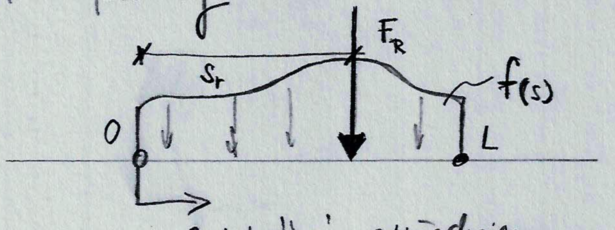
SPOJITÉ ZATÍŽENÍ

1) Určete reakce a polohu náhradního břemene

- náhradní břemeno je umístěno v těžišti zatěžovací plochy:



moment od spojitého z.: $M_0^f = -\underbrace{\int_0^L f_{(s)} \cdot s \, ds}$

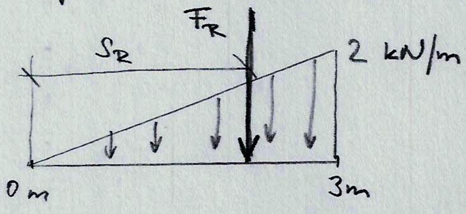
sečtou se "malé" dílky zatížení násobené jejich vzdáleností od počátku

moment od výsledné síly: $M_0^F = - F_R \cdot s_R$

$$M_0^f = M_0^F$$

$$-\int_0^L f_{(s)} s \, ds = - F_R \cdot s_R \Rightarrow s_R = \frac{\int_0^L f_{(s)} s \, ds}{F_R}$$

například pro trojúhelníkové zatížení:



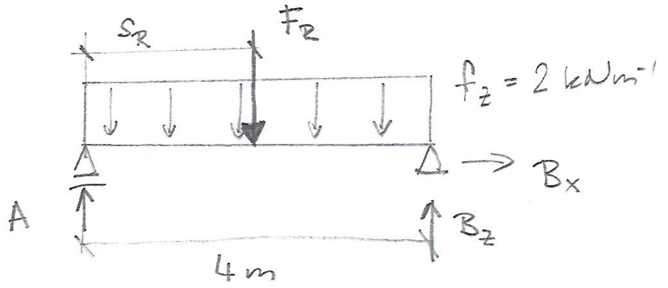
$f_{(s)} = \frac{2}{3} s \;;\; s \in \langle 0 ; 3 \rangle$

F_R = plocha pod grafem = $= 2 \cdot 3 \cdot \frac{1}{2} = 3 \text{ kN}$

$$\int_0^L f_{(s)} s \, ds = \int_0^3 \frac{2}{3} s^2 \, ds = \frac{2}{3} \int_0^3 s^2 \, ds = \frac{2}{3} \left[\frac{s^3}{3} \right]_0^3 = \frac{2}{3} \left[\frac{27}{3} \right] = \frac{2}{3} \cdot 9 = 6$$

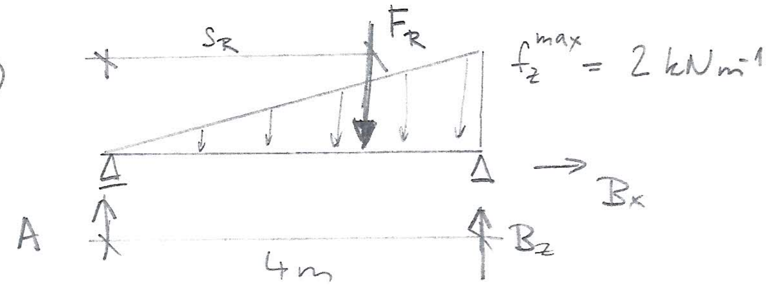
$$s_R = \frac{\int_0^L f_{(s)} s \, ds}{F_R} = \frac{6}{3} = 2 \text{ m}$$

a)



$\Rightarrow$
$B_x = 0$
$B_z = 4$ kN
$A = 4$ kN
$s_R = 2$ m
$F_R = 8$ kN

b)



$F_R = \frac{1}{2} \cdot 2 \cdot 4 = 4$ kN

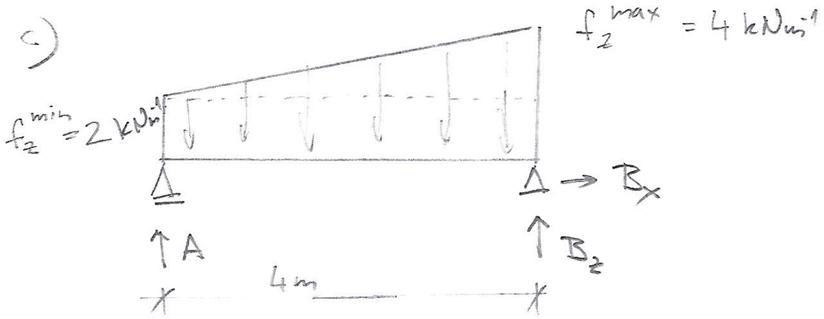
$\Rightarrow$ $s_R = \frac{2}{3} \cdot 4 = 2,\overline{66}$ m

$\curvearrowleft a$: $4B = \frac{8}{3} \cdot 4 \Rightarrow B = 2,\overline{66}$ kN

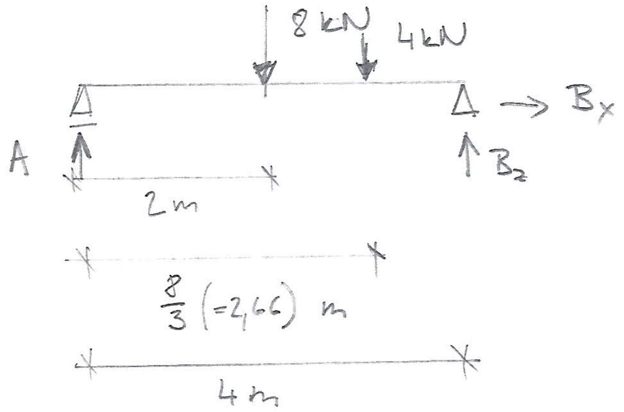
$\uparrow$: $A = 1,\overline{33}$ kN

$\rightarrow$: $B_x = 0$

c)



$\Downarrow$



$\rightarrow$: $B_x = 0$

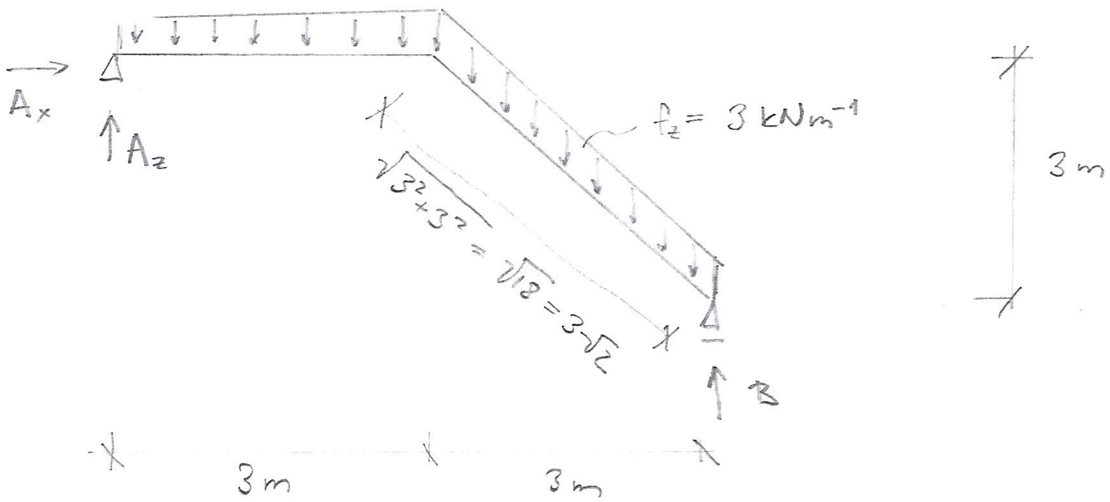
$\curvearrowleft a$: $8 \cdot 2 + 4 \frac{8}{3} = 4 B_z$

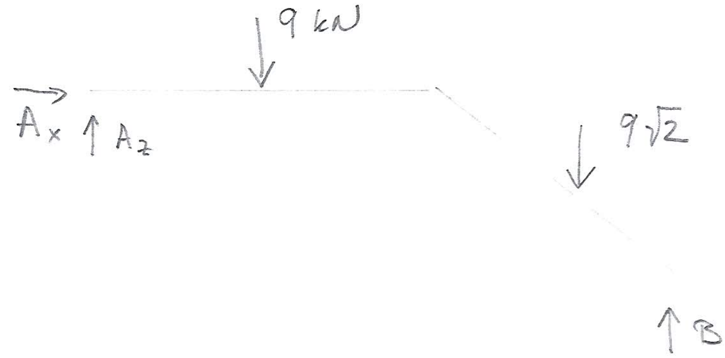
$B_z = \frac{20}{3} = 6,\overline{66}$ kN

$\uparrow$: $A = 8 + 4 - 6,\overline{66} = 5,\overline{33}$ kN

$\Rightarrow$ platí princip superpozice (sčítání jednotlivých stavů)

2) Určete reakce: a) vlastní tíha





$A_x = 0$

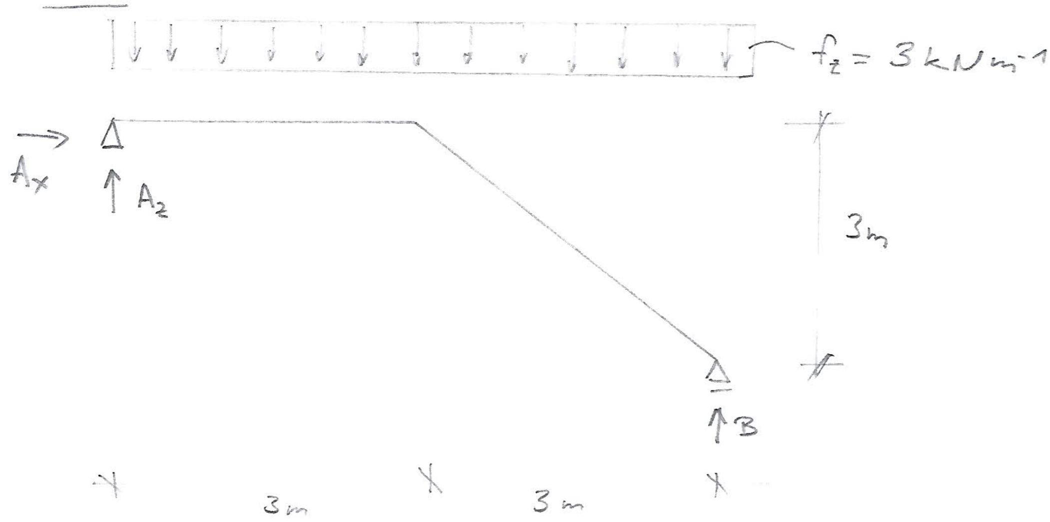
$\curvearrowleft a: \quad 6B = 4{,}5 \cdot 9\sqrt{2} + 1{,}5 \cdot 9$

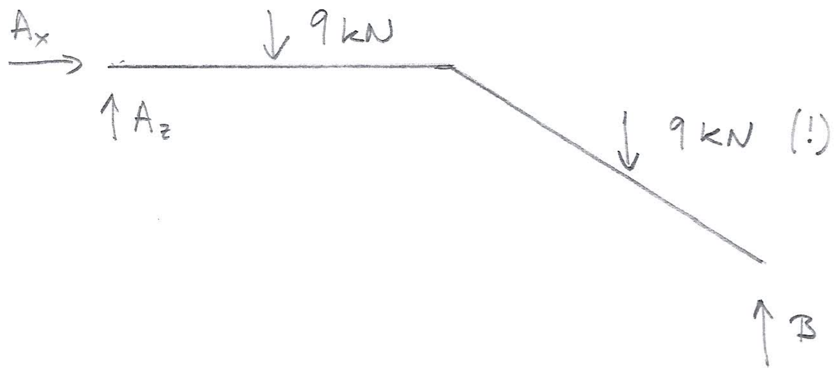
$B = 11{,}8\ kN$

$\uparrow \quad A_z = 9 + 9\sqrt{2} - 11{,}8$

$A_z = 9{,}93\ kN$

b) snih





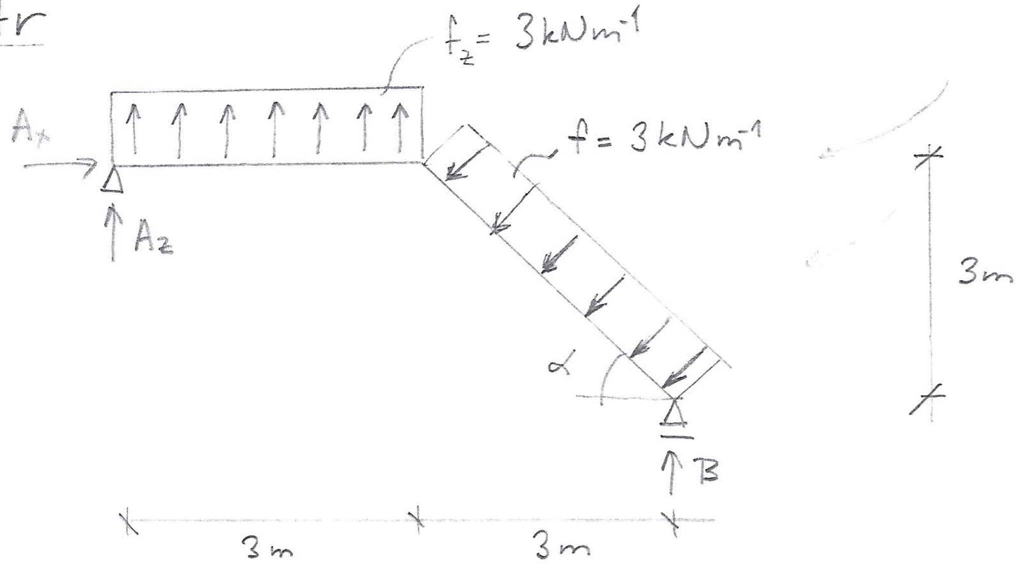
$A_x = 0$

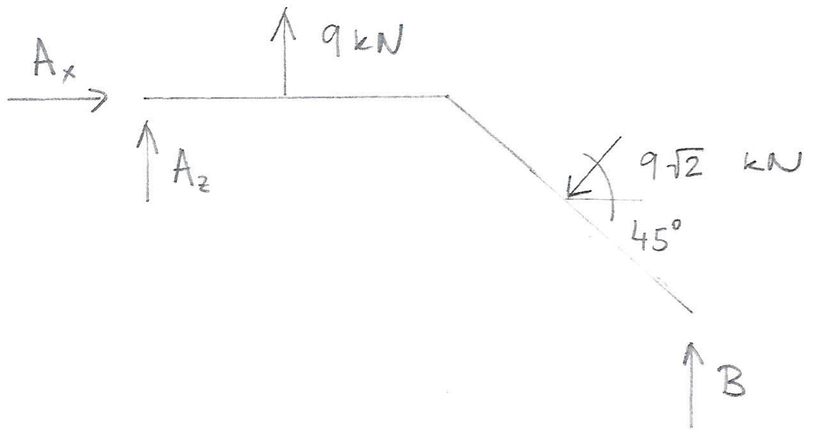
$\curvearrowleft a: \quad 6B = 4{,}5 \cdot 9 + 1{,}5 \cdot 9$

$B = 9\ kN$

$\uparrow: A_z = 9\ kN$

c) vítr





$\rightarrow: A_x = 9\sqrt{2}\cdot\frac{\sqrt{2}}{2} = \underline{9\ kN}$

$\curvearrowleft a: 6B = -1{,}5\cdot 9 + 4{,}5\cdot 9\sqrt{2}\cdot\frac{\sqrt{2}}{2} + 1{,}5\cdot 9\sqrt{2}\cdot\frac{\sqrt{2}}{2}$

$\underline{B = 6{,}75\ kN}$

$\uparrow: A_z = -9 + 9\sqrt{2}\frac{\sqrt{2}}{2} - 6{,}75$

$\underline{A_z = -6{,}75\ kN}$

I) Nejhorší kombinace

- přitěžuje se bezpečnostními koeficienty:
  a) stálé zatížení $\gamma_G = 1{,}35$ (vl. tíha)
  b) proměnné zatížení $\gamma_Q = 1{,}5$ (sníh, vítr, lidé!...)

$A_z^{max} = \gamma_G\cdot A_z^{vl.tíha} + \gamma_Q\cdot A_z^{sníh} = 1{,}35\cdot 9{,}93 + 1{,}5\cdot 9 = \underline{26{,}91\ kN}$

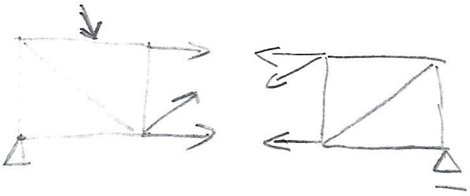
$A_z^{min} = A_z^{vl.tíha} + \gamma_Q\cdot A_z^{vítr} = 9{,}93 + 1{,}5\cdot(-6{,}75) = \underline{-0{,}195\ kN}$

podpora A musí být kotvená!

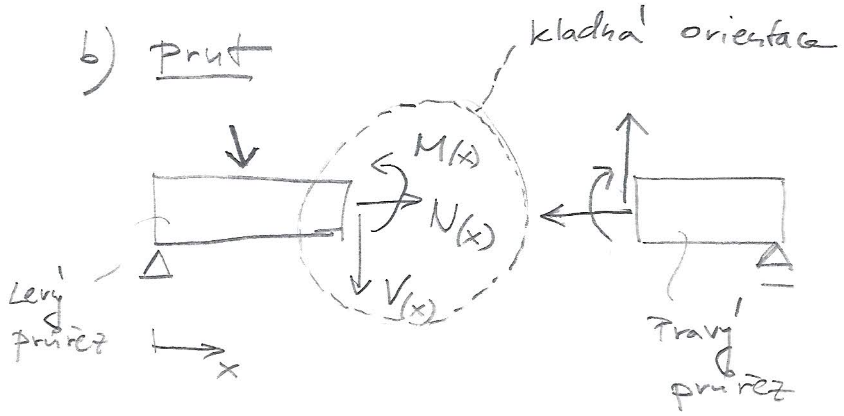
Vnitřní síly v prutu

- analogie vnitřních sil v příhradě:

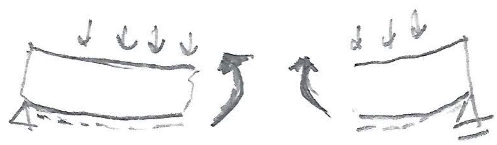
a) příhrada:



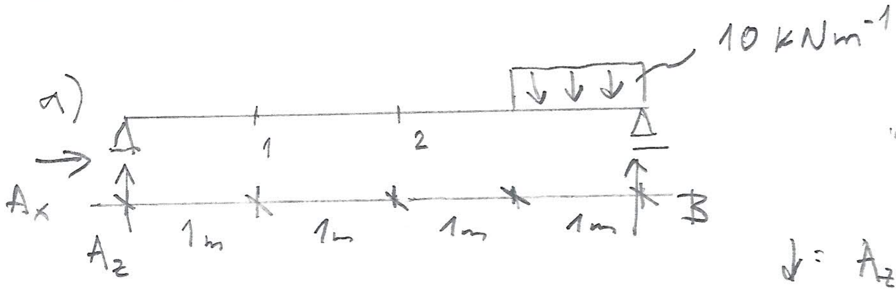
b) prut



$N(x)$ … normálová síla [N]; kladná orientace → tažený prut
$V(x)$ … posouvací (smyková) síla [N]
$M(x)$ … ohybový moment [Nm]; kladný, pokud napíná spodní vlákna



1) Určete vnitřní síly (N, V, M) v průřezech 1 (a 2):



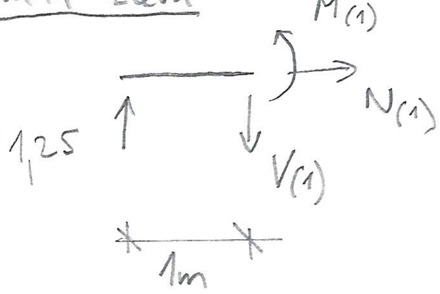
$\rightarrow: A_x = 0$

$\circlearrowleft_0: 4B = 3{,}5 \cdot 10$

$\underline{B = 8{,}75\ kN}$

$\downarrow: A_z = 10 - 8{,}75 \Rightarrow A_z = 1{,}25\ kN$

rovnováha zleva:



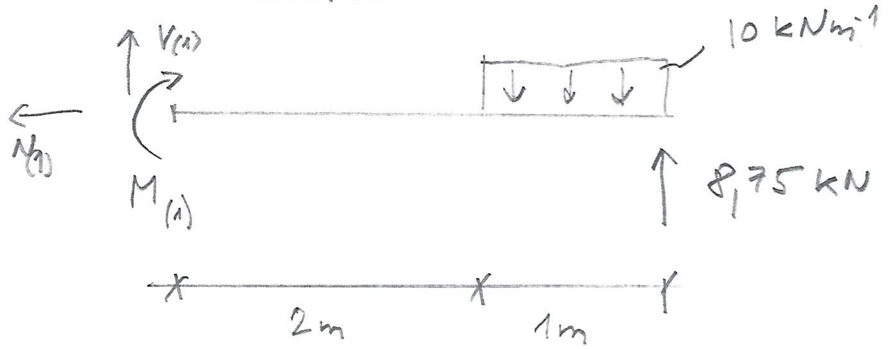
$\rightarrow: N_{(1)} = 0$

$\downarrow: V_{(1)} - 1{,}25 = 0 \Rightarrow V_{(1)} = 1{,}25\ kN$

$\circlearrowleft_1: M_{(1)} - 1{,}25 \cdot 1 = 0$

$\Rightarrow M_{(1)} = 1{,}25\ kNm$

rovnováha zprava:

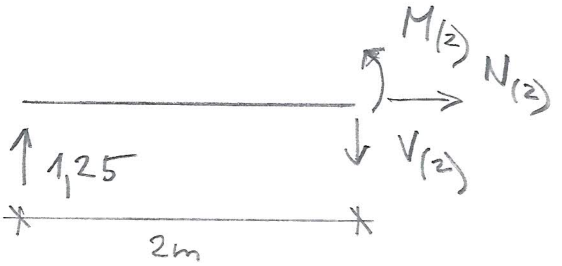


$\rightarrow$ $N_{(1)} = 0$

$\downarrow$: $10 - 8{,}75 - V_{(1)} = 0 \Rightarrow V_{(1)} = 1{,}25$ kN

$\circlearrowleft_1$: $-M_{(1)} - 10 \cdot 2{,}5 + 3 \cdot 8{,}75 = 0 \Rightarrow M_{(1)} = 1{,}25$ kNm

b)

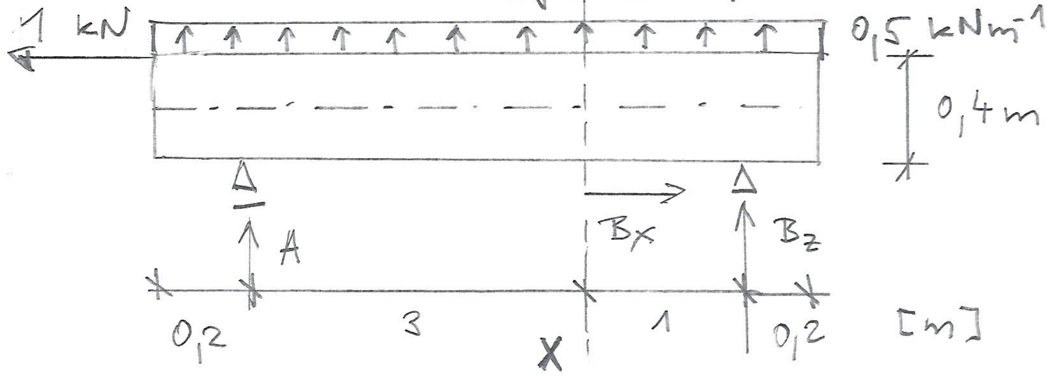


$\rightarrow$: $N_{(2)} = 0$

$\downarrow$: $V_{(2)} = 1{,}25$ kN

$\circlearrowleft$: $M_{(2)} = 1{,}25 \cdot 2 = 2{,}5$ kN

2) Určete vnitřní síly v průřezu X:

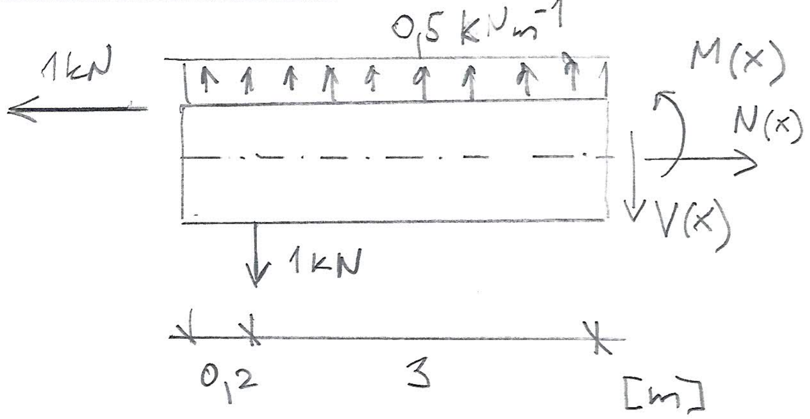


$\circlearrowleft_b$: $\underbrace{1 \cdot 0{,}4}_{\text{vztaženo k místu podpory}} - 4{,}4 \cdot 0{,}5 \cdot 2 - 4A = 0 \Rightarrow A = -1$ kN

$\downarrow$: $B_z = -1{,}2$ kN

$\rightarrow$: $B_x = 1$ kN

Rovnováha zleva:

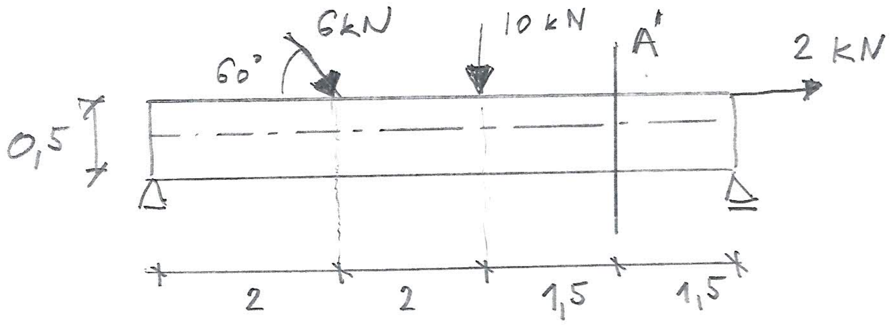


$\rightarrow N_{(x)} - 1 = 0 \Rightarrow N_{(x)} = 1\ kN$

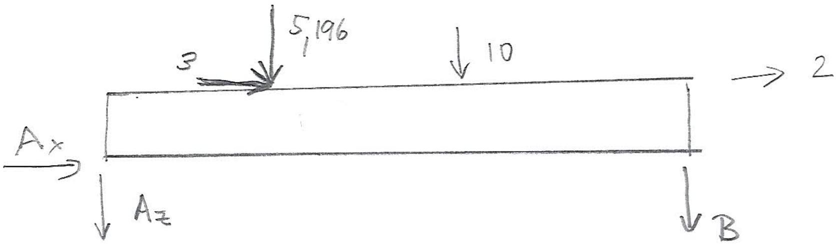
$\downarrow : V_{(x)} + 1 - 0{,}5 \cdot 3{,}2 = 0 \Rightarrow V_{(x)} = 0{,}6\ kN$

