension,

$$\varepsilon_{\mu t} = 0.5I_\varepsilon/(1 - 2\nu) + 0.5\sqrt{J_\varepsilon/(1 + \nu)}$$

-  $\varepsilon_{\mu c}$  is the equivalent strain in compression,

$$\varepsilon_{\mu c} = 0.2I_\varepsilon/(1 - 2\nu) + 1.2\sqrt{J_\varepsilon/(1 + \nu)}$$

$I_\varepsilon$  and  $J_\varepsilon$  are the first strain tensor invariant and the second deviator strain tensor invariant, respectively.  $Y_{\mu t}$  and  $Y_{\mu c}$  evolve monotonically (whatever is the loading path), from a threshold ( $Y_{\mu t0}$ ,  $Y_{\mu c0}$ ) to maximum values at failure. According the aforementioned, the variable which pilots damage is obtained by:  $Y_\mu = rY_{\mu t} + (1 - r)Y_{\mu c}$ ;  $r$  is the triaxial factor  $r = \sum \langle \sigma_i^e \rangle + \sum | \sigma_i^e |$ , where  $\sigma_i^e$  is the principal effective stress. Finally, through the evolution of  $\varepsilon_{\mu t}$  and  $\varepsilon_{\mu c}$  the evolution of  $D_\mu$  is associated to two loading surfaces:

$$f_{\mu t} = \varepsilon_{\mu t} - Y_{\mu t} = 0 \text{ and } f_{\mu c} = \varepsilon_{\mu c} - Y_{\mu c} = 0.$$

Using the  $D_\mu$  value, the equation  $\sigma = (1 - D_\mu)\Lambda_0 : \varepsilon$  is the only one necessary to describe the behaviour. Simplified FE description is based on the use of multifiber beam elements for beams and columns and equivalent lattice model for walls. A regularization method is introduced to limit dependency on the mesh size during strain and damage localization phenomena [3]. Propositions for damping description are discussed [2]. Applications on shaking table tests justify the various concepts and assumptions presented.

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## **Finite Element Simulation of Sandwich Panels of Laminated Plaster and Rockwool under Mixed Mode Fracture**

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Sandwich panels of laminated gypsum and rock wool have shown large pathology of cracking due to excessive slabs deflection. Currently the most widespread use of this material is as vertical elements of division or partition, with no structural function, what justifies that there are no studies on the mechanism of fracture and mechanical properties related to it. Therefore, and in order to reduce the cracking problem, it is necessary to progress in the simulation and prediction of the behaviour under tensile and shear load of such panels, although in typical applications have no structural responsibility. To carry out this research, the behaviour of this material can be considered quasi-brittle and, based on this idea, in this work has been studied using a cohesive crack model that has been applied to other quasi-brittle materials, such as concrete, and has provided very satisfactory results.

This communication presents the work carried out to study the mechanical and resistant behaviour under normal and shear load taking into account the size effect of the specimen of plaster and rock wool. The authors designed an experimental campaign under mixed mode composed by testing specimens of different sizes. Assymetrical three-point bending tests have been performed on notched specimens, geometrically similar and of different size, to obtain load-displacement and load-crack mouth opening displacement curves. Previously a series of experimental tests were carried out to characterize a sandwich panel of laminated gypsum and rock wool, and each of its components: plasterboard, rock wool and paper. We designed the experimental campaign to obtain the strength properties of the studied materials, and its specific fracture energy, GF, as well as the complete curves of applied load versus displacement. From the experimental results it can be observed that the fracture energy is strongly influenced by the thickness of the wool, rather than the plate.

To numerically simulate the mixed-mode fracture behaviour of the panels we have used a finite element model with embedded crack, based on the cohesive crack model, using as input the experimental parameters obtained in the experimental campaign, obtaining a good adjustment. Based on these results we analyze the mixed-mode fracture behaviour of the material and the size effect of the panels. Finally, the authors have used this model to study the limitation of the maximum deflection of slabs in order to avoid its cracking.

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## Fracture Modelling of Adhesive Bonds Subjected to Multiaxial Loading

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Due to their light weight and high strength and toughness, composites materials are becoming increasingly popular for the design of many engineering components in areas as diverse as tidal and wind energy, jet-engines, automobiles, etc Due to the complexity of one-piece component manufacturing, the need to develop good joining techniques is becoming more pressing. As opposed to more widespread joining techniques such as bolted joints, adhesive joints do not need holes machining thus limiting the addition of high stress concentrations and reducing the component exposure to failure. Furthermore, they are lighter and more economical. Despite having been studied extensively, predicting their resistance to failure remains exceedingly challenging [1]. Some success towards failure prediction of thin brittle adhesive joints has been achieved using the popular cohesive zone element method in finite element analysis. Interface elements however remain poorly adapted to the description of crack propagation in thick ductile adhesive as they require a-priori knowledge of the crack path and, even when an attempt is made to take the joint ductility into account [2, 3], do not allow appropriate description of constraint effects [1, 2]. This is a serious impediment to a more systematic use of adhesively bonded joints within composites-based engineering component design. Indeed, in many practical applications the manufacturing tolerance does not allow meeting the thin adhesive requirements.

In the present study an evolution of the interface element / continuum damage mechanics (CDM) hybrid approach to failure in thick ductile adhesive joints presented in [4] is used. The adhesive plasticity is taken into account through the use of a classical Drucker-Prager yield criterion. The material hardening behaviour is assumed to be piecewise-linear (the material behaviour in pure tension and pure compression is obtained from experimental testing and given as an input to the model). The material is degraded anisotropically through a 2nd order damage tensor. As in CDM, the material degra-

ation is triggered when the equivalent plastic strain reaches a certain threshold whilst, similarly to cohesive zone modelling, crack propagation is controlled by an energy term that ensures that the model is mesh-independent. The two model parameters controlling the damage development in the joint are determined through an inverse method. The validity of the the modelling approach and of the parameter identification procedure are confirmed by comparing the model predictions with experimental results obtained from double lap shear tests performed on 1 mm and 2 mm bondline specimens under different level of through thickness compression [5].

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## Cover Cracking of the Reinforced Concrete due to Rebar Corrosion Induced by Chloride Penetration

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Durability of reinforced and precast concrete is not an innovative aspect in the scientific-technical world and it is concerned with in numerous codes, standards and recommendations. The reparation of the durability problems is very expensive. For example, according to the Federal Highway Administration, approximately 30% of the American bridges were either structurally deficient or functionally obsolete in 2001. [1]. Among degradations sources we can consider: corrosion (either due to carbonation or chloride ingress), freezing and thawing cycles, abrasion, thermal gradients, chemical attacks, dissolutions processes and alkali-aggregate reactions [2]. Evidently every isolated process can interact with each other increasing structural damage this way [3].

This paper is focused on the problem of the chloride-induced corrosion of the rebar in reinforced concrete, with special application to the slabs and decks of the bridges. High superficial concentrations may be usual in these structures (marine environments or de-icing salts in roadway bridges, e.g.). Like any aggressive agent such as water, gases or other dissolved ions, chloride induced deterioration is very conditioned by possibilities of transport through concrete mass.

In the case of models for the chloride induced corrosion, the common service life involves two time periods: the first is the time for chloride diffusion and the second is the time for corrosion damage. According to Monte Carlo analysis, failure probability is based on time depending on materials properties and stochastic distribution of external loads: cover depth, chloride content analysis, surface chloride concentration, apparent diffusion coefficient based on Fick's second law (because of chemical reactions taking place between the penetrating agent and concrete, see ASTM C1556-04), chloride corrosion initiation concentration (may depends on composition of

concrete and environmental parameters) and time to corrosion damage [1].

This paper presents a model for the chloride diffusion in the concrete taking into account the pore structure of the concrete, the internal humidity and the temperature. The model accomplishes the concentration of the chloride, the gradient of the concentration, the temperature and the humidity. The model also models the presence of cracks in the concrete. Based on the results of chloride diffusion a corrosion rebar is assumed and the radial expansion of the corroded reinforcement simulated. The possible cracking around the rebar is modeled with an embedded cohesive model [4]. The model has been implemented in the FEAP Finite element code. The steel radial expansion is simulated by means of a temperature increase.

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## Experimental Validation of a Fracture Model for Pearlitic Steel Bars Based on the Cohesive Zone Model

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Steel is, together with concrete, the most widely used material in civil engineering works. Not only its high strength, but also its ductility is of special interest, since it allows for more energy to be stored before failure. A better understanding of the material behaviour before failure may lead to better structural safety strategies.

The characterisation of metallic materials is usually carried out as described by ISO 6892-1 standard, which defines the tensile testing method for metallic materials. When the maximum load point is reached, necking process begins. This makes difficult to study the behaviour of the material from that moment until failure.

On another point, metallic cylindrical specimens tested under tension usually show a cup-cone surface after failure and the failure mechanism is usually explained with the theory of nucleation, growth and coalescence of microvoids. Based on this theory, many numerical models have been developed, with a special mention to the Gurson-type models [1]. These models simulate mathematically the physical growth of microvoids, leading to a progressive development of the internal damage that takes place during a tensile test. In these models, the damage starts to develop in very early stages of the test.

Nevertheless, cylindrical specimens made of pearlitic steel rods used for manufacturing prestressing steel wires do not show a cup-cone fracture surface. These specimens show a flat fracture surface with a dark region in the centre of it. Experimental results obtained by the authors [2, 3] suggest that, in the case of this material, failure takes place as a result of a ductile-brittle transition process. Therefore, in this type of materials, a quasi-brittle fracture is developed as a consequence of a decohesion process, with the dark region acting as a circular crack perpendicular to the loading direction.

In the authors' opinion, using a cohesive model as a failure criterion is interesting in this case, since a cohesive model only requires two parameters to be defined, with the fracture energy being one of them, which can be obtained experimentally. In addition to this, given that it is known that the stress triaxiality has a strong influence on the fracture of ductile materials, a cohesive model whose parameters are affected by the value of the stress triaxiality can be considered.

This work presents the experimental validation of a fracture model for steel specimens in a tensile test, based on a cohesive behaviour and taking into account the effect of stress triaxiality. Experimental tests have been carried out on cylindrical specimens of three different diameters: 3, 6 and 9mm. These tests have been reproduced numerically using the aforementioned cohesive element. Results from the numerical simulations have been compared with the experimental results, showing good agreement with them.

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## Stochastic Crack Formation in Reinforced Concrete Tension Bars

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Estimation and control of crack width is essential for a proper design of reinforced concrete structures. Some well-known analytical approaches have been derived for crack width estimation [1, 2]. Although they are widely used in engineering practice they suffer from drawbacks, e.g. nonlinear behavior of the reinforced concrete constituents concrete, reinforcement and bond are not considered. Furthermore, another important aspect is given by the fact that properties of structures and building materials may have considerable scatter. In particular this concerns the properties of concrete. These aspects constrain the value of the results of deterministic analytical calculations and encourage a numerical stochastic approach.

Such an approach has two major components: a deterministic base model and a framework to generate and evaluate a large number of samples while utilizing statistical characteristics. This paper discusses a deterministic numerical model for the cracking of reinforced tension bars based on the Finite-Element-Method which overcomes simplifications and restrictions of the conventional analytical models [3]. Major parameters are the limited tensile strength of concrete and the nonlinear bond between concrete and reinforcement. The model yields the formation of discrete cracks over all stages of the loading history of reinforced concrete tension bars beginning with first single cracks followed by the final cracking pattern up to the yielding of the reinforcement. The crack width for every single crack together with stresses, strains and slips are computed for all components at every stage.

A major influence parameter for crack formation is given by the tensile strength of concrete. Furthermore, this parameter is subject to a relatively high scatter. Thus, this paper discusses the influence of the stochastics of concrete tensile strength on the stochastics of crack width in a first approach of stochastic crack formation simulation. Stochastics of concrete tensile strength has two dimensions. A

sample of a concrete bar has varying tensile strength along its axis, i.e. a stochastic field which is characterized by statistical parameters like distribution type, mean value, correlation length and others. Additionally the bulk of samples has to be considered.

The paper discusses a method to generate a population of stochastic fields for concrete tensile strength according to prescribed statistical parameters and to combine them with the deterministic base model within the framework of the Monte-Carlo-Method [4]. The method is applied to reinforced concrete tension bars exposed to external loadings and restraints like temperature and shrinkage. A parametric study is performed assuming different types of distributions, standard deviations and correlation lengths for concrete tensile strength and investigating the effects on crack width distributions and crack width mean values, standard deviations and quantile values over the entire loading range.

The entire approach shall support an extended view of reliability and durability of reinforced concrete structures.

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## Modeling Anisotropic Ductile Damage in Sheet Metal Forming

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Metal sheets show significant orientation dependence in their mechanical properties due to the rolling process. In metal forming practice, this inherent anisotropy is mostly reflected using anisotropic plasticity models. Once used with ductile damage models such as Gurson-Tvergaard-Needleman (GTN) or Lemaitre's conventional damage model, the anisotropy of the fracture is then implied by anisotropy of plastic flow. This, however, does not reflect the real material behavior completely. Like in the case of plasticity, where preferred directions are developed with extensive plastic deformation of grains, the void sources, e.g. inclusions or second phases, within the material matrix also do not deform evenly at all directions during rolling. Let alone the rolling process, even the simplest uniaxial loading can change an initially spherical void into an ellipsoidal one on the course of deformation. The former example of the rolled sheet can be seen here to constitute an inherent source of anisotropy of damage and the latter one of evolving void morphology during deformation as an induced one. During metal working the initial and induced anisotropy of damage has significant consequences on the formability due to varying toughness of the sheet with loading direction.

In this work, we present a critical review of two distinct modelling approaches while incorporating damage anisotropy to enlarge the predictive bounds of conventional plasticity within the context of sheet metal forming practice. The former modeling approach falls in the class of continuum damage mechanics (CDM). Following internal variable thermodynamics formalism, the evolution of the damage variable, a second order tensor reflecting material deterioration, is formulated [1]. Possible empirically motivated damage evolutionary variants are given which do not necessarily admit regarding dissipation potentials. The latter approach is a micromechanically based porous plasticity due to [2]. Unlike GTN model, initial and induced void shapes are not necessarily spherical, i.e. void aspect ratio may deviate from unity. A strain dependent void nucleation criterion and a

critical porosity dependent void coalescence complement the model to account for all aspects of ductile damage composed of void nucleation, growth and coalescence.

Based on a corotational framework the models are implemented as user defined material subroutines, VUMAT, into ABAQUS/Explicit. The identification of the parameters for DC04 sheets is handled using a hybrid experimental and numerical inverse scheme. These involve, on the experimental side, smooth and notched tensile tests at various orientations with respect to the rolling direction as well as the in-plane torsion test. The numerical side involves simulation of the tests using the developed subroutines. As a pragmatic solution to the pathological mesh dependence and a means of controlling extent of localization accompanying softening regimes, in all of the simulations, the minimum mesh size at the process zone is limited. The parameter identification phase is followed by the validation studies using actual forming processes. For this purpose conventional deep drawing and non-conventional sheet-bulk forming processes are selected. Especially, the latter process gives account for strongly nonlinear strain paths. The results are drawn around model predictive performances, load path dependence of damage evolution, complementary parameter identification effort and computational cost.