$\curvearrowleft : M(x) + 1 \cdot 0{,}2 + 1 \cdot 3 - 0{,}5 \cdot 3{,}2 \cdot \frac{3{,}2}{2} = 0 \Rightarrow M_{(x)} = -6{,}4\ kNm$

PŘ: Určete vnitřní síly v průřezu A'



[m]



$\rightarrow$: $A_x + 3 + 2 = 0$

$\Rightarrow \underline{A_x = -5 \text{ kN}}$

$\curvearrowleft a$: $-3 \cdot 0{,}5 - 5{,}196 \cdot 2 - 10 \cdot 4 -$

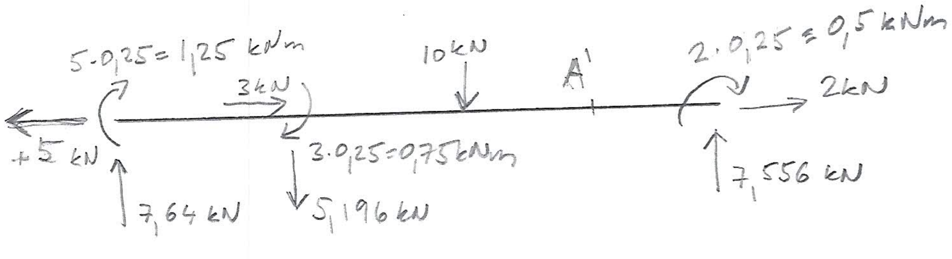
$-2 \cdot 0{,}5 - 7B = 0$

$\Rightarrow \underline{B = -7{,}556 \text{ kN}}$

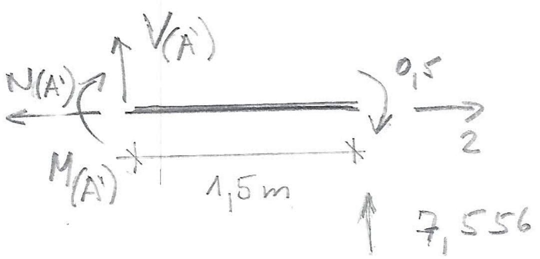
$\downarrow$ $A_z + B + 5{,}196 + 10 = 0$

$\Rightarrow \underline{A_z = -7{,}64 \text{ kN}}$

$\Rightarrow$ model (redukce zatížení a reakcí ke střednici):



$\rightarrow$ je jednodušší počítat vnitřní síly zprava:



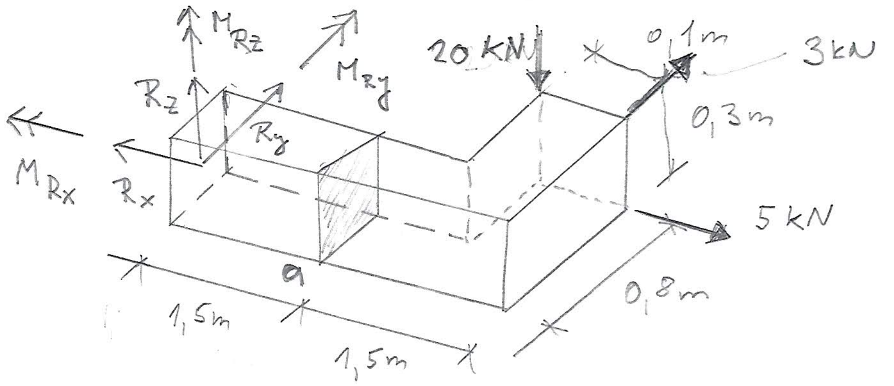
$\leftarrow$: $N_{(A')} - 2 = 0 \Rightarrow \underline{N_{(A')} = 2 \text{ kN}}$

$\uparrow$: $V_{(A')} + 7{,}556 = 0 \Rightarrow \underline{V_{(A')} = -7{,}556 \text{ kN}}$

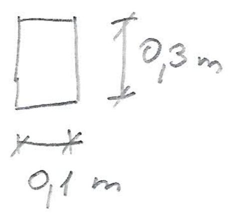
$\curvearrowleft A'$: $-M_{(A')} - 0{,}5 + 7{,}5556 \cdot 1{,}5 = 0$

$\Rightarrow \underline{M_{(A')} = 10{,}834 \text{ kNm}}$

PŘ: Určete vnitřní síly v řezu a



– průřez je konstantní



Reakce:

$-R_x + 5 = 0 \Rightarrow R_x = 5$ kN

$-R_y - 3 = 0 \Rightarrow R_y = -3$ kN

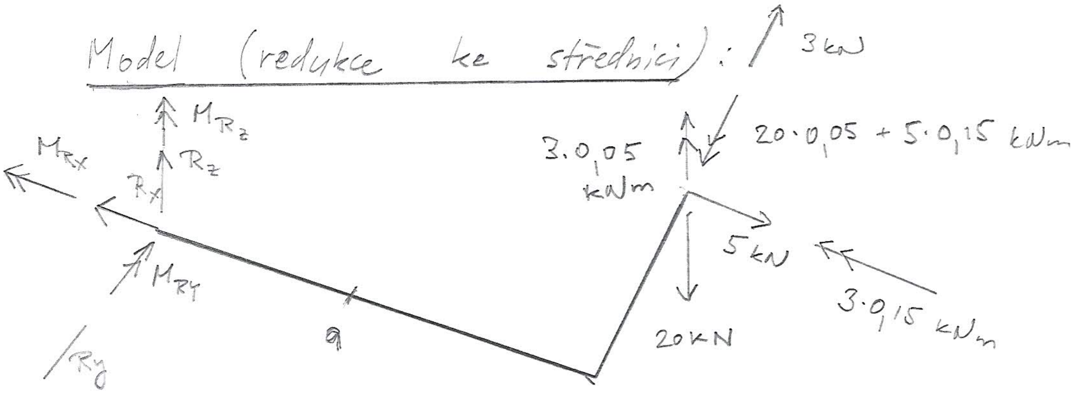
$-R_z + 20 = 0 \Rightarrow R_z = 20$ kN

$-M_{Rx} - 20 \cdot 0{,}75 - 3 \cdot 0{,}15 = 0 \Rightarrow M_{Rx} = -15{,}45$ kNm

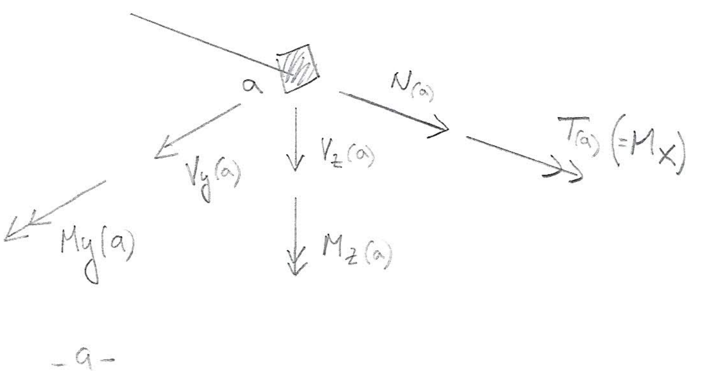
$-M_{Ry} - 20 \cdot 2{,}9 + 5 \cdot 0{,}15 = 0 \Rightarrow M_{Ry} = -57{,}25$ kNm

$-M_{Rz} - 3 \cdot 3 + 5 \cdot 0{,}75 = 0 \Rightarrow M_{Rz} = -5{,}25$ kNm

Model (redukce ke střednici):



Vnitřní síly zleva:



$N_{(a)} - R_x = 0 \Rightarrow \underline{N_{(a)} = 5 \text{ kN}}$

$V_{y(a)} - R_y = 0 \Rightarrow \underline{V_{y(a)} = -3 \text{ kN}}$

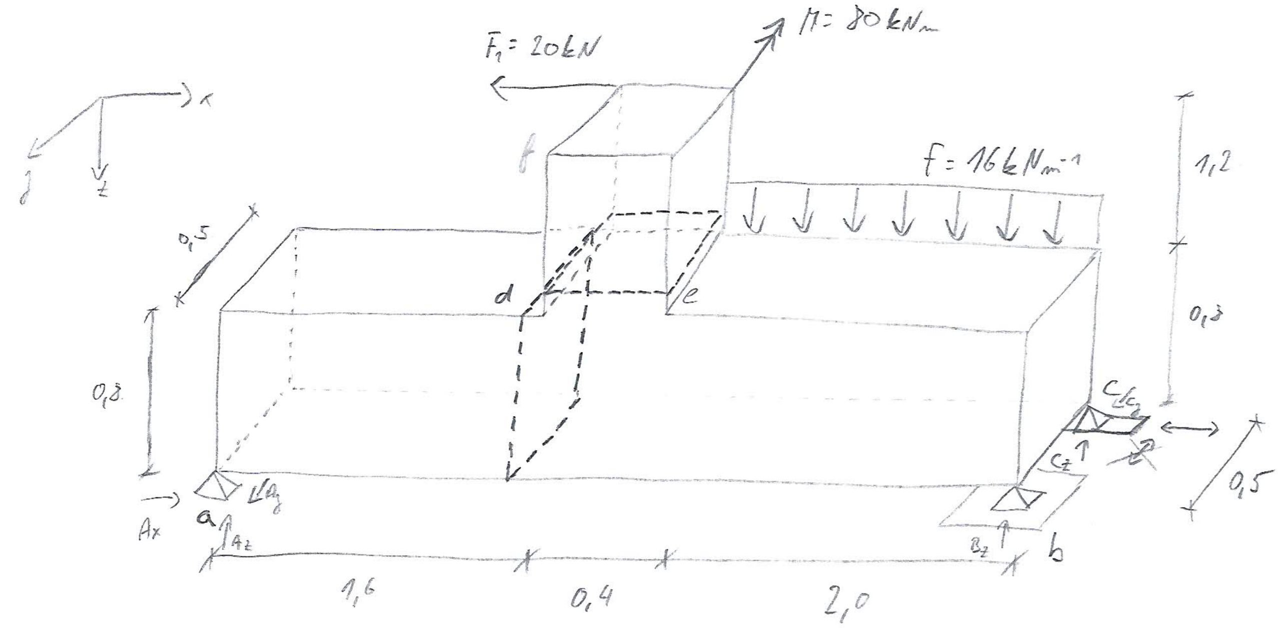
$V_{z(a)} - R_z = 0 \Rightarrow \underline{V_{z(a)} = 20 \text{ kN}}$

$T_{(a)} - M_{Rx} = 0 \Rightarrow \underline{T_{(a)} = -15{,}45 \text{ kNm}}$

$M_y(a) - M_{Ry} - R_z \cdot 1{,}5 = 0 \Rightarrow \underline{M_y(a) = -27{,}25 \text{ kNm}}$

$M_z(a) - M_{Rz} + R_y \cdot 1{,}5 = 0 \Rightarrow \underline{M_z(a) = -0{,}75 \text{ kNm}}$

VNITŘNÍ SÍLY VE 3D



$M_b: -A_z \cdot 4 + 16 \cdot 2 \cdot 1 + 20 \cdot 2 - 80 = 0 \longrightarrow A_z = -2\,kN$

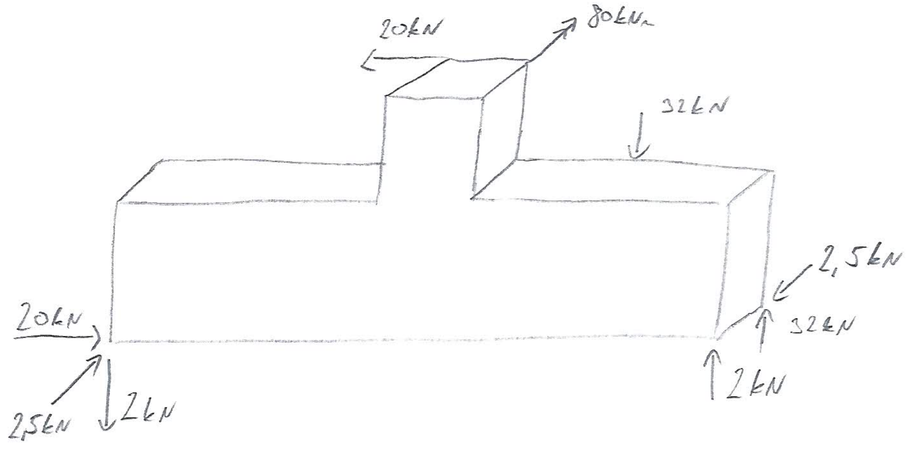
$M_c: \quad 2 \cdot 0{,}5 - B_z \cdot 0{,}5 = 0 \longrightarrow B_z = 2\,kN$

$\downarrow: \quad 2 - 2 - C_z + 2 \cdot 16 = 0 \longrightarrow C_z = 32\,kN$

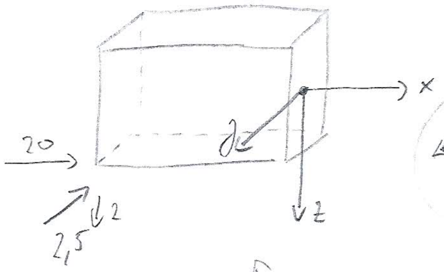
$\rightarrow: \quad A_x - 20 = 0 \longrightarrow A_x = 20\,kN$

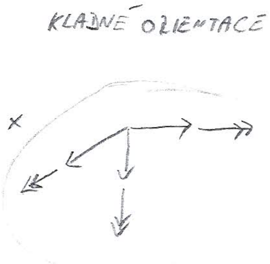
$M_c: \quad -A_y \cdot 4 - 20 \cdot 0{,}5 = 0 \longrightarrow A_y = -2{,}5\,kN$

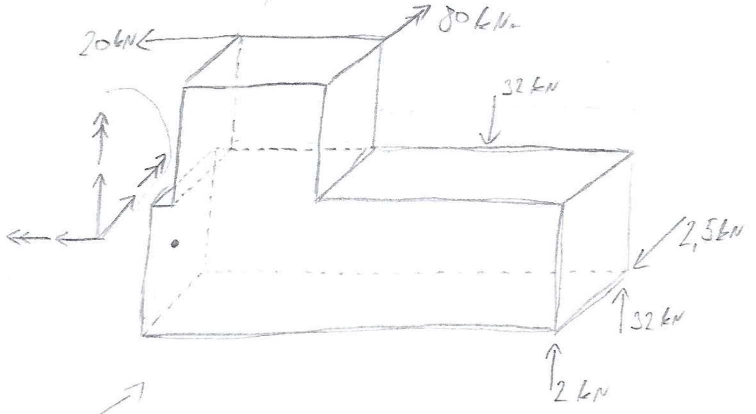
$\swarrow: \quad -2{,}5 + C_y = 0 \longrightarrow C_y = 2{,}5\,kN$



da







kladný řez : (veličiny orientované shodně s osami)

$N_{da} + 20 = 0 \rightarrow N_{da} = -20\,kN$

$V_{y,da} - 2,5 = 0 \rightarrow V_{y,da} = 2,5\,kN$

$V_{z,da} + 2 = 0 \rightarrow V_{z,da} = -2\,kN$

$M_{x,da} + 2\cdot\frac{0,5}{2} + 2,5\cdot\frac{0,8}{2} = 0 \rightarrow M_{x,da} = T_{da} = -1,5\,kNm$

$M_{y,da} + 20\cdot\frac{0,8}{2} + 2\cdot 1,6 = 0 \rightarrow M_{y,da} = -11,2\,kNm$

$M_{z,da} - 20\cdot\frac{0,5}{2} + 2,5\cdot 1,6 = 0 \rightarrow M_{z,da} = 1\,kNm$

záporný řez :

$N_{da} + 20 = 0 \rightarrow N_{da} = -20\,kN$

$V_{y,da} - 2,5 = 0 \rightarrow V_{y,da} = 2,5\,kN$

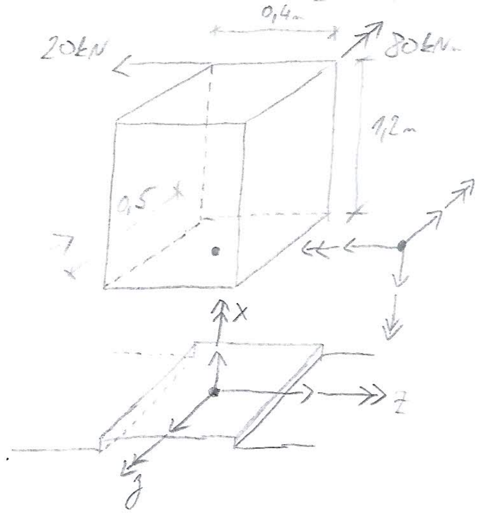
$V_{z,da} + 2 = 0 \rightarrow V_{z,da} = -2\,kN$

$M_{x,da} + 32\cdot\frac{0,5}{2} - 32\cdot\frac{0,5}{2} + 2\cdot\frac{0,5}{2} + 2,5\cdot\frac{0,8}{2} = 0 \rightarrow M_{x,da} = T_{da} = -1,5\,kNm$

$M_{y,da} + 80 - 20\left(1,2 + \frac{0,8}{2}\right) + 32\cdot 1,4 - 2\cdot 2,4 - 32\cdot 2,4 = 0 \rightarrow M_{y,da} = -11,2\,kNm$

$M_{z,da} + 20\cdot\frac{0,5}{2} - 2,5\cdot 2,4 = 0 \rightarrow M_{z,da} = 1\,kNm$

ef



záporný řez :

$N_{ef} = 0\,kN$

$V_{y,ef} = 0\,kN$

$V_{z,ef} + 20 = 0 \rightarrow V_{z,ef} = -20\,kN$

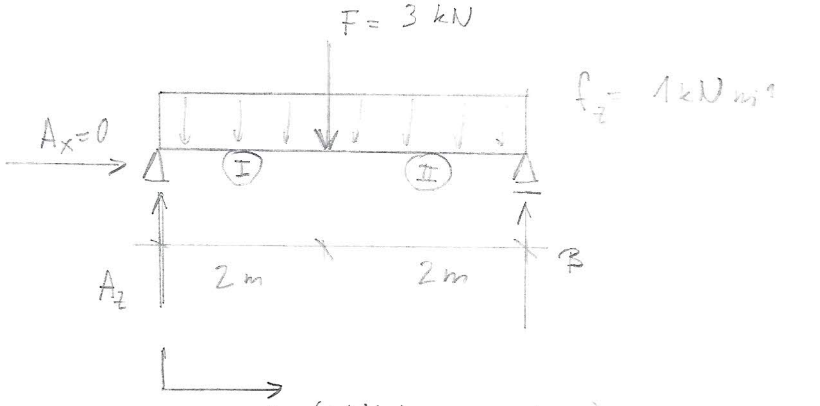
$M_{x,ef} - 20\cdot\frac{0,5}{2} = 0 \rightarrow M_{x,ef} = T_{ef} = 5\,kNm$

$M_{y,ef} + 80 - 20\cdot 1,2 = 0 \rightarrow M_{y,ef} = -56\,kNm$

$M_{z,ef} = 0\,kNm$

Průběh vnitřních sil po délce prutu

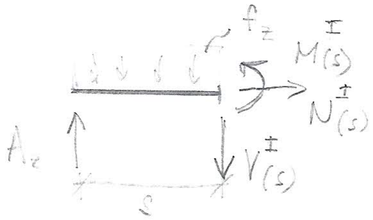
PŘ: Vykreslete průběh vnitřních sil na prutu



$A_z = B = 3{,}5$ kN

s (lokální souřadnice) ; $s \in \langle 0;4 \rangle$

I) 1. interval (k síle F, kde vzniká nespojitost) : $s^{I} \in \langle 0;2)$



$\rightarrow : N^{I}_{(s)} = 0$

$\downarrow : V^{I}_{(s)} - A_z + f_z \cdot s = 0$

$\Rightarrow V^{I}_{(s)} = A_z - f_z \cdot s = 3{,}5 - 1s$ [kN]

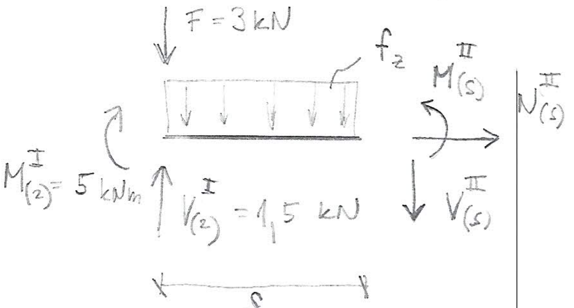
$\curvearrowleft : -A_z \cdot s + f_z \cdot s \cdot \frac{s}{2} + M^{I}_{(s)} = 0$

$\Rightarrow M^{I}_{(s)} = A_z \cdot s - \frac{1}{2} f_z s^2 = 3{,}5s - \frac{1}{2} 1 \cdot s^2$ [kNm]

$V^{I}_{(0)} = A_z = 3{,}5$ kN ; $V^{I}_{(2)} = A_z - f_z \cdot 2 = 3{,}5 - 2 = 1{,}5$ kN – lineární!

$M^{I}_{(0)} = 0$; $M^{I}_{(2)} = A_z \cdot 2 - \frac{1}{2} \cdot 1 \cdot 2^2 = 7 - 2 = 5$ kNm – kvadratický!

II) 2. interval (od síly F k podpoře B) : $s^{II} \in \langle 0;2 \rangle$



$\rightarrow : N^{I}_{(s)} = 0$

$\downarrow : -V^{I}_{(2)} + F + f_z \cdot s + V^{II}_{(s)} = 0$

$\Rightarrow V^{II}_{(s)} = \underbrace{V^{I}_{(2)}}_{\text{posouvací síla z předchozího intervalu}} - F - f_z \cdot s = 1{,}5 - 3 - 1 \cdot s = \underline{-1{,}5 - s}$ [kN]

$\curvearrowleft : -M^{I}_{(2)} - V^{I}_{(2)} \cdot s + F \cdot s + f_z \cdot s \cdot \frac{s}{2} + M^{II}_{(s)} = 0$

$\Rightarrow M^{II}_{(s)} = \underbrace{M^{I}_{(2)}}_{\text{moment na začátku}} + \underbrace{(V^{I}_{(2)} - F) \cdot s - \frac{1}{2} f_z s^2}_{\text{efekt krajních sil}}$

$M^{II}_{(s)} = 5 + (1{,}5 - 3) \cdot s - \frac{1}{2} \cdot 1 \cdot s^2 = 5 - 1{,}5s - 0{,}5s^2$ [kNm]

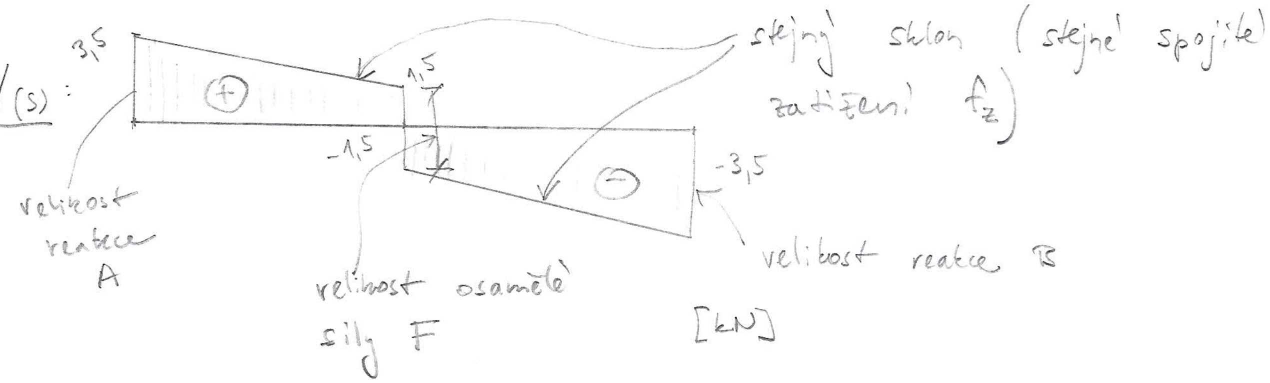
$$V^{II}_{(0)} = V^{I}_{(2)} - F - f_z \cdot 0 = 1{,}5 - 3 - 1 \cdot 0 = -1{,}5 \text{ kN}$$
$$V^{II}_{(2)} = V^{I}_{(2)} - F - f_z \cdot 2 = 1{,}5 - 3 - 1 \cdot 2 = -3{,}5 \text{ kN}$$

} lineární fce

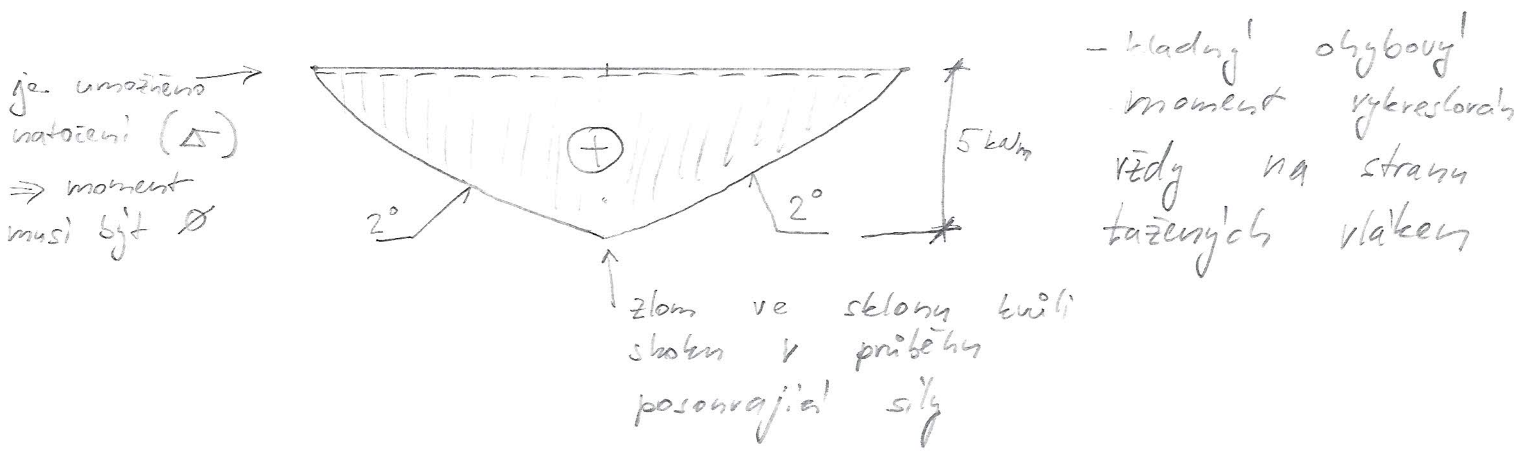
$$M^{II}_{(0)} = M^{I}_{(2)} + \left(V^{I}_{(2)} - F\right) \cdot 0 - \frac{1}{2} f_z \cdot 0^2 = 5 \text{ kNm}$$

kvadratický průběh

$$M^{II}_{(2)} = M^{I}_{(2)} + \left(V^{I}_{(2)} - F\right) \cdot 2 - \frac{1}{2} f_z \cdot 2^2 = 5 + (1{,}5 - 3) \cdot 2 - \frac{1}{2} \cdot 1 \cdot 2^2 = 0$$



$\underline{M_{(s)}}$:



Užití Schwedlerovy věty ke zjištění průběhu vnitřních sil

- souběžné zatížení (f_x) přispívá ke změně normálové síly:

$$\frac{dN}{dx} = -f_x$$

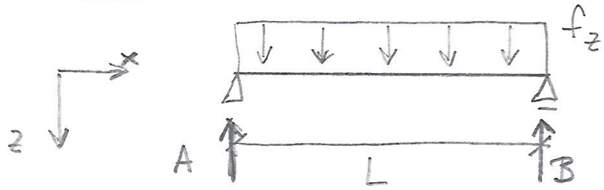
- kolmé zatížení (f_z) přispívá ke změně posouvací síly:

$$\frac{dV}{dx} = -f_z$$

- posouvající síla přispívá ke změně momentu:

$$\frac{dM}{dx} = V$$

PŘ:



$$A = B = \frac{f_z \cdot L}{2}$$

$V = -\int f_z \, dx = -f_z \cdot x + C_1$; C_1… hodnota na počátku (reakce)

$V = -f_z \cdot x + \frac{f_z \cdot L}{2}$ … lineární fce

kvadratická fce

$M = \int V \, dx = \int -f_z \cdot x + \frac{f_z \cdot L}{2} \, dx = -\frac{1}{2} f_z \cdot x^2 + \frac{1}{2} f_z \cdot L \cdot x + C_2$;

C_2… hodnota na počátku (zde $C_2 = 0$)

extrémy posouvací síly:

$\frac{dV}{dx} = 0$: … $-f_z = 0$ … neplatí nikdy, nemá lokální extrém

$V_{(0)} = \frac{f_z \cdot L}{2}$ $(= A)$; $V_{(L)} = -f_z L + \frac{f_z \cdot L}{2} = -\frac{f_z \cdot L}{2}$ $(= -A)$

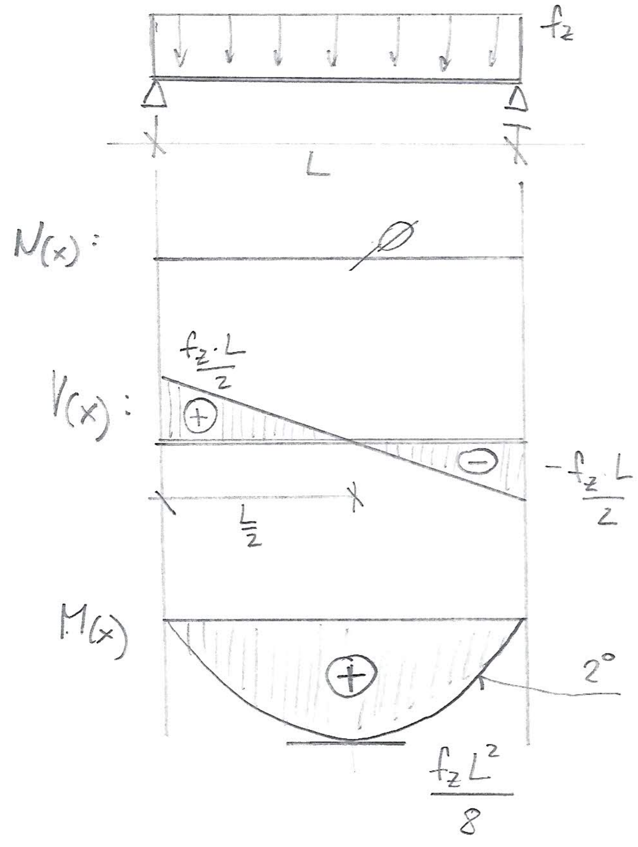
extrémy momentu:

- $\frac{dM}{dx} = 0$ (tam, kde je posouvací síla $= 0$)

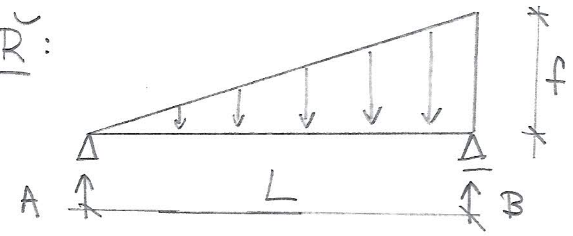
$$\frac{dM}{dx} = 0 : \quad -f_z \cdot x + \frac{f_z \cdot L}{2} = 0$$

$$f_z \cdot \left(\frac{L}{2} - x \right) = 0 \iff x = \frac{L}{2}$$

$$M\left(x = \frac{L}{2}\right) = -\frac{1}{2} \cdot f_z \cdot \left(\frac{L}{2}\right)^2 + \frac{1}{2} f_z \cdot L \cdot \frac{L}{2} = -\frac{1}{8} f_z L^2 + \frac{1}{4} f_z L^2 = \frac{f_z L^2}{8}$$



PŘ:



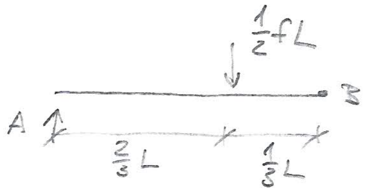
$$f_z(x) = \frac{f}{L} x$$

- výslednice $F = \int_0^L \frac{f}{L} x \, dx = \frac{f}{2L} \left[x^2\right]_0^L = \overbrace{\frac{1}{2} f L}^{= \text{plocha zatěžovacího diagramu}}$

- poloha výslednice (vzdálenost od A):

$$\underbrace{x_A \cdot F}_{\text{moment od } F} = \underbrace{\int_0^L x \cdot \left(\frac{f}{L} x\right) dx}_{\text{moment od spojitého zatížení}}$$

$$x_A = \frac{\int_0^L \frac{f}{L} x^2 \, dx}{F} = \frac{\frac{f}{L}\left[\frac{x^3}{3}\right]_0^L}{\frac{1}{2} f L} = \frac{\frac{f}{L} \cdot \frac{L^3}{3}}{\frac{1}{2} f L} = \frac{2}{3} L$$



- $\curvearrowleft b$: $A \cdot L = \frac{1}{2} f L \cdot \frac{1}{3} L$

$$A = \frac{1}{6} f L$$

- $V(x) = \int -f(x)\, dx = -\int \frac{f}{L} x \, dx = -\frac{f}{L} \cdot \frac{x^2}{2} + C_1 = -\frac{f}{2L} x^2 + \frac{1}{6} f L$

extrémy: $\frac{dV}{dx} = 0$: $-\frac{f}{L} x = 0 \iff x = 0$

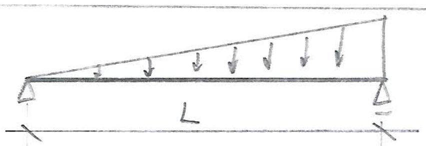
$$V(0) = \frac{1}{6} f L$$
$$V(L) = -\frac{f}{2L} \cdot L^2 + \frac{1}{6} f L = -\frac{fL}{2} + \frac{1}{6} f L = -\frac{1}{3} f L \quad (= -B)$$

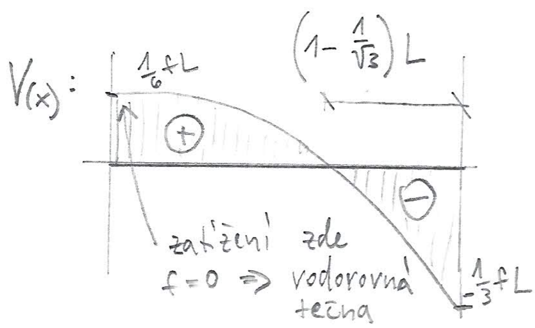
- $M(x) = \int V(x)\, dx = \int -\frac{f}{2L} x^2 + \frac{1}{6} f L \, dx = -\frac{f}{2L} \cdot \frac{x^3}{3} + \frac{1}{6} f L x + C_2$; $C_2 = \emptyset$

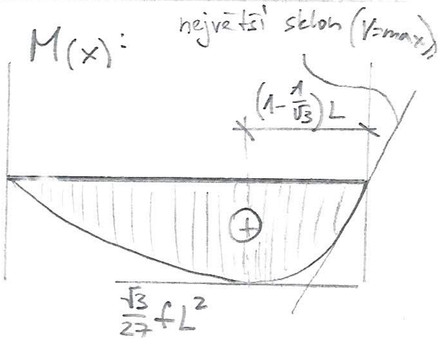
extrémy: $\frac{dM}{dx} = V = 0$: $-\frac{f}{2L} x^2 + \frac{1}{6} f L = 0$

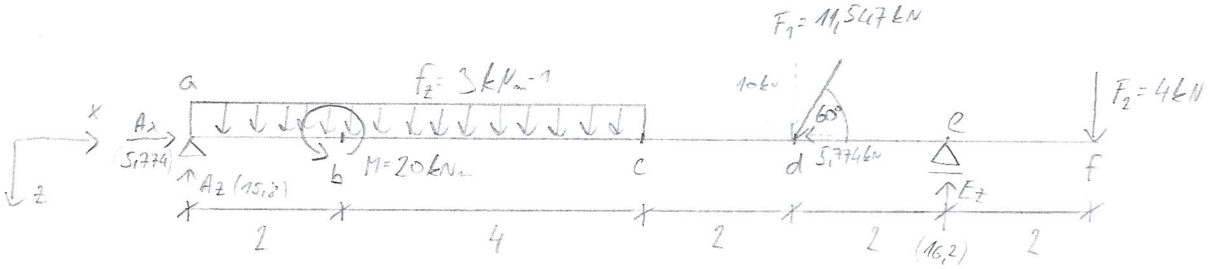
$$-\frac{f}{2L} x^2 = -\frac{1}{6} f L$$
$$x = \pm \sqrt{\frac{1}{3} L^2} \Rightarrow x = \frac{1}{\sqrt{3}} L = \frac{\sqrt{3}}{3} L$$

$$M\left(x = \frac{1}{\sqrt{3}} L\right) = \frac{1}{6} f L \cdot \frac{\sqrt{3}}{3} L - \frac{f}{6L} \left(\frac{\sqrt{3}}{3} L\right)^3 = \frac{\sqrt{3}}{18} f L^2 - \frac{3\sqrt{3}}{162} f L^2 =$$
$$= \frac{\sqrt{3}}{18} f L^2 - \frac{\sqrt{3}}{54} f L^2 = \frac{\sqrt{3}}{27} f L^2$$









$\rightarrow$: $A_x = 5{,}774$ kN

$\curvearrowleft a$: $-3\cdot6\cdot3 + 20 - 10\cdot8 + 10E_z - 4\cdot12 = 0 \longrightarrow E_z = 16{,}2$ kN

$\downarrow$: $-A_z + 3\cdot6 + 10 - 16{,}2 + 4 = 0 \longrightarrow A_z = 15{,}8$ kN

- postup zleva

$$\frac{dV(s)}{ds} = \frac{dV(x)}{dx} = -f_z$$

$\left(f_z > 0 \Rightarrow \frac{dV}{dx} < 0 \Rightarrow V \text{ klesající a naopak...} \right.$

$\left. \frac{df_z}{dx} > 0 \text{ (rostoucí)} \Rightarrow \frac{d^2V}{dx^2} < 0 \Rightarrow V \text{ konkávní a naopak...} \right)$

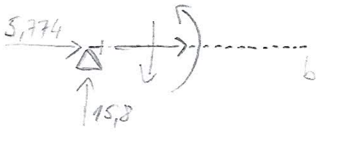
$$\frac{dM(s)}{ds} = \frac{dM(x)}{dx} = V$$

$\left(V > 0 \Rightarrow \frac{dM}{dx} > 0 \Rightarrow M \text{ rostoucí a naopak...} \right.$

$\left. f_z < 0 \Rightarrow \frac{dV}{dx} > 0 \text{ (rostoucí)} \Rightarrow \frac{d^2M}{dx^2} > 0 \Rightarrow M \text{ konvexní a naopak...} \right)$

vyjadřuje místo $\Rightarrow M$: ⊖ ⊕

ab

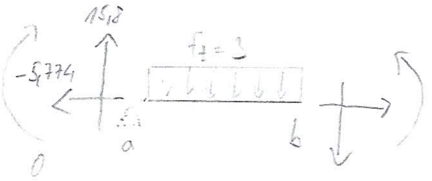


$N_{ab} + 5{,}774 = 0 \longrightarrow N_{ab} = -5{,}774$ kN

$V_{ab} - 15{,}8 = 0 \longrightarrow V_{ab} = 15{,}8$ kN

$M_{ab} = 0$ kNm

znaménka $\rightarrow$ konvence



$N_{ba} - 0 - (-5{,}774) = 0 \longrightarrow N_{ba} = -5{,}774$ kN $= N_{ab}$ – protože v průběhu intervalu ab není žádná změna v normálném směru

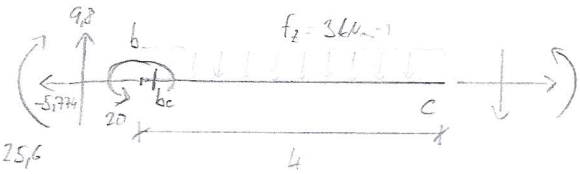
hodnota z předchozího intervalu

$$V_{ba} = \int -3\,dx = -3x + C = -3x + V_{ab} = -3\cdot2 + 15{,}8 = 9{,}8 \text{ kN}$$

$$M_{ba} = \int(-3x + 15{,}8)\,dx = -\frac{3x^2}{2} + 15{,}8x + C = -\frac{3\cdot2^2}{2} + 15{,}8\cdot2 + \overset{M_{ab}}{0} = 25{,}6 \text{ kNm}$$

– u výpočtu pomocí integrálů používání ekvivalence $\rightarrow$ konvence

bc



$N_{bc} = N_{ba} = -5{,}774\,kN$

$V_{bc} = 9{,}8\,kN$

$M_{bc} = 25{,}6 - 20 = 5{,}6\,kNm$

ekvivalence → konvence

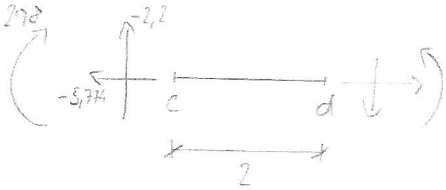
$N_c = N_{bc} = -5{,}774\,kN$

stačí jen jeden index (c), protože v bodě c není žádná nespojitost ve vnitřních silách

$V_c = \int -3\,dx = -3x + C = -3x + V_{bc} = -3\cdot 4 + 9{,}8 = -2{,}2\,kN$

$M_c = \int(-3x + 9{,}8)\,dx = -3\frac{x^2}{2} + 9{,}8x + \overset{M_{bc}}{C} = -3\frac{4^2}{2} + 9{,}8\cdot 4 + 5{,}6 = 20{,}8\,kNm$

cd

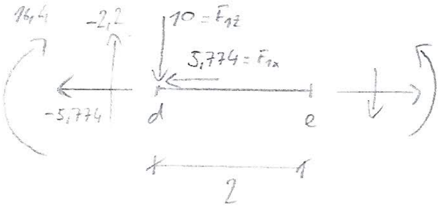


$N_{dc} = N_c = -5{,}774\,kN$

$V_{dc} = \int 0\,dx = C = V_c = -2{,}2\,kN$

$M_{dc} = \int -2{,}2\,dx = -2{,}2x + C = -2{,}2x + M_c = -2{,}2\cdot 2 + 20{,}8 = 16{,}4\,kNm$

de



$N_{de} = N_{dc} + F_{1x} = -5{,}774 + 5{,}774 = 0\,kN$ (ekvivalence →)

$V_{de} = \int 0\,dx = C = V_{dc} - F_{1z} = -2{,}2 - 10 = -12{,}2\,kN$

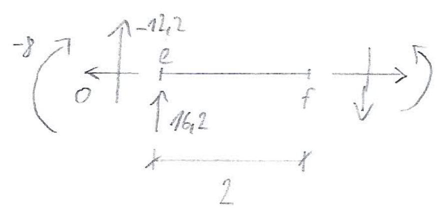
$M_{de} = \int -12{,}2\,dx = -12{,}2x + C = -12{,}2x + M_{dc} = -12{,}2\cdot 0 + 16{,}4 = 16{,}4\,kNm$

$N_{ed} = N_{de} = 0\,kN$

$V_{ed} = V_{de} = -12{,}2\,kN$

$M_{ed} = \int -12{,}2\,dx = -12{,}2x + C = -12{,}2x + 16{,}4 = -12{,}2\cdot 2 + 16{,}4 = -8\,kNm$

ef



$N_{ef} = N_{ed} = 0\,kN$

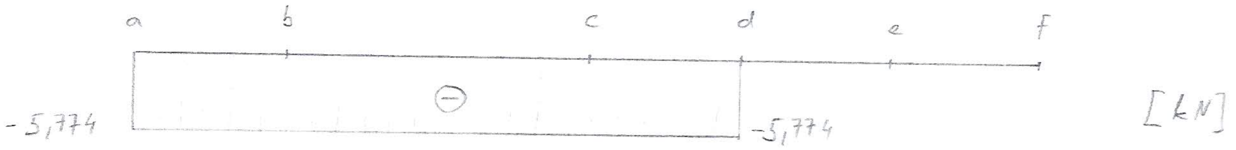
$V_{ef} = V_{ed} + 16{,}2 = -12{,}2 + 16{,}2 = 4\,kN$

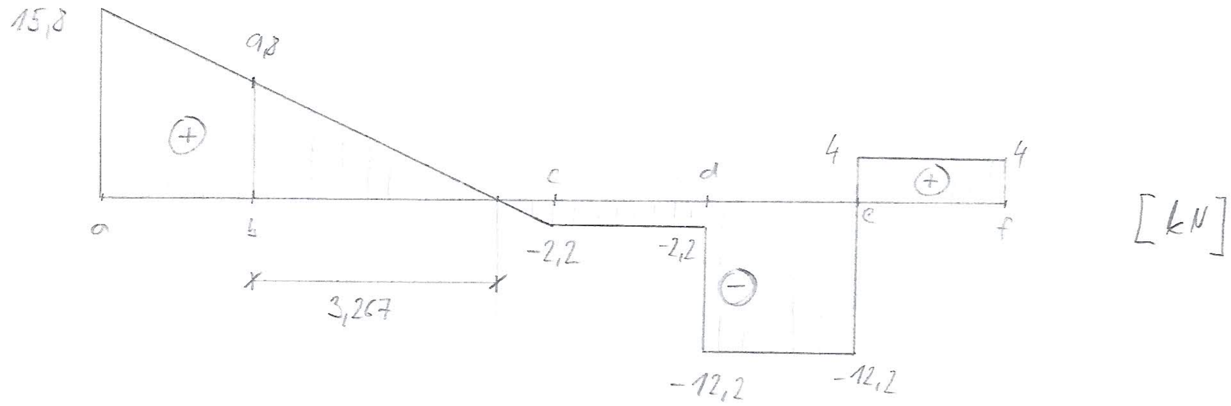
$M_{ef} = \int 4\,dx = 4x + c = 4x + M_{ed} = 4 \cdot 0 - 8 = -8\,kNm$

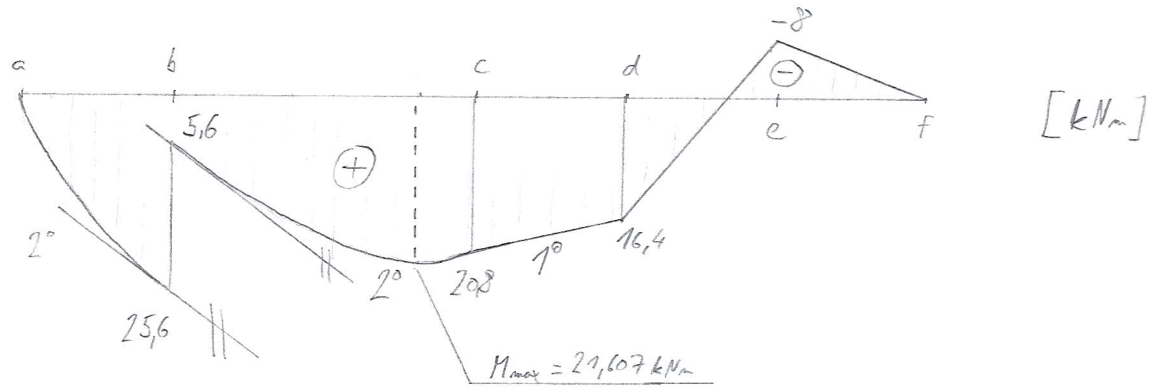
$N_{fe} = N_{ef} = 0\,kN$

$V_{fe} = V_{ef} = 4\,kN$

$M_{fe} = \int 4\,dx = 4x + C = 4x + M_{ef} = 4 \cdot 2 - 8 = 0\,kNm$



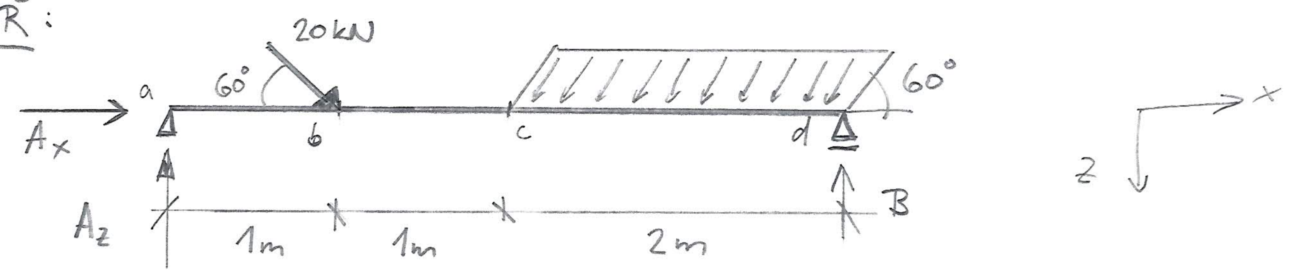




$M_{max} = \frac{dM}{dx} = V = 0 \quad \rightarrow \quad V^{bc} = -3x + 9{,}8 = 0 \quad \rightarrow \quad \underline{x = 3{,}267\,m}$

$M^{bc}(3{,}267) = M_{max} = -3\frac{x^2}{2} + 9{,}8x + 5{,}6 = \underline{21{,}607\,kNm}$

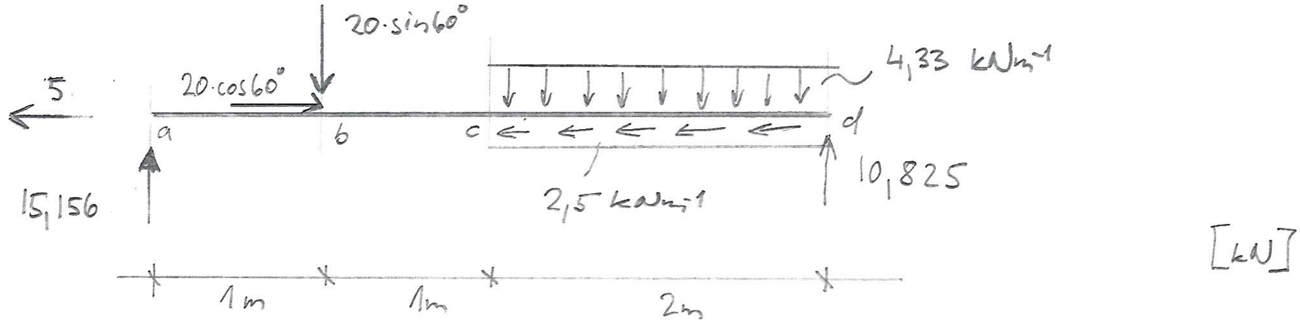
PŘ:



$\rightarrow: \; A_x + 20\cos 60° - 5\cdot\cos 60°\cdot 2 = 0 \quad \Rightarrow A_x = -5 \text{ kN}$

$\curvearrowleft a: \; -20\sin 60°\cdot 1 - 5\cdot\sin 60°\cdot 2\cdot 3 + D\cdot 4 = 0 \quad \Rightarrow D = 10{,}825 \text{ kN}$

$\uparrow: \; A_z + D - 20\sin 60° - 5\cdot\sin 60°\cdot 2 = 0 \quad \Rightarrow A_z = 15{,}156 \text{ kN}$



Normálová síla:

$N_{ab} = 5 \text{ kN}$

$N_{ba} = 5 \text{ kN}$

$N_{bc} = 5 - 20\cdot\cos 60° = -5 \text{ kN}$

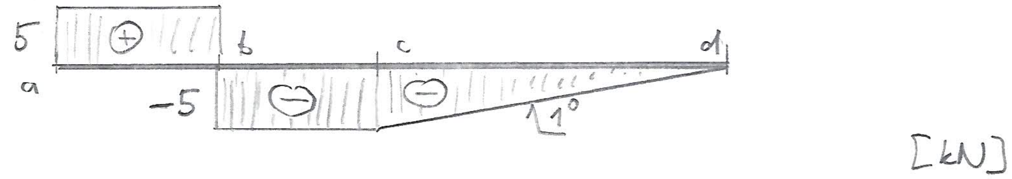
$N_{cb} = -5 \text{ kN}$

$N^{cd}_{(x)} = \int -f_x \, dx = -(-2{,}5)x + C_1 = 2{,}5x - 5 \quad \rightarrow$ lineární fce

$N_{cd} = N^{cd}_{(0)} = -5 \text{ kN}$

$N_{dc} = N^{cd}_{(2)} = 0 \text{ kN}$

$N_{(x)}$:



Posouvací síla: $V_{ab} = 15{,}156 \text{ kN} = V_{ba}$

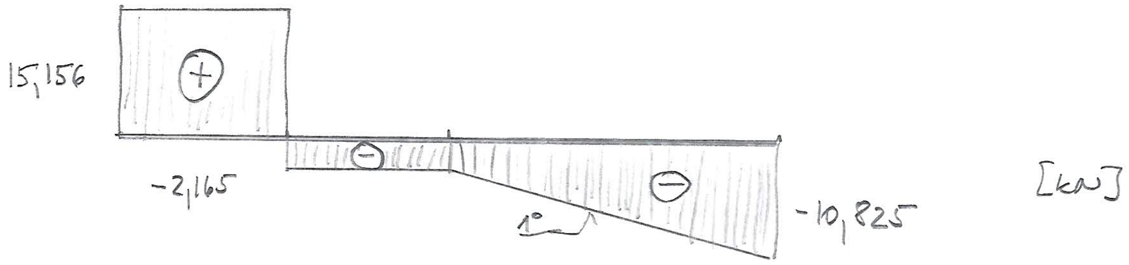
$V_{bc} = 15{,}156 - 20\cdot\sin 60° = -2{,}165 \text{ kN} = V_{cb}$

lineární fce

$$V_{(x)}^{cd} = \int -f_2 \, dx = \int -4{,}33 \, dx = -4{,}33x + C_2 = -4{,}33x - 2{,}165$$

$$V_{cd} = V_{(0)}^{cd} = -2{,}165 \text{ kN}$$

$$V_{dc} = V_{(2)}^{cd} = -10{,}825 \text{ kN}$$



Moment:

$$M_a = 0 \text{ kNm}$$

lineární fce →

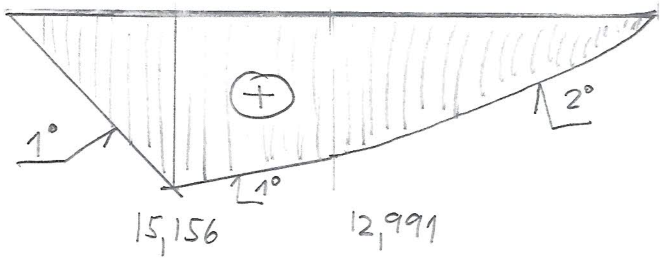
$$M_{(x)}^{ab} = \int V_{(x)} dx = \int 15{,}156 \, dx = 15{,}156x \rightarrow M_b = 15{,}156 \text{ kNm}$$