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## A Detailed Modeling for Fracture Simulation of Spot Welds in Advanced High Strength Steel DP600

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The aim of this article is to investigate the resistance spot welds fracture behavior under lap-shear condition and to develop an efficient constitutive model for the prediction of load bearing capacity and crack propagation in advanced high strength dual phase steel DP-600. The model is dedicated to future FE simulations of spot weld/bonded joint where a structural epoxy based adhesive optimized for crash situations and developed by SIKA AG will be used. Spot weld is the most common assembling technique in sheet metal joint, especially in automotive industry. The spot welds behavior, such as load bearing capacity, energy absorption, play a significant role in spot weld/bonded assembly sheets components. Therefore, the fracture prediction of single spot weld is indispensable for a whole car crash simulation. In spot welds, the heat affected zone and the fusion zone show different materials behavior due to the heat history of welding process. Hence, a detailed modeling including heat infected zone and fusion zone was developed. Corresponding flow stress scaling factors and Gurson model parameters are calibrated by inverse problems method in each zone. The model can successfully predict the location of initial crack and the load bearing capacity. The results can be directly used for calibrating spot weld/bonded model and simply connector element model in Abaqus code for a whole car crash simulation.

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# Nitsche Type Method for Handling the Interface Conditions in Equations of Elasticity

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This work deals with the crack problem simulation in dissimilar material. We propose a new numerical approach, based on Nitsche's variational formulation, which handles the interface conditions in the Navier-Lame equations between two or more subdomains, characterized by different material properties. The classical Nitsche formulation [2] was introduced several years ago to impose weakly essential boundary conditions in the scalar Laplace operator. Then, it has been worked out more generally to several physical fields and particularly to the Maxwell equations [1]. In order to obtain the Nitsche type variational formulation for the problem with dissimilar elastic material, we consider the deformation of axisymmetric domain  $\Omega_1 \cup \Omega_2$  which interpreters the dissimilar elastic plate, with initial crack  $C_1 \cup C_2$  and which is perfectly jointed on the interface  $B_2 \cup T_1$ . The plate is loaded by the opposed surface forces at  $B_1$  and  $T_2$  in the vertical directions, and fixed on  $R_1 \cup R_2$  (see Figure 1). The problem is defined as follows, for  $i = 1, 2$

$$\begin{cases} -\operatorname{div} \mathbb{S}_i = \mathbf{F}_i & \text{in } \Omega_i \\ \mathbf{u}_i = 0 & \text{on } R_i \\ \mathbb{S}_i = 0 & \text{on } C_i \cup L_i \\ \mathbb{S}_1 \cdot \mathbf{n} = \mathbf{G}_1 & \text{on } B_1 \\ \mathbb{S}_2 \cdot \mathbf{n} = \mathbf{G}_2 & \text{on } T_2 \end{cases} \quad (1)$$

where  $\mathbb{S}_i$  are the stress tensors,  $\mathbf{u}_i$  are the displacement field vector and  $\mathbf{F}_i$  is an acting force vector. Considering the interface between two sub domains  $\Omega_1$  and  $\Omega_2$ , we have to define the appropriate boundary *interface* conditions. The *interface* conditions refer to problem configuration, where the both of the half plates are perfectly jointed on the interface. In this case the interface transmission conditions refer to the fact that there is no shear stress on the interface and the displacement fields  $\mathbf{u}$  with the radial stress field  $\sigma_r$  are continuous across the interface between the sub domains  $\Omega_1$  and  $\Omega_2$ . The numerical results satisfy the interface conditions, where

the discontinuity in axial stress appears as a consequence of the dissimilarity of the material and at the opposite, the radial stress  $\sigma_{rr}$  is continuous through the interface. An example is shown in Figure 2.

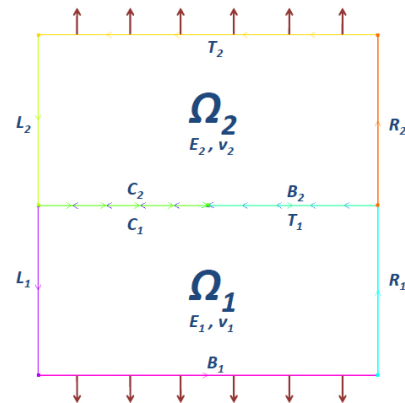


Figure 1: Cracked Domain - Dissimilar elastic plate.

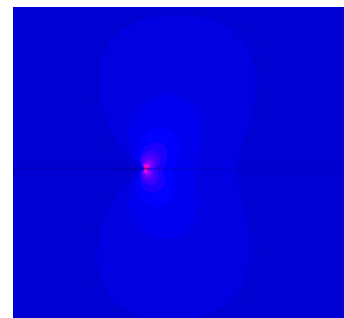


Figure 2: Stress field  $\sigma_{rr}$ .

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## Correlation of the Evolution of Cracks of 7075-T651 and Surface Integrity Caused by Various Machining Manufacture Processes

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Scratches and residual stress are unavoidable to appear on the surface of thin-plate structures in the machining manufacture process [1]. They may lead to the nucleation and growth of cracks when the structures undergo the cyclic load [2]. It is very important to investigate the correlation between the cutting parameters and the evolution of cracks. In this study, the surface integrity including the roughness and residual stress for Aluminum alloy 7075-T651 is discussed firstly considering various cutting parameters. Then, the Rios model is modified in which the effect of the residual stress is considered. It is used to study the damage evolution when the crack size is less than the size of grain [3]. The Walker model is used to study the evolution behavior of crack when its size is greater than the grains [4].

The depths of scratches are compared when the cutting speed are at 400m/min, 600m/min, 800m/min and 1000m/min. The results show that the fluctuation of depths of scratches is not great at the low cutting speed. However, the depth of some scratch is obviously much bigger than the others at the high cutting speed. The residual stress is mainly compressive stress.

The nucleation and growth of cracks are numerical studied by using the modified Rois model and Walker model in terms of the depths of scratches. At the low cutting speed, the difference of the depths of cracks is not obvious when the structure undergoes the cyclic load. Most of the cracks grow slowly. It is good for the damage tolerance of structures. However, some crack grows fast for the structure which was cut at high speed when it undergoes the same cyclic load. The depth of some crack is greater than the others. It is a bad case for the damage tolerance of structures.

The residual stress has obviously influence on the nucleation and growth of cracks. The growth rate increases nonlinearly when the residual stress varies from compressive stress to tensile stress. The

correlation between the lengths of cracks and initial depths of scratches and residual stress when the structure undergoes the same cyclic load is given. The corresponding mathematic description on this correlation is developed for the Aluminum alloy 7075-T651. It can be used to predict the lengths of cracks

Therefore, the choice of cutting parameters is directly dependent on the quality of the surface. The depths of scratches will lead to the nucleation and growth of the cracks when the structure undergoes the cyclic load. The residual stress is important for the evolution of cracks. The correlation between the evolution of cracks and surface integrity due to the machining manufacture can be used in the prediction of the damage tolerance of structures.

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## **Influence of Marginal Thickness and Convergence Angle of Taper Abutment on Resin-Bonded-Glass Models**

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In dentistry, many preparation guidelines for all-ceramic crowns are still based upon clinical experience and those developed for metal ceramic crowns that may not be applicable and need further experimental and numerical evaluation. Some specific features are requiring investigation in dental prosthetic devices, for example, configurations of the crowns' margin [1], marginal thickness [2, 3] and degree of convergence angle [4]. The goal of this study was to determine the influence of marginal thickness and taper abutment convergence angle on fracture resistance in resin bonded glass models. Extended finite element method (XFEM) has been adopted to examine models with variations of marginal thickness (0.8-1.2 mm), convergence angle (6-12°), and two cement methods (partially bonded and non-bonded). It has been shown that XFEM enables to model crack initiation and growth under the assumptions of linear elastic fracture mechanics (LFEM) [5].

The marginal thickness had a significant influence on the fracture load for a 0.4 mm variation of marginal thickness: approximate 240 N difference for partially bonded (PB) models with low elastic modulus material property of resin, 270 N difference for PB models with high elastic modulus resin, 630 N difference for non-bonded (NB) with low elastic modulus material models, and 730 N difference for NB with high elastic modulus resin. The taper abutment convergence angle had a lesser significant influence on the fracture load with a 6 degree variation: an average 66 N difference for PB with low elastic modulus resin, 120 N difference for PB with high elastic modulus resin, 174 N

difference for NB with low elastic modulus material, 300 N difference for NB models with high elastic modulus.

The models with a thicker in marginal thickness, a smaller taper abutment convergence angle, and partially bonded cement method are recommended to increase fracture resistance according to the XFEM fracture analyse investigated.

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## Improvement in the Fracture Numerical Simulation for Coupled CFD/CSD Blast and Impact Problems by Using Inter-Element Stabilization

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In this work a stabilized large deformation element suitable for real coupled fluid/solid simulations is presented. The element uses a mixed interpolation (Q1/P0): Standard continuous tri-linear finite element (FE) functions for the kinematic variables (displacements, velocities and accelerations), and a constant pressure per element (piecewise discontinuous pressures). It is well known that this type of element may show spurious pressure modes (chessboard mode) when is used to approximate incompressible fields (i.e. plastic flow, incompressible fluids, etc.). The mathematical explanation for such a behavior is the element inability of fulfilling the BB condition (the element is not div-stable). However, in Codina et al. [1], the P1/P0 element is stabilized by means of a variational multiscale method (VMS), and it is used to solve the Stokes problem (incompressible flow equations at very low Reynolds number).

Following the ideas of the cited reference, the authors of this work added to the standard large deformation Lagrangian FE (Galerkin) formulation, a stabilization contribution which is only evaluated over the inter-element boundaries. Such a term enforces in a weak manner the pressure continuity and, in that way, it adds control over the inter-element pressure jumps (in general this procedure may be used to stabilize elements with discontinuous pressures). The method is clearly consistent since at the continuous level the pressures are continuous, and the new term just enforces such a continuity at the discrete level. It is shown through numerical-experimental comparison that this stability improvement is essential to obtain smooth localization bands and, therefore, a stable fracture scheme.

The stabilized IEOSS-Q1/P0 solid element (Inter-Element Orthogonal Subgrid-Scale Stabilized Q1/P0 element) was embedded into an efficient finite element FSI (Fluid Structure Interaction) algorithm to deal with large deformation coupled blast and impact problems (See [1] for details).

Other ingredients of the formulation are: Several phenomenological material models (i.e. K&C concrete model [3], Johnson-Cook metal model [4], etc.) to compute the damage and fracture of reinforced concrete and steel structures, a general contact algorithm which uses bin technology to perform the node-face searching operations in a very efficient manner, and a cracking procedure to deal with the topology changes due to crack propagation and fragment formation.

Finally, it is important to mention that all the schemes, contact included, have been fully parallelized and coupled using a loose-embedded procedure with the well-established CFD (computational fluid dynamics) code FEFLO. Several real 3D coupled CFD/CSD cases with experimental comparison are presented to validate the scheme.

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**Minisymposium PFC:**  
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## Consistency of Phase-Field with Sharp Crack Evolutions in Brittle Fracture

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In the last decade the *phase-field* approach to fracture has gained a vast popularity. It is indeed an effective way to represent general cracks within an analytically and computationally convenient setting. The range of application of phase-field fracture is getting larger and today, beyond the quasi-static case, it includes dynamics and complex patterns in mixed-mode.

In the phase-field setting crack paths and crack surfaces are represented *indirectly* by means of an auxiliary variable, named the phase-field variable (the origin of this approach is indeed in the theory of phase transitions). Loosely speaking, this variable provides a *smeared* representation of the crack.

In particular both the elastic field and the phase-field function are continuous across the crack. It is not so in the *sharp* crack approach, where the fracture is modeled *directly* by means of curves (in the planar case) and surfaces (in the 3D case).

A "virtual" crack, defined with the aid of an auxiliary function, has several advantages: it is indeed possible to represent very general patterns, featuring kinks, bifurcations, echelon arrays etc. On the other hand, the phase-field variable typically requires a large number of degrees of freedom which lead to mesh refinement and adaptivity.

It is important to stress that phase-field models introduce an "artificial" length, say  $\epsilon$ , in the model. There is no clear physical meaning for this parameter, which should be understood, together with the regularizing term, as the size of the *smeared* representation of the crack.

It is well known that as the length  $\epsilon$  vanishes, the phase-field energy provides an approximation (in the sense of  $\Gamma$ -convergence [3]) of the sharp crack energy [1]. However, this argument is not enough in fracture mechanics, since the propagation of the crack is rather driven by the *energy release rate*, which is a *derivative* of the elastic energy. In general convergence of an energy does not imply convergence of its derivatives and further convergence

of derivatives does not imply convergence of the associated quasi-static evolutions. It is therefore interesting to investigate the relationship between the crack evolutions obtained within the sharp crack and the phase-field approach. This question has been addressed numerically, for a straight crack under tension [2], and theoretically, for the energy release on a smooth path [5]. In this direction our goal is to provide a rigorous connection between the phase-field evolutions and the sharp crack evolution, proving [4] the convergence of *energy*, *energy release* and *quasi-static evolutions* as the internal length  $\epsilon$  vanishes. To this end we consider the benchmark case of a straight crack together with the Ambrosio-Tortorelli [1] phase-field energy with an explicit choice of the field function.

To pursue this task it is technically convenient to have a new representation of the energy release, which allows to easily pass from the phase-field to the sharp crack setting.

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# A Variational Approach to the Numerical Simulation of Hydraulic Fracturing

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The growing controversy surrounding hydraulic stimulation (“fracking”) highlights the need for a better *predictive* understanding of crack propagation in *complex situations* potentially involving many interacting cracks propagating along unknown paths and interacting with pre-existing fracture networks. Indeed, recent observations challenge common assumptions in the numerical simulation of hydraulic fracturing where propagation is assumed planar and perpendicular to the minimum reservoir stress, itself aligned with the simulation grid.

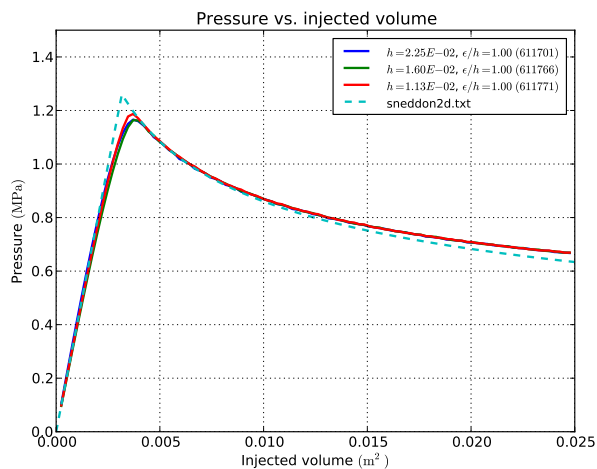


Figure 1: Injection pressure as a function of the injected volume. Closed form solution and numerical experiments

In this study, we follow the variational approach to fracture [3, 2], which we extend to account for hydraulic stimulation. The strength of this approach is to provide a rigorous and unified framework accounting for new cracks nucleation, existing cracks activation, and full path identification. Our numerical approach is based on a regularized model where fractures are represented by a smooth function which can be modified to account for pressure forces along crack boundaries, similar to that

in [1].