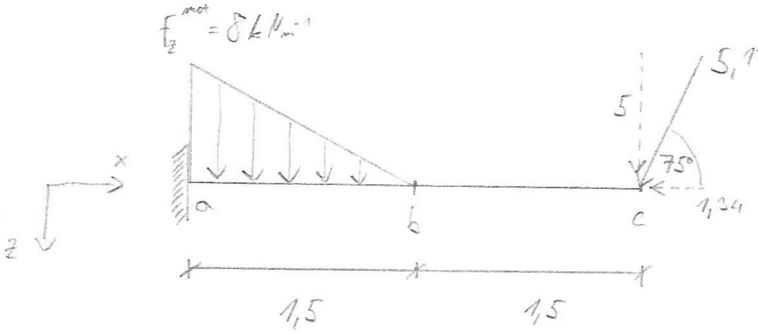
$$M_{(x)}^{bc} = 15{,}156 + (-2{,}165)x \rightarrow M_c = 12{,}991 \text{ kNm}$$

kvadratická fce →

$$M_{(x)}^{cd} = 12{,}991 + \int -4{,}33x - 2{,}165 \, dx = 12{,}991 - \frac{4{,}33}{2}x^2 - 2{,}165x$$

$$\rightarrow M_d = 0 \text{ kNm} \ ✓$$

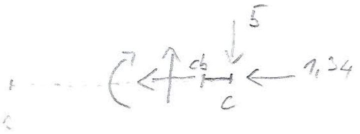




• postup zprava → nemusíme počítat reakce $\Rightarrow \frac{dV}{dx} = +f_z \rightarrow V = \int f_z \, dx$

$\frac{dM}{dx} = -V \rightarrow M = \int -V \, dx$

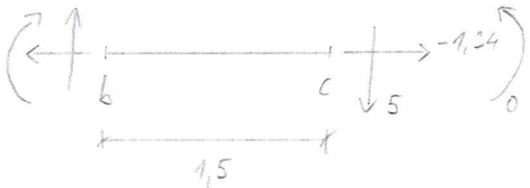
bc



$N_{cb} = -1{,}34\,kN$

$V_{cb} = 5\,kN$ tato síla to otáčí ve stejném směru jako vnitřní moment nalevo – to je nepřípustné, musí být naopak a odečítat → (−)

$M_{cb} = \int -5\,dx = -5x + C = -5 \cdot 0 + 0 = 0\,kNm$

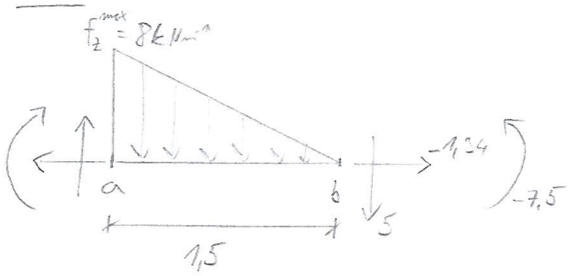


$N_b = N_{cb} = -1{,}34\,kN$

$V_b = V_{cb} = 5\,kN$

$M_b = \int -5\,dx = -5x + C = -5x + M_{cb} = -5 \cdot 1{,}5 + 0 = -7{,}5\,kNm$

ab



$N_{ab} = N_b = -1{,}34\,kN$

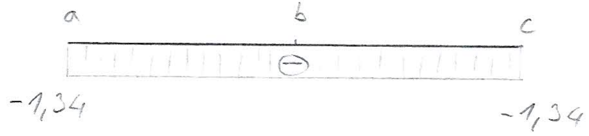
$f_z(x) = \frac{8}{1{,}5} x = 5{,}333\,x$

$\hookrightarrow V_{ab} = \int +5{,}333x\,dx = 5{,}333 \frac{x^2}{2} + \overset{V_b}{C} = 5{,}333 \frac{1{,}5^2}{2} + 5 = 11\,kN$

$M_{ab} = \int -\left(5{,}\overline{33} \frac{x^2}{2} + 5\right) dx = -5{,}\overline{33} \frac{x^3}{6} - 5x + \overset{M_b}{C} =$

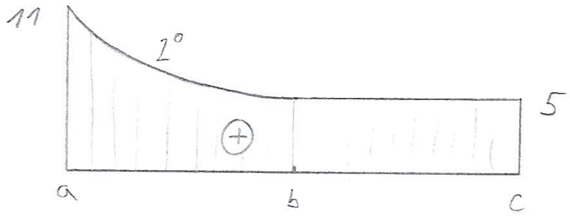
$= -5{,}\overline{33} \frac{1{,}5^3}{6} - 5 \cdot 1{,}5 - 7{,}5 = -18\,kNm$

N



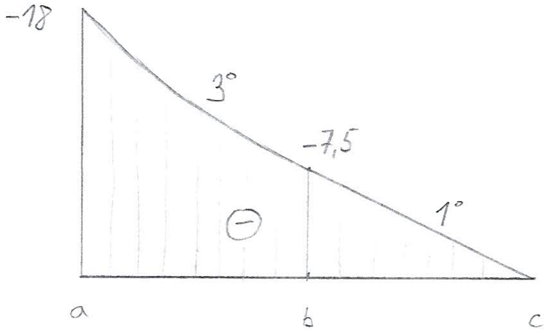
[kN]

V

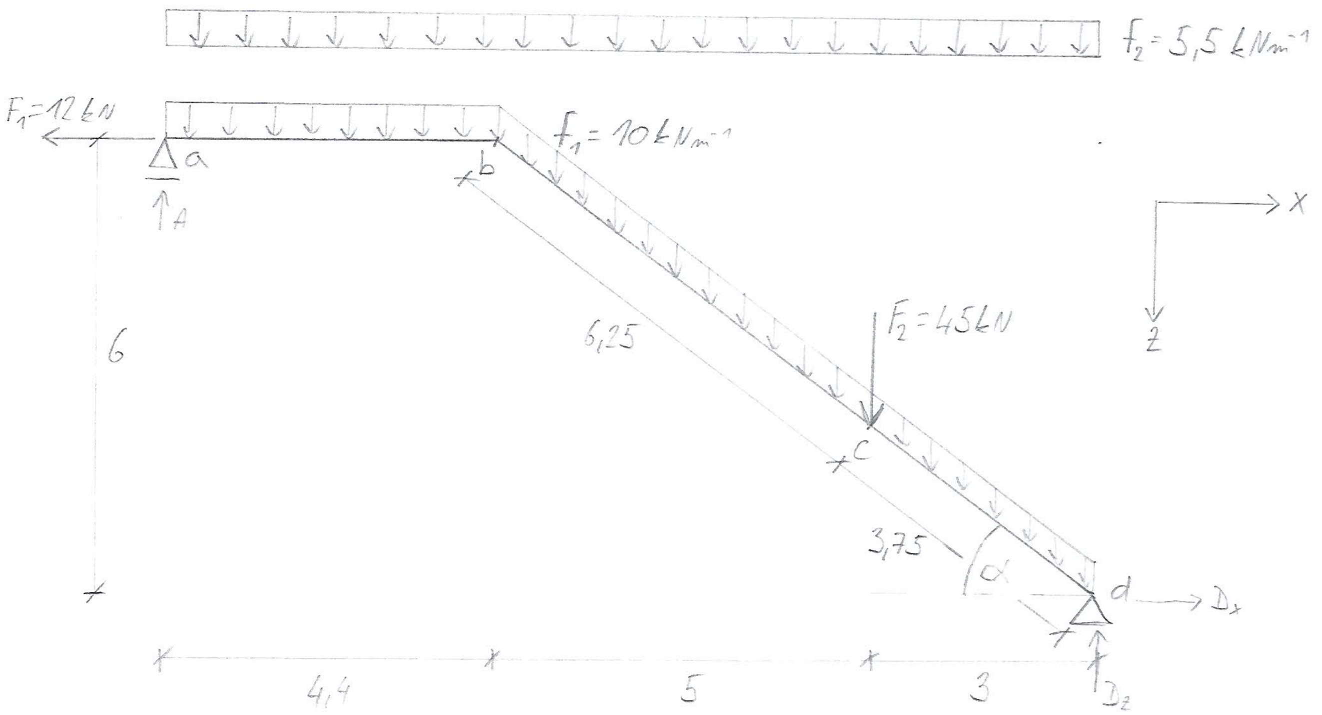


[kN]

M



[kNm]

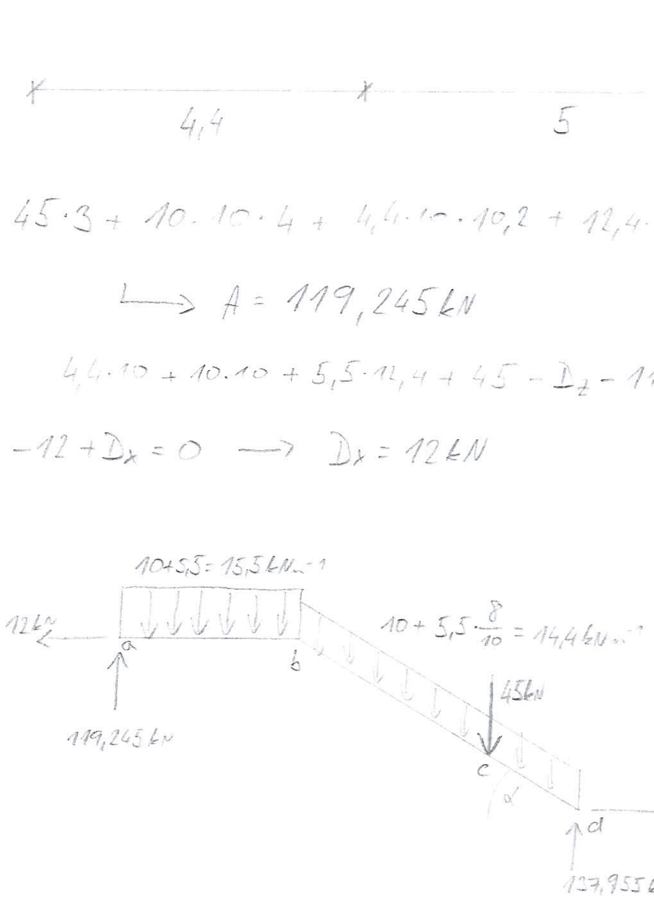


$\curvearrowleft M_a: \; 45\cdot 3 + 10\cdot 10\cdot 4 + 4{,}4\cdot 10\cdot 10{,}2 + 12{,}4\cdot 5{,}5\cdot 6{,}2 + 12\cdot 6 - A\cdot 12{,}4 = 0$

$\longrightarrow A = 119{,}245\,kN$

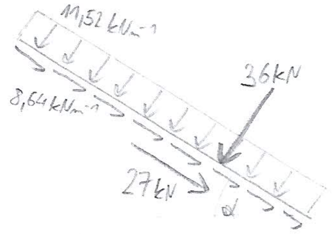
$\downarrow: \quad 4{,}4\cdot 10 + 10\cdot 10 + 5{,}5\cdot 12{,}4 + 45 - D_z - 119{,}245 = 0 \longrightarrow D_z = 137{,}955\,kN$

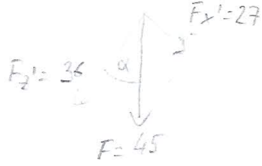
$\longrightarrow: \; -12 + D_x = 0 \longrightarrow D_x = 12\,kN$



$\cos\alpha = 0{,}8$

$\sin\alpha = 0{,}6$





lokální = natočené

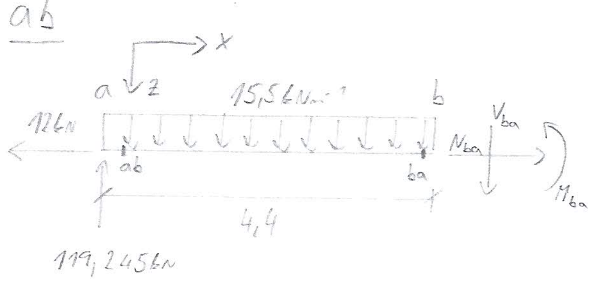
$F_{z'} = \cos\alpha \cdot F = 36\,kN$

$F_{x'} = \sin\alpha \cdot F = 27\,kN$

$f_{z'} = \cos\alpha \cdot 14{,}4 = 11{,}52\,kNm^{-1}$

$f_{x'} = \sin\alpha \cdot 14{,}4 = 8{,}64\,kNm^{-1}$

ab



$N_{ab} = 12\ kN$

$V_{ab} = 119{,}245\ kN$

$M_{ab} = 0\ kNm$ (kloub)

$N_{ba} = N_{ab} = 12\ kN$

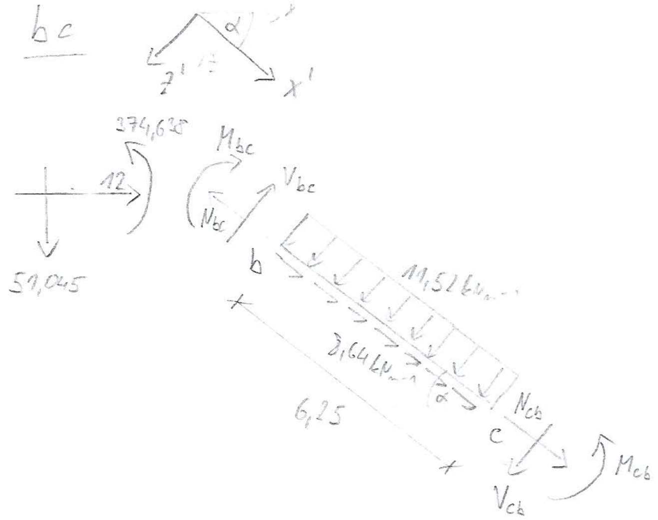
$V_{ba} = \int -15{,}5\, dx = -15{,}5x + V_{ab} = -15{,}5 \cdot 4{,}4 + 119{,}245 =$

$= 51{,}045\ kN$

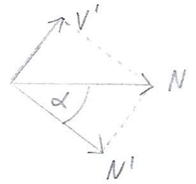
$M_{ba} = \int (-15{,}5x + 119{,}245)\, dx = -15{,}5 \frac{x^2}{2} + 119{,}245x + 0 =$

$= 374{,}638\ kNm$

bc

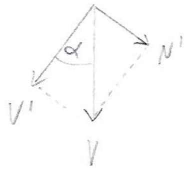


• transformace



$N' = N \cdot \cos\alpha$

$V' = -N \cdot \sin\alpha$ (V' jde nahoru)



$N' = V \cdot \sin\alpha$

$V' = V \cdot \cos\alpha$

$\Rightarrow N' = N \cdot \cos\alpha + V \cdot \sin\alpha$

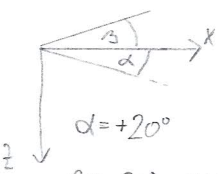
$V' = -N \sin\alpha + V \cdot \cos\alpha$

NEBO můžeme použít transformační matici $[T]$

$$[F'] = [T][F]$$

$$\begin{bmatrix} N' \\ V' \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} N \\ V \end{bmatrix}$$

POZOR!! kladný úhel je ve směru hodinových ručiček → prof. Kolek má v přednášce opačný úhel (proti směru ručiček), proto on má $[T] = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$



$\alpha = +20°$

$\beta = -20° = +340°$

• pozn:

$[F] = [T]^{-1}[F']$

• ale $[T]$ je ortogonální →

$[T]^{-1} = [T]^T$

↳ $[F] = [T]^T [F']$

$$N'_{ba} = 12 \cdot 0,8 + 51,045 \cdot 0,6 = 40,227 \text{ kN}$$

$$V'_{ba} = -12 \cdot 0,6 + 51,045 \cdot 0,8 = 33,636 \text{ kN}$$

$M'_{ba} = M_{ba}$ – moment se „prenaši", netransformije se

$$N_{bc} = N'_{ba} = 40,227 \text{ kN}$$

$$V_{bc} = V'_{ba} = 33,636 \text{ kN}$$

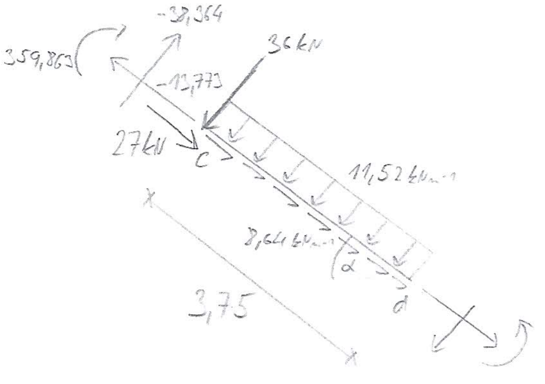
$$M_{bc} = M'_{ba} = M_{ba} = 374,638 \text{ kNm}$$

$$N_{cb} = \int -8,64\, dx = -8,64x + N_{bc} = -8,64 \cdot 6,25 + 40,227 = -13,773 \text{ kN}$$

$$V_{cb} = \int -11,52\, dx = -11,52x + V_{bc} = -11,52 \cdot 6,25 + 33,636 = -38,364 \text{ kN}$$

$$M_{cb} = \int (-11,52x + 33,636)\, dx = -11,52 \frac{x^2}{2} + 33,636x + \overset{374,638}{M_{bc}} = 359,863 \text{ kNm}$$

<u>cd</u>



$$N_{cd} = N_{cb} - 27 = -40,773 \text{ kN}$$

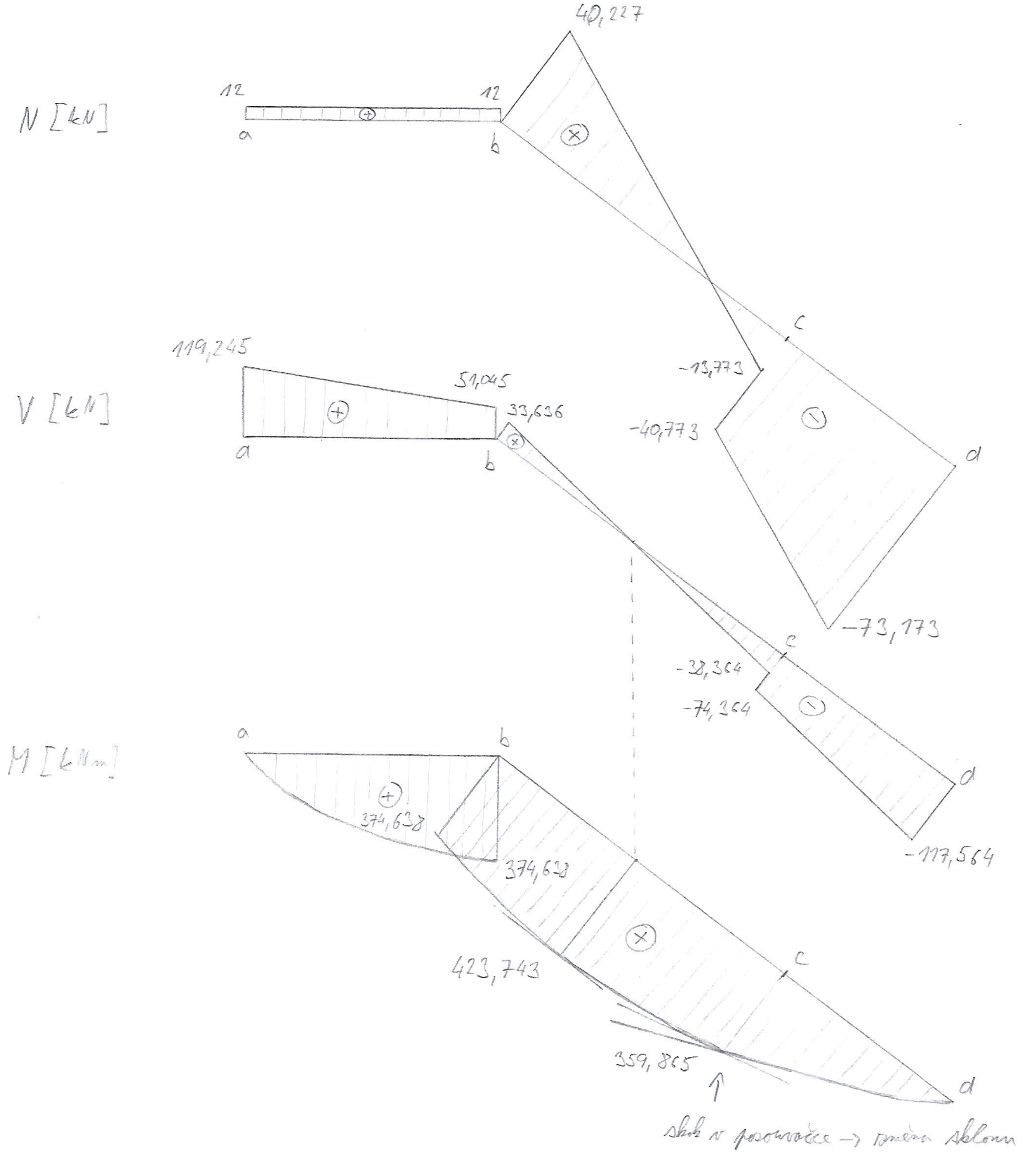
$$V_{cd} = V_{cb} - 36 = -74,364 \text{ kN}$$

$$M_{cd} = M_{cb} = 359,863 \text{ kNm}$$

$$N_{dc} = \int -8,64\, dx = -8,64x + N_{cd} = -8,64 \cdot 3,75 - 40,773 = -73,173 \text{ kN}$$

$$V_{dc} = \int -11,52\, dx = -11,52x + V_{cd} = -11,52 \cdot 3,75 - 74,364 = -117,564 \text{ kN}$$

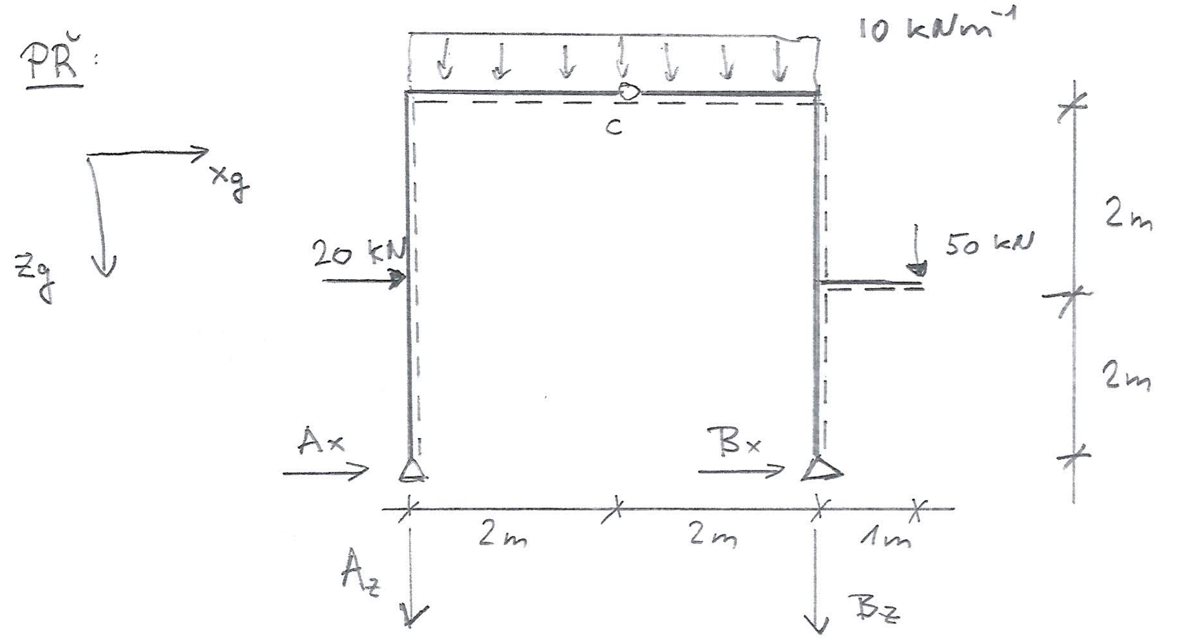
$$M_{dc} = \int (-11,52x - 74,364)\, dx = -11,52 \frac{x^2}{2} - 74,364x + \overset{359,863}{M_{cd}} = 0 \text{ kNm}$$



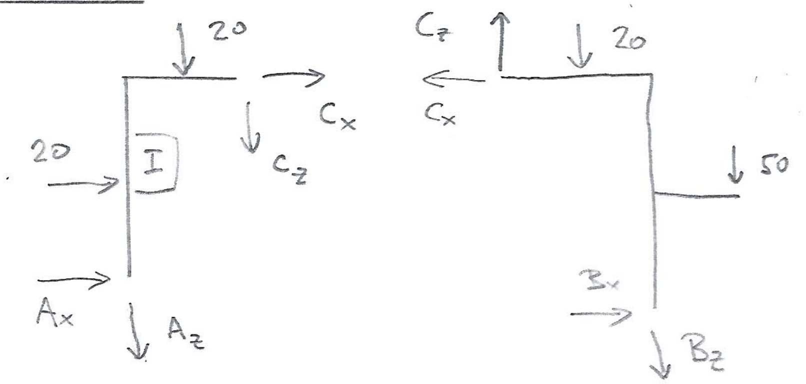
$$M_{max} = \frac{dM}{dx} = V = 0 \longrightarrow V_{cb} = -11{,}52\,x + 33{,}636 = 0 \longrightarrow x = 2{,}92\ m$$

$$M^{cb}(2{,}92) = -11{,}52\,\frac{x^2}{2} + 33{,}636\,x + 374{,}638 = 423{,}743\ kNm$$

PŘ:



Reakce:



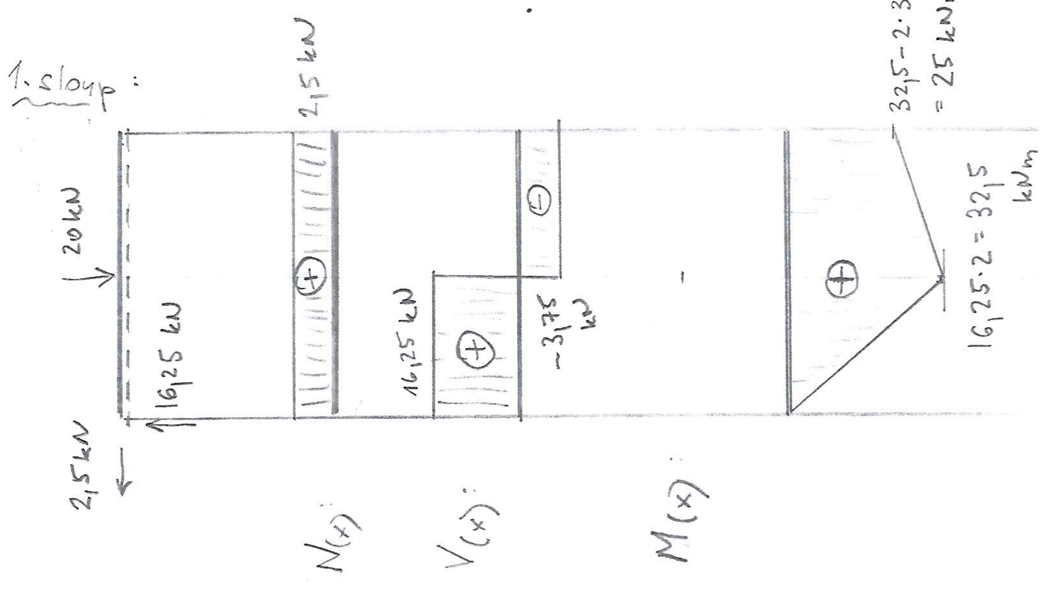
globální $\curvearrowleft A$: $-B_z \cdot 4 - 50 \cdot 5 - 10 \cdot 4 \cdot 2 - 20 \cdot 2 = 0 \Rightarrow B_z = -92{,}5$ kN

globální $\downarrow$: $A_z + B_z + 10 \cdot 4 + 50 = 0 \Rightarrow A_z = 2{,}5$ kN

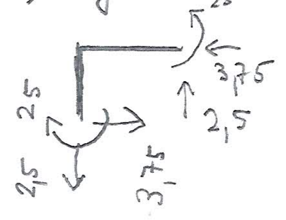
I $\curvearrowleft c$: $A_x \cdot 4 + A_z \cdot 2 + 10 \cdot 2 \cdot 1 + 20 \cdot 2 = 0 \Rightarrow A_x = -16{,}25$ kN

globální $\rightarrow$: $A_x + B_x + 20 = 0 \Rightarrow B_x = -3{,}75$ kN

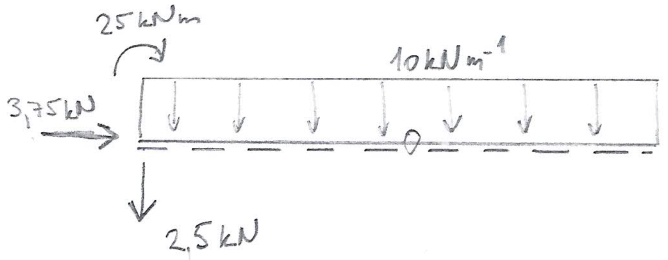
1. sloup:



⇒ styčník:



horní příčel :



$N_{(x)} = \text{konst} = -3{,}75 \text{ kN}$

$V_{(x)} = -2{,}5 - \int 10\,dx = -2{,}5 - 10x$

$V_{(0)} = -2{,}5 \text{ kN}$

$V_{(4)} = -2{,}5 - 10 \cdot 4 = -42{,}5 \text{ kN}$

$M_{(x)} = 25 + \int -2{,}5 - 10x \, dx$

$M_{(x)} = 25 - 2{,}5x - 5x^2$

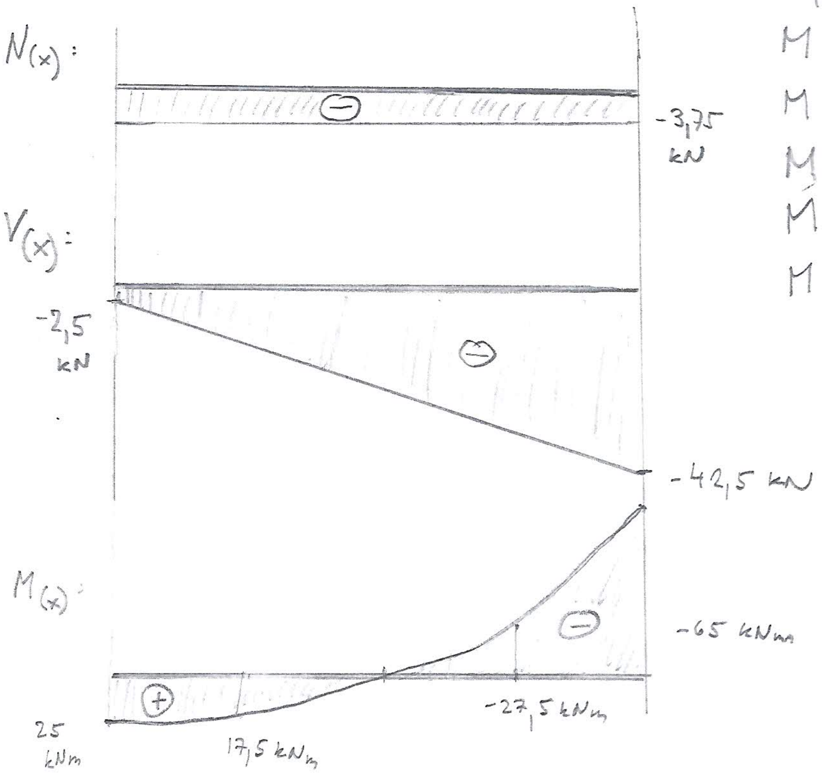
$M_{(0)} = 25 \text{ kNm}$

$M_{(1)} = 17{,}5 \text{ kNm}$

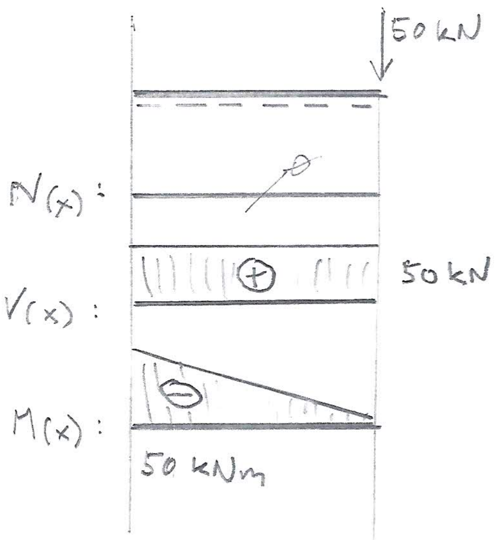
$M_{(2)} = 0$

$M_{(3)} = -27{,}5 \text{ kNm}$

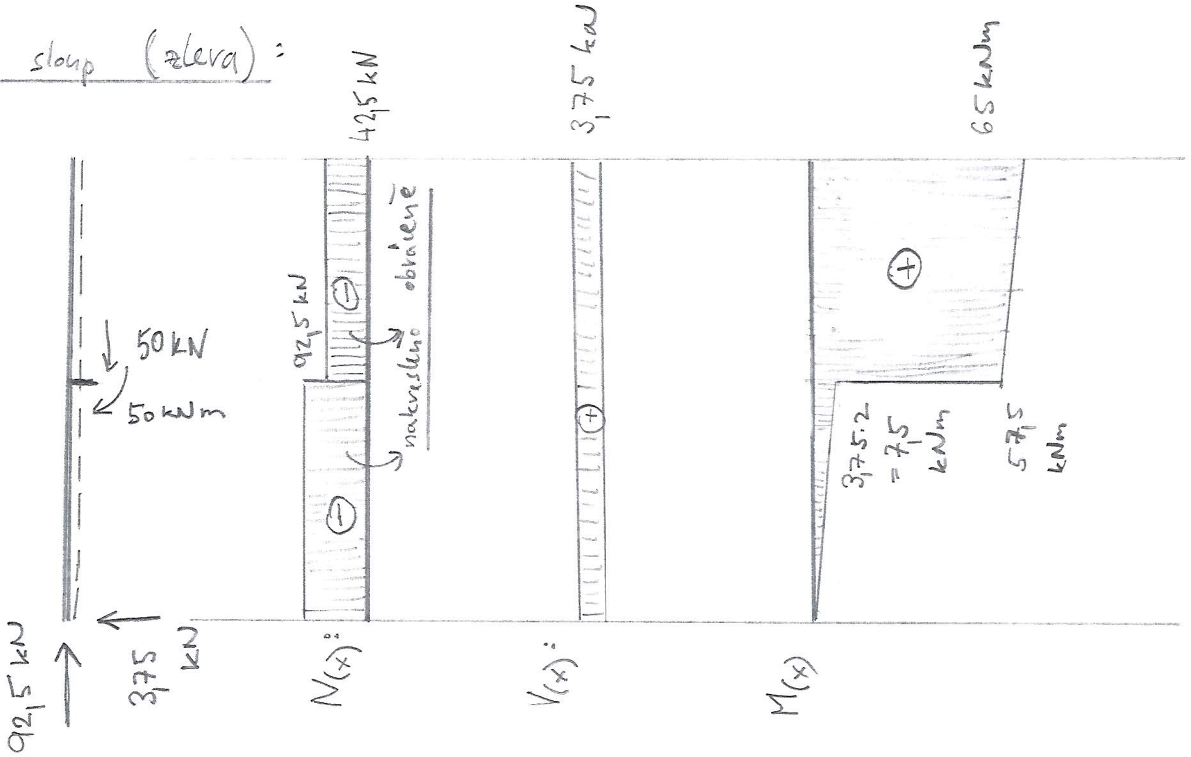
$M_{(4)} = -65 \text{ kNm}$

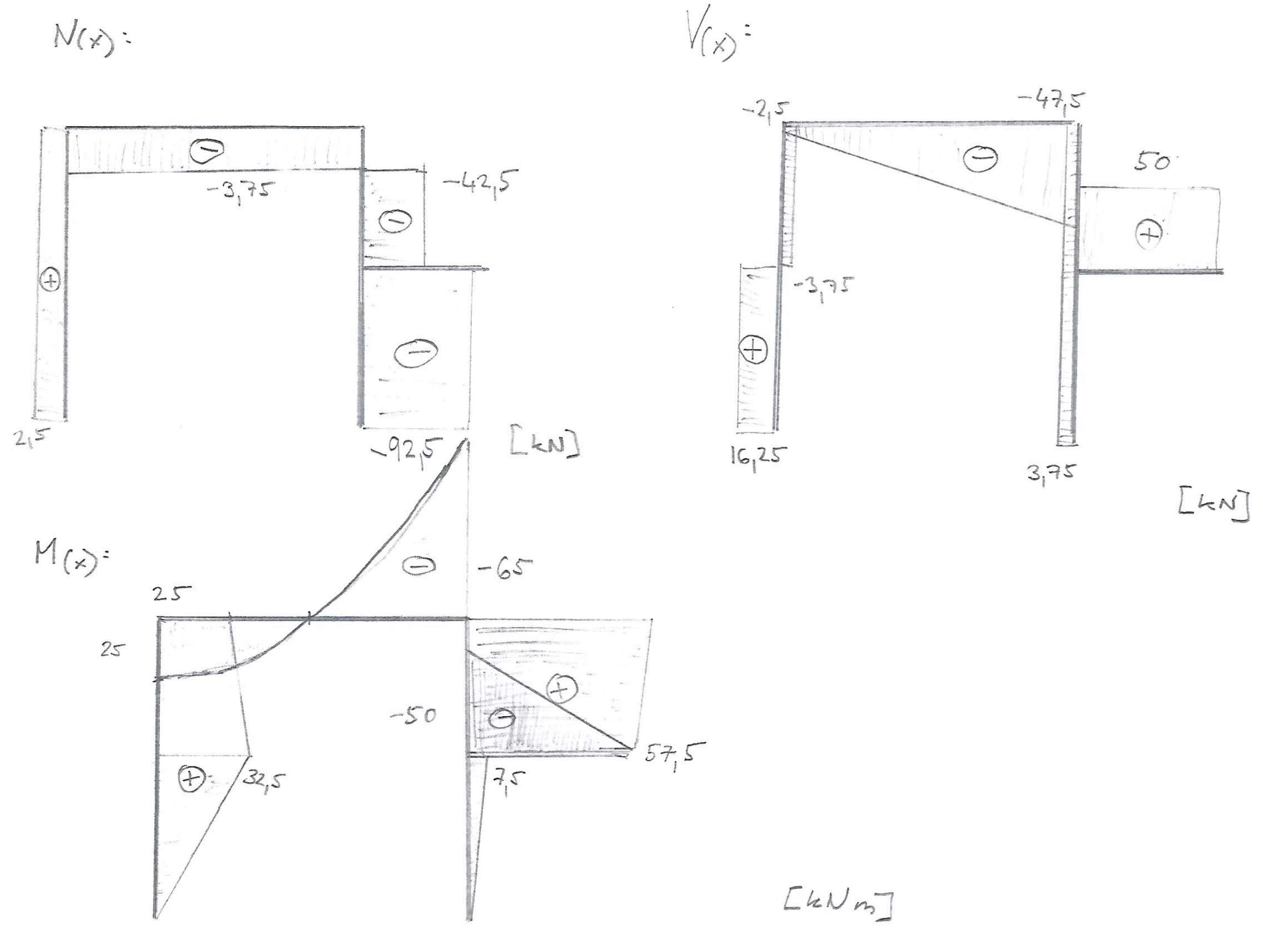


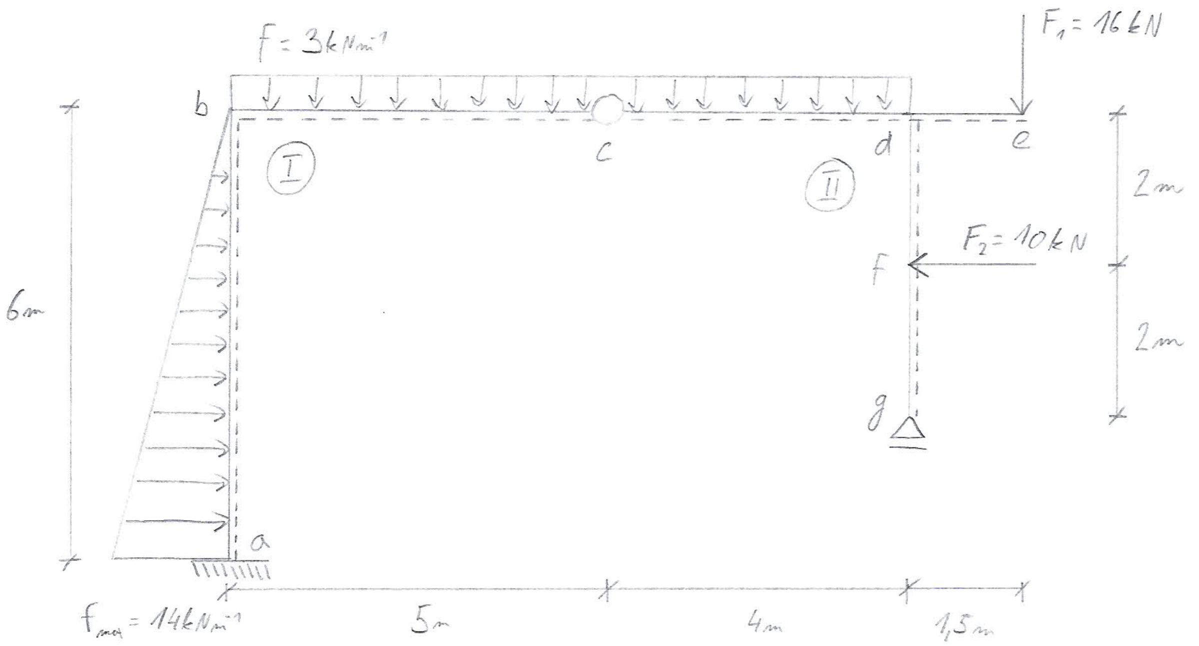
Malá konzola na pravém sloupu (postup zprava):

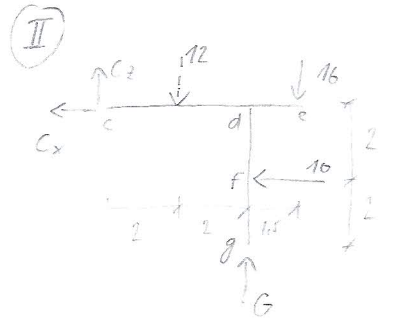


Pravý sloup (zleva):





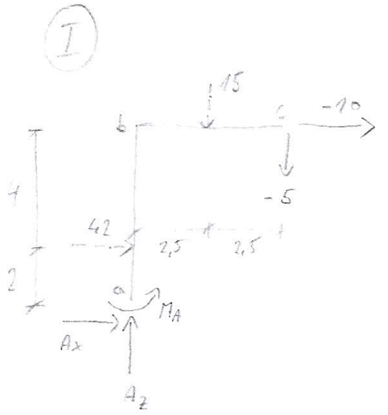




$\rightarrow: \; -C_x - 10 = 0 \;\longrightarrow\; C_x = -10\,kN$

$\circlearrowleft c: \; -12\cdot 2 - 16\cdot 5{,}5 - 10\cdot 2 + G\cdot 4 = 0 \;\longrightarrow\; G = 33\,kN$

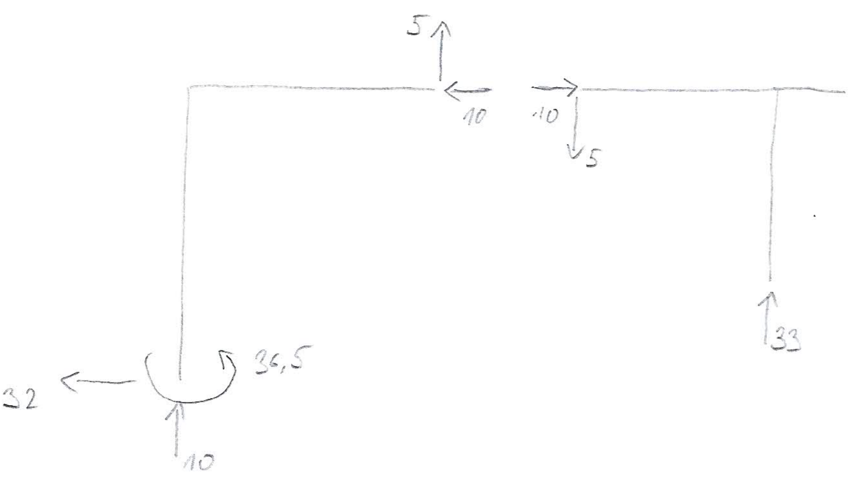
$\downarrow: \; -C_z + 12 + 16 - 33 = 0 \;\longrightarrow\; C_z = -5\,kN$



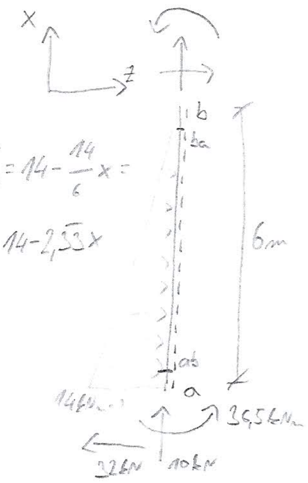
$\rightarrow: \; A_x + 42 + (-10) = 0 \;\longrightarrow\; A_x = -32\,kN$

$\downarrow: \; -A_z + 15 + (-5) = 0 \;\longrightarrow\; A_z = 10\,kN$

$\circlearrowleft c: \; 15\cdot 2{,}5 + 42\cdot 4 - 32\cdot 6 - 10\cdot 5 + M_A = 0 \;\longrightarrow\; M_A = 36{,}5\,kNm$



ab



$F_z = 14 - \frac{14}{6}x =$

$= 14 - 2{,}\overline{33}x$

$N_{ab} = -10\,kN$

$V_{ab} = 32\,kN$

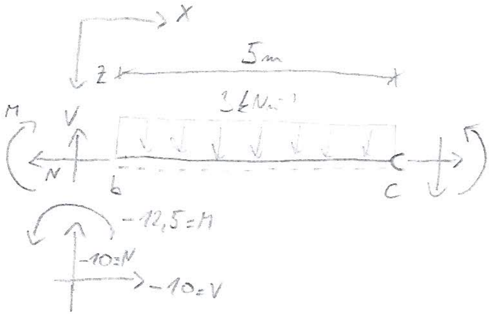
$M_{ab} = -36{,}5\,kNm$

$N_{ba} = N_{ab} = -10\,kN$

$V_{ba} = \int -(14 - 2{,}\overline{33}x)\,dx = -14x + 2{,}\overline{33}\frac{x^2}{2} + V_{ab} = -14\cdot 6 + 2{,}\overline{33}\cdot\frac{6^2}{2} + 32 = -10\,kN$

$M_{ba} = \int\left(-14x + 2{,}\overline{33}\frac{x^2}{2} + 32\right)dx = -14\frac{x^2}{2} + 2{,}\overline{33}\frac{x^3}{6} + 32x + M_{ab} = -7\cdot 6^2 + 2{,}\overline{33}\cdot\frac{6^3}{6} + 32\cdot 6 - 36{,}5 = -12{,}5\,kNm$

bc



$N_{bc} = V_{ba} = -10\,kN$

$V_{bc} = -N_{ba} = 10\,kN$

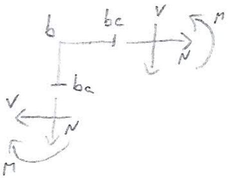
$M_{bc} = M_{ba} = -12{,}5\,kNm$

• je možná transformace $\alpha = 90°$

$N' = \cancel{\cos}^{0}\alpha\, N + \cancel{\sin}^{1}\alpha\, V = V$

$V' = -\cancel{\sin}^{1}\alpha\, N + \cancel{\cos}^{0}\alpha\, V = -N$

• nebo souřadnice ve styčníku



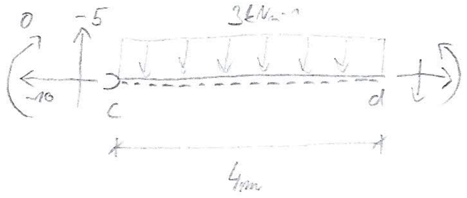
$N_c = N_{bc} = -10\,kN$

$V_c = \int -3\,dx = -3x + V_{bc} = -3\cdot 5 + 10 = -5\,kN$

$M_c = \int(-3x + 10)\,dx = -3\frac{x^2}{2} + 10x - 12{,}5 = -3\frac{5^2}{2} + 10\cdot 5 - 12{,}5 = 0\,kNm$

• v bodě c žádná změna sil (nespojitost) → není třeba cb, cd

cd



$N_{dc} = N_c = -10\,kN$

$V_{dc} = \int -3\,dx = -3x + V_c = -3\cdot 4 - 5 = -17\,kN$

$M_{dc} = \int(-3x - 5)\,dx = -3\frac{x^2}{2} - 5x + 0 = -3\cdot\frac{4^2}{2} - 5\cdot 4 = -44\,kNm$

de

• ve styčníku v bodě d neznáme všechny síly (od sloupu) → postup zprava

$N_{ed} = 0\,kN$

$V_{ed} = 16\,kN$

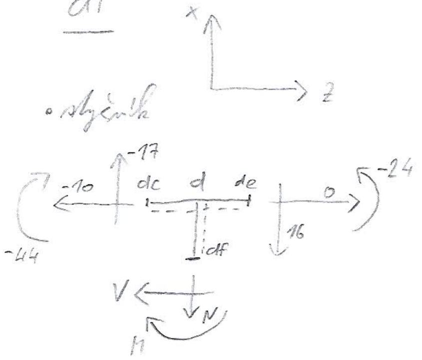
$M_{ed} = 0\,kN$

$N_{de} = N_{ed} = 0\,kN$

$V_{de} = V_{ed} = 16\,kN$

$M_{de} = \int -16\,dx = -16\cdot x + M_{ed} = -16\cdot 1{,}5 + 0 = -24\,kNm$

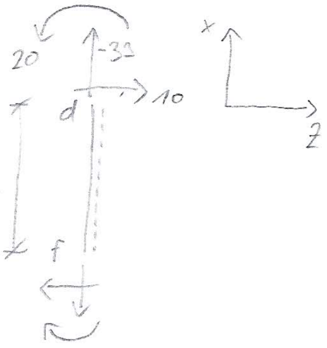
df



$N_{df} = V_{dc} - V_{de} = -17 - 16 = -33\,kN$

$V_{df} = -N_{dc} + N_{de} = -(-10) + 0 = 10\,kN$

$M_{df} = -M_{dc} + M_{de} = 44 - 24 = 20\,kNm$

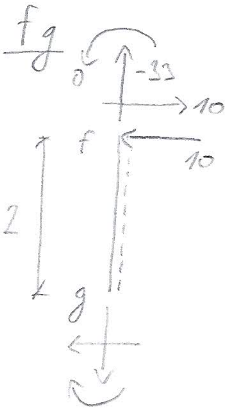


• jdeme směrem dolů i.e. proti směru osy x → postup zprava

$N_{fd} = N_{df} = -33\,kN$

$V_{fd} = V_{df} = 10\,kN$

$M_{fd} = \int -10\,dx = -10x + M_{df} = -10 \cdot 2 + 20 = 0\,kNm$



$N_{fg} = N_{fd} = -33\,kN$

$V_{fg} = V_{fd} - 10 = 10 - 10 = 0\,kN$

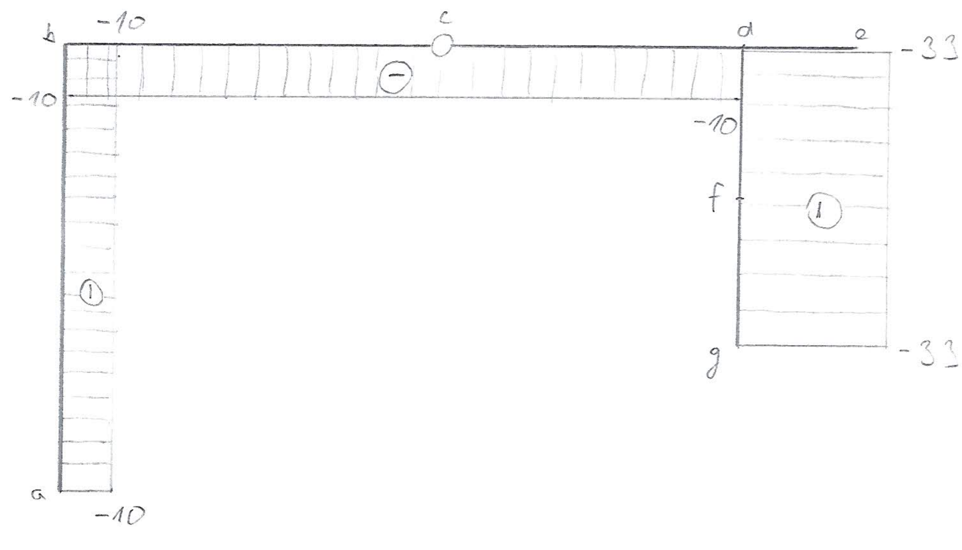
$M_{fg} = \int 0\,dx = C = M_{fd} = 0\,kNm$