We study a proof of concept problem where pressure is assumed constant throughout the fracture system. For simple geometries, a closed form solution can be derived and compared with numerical simulations. We present experiments highlighting the ability of this approach to deal with interacting cracks.

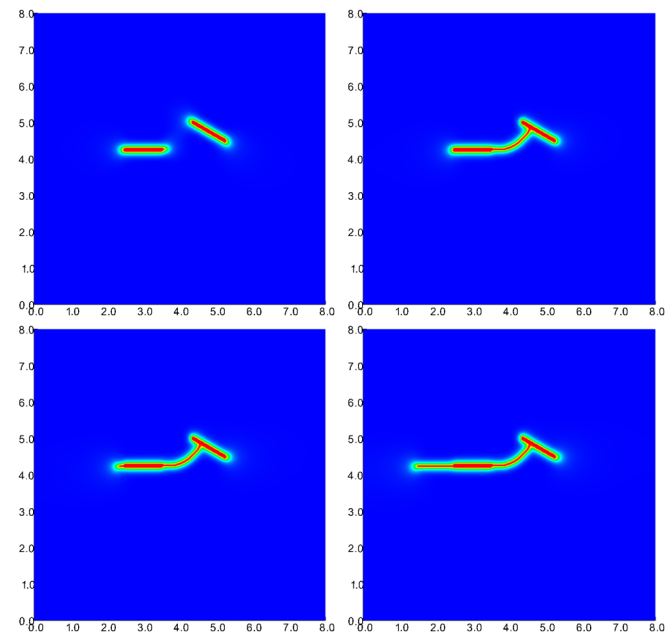


Figure 2: Interaction between two pressurized cracks.

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## Crack Kinking in Brittle Materials

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Although they are rooted in the mechanics of solids and in its history, most crack propagation criteria reveal incorrect safe equilibrium domains. With the obvious exception of the Maximum Energy Release Rate, all of them show unacceptable energy dissipation in mixed mode crack growth. They convey important physical information and allow a degree of freedom in material modeling, in the same spirit of the elastic limit surfaces in the mechanics of materials. Keeping such a degree of freedom in modeling brittle material behavior while preserving the mandatory requirements in terms of energy and of safe equilibrium domain is investigated in the light of a recently proposed incremental description of (linear elastic) fracture mechanics [1, 2].

Furthermore, a sheet of material is considered. It is loaded by uniaxial tensile stress and contains a random distribution of flaw orientations, with the flaws thought of as flat pre-cracks of comparable length, and with all crack planes being oriented perpendicular to the faces of the sheet. Intuition suggests that the most likely flaw to initiate fracture, which will be termed the “most dangerous defect”, lies orthogonally to the major load axis. The second purpose of the present paper is to show that such an assumption is incorrect [3]. The “most dangerous defect” will not be oriented perpendicular to the stress direction, and the first increments of crack growth will also not be oriented perpendicular to the stress direction (nor will they be co-planar with the orientation of the most critical flaw).

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## Numerical Simulation of Fracture in Viscoelastic Materials Based on Material Forces

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The crack growth mechanism of elastomers is of great importance and interest in engineering applications, however, the correlation between numerical and theoretical studies is not well established due to the complexity of the problem. The contribution presents an r-adaptive crack propagation scheme for the description of the viscoelastic fracture response of rubber-like materials at large strains. A similar algorithm is proposed by Miehe et al. [1]. The approach is extended for a generalized finite inelastic continuum by Kaliske et al. [2]. Key feature of this procedure is restructuring the overall system by duplication of crack front degrees of freedom based on minimization of the overall energy via the Griffith criterion. Use of the presented framework enables to study fracture behaviour of elastomers at different deformation rates. The experimental evidence from previous studies favors that the fracture toughness of non-strain-crystallising elastomers shows strong rate-dependency and the energy release rate versus the rate of tearing or crack propagation relation appears to be a fundamental material property [3]. Therefore, in this contribution, a dynamic fracture criterion, which is a function of the rate of crack growth, is shown to be more adequate in numerical simulations.

In addition, in previous studies, it is shown that the fracture energy per unit area of crack advancement appears to be the result of two contributions in terms of the change in elastic energy and in terms of the viscous dissipation by a configurational change [4]. In other words, the elastic part is the intrinsic strength of the interface which does not depend on the crack growth rate, whereas the second part, which reflects energy dissipated by viscosity, is a function of the crack growth rate. The separation of the fracture energy is obtained by the global energy momentum balance. To this end, a consistent thermodynamic framework for the combined configurational motion in viscoelastic continua at the finite strain regime is discussed. For the sake of simplicity, all results are obtained neglecting thermo-

mechanical phenomena and inertia effects. In addition, branching instability triggered by a significantly increased deformation speed is investigated. A crack speed limiter, which is a constant branching velocity, is used in order to explain the phenomenon in elastomers.

The Bergström-Boyce model is considered to introduce hyperelastic and finite viscoelastic behaviour of rubber-like bulk material. The crack driving force and the crack direction are predicted by the material force approach. The predictive capability of the proposed method is demonstrated by representative numerical examples. In conclusion, experimental and numerical results are discussed in order to clarify the relation between the tearing phenomenon and material force approach in viscoelastic materials.

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## Variational Models for Crack Growth

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We present an overview on variational models for evolution problems in fracture mechanics in the framework of Griffith's theory [4]. The variational approach to quasistatic crack growth was introduced by Francfort and Marigo [3]. One of its most important features is that the crack path is a priori unknown. Typically the solution is obtained through an approximation procedure based on time discretization. The approximate solutions solve suitable incremental minimum problems. The first complete mathematical analysis of a continuous-time formulation of such a model in the case of antiplane linear elasticity was given in [2]. Since then variational models have been extensively studied and proved to be suited for many applications.

Much attention has been lately devoted to models that allow following the evolution of local minimizers of the energy. For instance, in [6, 7] we consider the antiplane case and propose a model based on the method of vanishing viscosity, which enforces local minimization and provides also an energy equality. We are interested in an irreversible quasistatic evolution, that is, we want to find an increasing family of cracks such that at each instant Griffith's criterion holds. Under suitable regularity assumptions on the crack, this criterion can be expressed in terms of the energy release rate, i.e., the opposite of derivative of the elastic energy with respect to the crack's elongation. Griffith's criterion [4] states that

1. the crack must increase in time,
2. the energy release rate cannot exceed a threshold depending on the material's toughness,
3. the crack can grow only if the energy release rate equals that threshold.

To prove the existence of such an evolution we need the class of admissible cracks to be sequentially compact (with respect to a convergence of sets) and the energy release rate to be (lower semi-) continuous with respect to the same convergence. These two properties are satisfied for instance by a suitable

class of  $C^{1,1}$ -regular cracks, with a uniform bound on the curvature. In [5] we proved the relation between the stress intensity factor and the energy release rate under this regularity assumption.

Let us mention also that in [1] the existence of the stress intensity factor is proved for non-smooth fractures, and in [8] the class of admissible fractures allows for branching and kinking.

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## A Variational Algorithm for Crack Evolution in Plane Problems

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The analysis of fracture evolution in computational mechanics requires that to the continuum model be added a) a discontinuity in the displacement field and b) an algorithm for detecting the fracture path. It is well known that such a problem is functional set in the space  $BD$ , so that usual interpolation criteria are not sufficient to guarantee convergence of the numerical algorithm.

A successful approach to the FE modeling of discontinuities is the so called Strong Discontinuity Approach that introduces enhanced interpolation functions in the elements able to model a displacement jump. Usually the direction of the discontinuity line is defined by some criterion based on the stress state at the Gauss points, and the discontinuity crosses entirely the element [1, 2, 3]. Stress locking can arise as a consequence, so that fine meshes are required. Moreover, mesh independency is still a matter of discussion in many cases [4].

A celebrated variational model for fracture mechanics has been provided by Francfort and Marigo [5] that includes as result of the optimization process the crack path. It is based on the introduction of a variational principle that includes bulk as well as surface energy.

Motivated by the previous observations, the paper proposes a new algorithm for fracture evolution applied to plane problems, whose main points are:

1. A variational principle is introduced as an extension of the Hu-Washizu three-fields principle, with the addition of an enhanced discontinuous displacement field and of a properly defined dissipation function on the discontinuity surface.
2. B-splines interpolation is used rather than usual FE. B-splines are not interpolatory except at the boundary points, so that it is easier to introduce discontinuities simply modifying the structure of the knot vector that rules the interpolating functions.
3. The crack path is defined as a multipatch Bspline curve, each patch corresponding to a load step, whose control points are obtained as a solution of

the optimization of the variational principle, as proposed in [5]. Indeed, in this way the variation of the discontinuity path is easily calculated and can be used as an additional unknown of the problem.

The paper describes the main ideas of the method and the results that can be achieved. Problematics regarding its efficiency and generalization are left to future developments of the research.

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## $C^1$ -Continuous Crack Propagation Across Quadratic Elements

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*In anticipation of publication and presentation at COMPLAS 2013, intermediate results will be shown at CFRAC 2013 under the same title.*

Crack tracking algorithms are used in various material models in order to capture localized deformation patterns properly and to obtain mesh objective results. Different types of algorithms exist, each with its own complexity of implementation, computational costs and robustness [1]. Recently, a local crack tracking algorithm was proposed in order to improve the numerical modeling of tension and mixed-mode cracking problems in quasi-brittle materials within the framework of a standard smeared crack approach [2]. The algorithm is relatively simple and inexpensive, and is particularly suitable when using constant strain triangular elements with one-point integration. From the results a clear elimination of mesh-induced directional bias was observed.

In this work a new local crack tracking algorithm is presented, specifically developed for crack propagation in finite element discretizations that consist of higher order quadratic elements with full numerical integration. Generally, these elements perform better in describing more complex deformation modes and failure modes, giving them a recommended position in guidelines for engineers [3]. In addition, quadratic elements serve the formulation of smooth curved cracking paths. The new proposed method determines the crack propagation direction in a finite element by calculating a propagation vector field within that element. This vector field is obtained from the stress states in the integration points and the propagation direction with which the crack track enters the element. By taking into account the propagation direction in the entry point explicitly,  $C^1$  - continuity of the crack path can be obtained.

The new proposed crack tracking algorithm is applied within the context of a smeared crack model

and implemented in the Sequentially Linear Analysis method [4]. The algorithm is validated by means of several fracture tests with different failure modes.

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## PROCRACK - A Software Tool for Finite Element Simulation of Three-Dimensional Fatigue Crack Growth

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The lifetime and reliability of structural components under combined cyclic thermal and mechanical loading are crucially controlled by the occurrence and growth of fatigue cracks. The crack usually starts from an already existing defect and propagates in a subcritical manner until a critical crack size is attained. After that, the crack growth becomes unstable and leads to the failure of the component.

In order to ensure the safety and reliability of structural components, the prediction of fatigue crack growth is a crucial task, which has to be coped with by modern computational techniques. Due to the three-dimensional geometry and the complex loading conditions of cracked components, the finite element method (FEM) is the numerical tool of choice to solve the initial boundary value problems of fracture mechanics.

The program PROCRACK is a Python-based software for the automated finite element simulation of three-dimensional fatigue crack growth in pre-cracked structures [1]. The preprocessing of the computational model and its numerical analysis is performed by Abaqus. Therefore, the geometry of the investigated component and its thermomechanical loading can be arbitrary.

The modeling of the initial crack and the incremental crack extension is carried out exclusively in the Abaqus/CAE environment. The crack is described by its crack area at the geometric level and approximated by triangles. The crack front is simply discretized by geometric points connected by line segments. The fracture-mechanical parameters are computed at this reference points. It is easily possible to form a three-dimensional curved crack path due to the special arrangement of the triangles between the old and new crack front.

PROCRACK contains an algorithm for adaptive control of the discretization of the crack front and the crack area. A new point is created between two

reference points with increasing distance between them. An existing point is removed, if the distance becomes too small. In addition, it is possible to coarsen the fine discretization of the crack area in regions far behind the crack front to reduce the computational effort.

The fracture mechanics analysis is carried out within the framework of the classical linear elastic fracture mechanics. The singular stress field along the three-dimensional crack front is characterized by the stress intensity factors  $K_I$ ,  $K_{II}$  and  $K_{III}$ . The  $K$ -factors are the fundamental parameters for the analysis of fatigue crack propagation and have to be calculated numerically. To this end, PROCRACK uses the submodel technique, which enables much better accuracy. The tube-shaped domain surrounding the current crack front of the component is created as an external part and analyzed separately with the finite element method. The interaction integral is used for calculating the stress intensity factors.

The PARIS law and the NASGRO equation are available for the simulation of the crack growth propagation in PROCRACK. In every adaptive crack growth step, the updated crack front position is computed and the mesh in the crack region is automatically adapted.

Some application examples show the capability and performance of the simulation tool PROCRACK. Due to the use of the open-source computer language Python an easy extensibility and maintenance of the simulation program are ensured.

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## **3D Cellular Automata Finite Element Method With Explicit Microstructure: Modeling Quasi-Brittle Fracture Using Mesh-Free Damage Propagation**

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Quasi-brittle materials have heterogeneous structures, typically with brittle constituents. This important class of structural materials includes concrete, nuclear graphite, ceramic-matrix composites such as SiC-SiC fiber or zirconia toughened alumina, geological structures like rocks and tectonically faulted formations and also bio-medical materials such as bone and bone replacements. Their damage or defect tolerance is much less than engineering metal alloys, but can be quite significant compared to fully brittle materials such as monolithic ceramics. They differ in their length-scales of both their structures and the distributions of damage, and have varying degrees of brittleness.

Quasi-brittle fracture is an emergent characteristic, and this cannot be treated satisfactorily with the numerical methods based on macromechanics. Because of their complex microstructure, the continuum approach can be too simple for these materials, and needs a finer discretization to obtain satisfactory results. In numerical terms, this means that the computational cost of advanced methods, such as cohesive elements or embedded cracks, is often too high for engineering scale problems.

Considering those arguments, in this paper we use the Cellular Automata integrated with Finite Element method to account for the effect of microstructure on quasi-brittle properties within the finite element simulation [1]. Here the microstructure is modeled explicitly by subdividing a finite element into small elements called cells. The heterogeneous microstructure is created from key cells, called seeds, from which particle-like regions may be grown with defined characteristics; by this topological approach we obtain sets of cells with variable properties to model the microstructure (rules are enforced during the selection of the seeds to avoid overlap between particles). Graded microstructures, textures and particle anisotropy can be readily simulated in microstructures with multiple phases. The influen-

ce of the initial finite element mesh is erased during the development of the microstructure.