$N_{gf} = N_{fg} = -33\,kN$

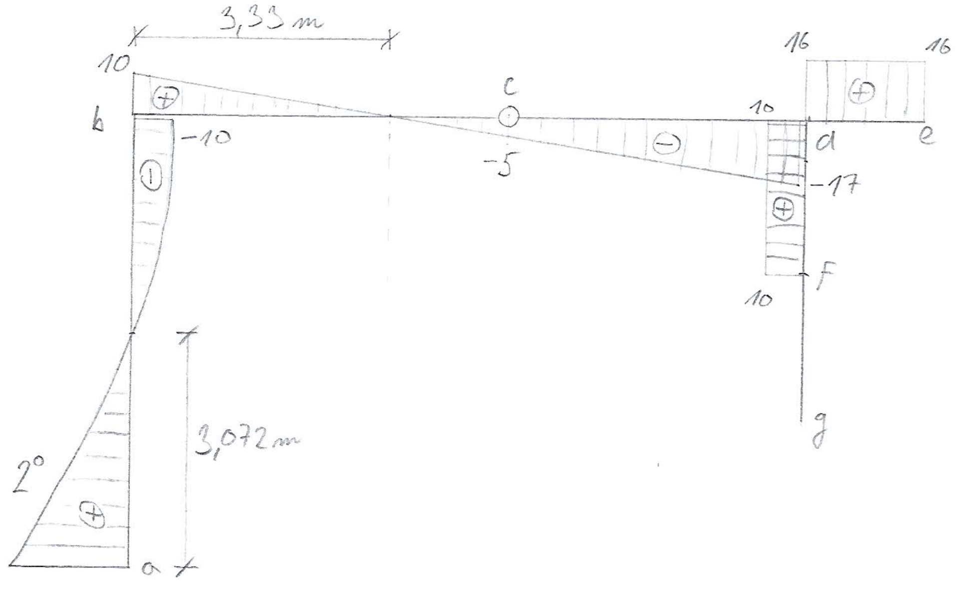
$V_{gf} = V_{fg} = 0\,kN$

$M_{gf} = M_{fg} = 0\,kNm$

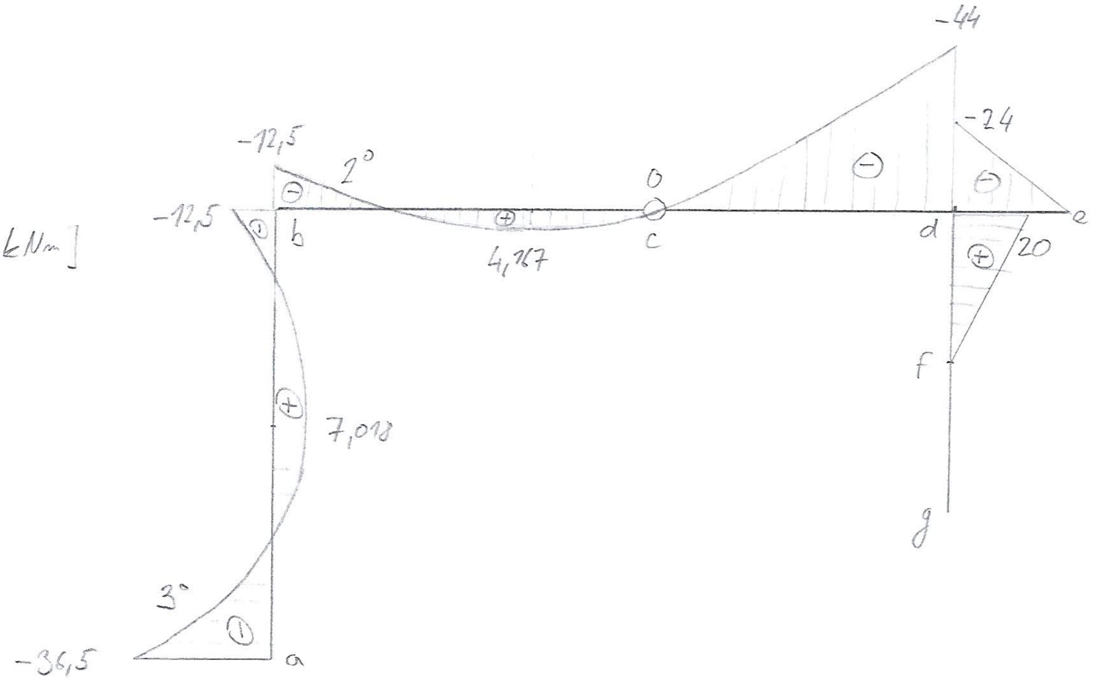
N [kN]



$V\,[kN]$



$M\,[kNm]$



$$M_{max} = \frac{dM}{dx} = V = 0$$

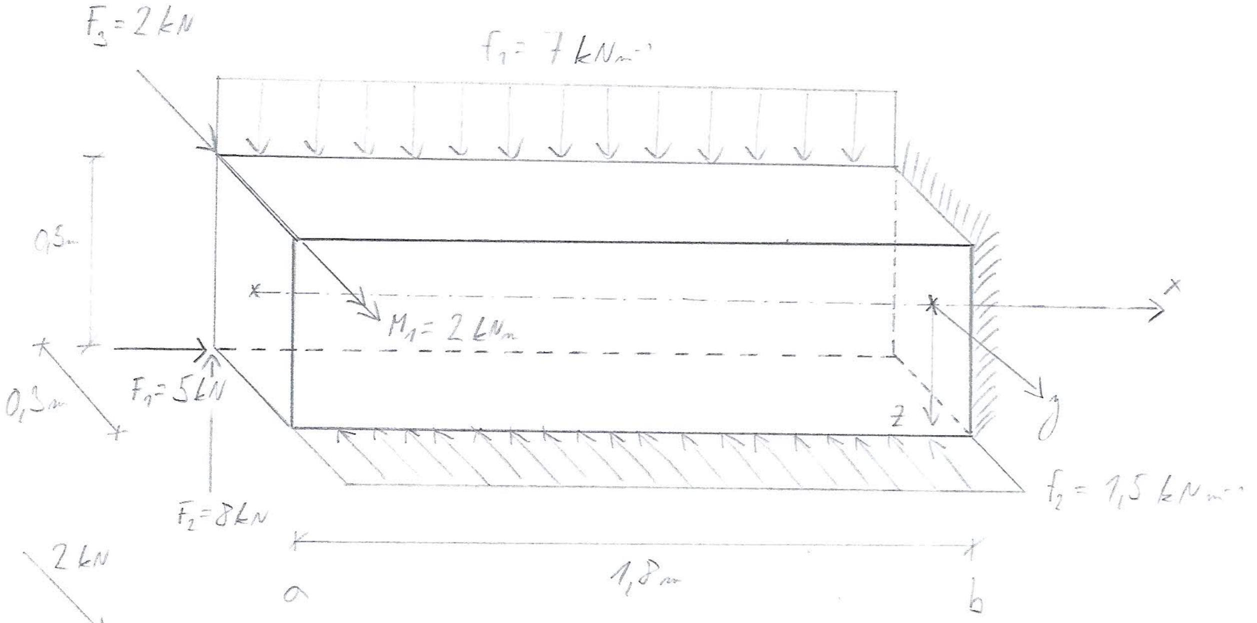
$$V_{ab} = -14x + 2{,}33\frac{x^2}{2} + 32 = 0 \longrightarrow \cancel{x_1 = 8{,}93\,m} \quad x_2 = 3{,}072\,m$$

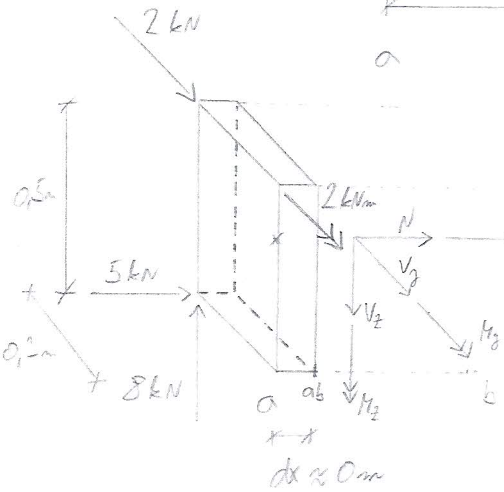
$$V_c = -3x + 10 = 0 \longrightarrow x = 3{,}3\overline{3}\,m$$

$$M^{ab}(3{,}072) = -14\frac{x^2}{2} + 2{,}3\overline{3}\frac{x^3}{6} + 32x - 36{,}5 = 7{,}018\,kNm$$

$$M^{c}(3{,}\overline{3}) = -3\frac{x^2}{2} + 10x - 12{,}5 = 4{,}167\,kNm$$

$$\hookrightarrow M_{max} = 7{,}018\,kNm$$





$N_{ab} = -5\ kN$ (musí odečíst tlačí síla $F_1 \rightarrow N$ musí být $\ominus$)

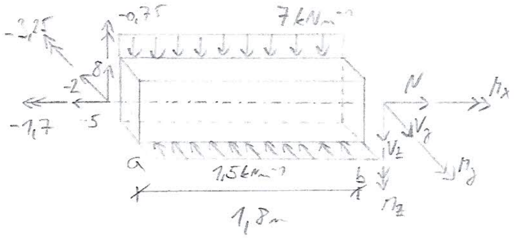
$V_{y,ab} = -2\ kN$

$V_{z,ab} = 8\ kN$

$M_{x,ab} = -8 \cdot 0{,}15 - 2 \cdot 0{,}25 = -1{,}7\ kNm = T_{ab}$

$M_{y,ab} = -2 - 5 \cdot 0{,}25 = -3{,}25\ kNm$

$M_{z,ab} = -5 \cdot 0{,}15 = -0{,}75\ kNm$



$N_{ba} = N_{ab} = -5\ kN$

$V_{y,ba} = \int +1{,}5\, dx = 1{,}5x + \overset{V_{y,ab}}{C} = 1{,}5 \cdot 1{,}8 - 2 = 0{,}7\ kN$

(f_2 jde proti $V_y \rightarrow$ to je správně, odečítá $\rightarrow \int \oplus \ldots$)

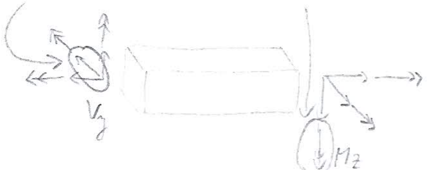
$V_{z,ba} = \int -7\, dx = -7x + \overset{V_{z,ab}}{C} = -7 \cdot 1{,}8 + 8 = -4{,}6\ kN$

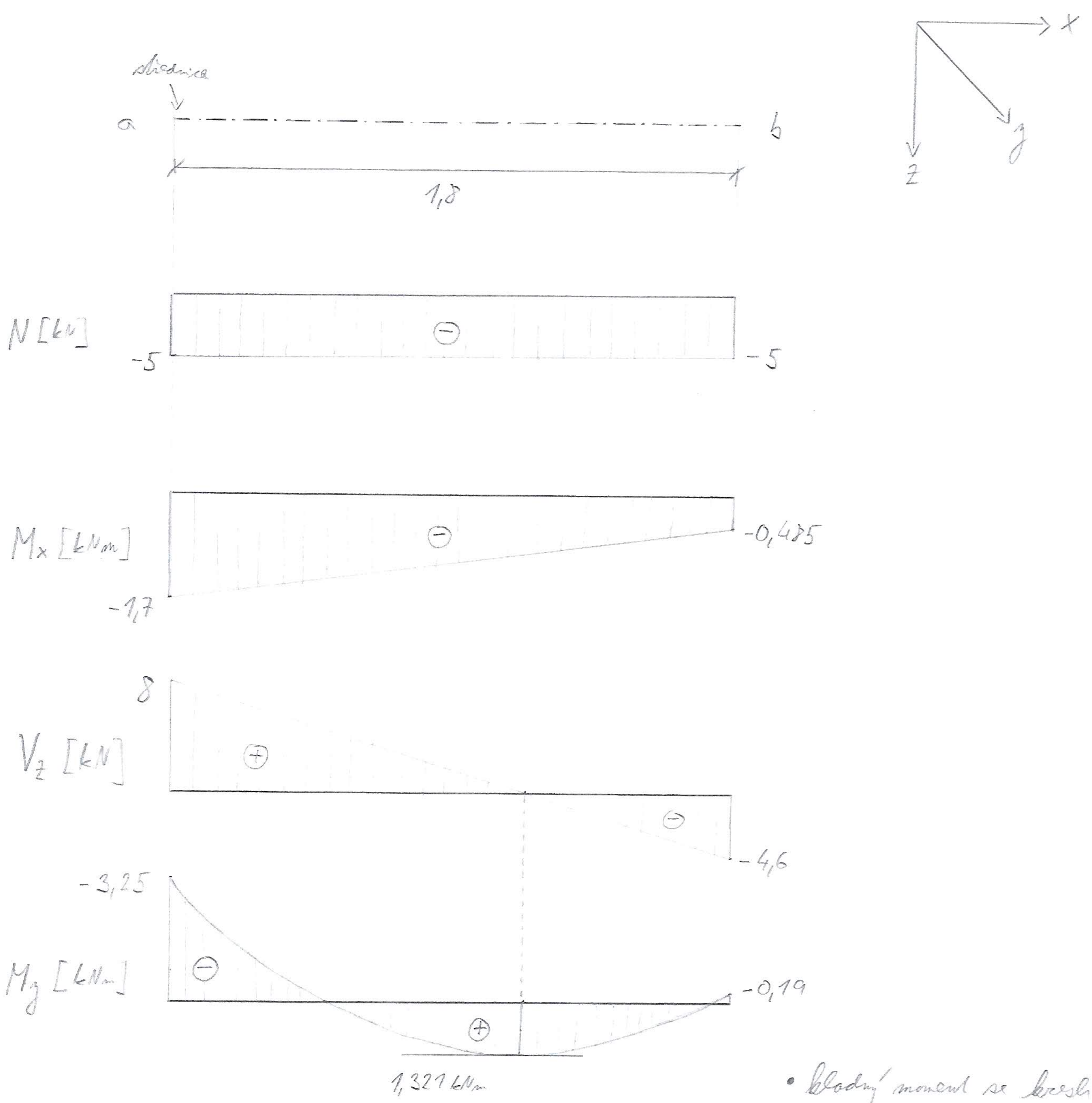
$M_{x,ba} = \int (+7 \cdot 0{,}15 - 1{,}5 \cdot 0{,}25)\, dx = 7 \cdot x\, 0{,}15 - 1{,}5 \cdot x \cdot 0{,}25 + \overset{M_{x,ab}}{C} = 7 \cdot 1{,}8 \cdot 0{,}15 - 1{,}5 \cdot 1{,}8 \cdot 0{,}25 - 1{,}7 = -0{,}455\ kNm$

$M_{y,ba} = \int +V_{z,ba}\, dx = \int (-7x + 8)\, dx = -7\frac{x^2}{2} + 8x + \overset{M_{y,ab}}{C} = -7 \cdot \frac{1{,}8^2}{2} + 8 \cdot 1{,}8 - 3{,}25 = -0{,}19\ kNm$

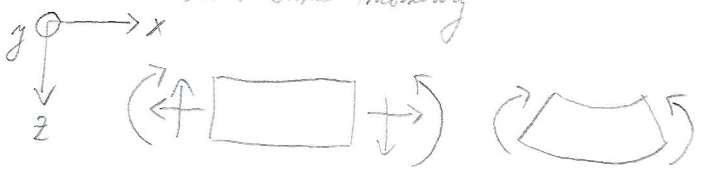
$M_{z,ba} = \int -V_{y,ba}\, dx = \int -(1{,}5x - 2)\, dx = -1{,}5\frac{x^2}{2} + 2x + \overset{M_{z,ab}}{C} = -1{,}5\frac{1{,}8^2}{2} + 2 \cdot 1{,}8 - 0{,}75 = 0{,}42\ kNm$

obecně, když letí síla ve směru tohoto momentu, ale musí odečíst $\rightarrow \ominus$



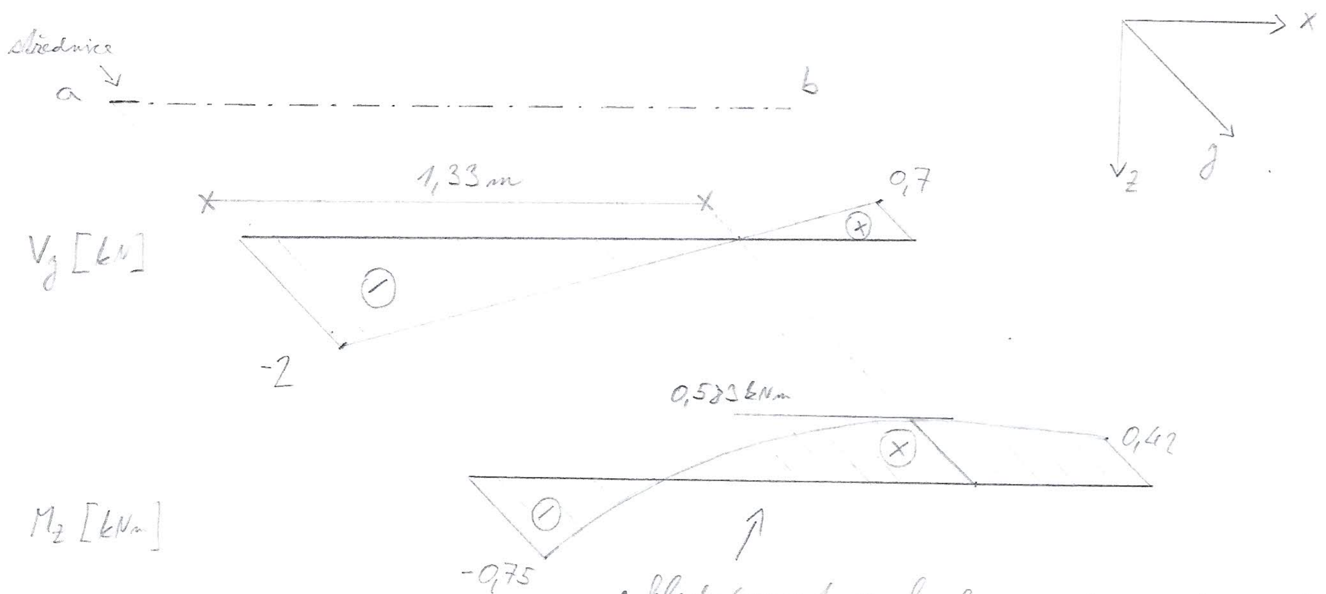


• kladný moment se kreslí na stranu tažených vláken prutu namáhaným konvenčními momenty

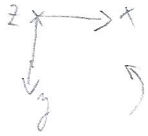


$$M_{y,max} = \frac{dM_y}{dx} = V_z = 0 \quad \longrightarrow \quad V_{z,ba} = -7x + 8 = 0 \quad \longrightarrow \quad x = 1,143 \, m$$

$$M_y^{ba}(1,143) = M_{y,max} = -7\frac{x^2}{2} + 8x - 3,25 = 1,321 \, kNm$$



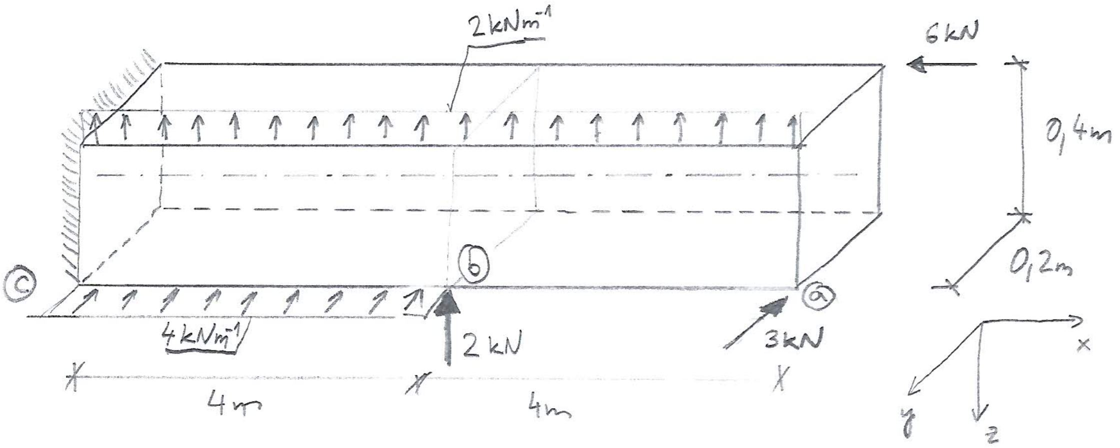
• kladný moment se kreslí na stranu tažených vláken prutu namáhaného konvexním momentem



$$M_{z,max} = \frac{dM_z}{dx} = -V_y = 0 \longrightarrow V_{y,ba} = 1,5x - 2 = 0 \longrightarrow x = 1,333\,m$$

$$M_z^{ba}(1,333) = M_{z,max} = -1,5\frac{x^2}{2} + 2x - 0,75 = 0,583\,kNm$$

PŘ: VYKRESLETE VNITŘNÍ SÍLY NA 3D KONSTRUKCI

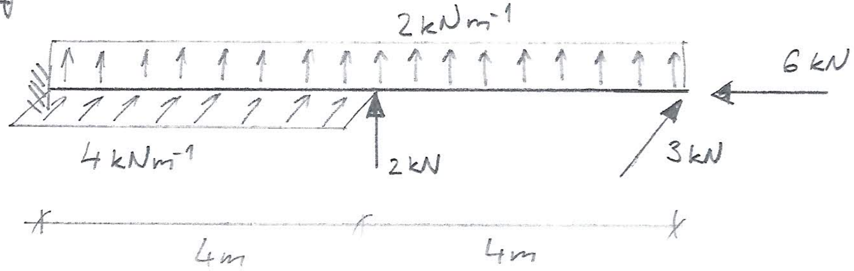


- aby platila zavedená znaménka vnitřních sil musí se používat pravotočivý souřadný systém, osa $\vec{x}$ jde s prutem, $\vec{z}$ směřuje dolů

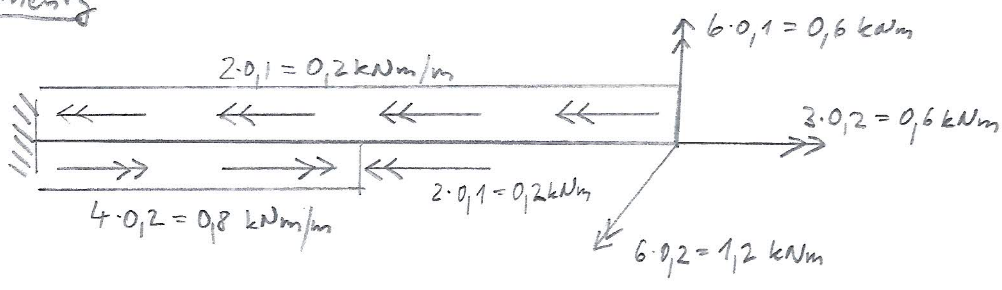
- půjdeme "zprava", aby se nemusely počítat reakce

- redukce ke střednici:

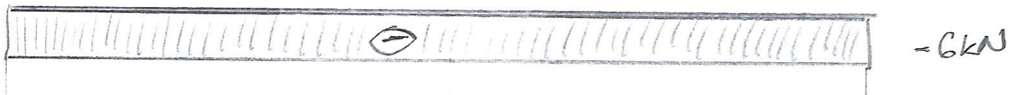
a) síly



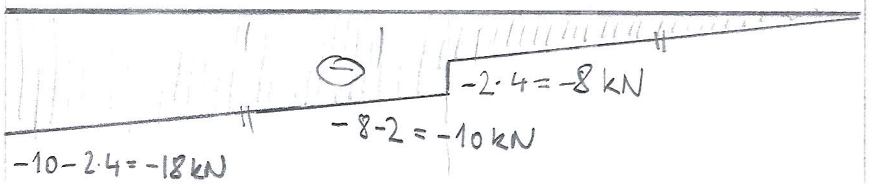
b) momenty



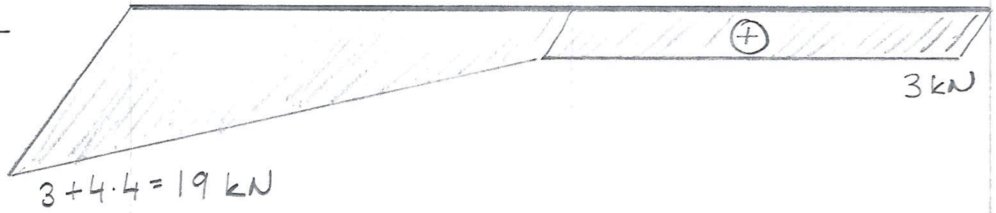
N_x:



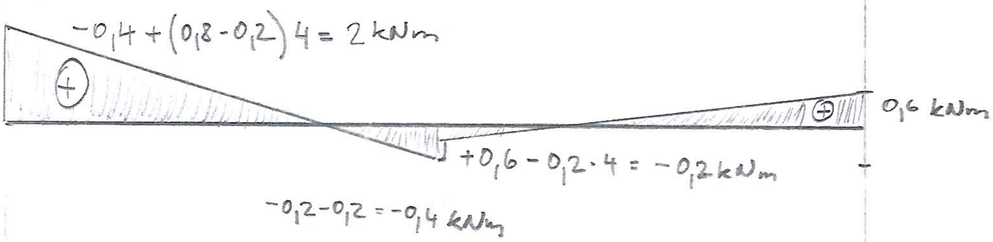
V_z:



V_y

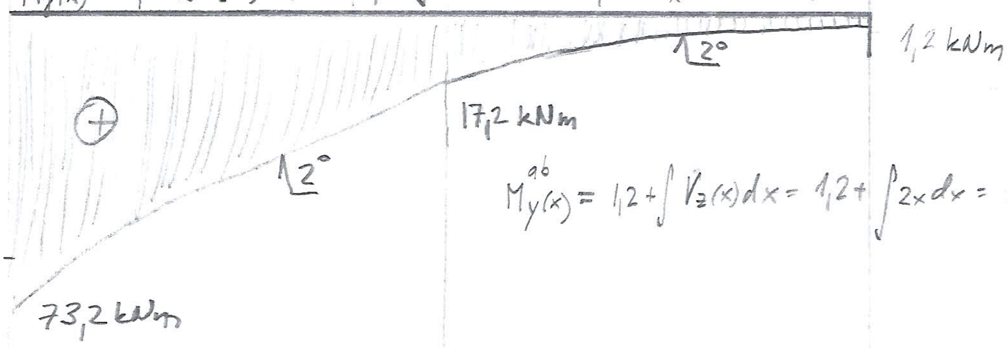


M_x:



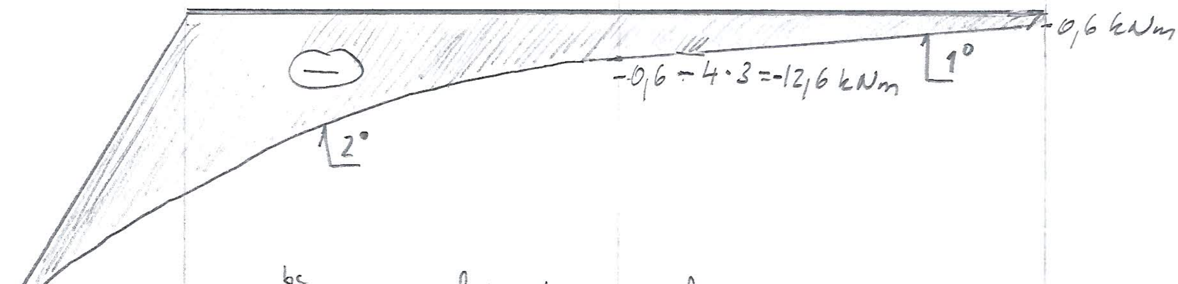
M_y:

$M_y^{bc}(x) = 17{,}2 - \int V_z(x)\,dx = 17{,}2 + \int 10 + 2x\,dx = 17{,}2 + 10x + x^2$



$M_y^{ab}(x) = 1{,}2 + \int V_z(x)\,dx = 1{,}2 + \int 2x\,dx = 1{,}2 + x^2$

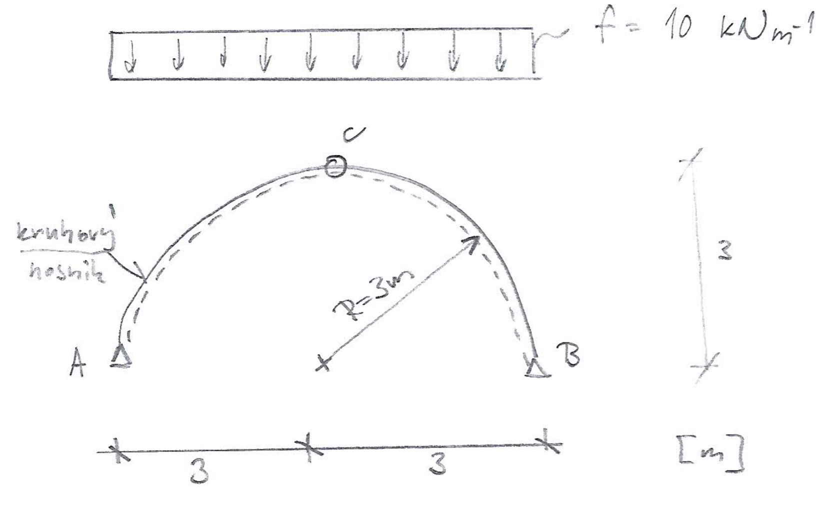
M_z:



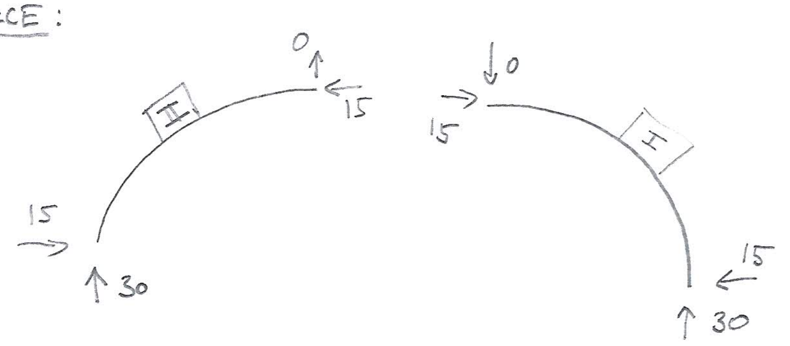
$M_z^{bc}(x) = -12{,}6 + \int V_y(x)\,dx = -12{,}6 - \int 3 + 4x\,dx = -12{,}6 - 3x - 2x^2$

-56,6 kNm

PŘ: PRŮBĚHY VNITŘNÍCH SIL NA ZAKŘIVENÉ KONSTRUKCI

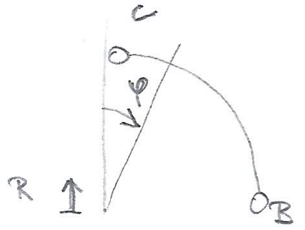


REAKCE:



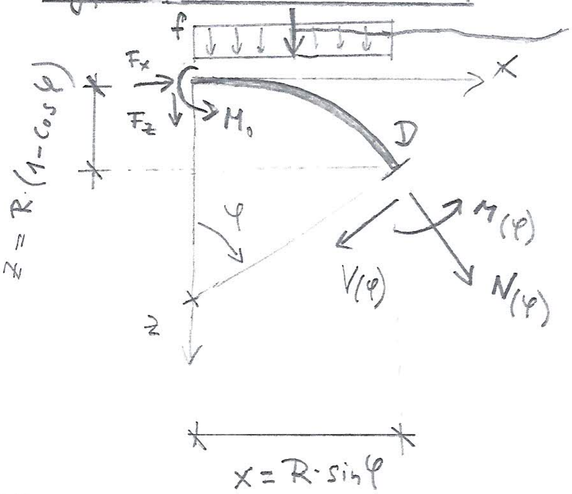
zavedení polárního souřadného systému

- body konstrukce popíšeme pomocí úhlu φ a poloměru R



- symetrické, lze řešit jen 1/2

výpočet vnitřních sil



náhradní břemeno: $f \cdot R \cdot \sin\varphi$

$\boxed{N(\varphi)}$

$\searrow: \; 0 = F_x \cdot \cos\varphi + F_z \cdot \sin\varphi + (f \cdot R \cdot \sin\varphi) \cdot \sin\varphi + N(\varphi)$

$N(\varphi) = -F_x \cdot \cos\varphi - F_z \cdot \sin\varphi - f \cdot R \cdot \sin^2\varphi$

$N(\varphi) = -15 \cdot \cos\varphi - 30 \cdot \sin^2\varphi$

$\boxed{V(\varphi)}$

$\swarrow$: $0 = -F_x \cdot \sin\varphi + F_z \cdot \cos\varphi + (f \cdot R \cdot \sin\varphi) \cdot \cos\varphi + V(\varphi)$

$$\boxed{V(\varphi) = F_x \sin\varphi - F_z \cos\varphi - f \cdot R \cdot \sin\varphi \cos\varphi}$$

$$V(\varphi) = 15 \cdot \sin\varphi - 30 \cdot \sin\varphi \cos\varphi$$

$\boxed{M(\varphi)}$

$\circlearrowleft D$: $M(\varphi) - F_x \cdot \overbrace{R \cdot (1-\cos\varphi)}^{z} + \overbrace{(f \cdot R \cdot \sin\varphi)}^{\text{náhradní břemeno}} \cdot \overbrace{\left(\frac{R \cdot \sin\varphi}{2}\right)}^{\frac{x}{2}} + M_0 + F_z \cdot \overbrace{R \cdot \sin\varphi}^{x} = 0$

$$\boxed{M(\varphi) = F_x \cdot R \cdot (1-\cos\varphi) - F_z \cdot R \cdot \sin\varphi - M_0 - \frac{1}{2} f \, (R \cdot \sin\varphi)^2}$$

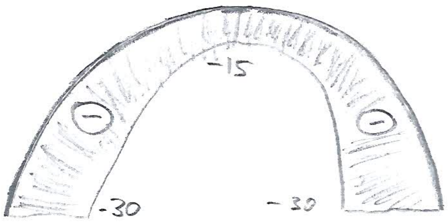
$$M(\varphi) = 45(1-\cos\varphi) - 45 \cdot \sin^2\varphi$$

<u>Extrémy momentu</u> $(Q = 0)$

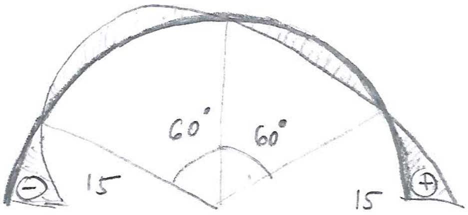
$$0 = 15 \cancel{\sin\varphi} - 30 \cdot \cancel{\sin\varphi} \cos\varphi$$

$$\varphi = \cos^{-1}\left(\frac{15}{30}\right) = 60^\circ \longrightarrow M(\varphi = 60^\circ) = \underline{-11{,}25 \text{ kNm}}$$

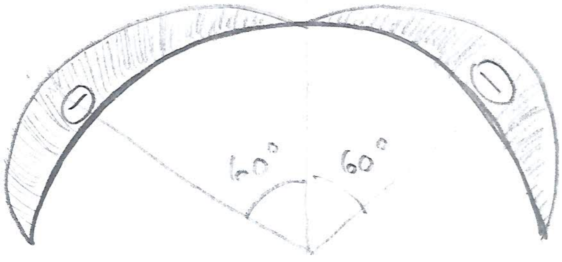
<u>$N(\varphi)$</u>:



<u>$V(\varphi)$</u>:

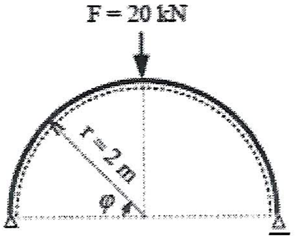


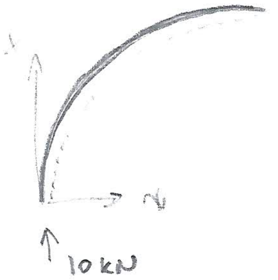
<u>$M(\varphi)$</u>:



PŘ: Vzorová zkouška

2. Určete průběhy vnitřních sil V a M na obloukovém nosníku.
   Funkce průběhů vyjádřete v závislosi na úhlu φ v intervalu $\varphi \in \langle 0, 90° \rangle$
   Určete polohu a velikost extrémního ohybového momentu. Průběhy vykreslete po celé konstrukci. $\boxed{8\text{ b.}}$





$$N(\varphi) = -F_x \cdot \cos\varphi = -10 \cdot \cos\varphi$$

$$V(\varphi) = F_x \cdot \sin\varphi = 10 \sin\varphi$$

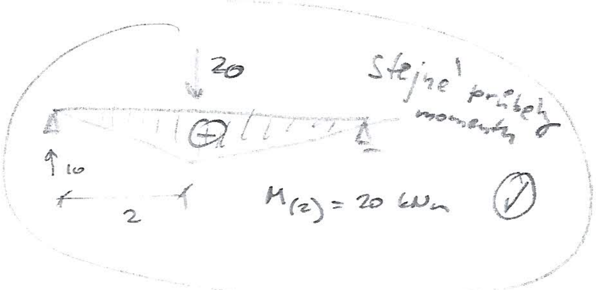
$$M(x) = F_x \cdot R \cdot (1-\cos\varphi) = 20 \cdot (1-\cos\varphi)$$

$M_{(\varphi)}^{extr.} \Leftrightarrow V_{(\varphi)} = 0: \quad 10 \cdot \sin\varphi = 0$

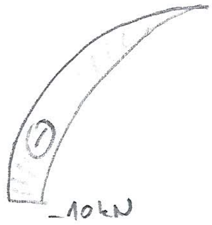
$\varphi = 0°$

$M_{(0)} = 0$ (= minimum)

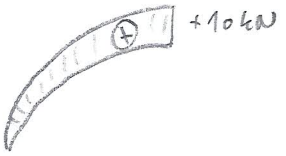
$M_{(90°)} = 20$ kNm



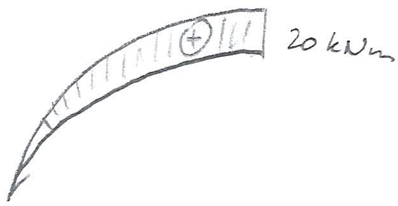
$N(\varphi)$:



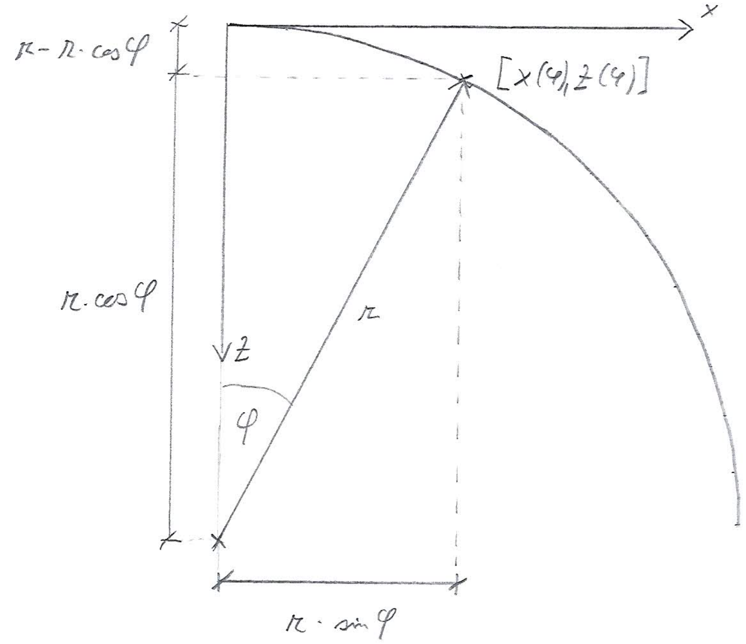
$V(\varphi)$:



$M(\varphi)$:



Kružnicový nosník

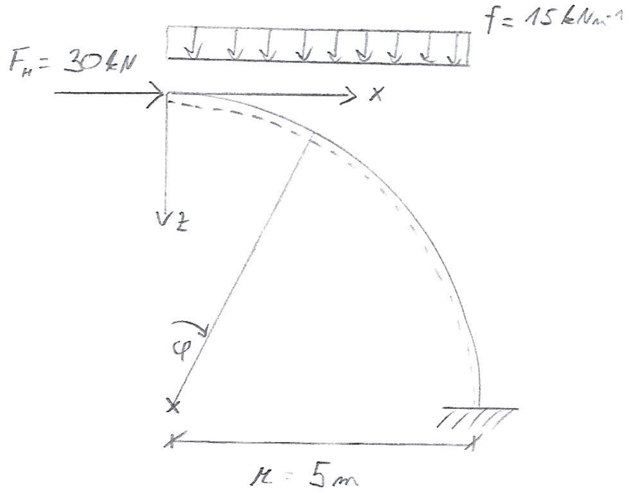


- lepší používat polární souřadnice → bod je určen úhlem φ a vzdáleností r

- souřadnice libovolného bodu na kružnici jsou pak určeny v závislosti na φ jako

$$x(\varphi) = r \cdot \sin\varphi$$

$$z(\varphi) = r - r \cdot \cos\varphi = r(1-\cos\varphi)$$



- jdeme zleva → nepočítáme reakce

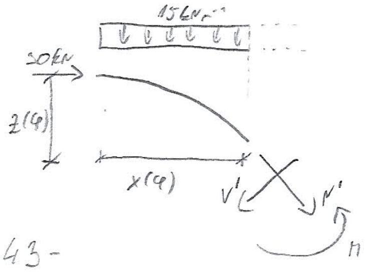
$$x(\varphi) = 5 \sin\varphi$$

$$z(\varphi) = 5(1-\cos\varphi)$$

$$\begin{bmatrix} N'(\varphi) \\ V'(\varphi) \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} N(\varphi) \\ V(\varphi) \end{bmatrix}$$

$$N'(\varphi) = \cos\varphi\, N(\varphi) + \sin\varphi\, V(\varphi)$$

$$N'(\varphi) = -\cos\varphi \cdot 30 - \sin\varphi \cdot f \cdot x(\varphi) = -\cos\varphi \cdot 30 - \sin\varphi \cdot 15 \cdot 5 \sin\varphi = -\cos\varphi \cdot 30 - \sin^2\varphi \cdot 75$$



- vnější zatížení jdou ve směru s vnitřními silami → musí ale odolávat, proto v dolu ⊖

$$N'(\varphi) = -\cos\varphi \cdot 30 - \sin^2\varphi \cdot 75$$

$N'(0°) = -30\,kN$ $\quad N'(22{,}5°) = -38{,}7\,kN$ $\quad N'(45°) = -58{,}713\,kN$

$N'(67{,}5°) = -75{,}497\,kN$ $\quad N'(90°) = -75\,kN$

$$V'(\varphi) = -\sin\varphi \cdot N(\varphi) + \cos\varphi \cdot V(\varphi)$$

$$V'(\varphi) = +\sin\varphi \cdot 30 - \cos\varphi \cdot f \cdot x(\varphi) = \sin\varphi \cdot 30 - \cos\varphi \cdot 15 \cdot 5 \cdot \sin\varphi = \sin\varphi \cdot 30 - 75\cos\varphi\sin\varphi$$

- síla 30 ji se rozložila do opačného směru, to je v pořádku, proto ji nechá mít ⊕
- spojité zatížení ji se rozložil do stejného směru, musí odečítat, proto ji nechá mít ⊖

$$V'(\varphi) = \sin\varphi \cdot 30 - 75 \cdot \cos\varphi \cdot \sin\varphi$$

$V'(0°) = 0\,kN$ $\quad V'(22{,}5°) = -15{,}036$ $\quad V'(45°) = -16{,}287\,kN$

$V'(67{,}5°) = 1{,}12\,kN$ $\quad V'(90°) = 30\,kN$

$$M(\varphi) = 30 \cdot z(\varphi) - f \cdot x(\varphi) \cdot \frac{x(\varphi)}{2} = 30 \cdot 5(1-\cos\varphi) - 15 \cdot 5\sin\varphi \cdot \frac{5\cdot\sin\varphi}{2}$$

$$M(\varphi) = 150 \cdot (1-\cos\varphi) - 187{,}5\sin^2\varphi$$

$M(0°) = 0\,kNm$ $\quad M(22{,}5°) = -16{,}041\,kNm$ $\quad M(45°) = -49{,}816\,kNm$

$M(67{,}5°) = -67{,}444\,kNm$ $\quad M(90°) = -37{,}5\,kNm$

• extrémy

$$\frac{dN'(\varphi)}{d\varphi} = +30\sin\varphi - 2\cdot 75 \cdot \sin\varphi\cos\varphi = 0$$

$$\sin\varphi\,(30 - 150\cdot\cos\varphi) = 0$$

$\sin\varphi = 0 \rightarrow \varphi_1 = 0°$

$30 - 150\cdot\cos\varphi = 0$

$\cos\varphi = \frac{30}{150} = 0{,}2$

$\varphi_2 = 78{,}463$

$N'(0) = -30\,kN$

$N'(78,463°) = -78\,kN$

$$\frac{dV'(\varphi)}{d\varphi} = 30 \cdot \cos\varphi - 75 \cdot (-\sin\varphi) \cdot \sin\varphi - 75 \cos\varphi \cdot \cos\varphi = 0$$

$$30 \cos\varphi + 75 \sin^2\varphi - 75 \cos^2\varphi = 0$$

$$30 \cos\varphi + 75(1-\cos^2\varphi) - 75 \cos^2\varphi = 0$$

$$30 \cos\varphi + 75 - 75 \cos^2\varphi - 75 \cos^2\varphi = 0$$

$$-150 \cos^2\varphi + 30 \cos\varphi + 75 = 0$$

$$\cos\varphi_{1,2} = \frac{-30 \pm \sqrt{30^2 - 4(-150) \cdot 75}}{2 \cdot (-150)} = \begin{cases} -0,614 \rightarrow \cancel{\varphi_1 = 127,9°} \\ 0,814 \rightarrow \varphi_2 = 35,497° \end{cases}$$

$V'(35,497°) = -18,036\,kN$

$$\frac{dM(\varphi)}{d\varphi} = +150 \sin\varphi - 2 \cdot 187,5 \sin\varphi \cdot \cos\varphi = 0$$

$$\sin\varphi\,(150 - 375 \cos\varphi) = 0 \quad \begin{cases} \sin\varphi = 0 \rightarrow \varphi_1 = 0° \\ 150 - 375 \cos\varphi = 0 \end{cases}$$

$$\cos\varphi = \frac{150}{375} = 0,4$$

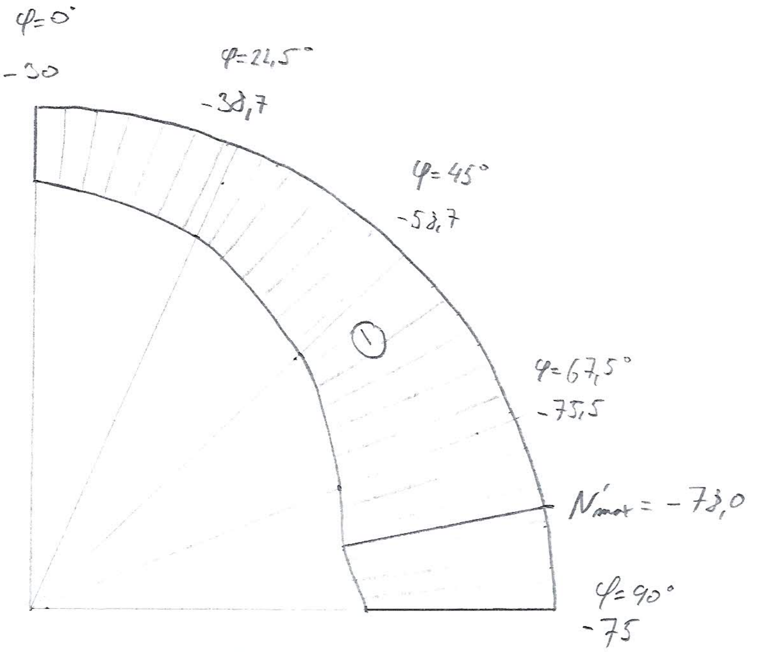
$$\varphi_2 = 66,422$$

$M(0°) = 0\,kNm$

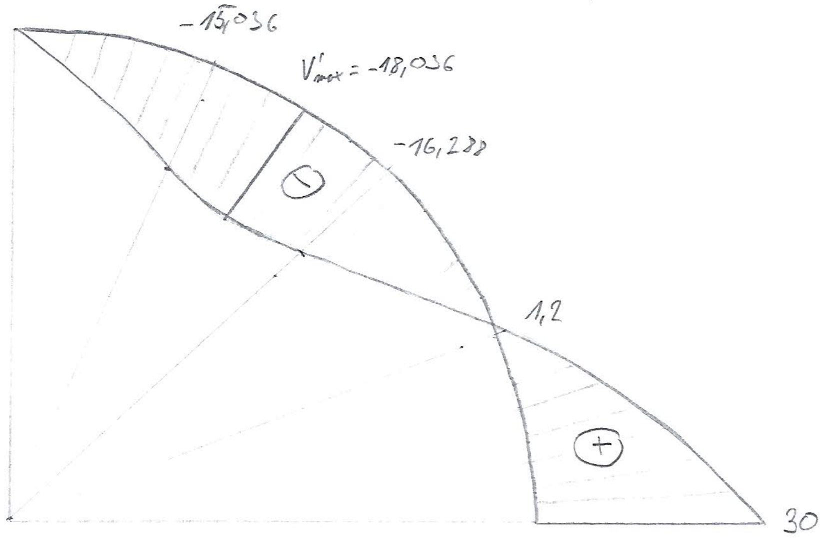
$M(66,422°) = -67,5\,kNm$

• u kruhových nosníků vykreslujeme vždy na oblouk

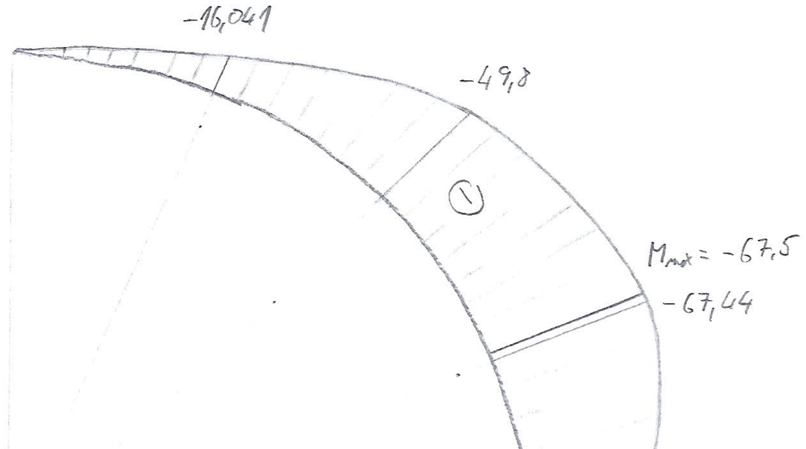
N [kN]



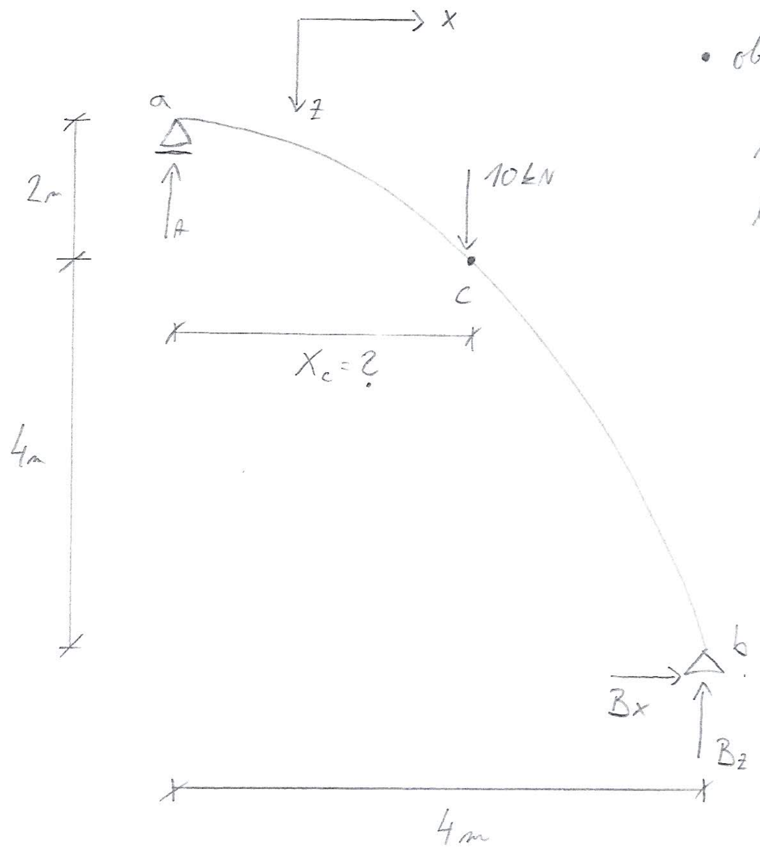
V [kN]



M [kNm]



Parabolický nosník



• obecná rce paraboly: $(x-m)^2 = 2p(z-n)$

souřadnice vrcholu a: $m = 0 \quad n = 0$

libovolný (známý) bod b: $x = 4 \quad z = 6$

$$(4-0)^2 = 2p(6-0) \rightarrow 2p = \frac{4^2}{6} = \frac{8}{3}$$

$$\hookrightarrow \quad x^2 = \frac{8}{3} z \rightarrow \underline{z = \frac{3}{8} x^2}$$

• x-ová souřadnice bodu c $[x_c, 2]$

$$2 = \frac{3}{8} x^2$$

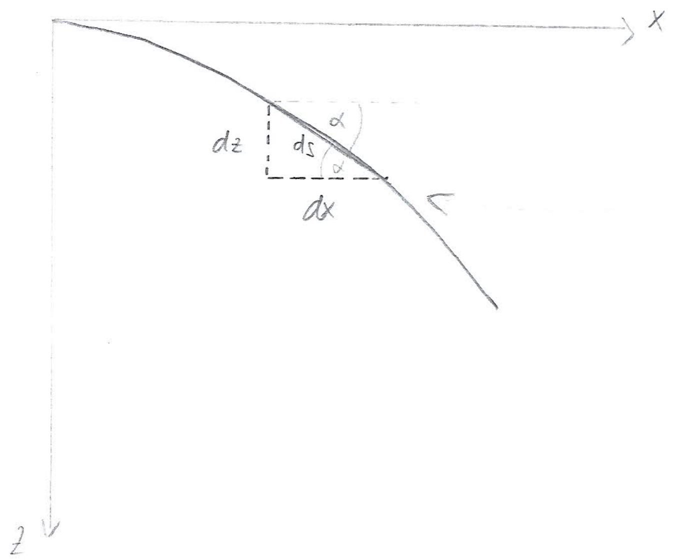
$$\frac{16}{3} = x^2 \quad \begin{cases} x_1 = \frac{4}{\sqrt{3}} \\ x_2 = -\frac{4}{\sqrt{3}} \end{cases}$$