This method provides two sets of elements representing the finite element model and the microstructure. The first is used to link the engineering scale problem with the microstructure, obtaining the stress and strain fields of the macromechanical problem. With those, we compute the micro-mechanical fields using the second set of elements, which describes explicitly the microstructure. We use a mesh-free approach for the damage development through the microstructure [2]. The material properties of the finite elements are recomputed according to the microstructure damage, hence the redistribution of strain and stress with crack propagation and damage is computed; factors such as the effects of size on crack path and crack stability are therefore addressed. The fracture path is completely free with respect to the finite element mesh. Consequently, very complex fracture behavior can be modeled, such multiple or discontinuous cracks.

In summary, by this method quasi-brittle fracture can develop freely through the microstructure, improving the accuracy and computational cost of the calculations at engineering length-scales in complex microstructures. This work is part of a larger project, which aims to experimentally validate the simulation of heterogeneous microstructures in structural integrity problems.

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## A Modelling Framework for Three-Dimensional Brittle Fracture

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This paper presents a finite element based numerical framework for the predictive modelling of three dimensional crack propagation in brittle solids. The presentation briefly sets out the theoretical basis for determining the initiation and direction of propagating cracks, based on the concept of configurational forces. Attention is focussed on resolution of cracks by the finite element mesh. Cracks are restricted to the element faces and the mesh is adapted in order to align element faces with the predicted crack path. A local mesh improvement procedure is developed to maximise mesh quality in order to improve accuracy and solution robustness and to reduce the influence of the initial mesh on the direction of propagating cracks. The performance of this modelling approach is demonstrated on three numerical examples that qualitatively illustrate its ability to predict complex crack paths. All problems are three-dimensional, including a torsion problem that results in the accurate prediction of a doubly-curved crack. In order to trace the dissipative load-displacement path, fully consistent with the assumption of quasi-static crack propagation, an arc-length scheme is adopted with a control function taken as an increment in change in crack surface area. Finally, the influence of hp-adaptivity is studied and the smoothing influence on the load-displacement response is demonstrated.

The approach taken in this paper is principally based on the principle of global maximum energy dissipation for elastic solids, with configurational forces determining the direction of crack propagation. This has been successfully adopted by a number of other authors, but here we mainly follow the work of Gürses and Miehe [1]. Such an approach for predicting the crack path can be coupled with local r-adaptivity to mitigate the influence of the mesh. In this paper problem tailored mesh improvement technique based on a volume- length quality measure is adopted. The fracture criterion is based on the Griffith force work conjugate to crack area increase. The governing equations are solved monolithically for

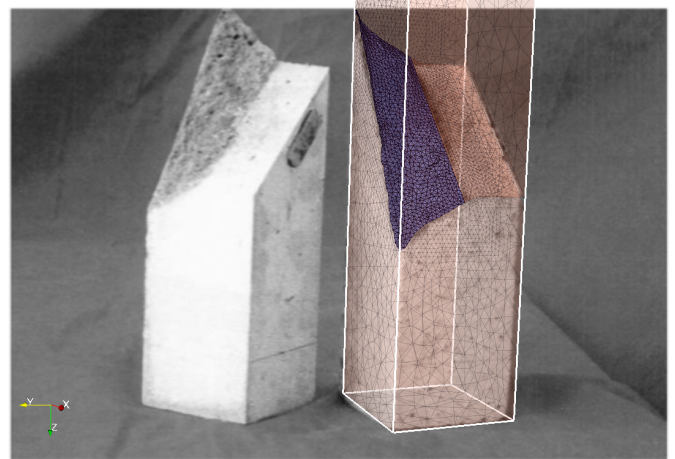


Figure 1: Crack surface and picture of fractured sample under torsion

material and spatial displacements.

### Examples

Three numerical examples are presented for crack propagation in three-dimensions that demonstrate the ability of the formulation to accurately predict crack paths, as well as demonstrate mesh independence and the influence of both mesh adaptivity and controlling mesh quality on the solution obtained.

### Acknowledgements

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## Crack Front Tracking Using Perturbation Approaches

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The crack path depends in general on the geometry of both the specimen and the crack (position, size, shape). Numerical tools based on the resolution of the entire elasticity problem such as finite elements, its more sophisticated X-FEM extension, phase-field methods, variational approaches, etc... encounter a growing success and are able to track the successive positions of the crack in problems of increasing complexity (as this conference will certainly demonstrate!). Nevertheless, when the scales are disparate, it becomes extremely difficult to reconcile the crack microscopic and the specimen macroscopic aspects. One possibility is to simplify the structural aspects and to concentrate on the local deformations of the crack front by assuming that the solid is infinite subjected to remote loading. The 3D perturbation approach initiated by Rice [1] gives the first-order variation of the stress intensity factors arising from a small arbitrary coplanar perturbation of the front and avoids the tedious (and sometimes intractable) calculation of the full mechanical fields. Initially, it was developed for the half-plane crack embedded in an infinite solid. Further, it has been extended to more and more realistic cases [2]. In order to deal with large crack front deformations, Bower and Ortiz [3] followed by Lazarus [4], developed a powerful numerical method based on the iteration of the linear scheme. The efficiency of this method arises from the need for the sole 1D meshing of the crack front.

The aim of my presentation is first to present the originality and limitations of those methods, then to present some results obtained recently concerning the crack front pinning by more tough obstacles [5, 6] and the crack front deformations undergone during the coalescence of several cracks [7]. A challenge for the future is to develop numerical tools which are able to deal accurately with both smallscale deformations of the crack and largescale boundary conditions.

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## Minimum Theorems in 3D Incremental LEFM: Theory and Numerical Tests

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The crack propagation problem for linear elastic fracture mechanics has been studied by several authors exploiting its analogy with standard dissipative systems theory (see e.g. [1, 2, 3]). In a recent publication [4] minimum theorems were derived in terms of crack tip “quasi static velocity” for two-dimensional fracture mechanics. They were reminiscent of Ceradini’s theorem [5] in plasticity.

Following the cornerstone work of Rice [6] on weight function theories, Leblond and coworkers [7, 8] proposed asymptotic expansions for Stress Intensity Factors (SIFs) in three dimensions. As formerly in 2D, expansions can be given a Colonnetti’s decomposition [9] interpretation. In view of the expression of the expansions proposed in [7, 8] however, symmetry of Ceradini’s theorem operators was not evident and the extension of outcomes proposed in [4] not straightforward. Following a different path of reasoning, minimum theorems have been finally derived.

Moving from well established theorems in plasticity, algorithms for crack advancing have been derived. Their performance is here presented within a set of classical benchmarks.

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## Nearly Non-Spurious Oscillations Time Scheme in Finite Element Analysis of Non-linear Wave Propagation and Dynamic Fracture Mechanics

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In this contribution, the near non-spurious oscillations time integration scheme [1, 2] for finite element numerical solution of transient problems in solids [3] is presented and tested in linear and mainly non-linear wave propagation of stress discontinuities in solids [4] and dynamic fracture mechanics [5]. Special attention is paid to numerical solution of propagation of stress discontinuities, wave-fronts interactions (loading and unloading effects) and also problems with reflections due to boundaries. Implementation into the Tahoe program [6], behaviours and properties, accuracy and stability analysis of the numerical method based on the classical (Lagrangian) linear finite element space discretization and the special front shock capturing explicit time scheme are mentioned and commented in details.

The nominated explicit near non-spurious oscillations method [1,2] is used in numerical evaluation of dynamic stress factor intensity for stationary cracks and also in numerical solution of dynamic crack propagation [5], where crack growth is modelled by cohesive type finite element frameworks (for examples, Xu–Needleman model [8], Tvergaard–Hutchinson model [9], etc.). The crack speeds and crack paths are compared with experimental and numerical results of crack propagation under dynamic tension and shear failure, and mainly in asymmetric impact of an edge cracked plate.

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## Complex Crack Patterns: Transverse Fractures and Delamination in Thin Film Systems

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The appearance of complex crack patterns is a peculiar feature of thin film systems undergoing tensile loads. The elastic energy release is as a means to relieve internal stresses and pays for the opening of new free surfaces.

Typically, cracks develop away from strong singularities in smooth domains in a brutal fashion and produce a vast panoply of crack patterns, which, themselves, become fundamental unknowns of the problem. Dealing with cracks which are free to appear anywhere at any time in the sound body, arranged in geometries of arbitrary complexity renders the description and evolution of crack patterns challenging both from a theoretical and numerical viewpoint.

In the framework of variational fracture mechanics [1], a dimension reduction result [2] shows that such systems naturally discriminate between transverse cracking modes (channeling cracks) and interface cracks (delamination). We propose a two-dimensional formulation the fracture problem for thin film systems based on variational approximation [3] of the system. The elastic displacement field and the crack pattern are sought among minimizers a reduced energy with no a priori geometric restriction on the crack pattern. This problem is solved numerically via an elliptic approximation. Cracks, identified by the localization in narrow bands of a continuous scalar field, and debonded regions evolve quasi-statically under increasing tensile loads, minimizing the energy of the system under irreversibility constraint preventing unphysical self-healing. We investigate periodic and sequential appearance of cracks in uniaxial traction tests, extended spontaneous peripheral debonding, geometrically periodic structures in two-dimensional domains and crack evolution in complex domains. The

outcome of numerical experiments is consistent on the mechanical ground and with experimental evidence. Numerical experiments range from regimes of isolated channeling transverse cracks without debonding to that of extensive debonding without transverse fracture. Between the two, strong coupling between the fracture modes exists, revealing the presence of a new internal characteristic length. It is associated to the coupled evolution of parallel cracks connected by debonded regions, which opens the way to the explanation of more complex spiraling and oscillating patterns.

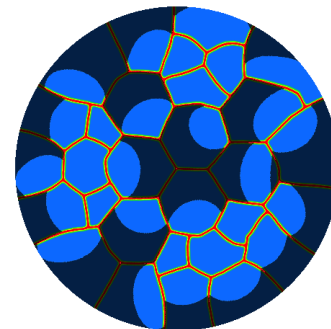


Figure 1: Transverse cracks and debonded areas develop in a disk loaded by tensile inelastic stresses.

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## Overall Elastoplastic Behaviour of a Cohesive Medium

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Cohesive Zone Models (CZMs) have gained much popularity thanks to their promising applications in numerical damage and failure simulations. Their numerical implementation in finite element schemes is based on embedding surface elements between two adjacent bulk elements. The traction between the two opposed elements is linked to the corresponding opening by a cohesive relationship [1]. Despite all the achievements in this field, some mechanical and numerical aspects still remain interesting research topics: the pathological mesh influence, the suitable calibration of the cohesive law, the link between cohesive parameters and the overall response, etc. The overall behaviour of an isotropic and elastic cohesive medium has been studied in [2] by means of micromechanical approaches. The obtained results rigorously provide some answers to the so-evoked issues. Recently, extension with a damageable interface has been done [3].

This present work is an extension of the obtained models to the case of plastic media. It is devoted to the development of the overall behaviour of an isotropic elasto-plastic medium containing elasto-plastic penny-shaped cohesive inclusions randomly distributed in space and in orientation. This configuration corresponds to a representation of cohesive-volumetric discretizations with CZMs embedded along each edges of finite elements.

The homogeneous overall behaviour of the non-linear medium is then estimated using the variational approach of [4] based on the introduction of a linear comparison composite (LCC). A convenient estimate of the LCC properties (Hashin Shtrikman scheme [5]) and an asymptotic analysis lead to a micromechanical plastic model based on the cohesive zone approaches.

The interests of this new model are: 1/ its ability to be applied whatever the loading triaxiality, 2/ its

ability to exhibit the influence of the Poisson coefficient on the overall response.

Moreover, explicit relationships between the local cohesive parameters and the overall material properties are obtained. Rigorous criteria allowing insensitive CMZs to the mesh size are thus derived. These criteria depend on the mesh size, on an elastic tolerance, on the triaxiality ratio and on the overall damageable elasto-plastic behaviour of the cohesive-volumetric medium.

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## Modeling Failure Wave Propagation in Impacted Glass Rods with OTM and Eigenerosion Schemes

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In impact experiments involving slender rods, the stress state of the portion of the specimen in proximity of the impact face varies from one-dimensional strain state to a one-dimensional stress state within a two-diameters zone. The one-dimensional strain state at the impact face is originated by the shock wave that compresses the material behind the wave front which cannot expand due to the confinement of the impacting rod. When the wave front moves further, the lack of lateral confinement allows the rod to expand outwards radially releasing lateral waves. Beyond the transition zone, experiments have revealed the evidence of failure occurring behind a traveling boundary (failure wave) that follows the shock front [1]. Such a boundary resembles a phase boundary which separates an essentially intact material ahead of the wave front from a comminuted material behind the wave front. The failure is observed to be explosive in nature, leading to apparent radial expansion as comminuted material moves outwards [1].

Starting from the '90s, the experimental observations of impact damage in brittle materials have inspired a considerable theoretical and numerical modeling activity. Earlier numerical models were based on continuum damage theories describing the degradation of the material stiffness and strength as the result of inelastic deformation caused by microcracking. More recently, numerical simulations have been using widely cohesive fracture models, based on explicit representation of cracks nucleation, propagation, and coalescence of discrete cracks. Alternative approaches are based on molecular dynamics and meshfree methods.

Among other meshfree approaches, the Optimal-Transportation MeshFree (OTM) method of Li et al. [2] is particularly suitable to be combined with the eigenerosion approach to fracture [3, 4]. In OTM, spatial discretization is based on material point sam-

pling and integration and maximum-entropy (max-ent) interpolation, entirely defined by the current nodal set positions.

In the present work, we aim at simulating the bar to bar impact experiments on glass documented in [1] by using the capabilities of the OTM–eigenerosion method [2, 4]. In particular, we show that the method is able to describe with accuracy the structure of the shock and failure waves observed in the experiments. The numerical simulations are also able to predict the actual speed of the failure wave and its dependency on the magnitude of the impact pressure.

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**Minisymposium PND:**

**Phase-Field and Nonlocal Damage Approaches  
to Fracture**

*Organized by Christian Miehe and Nicolas Moës*



## Phase-Field Modeling of Brittle Fracture: Geometrically Complex Crack Paths Beyond the Principle of Local Symmetry

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During the last decade, the phase-field method has emerged as a powerful method to simulate crack propagation in different geometries and loading conditions [1]. This method is rooted in continuum models of phase transformations, which exploit a coarse-grained scalar order parameter  $\phi$  to distinguish between different thermodynamic phases (e.g. solid and liquid in the solidification context where the term “phase field” was first coined). In a fracture context,  $\phi$  has been re-interpreted as a phenomenological measure of damage [2], which varies smoothly in space between two values corresponding to the intact and broken states of the material. Furthermore, coupled dynamical equations for the phase and displacement fields can be derived variationally from an energy functional with both elastic strain and surface energy contributions. This approach incorporates both the short scale physics of materials failure and macroscopic elasticity within a set of self-consistent of equations. In addition, those equations scan be simulated on massively parallel computer architecture to describe geometrically complex dynamical phenomena such as crack nucleation, crack kinking and branching, and crack-front segmentation in three dimensions.

This talk will discuss insights gained from the mathematical analysis of the phase-field model of brittle fracture [3] and three-dimensional (3D) simulations of this model under the superposition of tensile and anti-plane shear loading (mode I+III) [4]. The model analysis allow us to relate quantitatively the phase-field approach to the standard continuum theory of linear elastic fracture mechanics (LEFM), which describes crack propagation in terms of a Griffith-like energetic condition ( $G = G_c$ ) and the principle of local symmetry (PLS), which assumes that smooth cracks in isotropic media propagate with a vanishing mode II stress intensity factor ( $K_{II} = 0$ ). The analysis of the phase-field model

provides an explicit derivation of a crack propagation law, which reduces to the PLS in isotropic media and can be extended to a condition for a finite  $K_{II}$  in anisotropic media. The phase-field simulations of mode I+III cracks reveal that planar crack propagation is linearly unstable against helical deformations of the crack front, which evolve nonlinearly into a segmented array of finger-shaped daughter cracks. Furthermore, facet coarsening is found to be strikingly analogous to the coarsening of finger patterns in nonequilibrium growth phenomena. Those results are discussed in the light of a subsequent analysis of this instability in the framework of LEFM [5]. An important conclusion is that the maximum growth rate of this instability becomes arbitrarily large on arbitrarily short scale if one uses the PLS. Hence, the PLS generally needs to be modified to regularize this unphysical ultraviolet divergence by including the short scale stabilizing effect of cohesive forces.

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## On the Interpretation of Parameters in Phase Field Fracture Models

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In phase field fracture models, the crack set of a fractured body is approximated by the zero set of the phase field order parameter. This scalar crack field interpolates smoothly between broken and undamaged material, where the crack field takes the values zero and one, respectively. By means of a degradation function, the crack field is coupled to the elastic stiffness tensor of the material in order to model the change in stiffness between broken and undamaged material. The width of the transition zone between broken and undamaged material is controlled by a length parameter  $\epsilon$ , so that sharp cracks are recovered in the limit  $\epsilon \rightarrow 0$ . Several issues connected with the interpretation of the crack field and the parameters of the phase field model, as well as the impacts of different formulations of crack evolution laws are addressed in this work. The study comprises analytical considerations as well as finite element simulations of the phase field model.

In [1], the crack evolution is obtained by a global minimization of the total energy of the phase field model, which is a function of the strain tensor and the crack field. However, this ansatz is plagued by certain difficulties concerning the handling of traction loads or volume forces and the occurrence of unphysical size effects, see [2]. More commonly, the crack field is assumed to follow a Ginzburg-Landau type evolution equation, which must be modified properly in order to consider the irreversibility of fracture processes, see e.g. [3] or [4]. As long as local and global minimizers of the phase field energy functional coincide, both strategies render the same crack evolution. However, different crack evolutions may be observed, if this is not the case.

From a conceptual point of view, phase field fracture models are very similar to gradient damage models. The main differences lie in the interpretation of the damage or crack field and the intrinsic length scales and in the formulation of appropriate irreversibility constraints. In damage models the damage field describes the development of micro cracks and voids in a homogenized macro-

scopic sense. The characteristic length scale of a gradient damage model is anticipated to be connected with the maximum size of material inhomogeneities. In contrast, the crack field and the regularization length of a phase field fracture model are generally regarded as purely auxiliary quantities, which approximate the sharp crack limit.

From another point of view, the role of the regularization parameter is interesting, because it affects the maximal stress response in the phase field model with Ginzburg-Landau evolution equation, before the onset of fracture. Thus, in conjunction with the other material parameters, the length parameter defines a fracture strength in the phase field model, which is able to reproduce crack nucleations in originally undamaged material.

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## Using Massively Parallel Processors to Simulate Crack Propagation Problems with the Phase-Field Formulation

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Recently, the regularized variational fracture framework [1], or termed the phase field approach [2, 3], has emerged as an efficient method to solve fracture problems with complicated crack patterns. Phase field models for fracture employ a continuous field variable to indicate cracks. The ideal crack with zero thickness is represented with a smooth field characterized by a regularization parameter.

The numerical implementation of such models is sensitive to the choice of this parameter in conjunction with the mesh size, as the mesh has to be fine enough to resolve high gradients of the crack field appearing in the transition zones. This is one of the main computational limit and challenge of the implementation.

Recently, graphics processing units (GPUs) have had great success in accelerating many numerical computations. Modern GPUs bring remarkable computational performance at a fraction of the cost and power consumption in an old-style cluster of central processing units (CPUs). While a CPU core performs a single instruction at a time, a GPU can execute hundreds. Therefore, GPUs are ideally suited to data-parallel computations with high arithmetic intensity.

Computations on an unstructured grid follow a typical pattern in which data is first gathered, calculations are performed, and the resulting data is reduced and scattered. Therefore, two key challenges result from such an algorithm: the amount of data to be read and written to global memory is typically quite large in comparison with the amount of floating point operations to perform, and the reduction step is made difficult by the concurrent execution of many threads on the GPU. However, it has been shown that in fact excellent performance can still be obtained from the hardware.

In this presentation, we discuss the use of a single GPU to accelerate the explicit integration of the rate-dependent phase-field model [2, 3] using a

standard finite element method (FEM) on unstructured meshes.

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## A Computational Study of Fracture in Multilayer Ferroelectric Actuators

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Multilayer ferroelectric actuators are ideal candidates for numerous applications in smart structures and adaptive systems in view of their small sizes, low driving voltages and quick response times. Applications include microprecision cutting machines, inkjet printer heads, laser printers, optical disk drives and laser tuning, to mention a few. However, the inherent brittleness of ferroelectrics is a serious obstacle to their reliable operation in devices. In addition, the structures of multilayer actuators often employ internal electrodes, which terminate inside the ferroelectric ceramic. In the vicinity of each terminated electrode edge, a non-uniform electric field is induced by the driving voltage, producing an incompatible strain field. A complex stress field arises from this incompatibility, which may lead to crack initiation and propagation around the electrode edge. Therefore, it is necessary to understand the fracture behavior of multilayer ferroelectric actuators to assure optimum reliability of the systems and guide the design. Previously, the fracture simulations were performed to analyze the electromechanical fields near the electrode edge. Based on this analysis, some design criteria can be proposed regarding the geometry of the actuators and electrodes to reduce the probability of fracture from the electrode edge. However, these models do not study the crack propagation mechanisms of the actuators and these mechanisms are still unclear due to the complex interactions between the propagating cracks, electromechanical fields and microstructure of the material near the electrode edge. We have recently introduced phase-field models for the coupled microstructure and fracture evolution in ferroelectric single and polycrystals [1, 2, 3, 4]. The potential of this phase-field approach to capture the complex interactions between the crack and the material microstructure motivates us to employ it for the fracture analysis of multilayer ferroelectric actuators. In particular, the objective of this paper is to study the crack initiation at the electrode edge during the poling process. Considering different

bonding conditions between the ceramic and electrode layers, different crack initiation patterns are obtained, which are useful to understand the fracture processes in this type of actuators. Three extreme conditions are considered, which are the fully cofired, partially cofired, and separated layers. The crack initiation patterns can be either delimitation along the electrode-ceramic interface or oblique cracking from the electrode into the material. The calculations suggest a mechanism explaining the experimentally observed crack branches near the electrode edges. The effects of the ceramic layer thickness and length of the internal electrode on the crack initiation are also evaluated [5].

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## Phase Field Modeling of Brittle and Ductile Fracture at Finite Strains. Formulation of Failure Criteria and Multi-Physics Extensions

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The computational modeling of failure mechanisms in solids due to fracture based on sharp crack discontinuities suffers in dynamic problems with complex crack topologies including branching. This can be overcome by a diffusive crack modeling based on the introduction of a crack phase field as proposed in [1, 2, 3]. Recently, we extended these models of *brittle* crack propagation towards the analysis of *ductile fracture* in inelastic solids undergoing finite strains, including thermo- and chemo-mechanical coupling scenarios. In particular, we propose a formulation that is able to predict brittle-to-ductile failure mode transitions in *metals* and *polymers* under dynamic loading at finite elastic-plastic strains. The proposed model is able to reproduce classical impact tests of metal specimens, which show brittle-to-ductile failure mode transition for increasing impact velocity. Similar failure mode transitions occur in glassy polymers due to the competition of shear yielding and crazing.

We outline a thermodynamically consistent framework for continuum phase field models of crack propagation in brittle elastic and ductile elastic-plastic solids and consider their robust numerical implementations by multi-field finite element methods. We start our investigation with an intuitive and descriptive derivation of a regularized crack surface density function. This function provides the basis for the construction of suitable dissipation functions, which govern the degrading stress response in ductile materials, the evolution of plastic strains and temperature as well as the crack phase field.

We then introduce local history fields which contain suitably defined crack sources based on *alternative failure criteria*. It is shown that these local variables drive the evolution of the crack phase field. The introduction of the history fields inspires the construction of extremely robust operator split schemes of phase-field-type fracture which successively update in a typical time step the history fields, the crack

phase field and finally the deformation, temperature or concentration fields of typical multi-physics applications. The performance of the new phase field formulation of fracture is demonstrated by means of representative simulations of failure in metals, rubber and glassy polymers.

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## Advanced Nonlocal FE Modelling to Assess Crack Properties

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In engineering practice, the durability of concrete structures is usually, as in European standard (EC2) dealt with crack opening and crack interdistance in order to limit the corrosion risk for rebars due to the penetration of chemical agents coming from the environment of the structure.

Regarding numerical modelling of concrete structures, two main classes of approaches do exist. On one hand the discontinuous ones such as X-FEM are based on crack propagation criteria for which the crack opening is part of the constitutive law. On the other hand, continuous ones such as damage models are capable to simulate both the initiation and the propagation of cracking. However they do not provide any information on crack properties (path and opening) and they usually suffer from mesh dependency if not properly regularized.

An advanced non local approach has been recently developed by Giry et al. [1] to overcome the main drawbacks of the standard method, i.e. bad estimation of the location for the damage initiation close to a notch tip, attraction of the damage field by boundaries and spurious damage diffusion at complete failure. These authors have proposed to account for the distant stress field in the nonlocal interactions. Since they are reduced to zero across a crack at complete failure (null stress), the strain localises in a single finite element that is more realistic. However the dissipated energy is mesh-independent due to the regularisation effect prior to the failure.

Besides Dufour et al. [2] and Bottoni and Dufour [3] have proposed two methods to estimate the crack path from continuous fields. The latter one is based on the 2D topological search of the ridge of a field characterising the crack. If the damage field from the original non local approach was the natural candidate, it has not been possible to use it. Indeed the diffusion of damage yields a plateau with a unit damage from which it is not possible to extract the maximum value (i.e. the ridge).

Finally a method has been developed by Dufour et al. [4] to estimate the crack opening by equalling the convoluted product of the strain issued from a FE computation and the one from an analytical strong discontinuity approach. Part of the estimation error finds its root in the shape of the non local strain field across the crack. For this reason, the equivalence is originally performed in average only since local values provide a poor estimation.

In this contribution, we investigate the improvements that the stress based non local regularisation provides to the estimation of the crack path and the crack opening. The latter one is compared to experimental results measured by digital image correlation on a three-point bending concrete beam. Since both the strain field and the damage field do not spread over the domain upon failure, the first one is localised in a single element and the latter one presents a ridge with a maximum value. Therefore, the damage field can be used to estimate the crack path and a local equivalence is found with respect to strong discontinuity approach to accurately estimate the crack opening.

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## A New Combined Strategy for Modelling Failure: From Smoothed Displacements to Cohesive Cracks

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To achieve an accurate description of a whole failure process, combined strategies coupling the two traditional approaches (damage and fracture mechanics) can be used. This work addresses a new contribution in this direction: a non-local continuum damage model with smoothed displacements is combined with cohesive cracks thus allowing the entire description of failure processes.

To describe the first stages of the failure process, a damage model is employed. To avoid the characteristic problems of local descriptions such as mesh-sensitivity, a gradient-enhanced model [1] with regularised displacements is used. As analysed in [2], different boundary conditions may be prescribed for this non-local field. Here, combined boundary conditions are imposed: Dirichlet boundary conditions are prescribed for the normal component of the displacement field whereas non-homogeneous Neumann boundary conditions are imposed for the tangential ones.

To describe the last stages of the process, when the material is physically separated, this regularised description is combined with cohesive cracks. When dealing with this transition (from regularised damage to evolving cracks) different difficulties may arise. In this work, focus is in the way as this transition is carried out.

On the one hand, the properties of the cohesive crack should be defined. In order to estimate the fracture energy, and thus, of the crack stiffness, a new methodology is used here. The main idea is based on the assumption that the energy which would be dissipated by a continuum approach is conserved if a combined strategy is employed, see [3]. In order not to overestimate the transferred energy, here we propose to take into account the unloading behaviour of each point across the damage band.

On the other hand, the crack-path direction should be defined. Since linear elastic fracture mechanics

cannot be used in a regularised continuum, a new criterion is used here: the evolving crack advances following the direction dictated by the medial surface [4] of the regularised damaged profile. That is, a geometric tool widely used in image processing and computer vision is used here to locate cracks. By means of this technique, the crack is forced to evolve through the middle of the damaged bulk.

The proposed methodology is applied to different three-dimensional problems illustrating that no mesh-dependent results are obtained.

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## Fracture of Concrete: Nonlocal Damage Laws and the Fictitious Crack Model

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For some concrete structures submitted to mechanical loadings, the safety assessment requires to predict potential crack propagations. To this end, the fictitious crack approach introduced by Hilleborg thirty years ago and based on cohesive laws brings acceptable results regarding the crack length and the overall structural response [1], thanks to a proper energy balance between strain energy and dissipation through micro and macro-cracks. However, the prediction of crack paths may remain difficult and/or cumbersome on a finite element level when dealing with 3D simulations despite constant progresses reported in the literature. An alternative relies on the smeared crack approach where Continuum Damage Mechanics provides the setting to predict damage levels (*i.e.* crack lengths) as well as damage patterns (*i.e.* crack paths). But the inherent ill-posedness of softening laws – resulting namely in spurious mesh dependency and wrong dissipation assessment – has to be given a proper answer such as the introduction of a nonlocal interaction between neighbor material points.

The idea developed in the presentation consists in reconciling the fictitious and the smeared crack approaches in order to gain the advantages offered by both. The starting points consists of a nonlocal damage law which models the coupling between elasticity and brittle isotropic damage and where the nonlocal interactions are taken into account through the gradient of the damage field, weighted by a parameter which introduces a nonlocal length scale. Focusing on a 1D problem enables to express under which conditions the damage model converges toward a cohesive zone model for vanishing nonlocal length scale [2]. In that way, the damage model appears as a spatial regularization of the cohesive one and relies on a small regularization parameter, the nonlocal length scale. In particular, there is not much ambition regarding the physics of the nonlocal interactions: the scope of the model is clearly structures the size of which is far larger than the nonlocal length scale. In return, the damage model is based on macroscopic parameters – *i.e.* readily accessible through

normalized experiments – such as the peak stress, the fracture energy and so on.

Regarding the extension of the convergence properties to 2D and 3D settings, numerical experiments show the importance of the initial damage threshold surface in the stress space (or equivalently in the strain space). In particular, the peak stress in bi-tension appears to be a crucial parameter for crack propagation: the usual tensile peak stress  $f_t$  does not seem sufficient, on the contrary of what happens with a cohesive law. Moreover, the damage surface should also distinguish tensile from compressive stress states in order to avoid spurious damage in compression, even in cases where only tensile damage is expected. Both observations have motivated the introduction of a damage threshold surface which fits experimental measures while preserving the former convergence results in the 1D setting.

A new layer has been introduced in the gradient damage model which enables to take into account (i) an asymptotic cohesive law and (ii) any damage threshold surface. 2D and 3D illustrations are conducted on the basis of a specific damage threshold function which meets the previous requirements and a cohesive softening function which fits most of the experimental results in [1]. They show the ability of the present formulation to predict crack path propagation under complex loading conditions.

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## Advances in Gradient-Enhanced Damage Models with Evolving Length Scale

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It is widely recognized that a constant length scale parameter in integral and differential non-local damage models leads to incorrect representation of failure mechanisms either by spurious damage growth [1] or by incorrect damage initiation and propagation [2, 3]. In this contribution I will discuss some recent developments in gradient-enhanced damage models which aim, through an evolving length scale parameter, at improving failure representation.

One of the shortcomings of integral and differential non-local damage models with constant length scale parameter is the spurious spreading of damage close to failure. This unphysical behavior can be corrected by considering an evolving length scale. Geers et al. [1] proposed an improvement of the gradient enhanced continuum damage model formulated by Peerlings et al. [4] in which the length scale parameter evolution is made a function of the current strain level. This improvement is achieved at the cost of one extra nodal degree of freedom in a finite element discretization. In this contribution, I will illustrate a simpler formulation [5] that retains the same number of degree of freedom as the original model [4] thus resulting in a very appealing alternative to the formulation by Geers et al. [1]. Representative examples will show the performance of the proposed approach.

Dufour and coworkers [6] have recently proposed an approach, based on an integral non-local damage model, that solves the problem of incorrect initiation and propagation of damage as discussed by Simone et al. [3]. I will present a similar approach in a differential non-local damage model [7].

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## Advanced Nonlocal Elastoplastic Constitutive Equations in the Framework of the Micromorphic Continua

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The primary objective of this talk is to develop a straightforward formulation of gradient-based nonlocal constitutive equations accounting for the full coupling between the plastic flow with mixed isotropic and kinematic hardening and the isotropic ductile damage under large plastic strains, using the general framework of micromorphic continua ([1], [2]). In the present formulation, three micromorphic phenomena are taken into account namely, the isotropic damage, the isotropic hardening and the kinematic hardening. The principle of virtual power accounting for these three micromorphic phenomena leads to three additional balance or micromorphic momentum equations. These three additional PDEs (Partial Differential Equations) together with the classical equilibrium equations are used to define highly nonlinear and fully coupled initial and boundary value problem (IBVP) with four functionals. When expressed in terms of strain-like state variables, the additional PDE associated to the micromorphic damage leads to a widely used nonlocal Helmholtz equation with its appropriate boundary condition [3].

On the other hand, using the thermodynamics of irreversible processes for micromorphic continua, fully coupled constitutive equations are obtained in terms of the micromorphic variables and their first gradients. For this end, new micromorphic state variables are added to the classical local state variables and used in the appropriate state and dissipation potentials to derive both the stress-like variables (state relations) and the strain-like flux variables (evolution relations). If, for the sake of simplicity, the micromorphic dissipations are neglected, it is shown that, each state variable undergoing the micromorphic aspect is written under the additive form of local and nonlocal contributions [4].

From the four strong forms obtained in terms of strain-like variables, the associated weak forms are deduced using the classical weighted residual and Galerkin method. The associated discretized weak forms are implemented in the commercial FE code

ABAQUS/Explicit via the construction of appropriate new elements having additional degrees of freedom, using the user-defined subroutine VUEL as well as the micromorphic constitutive equations through the VUMAT subroutine.

The simulations of some simple and more or less complex mechanical structures are conducted for various local and micromorphic material parameters and different mesh sizes in order to show the efficiency of the proposed formulation in giving, at convergence, mesh independent solutions of the IBVP [4].

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## Crack Periodicity in the Thermal Shock Setting: A Gradient Damage Model

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This presentation is devoted to the nucleation of cracks in the thermal shock problem. The underlying fracture model, that of Francfort and Marigo's theory, is governed by the minimization of the total energy composed of an elastic bulk term and a surface term proportional to the cracks length (in 2d) or surface area in 3d [2]. In this variational approach to fracture no a priori hypothesis on the crack nucleation or crack paths are made. This allows to study the topology of the cracks and especially their spacing. The numerical approach relies on minimizing a regularized functional where the cracks are represented by a smooth variable  $\alpha$ . The regularized model can also be seen as a gradient damage model where  $\alpha$  is the damage variable.

In a two-dimensional setting, starting at the thermal shock and for small times, a strip with diffuse damage propagates inside the body. Damage decreases from a maximal value at the surface towards zero, being homogeneous in the direction parallel to the surface of the thermal shock. The maximal value of damage at the surface increases with time. At some critical time  $t_b$ , the homogeneous solution bifurcates towards a solution including a set of periodically distributed damaged bands penetrating inside the body. The damage field grows until 1 (fully damaged material) in the mid-line of these zones. A set of periodically distributed cracks of equal length has formed and starts propagating inside the body. Some damage bands stop to propagate whereas the other ones continue penetrating inside the body.

At short times, we compare these results from those given by a *rate-independent gradient damage analysis* governed by the three variational principle of irreversibility, stability and energy balance [1, 3]. In the case of a sufficiently severe shock, we show that damage immediately occurs and that its evolution

follows first a fundamental branch without localization. Then it bifurcates into another branch in which damage localization will take place to finally generate cracks. The determination of the time and mode of that bifurcation allows us to explain the periodic distribution of the so-initiated cracks and to calculate the crack spacing in terms of the material and loading parameters.

For longer times we compare the crack selection mechanisms of the alternate minimization to theoretical and numerical results from the literature. The scale law for the spacing of cracks is captured. We have thus given a typical illustration of the strength of the numerical approach to fracture and its numerical implementation. Indeed the ability of this algorithm to capture crack nucleation as well as *Griffith like* crack propagation has been illustrated.

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## Damage-Based Fracture in Brittle Materials with Shape Optimization Methods

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Damage and fracture are almost always intimately linked together. For instance, in [2] a damage model is shown to provide crack-like results in some limit case while in [3] a purely fracture-dedicated model is implemented numerically with help of an auxiliary variable which has the effect of smearing the crack.

The model discussed in this talk is based on Griffith's concept of crack propagation, i.e., a balance between energy released by the crack and energetic cost to propagate the crack. A damage-based approach to fracture by means of shape optimization tools will be discussed and in particular we will

- show that 2D and 3D numerical solutions of the damage problem converge to crack-like solutions (in modes I, II and III).
- emphasize the role of optimization tools such as shape derivatives for damage and crack problems [2]; the notion of topological derivative to describe nucleation of cracks will also be introduced [1].
- describe the theoretical framework to justify mathematically the above numerically-observed convergence.
- discuss a new algorithm based on the sole notion of topological derivative and a level-set method to compute the evolution of damage and cracks.
- by means of topological derivative, discuss a nonlinear damage problem, where traction and compression cracks obey to different propagation laws.

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## Applying a Second Gradient Theory on Reinforced Concrete Structural Elements

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Strain localization is a common phenomenon in solids which have suffered severe loadings and can lead to fracture and failure of the medium. From a practical point of view, it is important to predict the possible occurrence of such phenomena (location threshold), but also to be able to simulate the behavior of a structure (e.g. reinforced concrete buildings, dams, nuclear power plants) beyond this point (post-localization behavior). It is well known that continuum mechanics models that do not have an internal length are not suitable for this type of engineering problems.

A rather natural way of introducing (indirectly) a length parameter in a continuum model is to somehow account for the microstructure of the material. The general class of so called microstructured models or higher order continuum models allows for the description of the kinematics of the microstructure by using an additional tensor in the displacement field. Higher order continuum theories can be traced back to the works of the Cosserat brothers and have been generalized and properly formulated by Mindlin and Germain using the method of virtual power.

The local second gradient model developed by Chambon et al. [1] can be seen as a particular case of a higher order continuum and has been often used to regularize problems involving localization in soils. In this work, it adopted to simulate localization problems in reinforced concrete structural elements. More specifically, a two dimensional nine-node second gradient finite element is used [2] and the constitutive laws for the first gradient part are based on damage mechanics [3,4].

Experiments on a reinforced concrete beam (three point bending test) and a reinforced concrete shear wall (static monotonic loading) were studied within the French National Program CEOS.fr (Behaviour and Assessment of special R.C. works - cracking & shrinkage), www.ceosfr.org. Global (force-displa-

cement curves) and local results (strains in concrete and steel, crack propagation using the Digital Image Correlation technique) were monitored and are compared with the numerical results of the second gradient approach.

The results show that the model is able to reproduce the force-displacement curve obtained experimentally. Damage localizes into bands whose width is controlled by the model parameters. The uniqueness of the solution is however not restored.

These results are encouraging and represent the first steps toward a wider use of the local second gradient method for concrete structures.

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## **Crack Propagation Modelling in Presence of Voids and Inclusions Using a Peridynamic Approach**

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The capability to predict damage and crack evolution by adequate numerical techniques is becoming increasingly important, both for safety and economy reasons in many industrial applications. The presence of voids and inclusions, due to manufacturing process, increases the difficulty of the analysis [1].

Classical mechanics methods, based on partial differential equations, cannot be directly applied to discontinuities and require auxiliary equations that govern damage initiation and progression.

A new approach, based on the non-local peridynamic theory [2], will be presented. In this theory each point of the domain of interest interacts with the other points that are within a finite distance (horizon). Each pair of interacting material points can be assumed to be connected through a bond, and they interact through a force defined by a specific function. The force function contains the whole constitutive information about the material and depends on the distance between the material points. The peridynamic equations of motion are formulated using spatial integral equations, as opposed to spatial partial differential equations such as those in classical-continuum theory. The fact that displacement derivatives do not appear in peridynamic equations permits this formulation to be valid whether discontinuities, such as cracks, voids or interfaces, are present or not.

Peridynamics appears to be more flexible than other computational techniques applied to mechanical problems of crack propagation such as interface elements or XFEM [3], in the sense that the crack is free to appear in every part of the structure, following only physical and geometrical constraints, does not require any a priori assumption for the definition of its propagation and does not seem to be more difficult to apply in 3-D than in the planar cases.

A distinguishing feature of this approach is its ability to treat the spontaneous formation of discontinuities at different locations together with

their mutual interaction and dynamic growth in a consistent framework. The method does not require a separate crack growth law to be provided that governs cracks and damage initiation, growth, arrest, branching and so on: these features emerge from the equation of motion and constitutive models.

The peridynamic method will be applied to model a 2D structure in presence of voids and inclusions under dynamic load conditions: a first time considering an isotropic material and then with a laminate composite material [4]. Results will be evaluated taking into account different mesh sizes, horizon dimension [5], crack initial lengths and crack orientations.

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## Upscaling Non-Local Interactions During Damage and Failure in Quasi-Brittle Materials

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Classical constitutive models for the description of progressive micro-cracking in quasi-brittle materials involve strain softening and a regularization technique for avoiding spurious strain and damage localization. Different continuum-based approaches have been promoted in the literature such as integral-type non-local models or gradient formulations. Such macroscale failure models have been applied on a wide range of problems involving the safety of concrete and reinforced concrete structures, fracture of sea ice, or the structural strength of composite elements.

These models exhibit, however, some inconsistencies such as (i) incorrect crack initiation, ahead of the crack tip; (ii) propagating damage fronts after failure due to non-local averaging, (iii) incorrect shielding effect with non-zero non-local interactions transmitted across a crack; (iv) deficiencies at capturing spalling properly in dynamics, with spalls of zero thickness when the expected spall size is below the internal length of the model [1,2].

In non-local models, the internal length is the parameter inside the weight function (or as a prefactor of the gradient term) that controls non-locality. There is today a consensus that this quantity might not be constant. Recently, two different modifications of non local integral damage models were considered: a different averaging process close to the boundary of the solid [1] in order to capture boundary effects and an evolving internal length in the course of damage [2]. The first one handles the deficiency observed in computation of spalling failure. The second one handles the incorrect crack initiation and also induces a correct shielding effect with zero interaction between material points located on both sides of a crack. The evolution of the internal length in these models remains, however, quite empirical. Changing geometry, e.g. from tensile to bending loads or from unnotched to notched specimens, results in the loss of predictive

capabilities of the classical non-local damage model [3].

The purpose of this paper is to present a new macroscopic approach to capture the evolving non-local interactions which are produced during damage and failure in quasi-brittle materials. The non-local interactions induced by the deformation of a domain around a point on a neighboring point are computed explicitly using a classical Eshelby problem. These interactions are then summed and converted into a weight function, which is implemented for non-local averaging of the variable that controls damage. Thus, weight functions centered at each point may be different from one another, depending on the distribution of damage, and on the proximity of a boundary. These weight functions evolve in the course of damage and failure.

This new interaction-based non local model is first validated on simple 1D problems and then used to reproduce experiments presenting failure and size effect for notched and unnotched concrete beams [3].

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## **The Thick Level Set Model: a Non-Local Damage Model Allowing Automatic Crack Placement Inside Localization Zones**

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When dealing with damage models in order to initiate and grow cracks, two issues at least must be taken into account. The first one is to avoid spurious strain localization (zero dissipation produced although damage has reached its maximum value). The second one, is to allow displacements to be discontinuous when the material stiffness is totally lost.

The first issue is usually taken into account by introducing a length scale in the model and rendering the model non-local. Regarding the second issue, crack placement is so far in the literature a numerical issue and not related to the non-local damage model.

In this paper, we improve the current state of affairs regarding the issues described above. The Thick Level Set (TLS) model ties damage to a level set and was initially introduced in [1,2]. An important advantage of this model is that the model is non-local only where needed in the domain (close to localization) allowing the major part of the domain to be dealt with purely local constitutive law. Somehow, local and non-local zones are different entities separated by a moving surface treated in the spirit of [3].

Regarding the second issue, the TLS models offers an automatic way to place cracks inside localized zone. The crack location is simply all points beyond the level set  $l_c$  (iso- $l_c$ ). There is thus no limitations on the crack shape. Merging and branching is also carried out automatically due to the topological flexibility of the level set representation.

The first numerical implementation of the TLS was performed in [1,4]. It relies on the use of the X-FEM [5] to allow finite element support cut by the crack to be enriched with a Heaviside function thus modeling crack displacement discontinuity. In the presentation, we will introduce a further refinement in the numerical implementation. Both crack lips (iso- $l_c$  of the level set) will be allowed to cut the

same finite element. This permits a crack placement rather insensitive to the mesh size (provided it is of course below the characteristic length scale of the material).

Numerical simulations will target 2D and 3D application in quasi-static analysis of quasi-brittle materials. Disymmetric tension-compression constitutive models will be used. CPU time and numerical robustness of the TLS will be discussed.

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**Minisymposium TCC:**

**Two-Scale Coupled Computational Approaches  
Towards Failure and Fracture**

*Organized by Marc G. D. Geers,  
Alfredo E. Huespe, Stephan Loehnert,  
Xavier Oliver and Peter Wriggers*



## Towards a Weakly Intrusive Space-Time Multi-Scale Strategy for the Prediction of Delamination Under Impact

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Composite laminated materials are increasingly employed in aeronautics but can be prone to extensive delamination when submitted to impact loads such as from bird strikes. For most practical purposes, current analysis tools allow to determine whether a given structure can sustain given impact loads while appropriate safety margins are considered. In order to improve a given design, further insight needs to be gained into the complex interactions associated with impact on composites. The need to be able to perform virtual delamination testing, that is to be able to predict the extension of damage under impact, becomes essential to engineering workflows. In that case the use of a meso-scale modeling scheme for laminates, where individual modeling of the plies and interfaces are introduced, seems desirable. However, the computational cost associated with such modeling schemes for large structures would be prohibitively high for the engineering practice, as the precise study of the damage and failure response requires the consideration of phenomena encompassing multiple spatial scales and temporal scales.

In order to construct an efficient numerical scheme, the basic idea is that while a rather detailed meso-scale model could be used to simulate delamination where needed, the rest of the structure could be described by a less detailed more economical macro-scale model. The paper will first discuss in broad terms the possibility to adapt a commercial software package (such as Abaqus) to deal as efficiently as possible with such a multi-scale scheme. Estimates of potential advantages of multi-scale strategies compared to monolithic solutions for industrial applications are also given.

In order to efficiently follow the delamination front propagation, the classical Domain Decomposition [1] would have to be coupled with a re-meshing

technique, which is costly and complex to implement. The paper will present the basis of a proposed less intrusive approach, called the Substitution method [3]. The method is designed in such a way that is possible to make use of an unchanged coarse macro model for the whole structure and to couple it with an evolutive meso-scale analysis where needed. The computational price to pay is that the method is “local” iterative.

Two versions of the Substitution method have been developed, based on two different formulations on how to couple the macro and meso models. A weakly intrusive version of the method developed in [1] has first been obtained. It leads to satisfying results but with a level of dissipated energy difficult to control. Therefore a second formulation based on [2] has been developed which avoids the precedent drawback. First simple applications of this method in the case of the propagation of delamination under impact should be presented during the conference.

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## Micro-to-Macro Transitions for Heterogeneous Material Layers Accounting for In-Plane Stretch

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Material layers within a macroscopic continuum play an important role in numerous engineering applications. Examples include adhesive bonding layers, laminated composite structures, geomaterials etc.. The material layer is generally weaker than the surrounding bulk causing the deformation to localise in the layer. The ability to model the response of the material layer is therefore critical in order to determine the behaviour of the overall continuum. The response of the material layer is dependent upon its microstructure and, in particular, heterogeneities. Microscopic heterogeneities include voids, inclusions and micro-cracks.

Homogenisation, as pioneered by Hill [1], provides a consistent methodology to link the macroscopic and microscopic scales and forms the basis for computational micro-to-macro transitions (see e.g. [2]).

The key contribution of this work is to propose a micro-to-macro transition approach for material layers which consistently accounts for in-plane stretch by endowing the macroscopic interface with its own energetic structure in the spirit of Gurtin and Murdoch [3]. The importance and relevance of considering materials layers using a multi-scale paradigm was commented upon in a recent overview paper by Geers et al. [4]

In summary (see [5, 6] for further details) the key objectives and contributions of this work are as follows:

- to state the governing equations;
- to consistently transfer the macroscopic kinematic interface data to the underlying microscopic problem;

- to determine the macroscopic kinetic quantities and the interface tangent modulus;
- to solve the three-dimensional, nonlinear microscopic problem using the finite element method;

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# Multiple Spatio-Temporal Scale Modeling of Failure in Composites Subjected to Cyclic Loading

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We present a multiscale modeling methodology for failure prediction in composite materials subjected to cyclic loading. These predictions involve solving a problem that is multiscale with respect to both space and time, and accordingly, the presented methodology is a multiple spatio-temporal method.

Failure prediction with multiple spatial scales is complicated by the size disparity between the microstructure where material damage initiates and the macrostructure across which damage propagates and induces ultimate failure, and further complicated by the existence of multiple failure modes, such as fiber fracture, diffuse matrix damage, fiber/matrix debonding, and delamination. These complexities were addressed by utilizing the eigendeformation-based reduced order homogenization method (EHM) [1]. Within EHM, a reduced order microscale problem is solved to determine the homogenized material response of a material point, and since the material response is nonlinear, we must repeat this process at every macroscale Gauss point for every loading increment, so in recognition of this prohibitive computational burden, we reduce the order of the microscale problem to gain computational efficiency. The geometry of the composite microstructure and the multiple types of failure modes incorporate naturally into the order reduction of the microscale problem.

The naive use computational homogenization based methods such as EHM leads to mesh dependency of the multiple spatial scale techniques. We overcome this mesh dependency by utilizing the extended finite element method to inject a macroscale crack when loss of ellipticity is detected at a material point. This approach attempts to address mesh dependency without the significant mesh refinements associated with nonlocal homogenization approaches.

The multiple spatio-temporal method incorporates the EHM ideas inside of an adaptive temporal ho-

mogenization technique. The methodology is designed to capture not only the multiple spatial scales but also the multiple temporal scales coming from the disparity between the length of a single loading cycle and the entire lifetime of a composite structure. The technique is based upon the fixed point temporal homogenization presented in [2]. Within the method, only a small subset of the total number of cycles over the lifetime of a structure are resolved. This reduction in the number of resolved cycles is necessary due to the computational intractability of explicitly simulating the millions of loading cycles in the designed lifetime of many composite structures. Obviously, the method is more accurate if more cycles are simulated, so adaptive step size criteria were designed to minimize error while maximizing computational efficiency. In addition, the technique presented in [2] has been made more efficient by now requiring only linear elastic simulations while resolving the load cycles. We explore the effect of this change and different choices of the fixed point operator in several example problems.

A suite of experiments was conducted on the CFRP, IM7/977-3. The specimens were subjected to both monotonic and fatigue loadings to test both the multiple spatial and temporal scale aspects of the methodology. The model shows good qualitative agreement with the experiments. We also present computational performance tests.

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## Multi-Scale Modeling of Fibrillation During Copper-Rubber Interface Delamination

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Stretchable electronic devices improve the design freedom, comfort and portability of electronic products. These devices typically consist of small rigid semiconductors which are interconnected with thin metal conductor lines. These interconnects are located on top of, or encapsulated in, a highly compliant substrate (typically a rubber material). It has been shown that the maximum stretchability that can be achieved for a given interconnect design is determined by stretching-induced interface failure [1]. To develop models capable of describing interface behavior, the interface properties need to be known. In earlier work it was observed that the main interface failure mechanism was fibrillation [2]. This mechanism involved the formation, elongation, rupture and delamination of rubber fibrils.

In a macroscopic approach, a significant mismatch was observed between the obtained interface parameters and the experimentally observed fibril micromechanics, most notably the observed maximum fibril length. Furthermore, the dissipation related to fibrillation was observed to be dependent on the test and loading conditions [3]. Evidently, this macroscopic approach renders the obtained interface properties intrinsically case specific.

To prevent large experimental effort to establish case-dependent interface properties, and to obtain a better understanding of the relation between the obtained interface parameters and the experimental observations, the micro-scale dissipation mechanisms need to be identified and taken into account explicitly in the corresponding models. Clearly, this requires a multi-scale approach.

On the macro-scale the predefined interface is modeled using cohesive zone elements. Instead of defining the traction-opening relation a priori, it is obtained from the underlying micro-model. In the micro-model, the fibrillation process is taken into account explicitly. At this level, relevant mechanisms and phenomena such as substrate roughness, and fibril debonding from the substrate are included. The actual bonding between the rubber

and copper is characterized by the intrinsic adhesion properties, i.e. adhesion energy and adhesion strength.

The micro-model results show the different stages of the fibrillation process. First, the fibril is formed. Then, upon increasing the load on the fibril, the fibril elongates. The elongation takes place partly by stretching of the material in the fibril, and partly by drawing material from the surrounding bulk into the fibril. Finally, the fibril debonds from the substrate.

The fibrillation process is mainly controlled by the adhesion strength of the interface between copper and rubber. The adhesion energy is shown to be less important. Although the rubber material is assumed to be fully hyperelastic, the loss of stored elastic energy is shown to contribute significantly to the overall work-of-separation.

The macroscopic cohesive zone traction-opening relation is obtained from the micro-model by the established coupling between the fibril micromechanics and the resulting macroscopic interface properties.

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## Overall Strength of Ductile Materials: Spheroidal Nanovoids Size Effects

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The elastic properties as well as the yield strength of a solid are significantly affected by the presence of surfaces and interfaces. Surface effects are attributed to the presence of few layers of atoms which experience a different local environment than atoms in the bulk and have a different equilibrium positions and energy. For nanosized objects, the interfacial effects become predominant since the area of surface per unit of volume is very high. Surface effects in standard continuum theories can be treated within the framework of the Gurtin and Murdoch (1975) stress interface model which assumes a jump of the traction vector while the displacement field is considered continuous across the surface [2]. The jump condition may comply with a generalized Young-Laplace equation. Recently, by making use of a plastic version of the Gurtin stress interface model (see for instance [3]) which relates the interfacial stress to the plastic deformation at the cavity surface, Dormieux and Kondo [1] generalize the well-known Gurson model in order to account for size effects induced by spherical voids.

The present study provides an extension of the model proposed by these last authors to the case of spheroidal cavities. It aims at investigating voids size effects on the macroscopic yield strength of plastic porous media containing nanosized spheroidal cavities (see [4]). To this end, we perform limit analysis of a confocal spheroidal unit cell, containing a confocal spheroidal cavity, and subjected to arbitrary mechanical loadings. The solid matrix is assumed to obey to the von Mises criterion with associated flow rule. The void size effects are captured by considering at the interface between the matrix and spheroidal cavities the plastic version of the Gurtin stress interface model. At this interface, the jump of the traction vector is related to the interfacial residual stress and interfacial plastic strain.

The salient features of the new model, namely

the effects of the void size on the macroscopic yield strength of the ductile material containing spheroidal nanocavities, are fully illustrated for axisymmetric loadings (see for instance [5]). It is shown that the macroscopic yield criterion of the nanoporous material exhibits unusual features such as (i) an increase of the yield stress when the void size decreases, (ii) asymmetry between the yield stress in uniaxial tension and compression. Moreover, significant size effects are observed for oblate nanovoids than for prolate ones.

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## Multiscale Computational Modelling of Inelasticity and Fracture in Fibre Networks

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Fibrous networks such as textiles, paper and biological materials, are conveniently modelled using discrete, regular networks of truss elements. Such models capture the discrete character of the microstructure in a natural fashion, including the relevant spatial (micro)scales. Furthermore, discrete events such as fibre fracture, bond breakage, fibre sliding, etc. are easily incorporated. However, for problems at technologically relevant (macro)scales, the computational cost of such detailed models may be prohibitive.

To deal with this difficulty, we employ a multiscale methodology which is based on the quasi-continuum method – a method which is well-known for atomistic lattices [1]. The quasi-continuum method allows one to reduce the resolution of the description where appropriate, so that realistic simulations become feasible. On the other hand, the underlying discrete network may be fully resolved in regions of interest. The transition between fully resolved and interpolated regions is seamless. The method is entirely computationally based and makes use solely of the underlying discrete microstructural model.

We extend the method to a more general class of microstructural models in which discrete points interact via discrete interactions which are not necessarily conservative [2, 3]. As a result, plasticity and/or damage of the fibres may be incorporated, as well as failure and frictional sliding of the bonds between the fibres. This allows one to formulate quite comprehensive microstructural models which, by virtue of the quasi-continuum approach, may nevertheless be applied in large-scale problems.

In order to provide the quasi-continuum method with this versatility, the conventional, energy based method is reformulated in terms of a virtual power balance. Like in the conventional method, interpolation is used to eliminate displacement degrees of freedom of the full system and thus reduce the system size. For systems with bond-based dissipation

mechanisms, the internal variables which characterise these mechanisms are also interpolated. The cost of constructing the remaining governing equations is reduced by employing a so-called summation rule. Here we use a rule which is inexpensive but nevertheless sufficiently accurate – and which may easily be integrated within the virtual power based quasi-continuum method.

The performance of the resulting method is illustrated by a number of examples in which the lattice points are connected by elastoplastic trusses or the bonds between fibres show frictional sliding. The latter allows one to model complete failure of the network by bond failure and subsequent sliding.

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## A Micro-Model of the Intra-Laminar Fracture in Fiber-Reinforced Composites Based on a Discontinuous Galerkin/Extrinsic Cohesive Law Method

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The hybrid discontinuous Galerkin (DG)/extrinsic cohesive law (ECL) method was recently proposed [1] to circumvent the drawbacks of the cohesive element methods. With the DG/ECL method, prior to fracture, the flux and stabilization terms arising from the DG formulation at interelement boundaries are enforced *via* interface elements in a way that guarantees consistency and stability, contrarily to traditional extrinsic cohesive zone methods. At the onset of fracture, the traction–separation law (TSL) governing the fracture process becomes operative without the need to modify the mesh topology since the cohesive elements required to integrate the TSL are already present. This DG/ECL method has been shown to be an efficient numerical framework that can easily be implemented in parallel with excellent scalability properties to model fragmentation, dynamic crack propagation in brittle and small-scale yielding materials, both for 3D problems and for thin structures [1, 2].

In this work, following the developments in [3], the DG/ECL method is extended to the study of composite materials failures at the micro-scale. The method is applied to study the transverse traction of composite materials in characteristic micro-volumes of different sizes. The method captures the debonding process, assimilated to a damaging process before the strain softening onset. It is shown that the density of dissipated energy resulting from the damage (debonding) remains the same for the different studied cell sizes. During the strain softening phase, a micro-crack initiates and propagates, in agreement with experimental observations. After strain softening onset, the extracted macroscale cohesive law, obtained by the method proposed in [4],

is ultimately shown to converge for the different cell sizes. The predicted behaviors are then compared to experimental results obtained from laminate tests, and are found to be in good agreement.

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## Discretization and Model Adaptivity for the Multiscale XFEM Simulation of Arbitrary Crack Geometries Using Unstructured Meshes

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Experiments of fracture processes in brittle materials such as ceramics are rather complex. Nevertheless, they show that in many cases fracture of brittle materials involves the development of smaller cracks around the main crack. The small cracks influence the direction of crack propagation as well as the critical load. Depending on their geometry and size they lead to crack shielding or crack amplification. One goal of numerical simulations is to include the effects of smaller cracks around a main crack.

Within finite element (FE) simulations the extended finite element method (XFEM) is established as a suitable method to model cracks almost independent of the FE discretization. However, due to high stresses and stress gradients even for moderately fine meshes rather high discretization errors may occur. To keep the computational cost low, an adaptive mesh refinement as proposed in [2] can be applied to various XFEM simulations. The refinement is based on a discretization error indicator using smoothed enhanced stresses. The stress smoothing technique is based on the well known Zienkiewicz and Zhu error estimator [3] modified to capture the stress singularities in the vicinity of crack fronts in linear elastic fracture mechanics.

To accurately and efficiently capture the interaction of small cracks with larger cracks a multiscale technique for the XFEM as proposed in [1] is implemented. Therefore, small cracks are modeled explicitly on a fine scale only. The projection method includes a displacement boundary projection from the coarse scale to the fine scale domain and a projection of the resulting stresses of the fine scale simulation to the coarse scale. Thus, the cracks modeled explicitly on the fine scale only are considered implicitly on the coarse scale. As a preceding step to the application of the multiscale projection method the coarse scale may be refined based on the mentioned discretization error indicator to achieve a finer coarse scale mesh around each main crack

front. Within the multiscale projection method the fine scale mesh is generated by a submeshing of each coarse scale element contained in the fine scale domain leading to fine scale element sizes that are typically multiple orders of magnitude smaller than the typical coarse scale element size without leading to numerical difficulties. This facilitates the accurate three dimensional simulation of the interaction of cracks that differ three or more orders of magnitude in size on standard computers.

The determination of the fine scale domain within the coarse scale domain is achieved by means of a model indicator. Based on the  $L_2$ -norm of the stress gradient within each coarse scale element the fine scale domain is determined. Using an additional discretization error indicator on the fine scale its mesh is adaptively refined to improve the accuracy of the fine scale simulation.

For the simulation of real engineering parts in general unstructured meshes are necessary. Additionally real crack geometries may have nearly arbitrary shapes. Therefore the presented methods are applied to a number of examples with unstructured meshes and more complex crack geometries demonstrating their suitability for real engineering applications.

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## An Adaptive Multiscale Strategy to Simulate Fracture of Heterogeneous Structures

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In order to simulate fracture in composite structures, one of the most promising approaches is to model the behaviour of the material at the scale of the material heterogeneities, which is usually called micro or meso-modelling. In a second step, these fine-scale features can be transferred to the scale of the structure by averaging techniques (on representative volume element or unit cells) or homogenisation. However, in the case of fracture, these upscaling methods cannot be used in the vicinity of cracks, as the separation of scales necessary for their application is lost.

In the literature, two schools of thought aim at alleviating this problem. The first one tries to extend the applicability of averaging techniques to fracture (e.g. reference [1] for special averaging techniques dedicated to established damage bands). The second one aims at analysing the zones where homogenisation fails directly at the microscopic scale (e.g. [2, 3]), in a concurrent framework (i.e. domain decomposition). Although the latter approach is more general, it is heavier in terms of computations, and requires the development of robust adaptivity procedures [3, 4, 5], which is the topic of this contribution.

We propose to capture the initiation of the damage mechanisms at the macroscale using a classical FE<sup>2</sup> approach [6]. In order to control the precision of the simulations, an error estimation for the upscaling strategy is carried out at each step of the time integration algorithm. Based on this estimation, the macro elements are refined hierarchically where needed. When the size of a macro-element becomes of the order of the statistical volume element used in the FE<sup>2</sup> method, the homogenisation step is bypassed. Instead, the corresponding process zone is modelled directly at the microscale and coupled to the homogenised region by a mortar-type gluing technique. In the presentation, we will emphasise some key points of the adaptive multiscale method, including:

- the error estimation technique for the chosen up-

scaling method

- the transfer of internal variables when adapting the macroscopic mesh

- the arc-length method, defined over multiple scales, allowing to regularise softening problems that are treated in quasi-statics.

The efficiency of the method will be demonstrated on examples of fracture in polycrystalline materials, for which the damage mechanisms are represented by intragranular plasticity and intergranular cohesive debonding.

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## Two-Scale Modeling of Material Failure Based on the Continuum Strong Discontinuity Approach

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Two-scale computational modeling of materials is a subject of increasing interest in computational mechanics. When dealing with materials displaying a spatially smooth behaviour, there is wide consensus on the suitable mechanical approaches to the problem. The so called FE<sup>2</sup> methods, based on the hierarchical, bottom-up one-way coupled, description of the material using the finite element method in both scales, and computational homogenization procedures at the fine scale, is nowadays one of the most popular approaches. At the heart of this direct computational homogenization procedures lies the notion of representative volume element (RVE) defined as the smallest possible region representative of the whole heterogeneous medium on average. When material failure is aimed at being captured, the problem exhibits additional complexity. Either if discrete approaches (based on non-linear softening cohesive models), or continuum approaches (strain localization-based or regularized models) are used, at the fine scale, to model material failure, the kinematic description of some, or both, scales can no longer be considered smooth, and the existence of the RVE becomes questionable on the grounds that, in this case, the material loses the statistical homogeneity inherent to the RVE concept. A crucial consequence of this is the lack of objectivity of the results with respect to the size of the RVE. In [1] a recent attempt to overcome this flaw, for regularized non-local models, can be found.

The present work is an attempt to address this issue in the setting of the Continuum Strong Discontinuity Approach (CSDA) to material failure developed by the authors in the past [2]. The essentials of the method are: 1) propagating strong discontinuities (displacement jumps representing the cracks) are captured at the microstructure using well-established continuum stress-strain local

constitutive models (typically continuum damage models). 2) it is proven that the application of standard stress homogenization procedures to the resulting micro-stress fields, translates into a macroscopic constitutive law that inherits a characteristic length from the microstructure, which depends, on one hand, on the amount of microscopic failure and, on the other, also on the size of the RVE. 3) the macroscopic finite element model is equipped with finite elements with embedded discontinuities, to capture the onset and propagation of the macroscopic discontinuities, which are consistent with that inherited characteristic length, imported from the corresponding RVE, 4) in this context complete insensitivity, with respect to the RVE size, of the macroscopic crack pattern and the structural response is achieved and 5) material failure properties, like the fracture energy, are consistently up-scaled.

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## Continuous-Discontinuous Computational Homogenization Framework for Modelling Micro-Scale Damage Towards Macroscopic Failure

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The aim of this research is to propose a novel multi-scale continuous-discontinuous computational homogenization framework, able to upscale microscopic localization towards the initiation of a macroscopic cohesive crack.

The stress-strain and the cohesive macroscopic constitutive responses are resolved through the incorporation of the underlying microstructural volume element (MVE), in which the damage evolution results in the formation of a strain localization band. The macro-scale kinematics entails a discontinuous displacement field and a non-uniform deformation field across the discontinuity. Proper scale transitions are formulated to provide a consistent coupling to the continuous micro-scale kinematics. To allow for the strain localization band development within the MVE with minimal interference of the boundary conditions, new percolation path aligned boundary conditions have been used, based on the projection of the boundary constraints in the direction of the developing localization band [1]. From the solution of the micro-mechanical boundary value problem, the macroscopic stress responses at both sides of the discontinuity are recovered, providing the cohesive tractions at the interface. The effective values of the displacement jump and of the deformation field discontinuity across the interface are derived from the same micro-scale analysis.

The macroscopic boundary value problem may be solved both with embedded discontinuities [2], or within an extended finite element (XFEM) approach [3].

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## Contact Analysis in Multi-Scale Computational Homogenization

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Several homogenization techniques have been developed the last decades, for study of heterogeneous materials [1]. In this work, principles taken from non-smooth contact mechanics have been incorporated in a multi-level computational homogenization scheme, for the study of composite materials.

In the majority of the presented multi-scale methods, a continuous micro structure is considered, which is usually simulated with a non linear damage or plasticity model. On the contrary, in this article a discrete contact model is used for the simulation of the non linear behaviour of the micro structure of a composite material. The degradation of the material properties due to the loss of contact and debonding between the connecting parts of the Representative Volume Element (RVE), is captured by the proposed method.

In this framework, a concurrent multi-level analysis between a continuous macro model and a discrete RVE, is considered. The macro model is a continuous rectangular body, consisting of rectangular plane stress elements. In each integration point of the macro model, a discrete micro model is accounted for. The micro model, consists of two or more discrete bodies in contact. For the solution of the contact problem an Augmented Lagrangian technique is used [2], for the simplest case of a node to node discretization of the contacting bodies, while friction is omitted.

The classical steps of the multi-scale computational homogenization [3] are followed herein. In particular, each strain vector on each Gauss point of the macro model, is the loading (linear boundary conditions) for each corresponding RVE, within a non linear incremental procedure. Then, non linear analysis of each RVE until convergence, results in a Consistent tangent stiffness and an average stress vector. The Consistent tangent matrix is used in the macro model for the creation of the macro tangential stiffness matrix, while the average stress of the

RVE is the macro stress in each Gauss point, used for the calculation of the internal force vector.

The contribution of the present work is related to the incorporation of a classical contact numerical scheme for the study of the RVE, in a multi-scale computational homogenization procedure. For the solution of the RVE, an optimization problem is written that corresponds to the minimization of the potential energy with the inequality constraints expressing the non-penetration between the contacting bodies [4]. The Augmented Lagrangian method within the Newton-Raphson incremental iterative procedure leads to the tangent stiffness matrix of the RVE. Then, partition of this matrix is considered, according to the descriptions given in [3], for the creation of the Consistent tangent stiffness matrix of the macro structure.

Examples of the described scheme demonstrate the appearance of debonding in RVEs, as well as the loss of stiffness in the macro structure when debonding in the RVE occurs.

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## Determining a Representative Sample for DEM Modeling of Concrete

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The discrete element method (DEM) is a powerful alternative to the finite element method (FEM) when advanced damage states and failure of concrete have to be studied [1]. The cohesive DEM model has to reproduce the macroscopic behavior of the solid material volume, which is not automatically satisfied because DEM model is governed by Newton's laws and not by equations of continuum mechanics. The local DEM properties must be identified from tension-compression tests.

The discrete elements considered here are rigid spheres of different sizes and masses constituting a disordered poly-disperse assembly generated by a geometric algorithm which pads by spheres a given tetrahedral mesh of the modeled structure [2]. Note that the characteristic size of discrete elements used here is not representative of concrete constituents such as aggregates. Thus, we are dealing with a higher scale model which aims at reproducing the macroscopic behavior of concrete. Cohesion-type interactions within each pair of neighboring discrete elements are defined by means of non-linear normal and tangential stiffnesses. Since the strain energy for a given cohesive spring-type link depends on the size of the interacting elements, the local interaction stiffnesses are not identical over the sample. "Micro-macro" relations are used to calculate these local stiffnesses in elastic regime from macroscopic elastic parameters, which are the Young's modulus and the Poisson's ratio. Those relations stem from homogenization models adapted to take into account both the relative disorder of the DE assembly and dependence of the interaction surface on the size of interacting elements. Thus after calibration, local parameters account for spatial distribution of DE elements, which is specific to each packing algorithm.

Contrary to FEM, whose convergence properties with mesh refinement are well established, it is problematic to prove the convergence for DEM we use because every modification of the DE mesh involves changes of its "properties", requiring *stricto sensu* a re-calibration of local parameters.

To appreciate the results of DEM calculations, two important questions have to be answered: 1) what is the minimum fineness of the DE assembly needed to guarantee that the DE model reproduces the material macroscopic behavior under different types of loading, and 2) how the DE assembly properties (fineness, compactness) and the packing method influence the DEM model behavior?

To get information about the consistency of our DEM model, we have conducted an exhaustive computational study by varying the DE mesh fineness and compactness, and the geometry of DE samples as well, and by subjecting the samples to compression and shear quasi-static loads. Mean values of Young's modulus, shear modulus and Poisson's ratio have been determined from the deformed state. For the DE sample, used for the local parameters identification, the values of global parameters  $E$ ,  $\nu$  and  $G$  are restituted almost exactly. When applying the same set of local parameters to other samples, the maximum error obtained for  $E$ ,  $\nu$  and  $G$  does not exceed 10% in the worst case, which allows avoiding systematic re-calibration of local parameters. The obtained results clearly indicate the minimum discretization needed for a DE sample to reproduce the macroscopic behavior of material, for both compression and shear types of loading.

By using indications of this study, we have simulated the non-linear behavior of a reinforced concrete beam under impact of a deformable projectile.

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## Two-Scale Model for Failure Analysis of Heterogeneous Materials: Numerical Validation

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In recent works ([1] and [2]), the authors have presented a new variational multi-scale formulation devised to modeling the failure of heterogeneous materials (*Failure-Oriented Multi-scale Formulation (FOMF)*).

Two well-separated length scales are considered in the *FOMF*. The macro model describes the failure processes that are taking place at the micro model by means of a cohesive interface which is mechanically characterized through a traction  $\mathbf{T}$  vs. separation  $\beta$  relation. The failure processes at the microscopic level are modeled using a Representative Volume Element (RVE).

The traction  $\mathbf{T}$  is defined as a function of  $\beta$  by means of a computational homogenization technique that considers two transfer operators: *i*) a non homogeneous strain injection operator that transfers the crack opening  $\beta$ , from the macro to the micro scale; *ii*) as consequence of a well established variational principle and the adopted approach, it is derived a stress homogenization operator which determines  $\mathbf{T}$  as a results of a homogenization of the stress field at the RVE level.

One of the main characteristics of this technique is its full variational consistency, as well as, that the vector  $\mathbf{T}$  is objective with respect to the micro-cell size taken to perform the material failure analysis at the microscopic level.

The numerical implementation of the multi-scale model is presented. At the macroscopic scale, an E-FEM technique (finite elements with embedded strong discontinuities) is adopted to simulate the cohesive interfaces. At the RVE scale, continuum damage or elasto-plastic models with softening are used. Then, strain localization solutions act as the precursor mechanism leading to failure. A smeared

crack approach is taken to circumvent the deficiencies of standard finite element approaches with this kind of continuum constitutive models.

Specific boundary conditions, imposed on the RVE, are given in order to satisfy: *i*) objective macroscopic relations ( $\mathbf{T}$  vs.  $\beta$ ) with respect to the micro-cell size, and, *ii*) full degradation of the RVE model in the sense that the homogenized material response reaches a completely exhausted state. Boundary conditions similar to that presented in [3] are taken in order to satisfy the second requirement.

In this paper, emphasis is given to the numerical assessments of the model. In particular, we compare and verify the numerical solution provided by the two-scale variational formulation with respect to a mono-scale *Direct Numerical Simulation (DNS)* approach.

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