$\rightarrow : \quad B_x = 0$

$\curvearrowleft a : \quad -10 \cdot \frac{4}{\sqrt{3}} + B_z \cdot 4 = 0 \rightarrow B_z = 5{,}774\,kN$

$\downarrow : \quad -A + 10 - 5{,}774 = 0 \rightarrow A = 4{,}226\,kN$

• v tomto případě je transformace složitější, raději nepracujeme s lineárními úseky, ale musíme postihnout kvadratické klesání



$z = g(x) \qquad$ např.: $z = \frac{3}{8} x^2$

$$ds = \sqrt{dx^2 + dz^2}$$

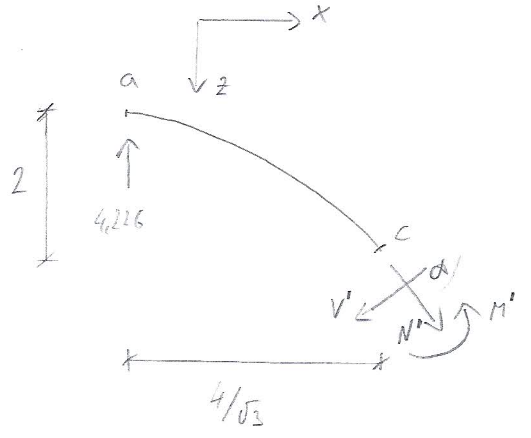
$$\frac{dz}{dx} = g' \rightarrow dz = g'dx$$

$$\hookrightarrow ds = \sqrt{dx^2 + (g'dx)^2} = dx\sqrt{1 + (g')^2}$$

$$\cos\alpha = \frac{dx}{ds} \Longrightarrow \cos\alpha = \frac{1}{\sqrt{1+(g')^2}}$$

$$\sin\alpha = \frac{dz}{ds} \Longrightarrow \sin\alpha = \frac{g'}{\sqrt{1+(g')^2}}$$

ac



- buď rozkladem vektorů nebo užitím [T] dostáváme výrazy pro redukované vnitřní síly

$$\begin{bmatrix} N' \\ V' \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} N \\ V \end{bmatrix}$$

$$N' = \cos\alpha \, \overset{\nearrow 0}{N} + \sin\alpha \, V$$

(průmět V do směru N' jde podle směru N', to je OK, proto ⊕)

$$N' = \sin\alpha \, V = \frac{g'}{\sqrt{1+(g')^2}} \cdot V$$

$$g: z = \frac{3}{8}x^2 \longrightarrow g' = \frac{dz}{dx} = \frac{3}{4}x$$

$$N' = \frac{\frac{3}{4}x}{\sqrt{1+\frac{9}{16}x^2}} \cdot V$$

$$N_a(x=0) = \frac{0}{1} \cdot V = 0 \, kN$$

$$N_{ca}\left(x=\frac{4}{\sqrt{3}}\right) = \frac{\frac{3}{4}\cdot\frac{4}{\sqrt{3}}}{\sqrt{1+\frac{9}{16}\cdot\frac{16}{3}}} \cdot 4{,}226 = 3{,}66 \, kN$$

$$V' = -\sin\alpha \, \overset{\nearrow 0}{N} + \cos\alpha \, V$$

$$V' = \cos\alpha \, V = \frac{1}{\sqrt{1+(g')^2}} \cdot V$$

$$V' = \frac{1}{\sqrt{1+\frac{9}{16}x^2}} \cdot V$$

$$V_a(x=0) = \frac{1}{1} \cdot 4{,}226 = 4{,}226 \, kN$$

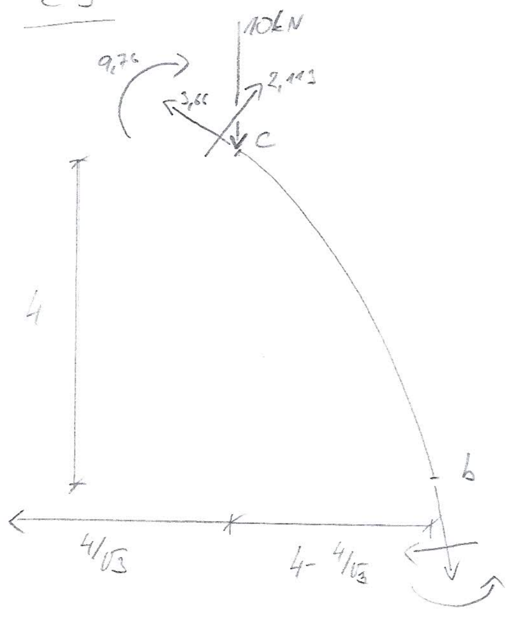
$$V_{ca}\left(x=\frac{4}{\sqrt{3}}\right) = \frac{1}{\sqrt{1+\frac{9}{16}\cdot\frac{16}{3}}} \cdot 4{,}226 = 2{,}113 \, kN$$

$M_a = 0 \, kNm$ (kloub)

M_{ca} – museli bychom integrovat V podle délky střednice ds ⟶ velmi složité, použijeme postup „síla × rameno“

$$M_{ca} = 4{,}226 \cdot x = 4{,}226 \cdot 4/\sqrt{3} = 9{,}76 \, kNm$$

cb



$$N_{cb} = (4{,}226 - 10)\sin\alpha = (4{,}226-10)\cdot\frac{\frac{3}{4}\cdot\frac{4}{\sqrt{3}}}{\sqrt{1+\frac{9}{16}\cdot\frac{16}{3}}} = -5\,kN$$

- rozložená síla 10 kN jde ve směru N_{cb}, ona má ale odolávat → ⊖
- hodnota x dosazená do sin α je opět $4/\sqrt{3}$, protože ta vzdálenost od počátku nám určuje sklon nosníku – proto NEmůžeme dosadit lokální souřadnici části cb, která je rovna 0 (tj. bychom se sklonem zase v počátku [0;0])

$$V_{cb} = (4{,}226-10)\cos\alpha = (4{,}226-10)\frac{1}{\sqrt{1+\frac{9}{16}\cdot\frac{16}{3}}} = -2{,}887\,kN$$

$$M_{cb} = M_{ca} = 9{,}76\,kNm$$

$$N_{bc} = (4{,}226-10)\sin\alpha = (4{,}226-10)\frac{\frac{3}{4}\cdot 4}{\sqrt{1+\frac{9}{16}\cdot 16}} = -5{,}478\,kN$$

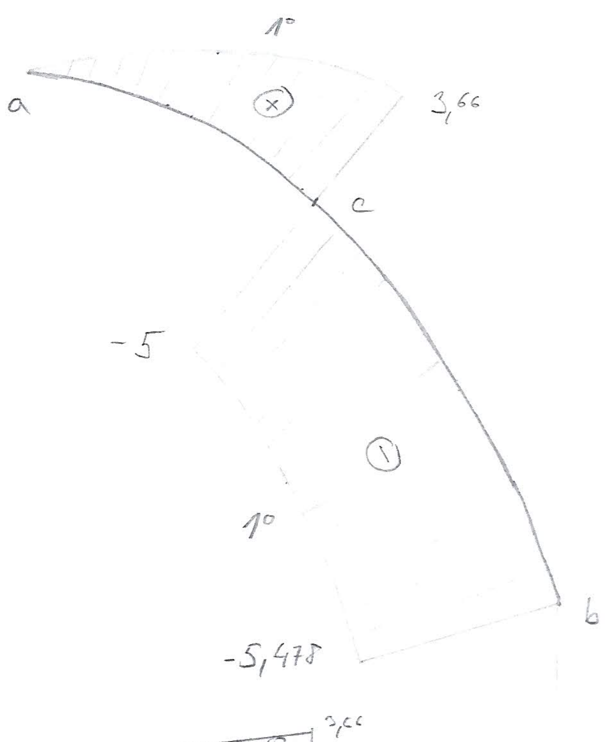
$$V_{bc} = (4{,}226-10)\cos\alpha = (4{,}226-10)\cdot\frac{1}{\sqrt{1+\frac{9}{16}\cdot 16}} = -1{,}826\,kN$$

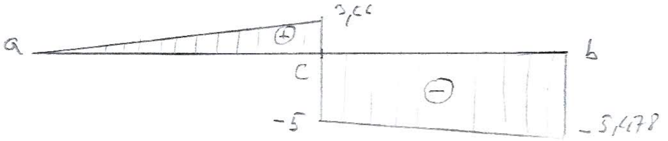
$$M_{bc} = \underbrace{4{,}226\cdot x}_{M_{ca}} - 10\left(x-\frac{4}{\sqrt{3}}\right) = 4{,}226\cdot 4 - 10\left(4-\frac{4}{\sqrt{3}}\right) = 0\,kN$$

jasné – blbě, spíš kontrola

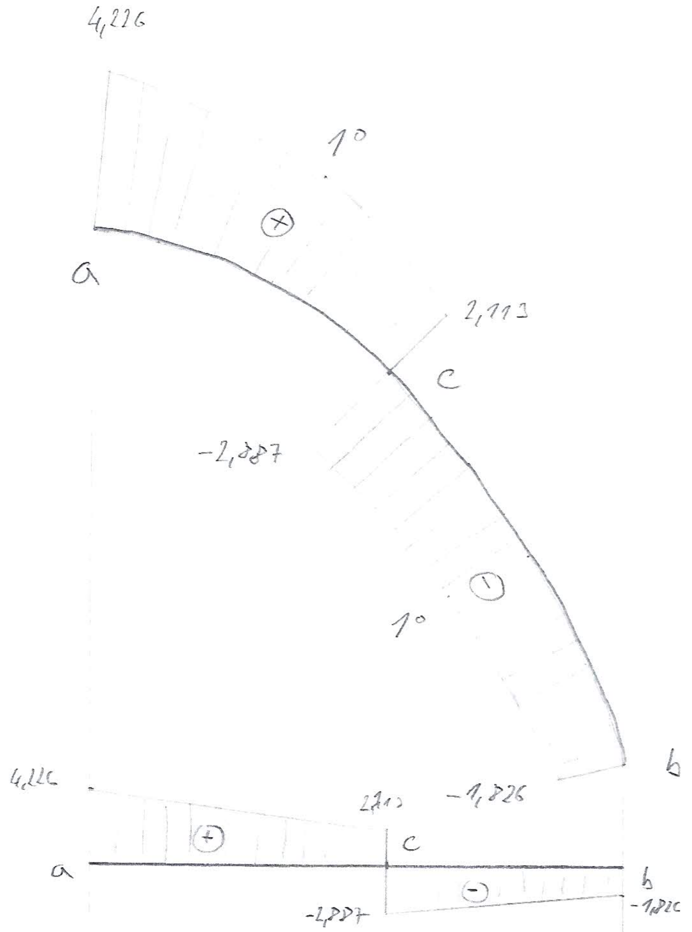
- u parabolických nosníků můžeme vykreslovat na průmět

N [kN]

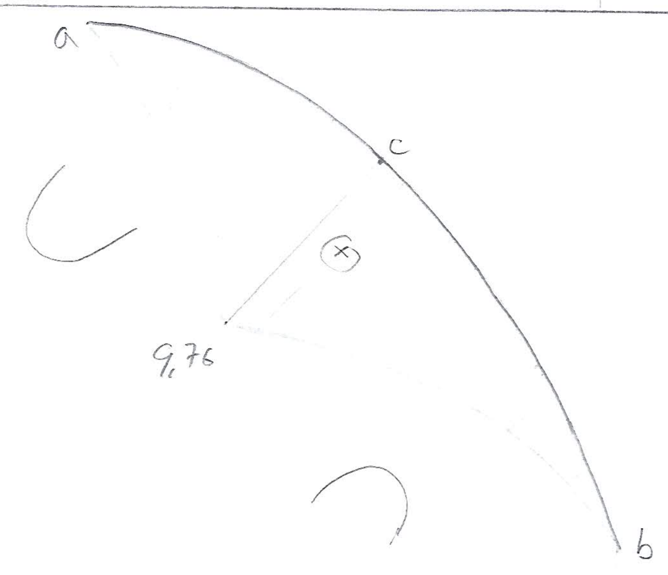


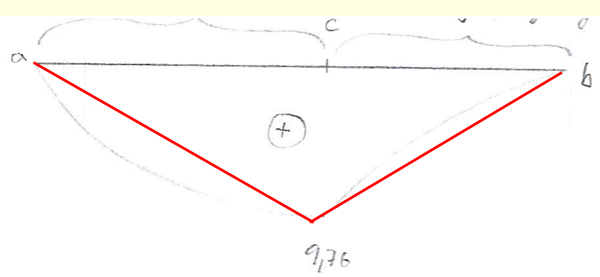


V [kN]

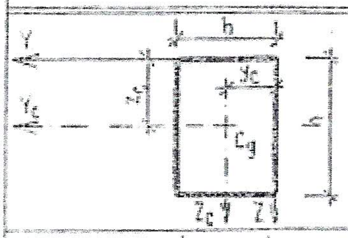
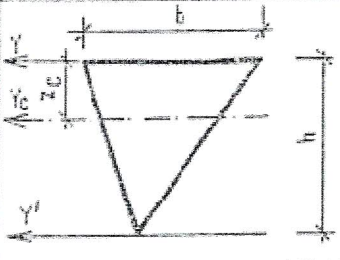
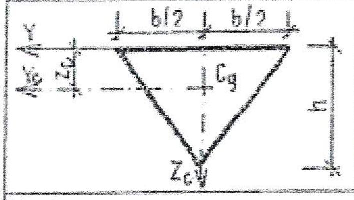
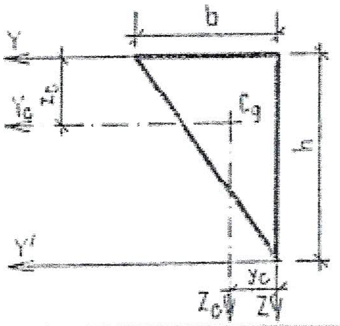
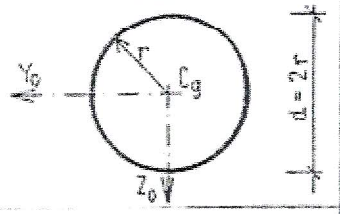
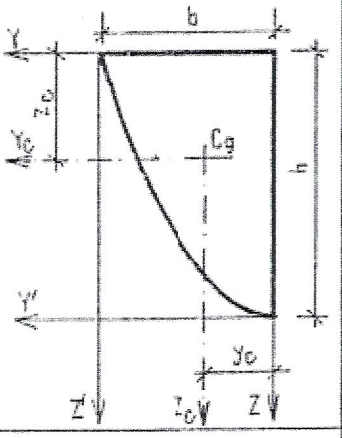

4,226
1°
a
2,113
c
-2,887
1°
b
4,226
2,113
-1,826
c
a
b
-2,887
-1,826

M [kNm]


a
c
9,76
b

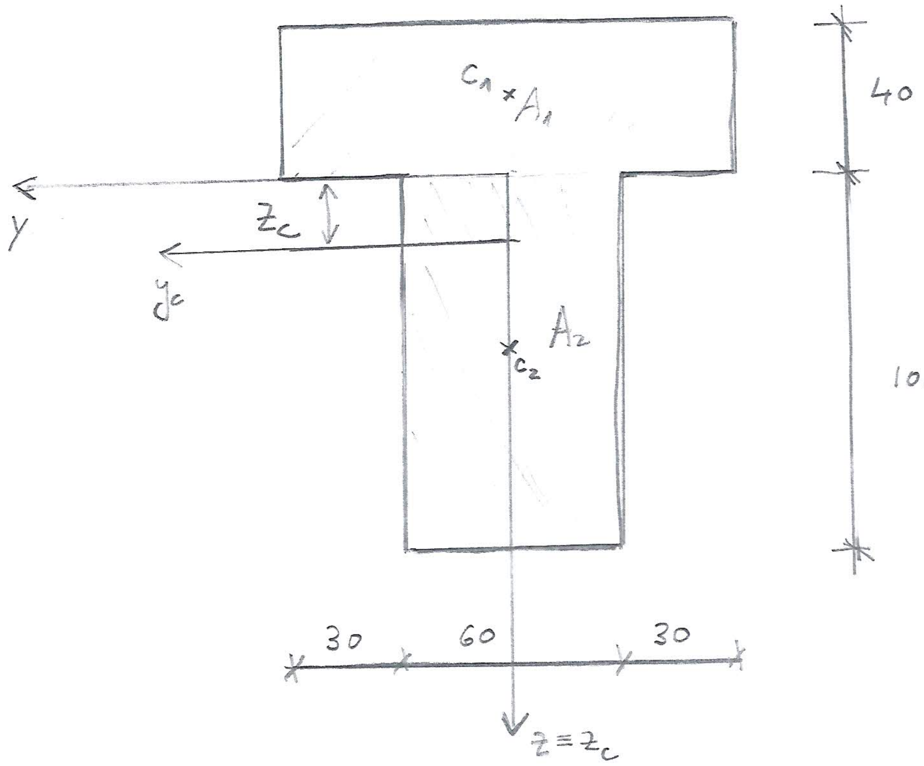

c
a
b
9,76

GEOMETRICKÉ CHARAKTERISTIKY ROVINNÝCH OBRAZCŮ TABULKA 3.

TVAR OBRAZCE	PLOCHA A	SOUŘADNICE TĚŽIŠTĚ $C_g(y_c, z_c)$	AXIÁLNÍ MOMENTY SETRVAČNOSTI I	DEVIAČNÍ MOMENTY D
	$A = bh$	$y_c = \frac{b}{2}$ $z_c = \frac{h}{2}$	$I_{Y_c} = \frac{bh^3}{12}$, $I_{Z_c} = \frac{hb^3}{12}$ $I_Y = \frac{bh^3}{3}$, $I_Z = \frac{hb^3}{3}$	$D_{YZ} = \frac{b^2h^2}{4}$ $D_{Y_cZ_c} = 0$
	$A = \frac{bh}{2}$	$z_c = \frac{h}{3}$	$I_{Y_c} = \frac{bh^3}{36}$ $I_Y = \frac{bh^3}{12}$ $I_{Y'} = \frac{bh^3}{4}$	
	$A = \frac{bh}{2}$	$z_c = \frac{h}{3}$	$I_{Y_c} = \frac{bh^3}{36}$, $I_{Z_c} = \frac{hb^3}{48}$ $I_Y = \frac{bh^3}{12}$	$D_{Y_cZ_c} = 0$
	$A = \frac{bh}{2}$	$y_c = \frac{b}{3}$ $z_c = \frac{h}{3}$	$I_{Y_c} = \frac{bh^3}{36}$, $I_{Z_c} = \frac{hb^3}{36}$ $I_Y = \frac{bh^3}{12}$, $I_Z = \frac{hb^3}{12}$ $I_{Y'} = \frac{bh^3}{4}$	$D_{Y_cZ_c} = -\frac{b^2h^2}{72}$ $D_{YZ} = \frac{b^2h^2}{24}$ $D_{Y'Z} = -\frac{b^2h^2}{8}$ ZNAMÉNKA!
	$A = \pi r^2 = \frac{\pi d^2}{4} \doteq$ $\doteq 3{,}1416\, r^2 =$ $= 0{,}7854\, d^2$		$I_{Y_c} = I_{Z_c} = \frac{\pi r^4}{4} = \frac{\pi d^4}{64} \doteq$ $\doteq 0{,}7854\, r^4 =$ $= 0{,}0491\, d^4$	$D_{Y_cZ_c} = 0$
	$A = \frac{2}{3}\, bh$	$y_c = \frac{3}{8}\, b$ $z_c = \frac{2}{5}\, h$	$I_{Y_c} = \frac{8}{175}\, bh^3 \doteq 0{,}0457\, bh^3$ $I_{Z_c} = \frac{19}{480}\, hb^3 \doteq 0{,}0396\, hb^3$ $I_Y = \frac{16}{105}\, bh^3 \doteq 0{,}1524\, bh^3$ $I_Z = \frac{2}{15}\, hb^3 \doteq 0{,}1333\, hb^3$ $I_{Y'} = \frac{2}{7}\, bh^3 \doteq 0{,}2857\, bh^3$ $I_{Z'} = \frac{3}{10}\, hb^3 \doteq 0{,}3000\, hb^3$	

PŘ: Jaká je plocha těžiště, momenty setrvačnosti a poloměry elipsy setrvačnosti?

[cm]



$A_1 = 120 \cdot 40 = 4800\,cm^2$

$A_2 = 60 \cdot 100 = 6000\,cm^2$

$A = 10\,800\,cm^2$

$C_1[0; -20]$

$C_2[0; 50]$

- symetrický obrazec (průřez) $\Rightarrow$ momenty setrvačnosti k ose symetrie = hlavní momenty; těžišťové osy $\equiv$ hlavní osy (musí jít s osou symetrie)

- $z_c = \dfrac{\sum S_{y_i}}{\sum A_i} = \dfrac{4800 \cdot (-20) + 6000 \cdot 50}{10\,800} = 18{,}889\,cm$

- vzdálenost těžišť jednotlivých obrazců od těžiště celku
  - $y_{c_1} = y_1 = 0$, $y_{c_2} = y_2 = 0$
  - $z_{c_1} = z_1 - z_c = -20 - 18{,}889 = -38{,}889\,cm$

    $z_{c_2} = z_2 - z_c = 50 - 18{,}889 = 31{,}111\,cm$

- momenty setrvačnosti k těžišťovým osám = vlastní moment jednotlivých částí ke svému těžišti + plocha části násobená druhou mocninou vzdálenosti vlastního těžiště od těžiště celku

$$I_y = I_{I_y} + A_I \cdot r_y^2$$

$$I_{yc} \equiv I_{yco} = \frac{1}{12}\cdot 120\cdot 40^3 + 4800\cdot(-38{,}889)^2 + \frac{1}{12}\cdot 60\cdot 100^3 + 6000\cdot(31{,}111)^2 =$$

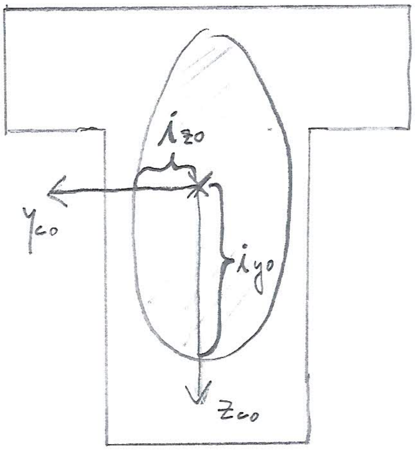
$$= 18\ 706\ 667\ \mathrm{cm}^4$$

(těžišťové osy jsou shodné s hlavními)

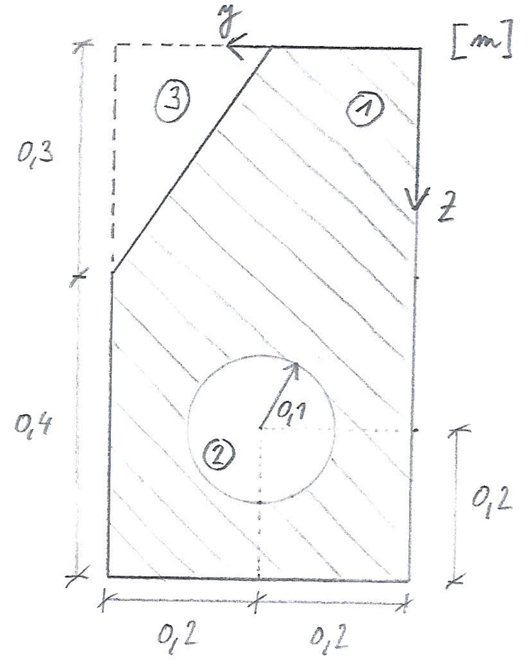
$$I_{zc} \equiv I_{zco} = \frac{1}{12}\cdot 40\cdot 120^3 + 4800\cdot 0^2 + \frac{1}{12}\cdot 100\cdot 60^3 + 6000\cdot 0^2 = 7\ 560\ 000\ \mathrm{cm}^4$$

- poloměry setrvačnosti: $i_{yo} = \sqrt{\frac{I_{yco}}{A}} = \sqrt{\frac{18\ 706\ 667}{10\ 800}} = 41{,}62\ \mathrm{cm}$

$$i_{zo} = \sqrt{\frac{I_{zco}}{A}} = \sqrt{\frac{7\ 560\ 000}{10\ 800}} = 26{,}457\ \mathrm{cm}$$



- nakreslete elipsu setrvačnosti k hlavním těžišťovým osám



① $A_1 = 0{,}7 \cdot 0{,}4 = 0{,}28\ m^2$

$y_1 = 0{,}2\ m$

$z_1 = 0{,}35\ m$

vzdálenost těžiště od počátku souřadnic

$S_{z1} = A_1 \cdot y_1 = 0{,}056\ m^3$

$S_{y1} = A_1 \cdot z_1 = 0{,}098\ m^3$

$I_{y1} = \frac{1}{12} b h^3 = \frac{1}{12}\, 0{,}4 \cdot 0{,}7^3 = 114{,}333 \cdot 10^{-4}\ m^4$

$I_{z1} = \frac{1}{12} b^3 h = \frac{1}{12}\, 0{,}4^3 \cdot 0{,}7 = 37{,}333 \cdot 10^{-4}\ m^4$

z tabulky (I_{yc}, I_{zc}, D_{yczc})

$D_{y1z1} = 0\ m^4$

② $A_2 = \pi \cdot 0{,}1^2 = 0{,}031\ m^2$

$y_2 = 0{,}2\ m$

$z_2 = 0{,}5\ m$

$S_{z2} = A_2 \cdot y_2 = 0{,}006\ m^3$

$S_{y2} = A_2 \cdot z_2 = 0{,}016\ m^3$

$I_{y2} = I_{z2} = \frac{1}{4} \pi r^4 = \frac{1}{4} \pi \cdot 0{,}1^4 = 0{,}785 \cdot 10^{-4}\ m^4$

$D_{y2z2} = 0\ m^4$

③

$A_3 = 0{,}3 \cdot 0{,}2 \cdot \frac{1}{2} = 0{,}03\ m^2$

$y_3 = 0{,}333\ m$

$z_3 = 0{,}1\ m$

$S_{z_3} = A_3 \cdot y_3 = 0{,}01\ m^3$

$S_{y_3} = A_3 \cdot z_3 = 0{,}003\ m^3$

$I_{y_3} = \frac{1}{36} b h^3 = \frac{1}{36} \cdot 0{,}2 \cdot 0{,}3^3 = 1{,}5 \cdot 10^{-4}\ m^4$

$I_{z_3} = \frac{1}{36} b^3 h = \frac{1}{36} \cdot 0{,}2^3 \cdot 0{,}3 = 0{,}667 \cdot 10^{-4}\ m^4$

$D_{y_3 z_3} = (+) \frac{1}{72} b^2 h^2 = \frac{1}{72} \cdot 0{,}2^2 \cdot 0{,}3^2 = 0{,}5 \cdot 10^{-4}\ m^4$

$\downarrow$

• u D_{yz} pozor na kvadrant, když jsme (po virtuálním posunutí) v II. kvadrantu → naopak než v tabulce

• těžiště:

• jsou to díry

$$y_c = \frac{\sum S_{z_i}}{\sum A_i} = \frac{0{,}056 - 0{,}006 - 0{,}01}{0{,}28 - 0{,}031 - 0{,}03} = \frac{0{,}04}{0{,}219} = 0{,}182\ m$$

$$z_c = \frac{\sum S_{y_i}}{\sum A_i} = \frac{0{,}079}{0{,}219} = 0{,}363\ m$$

• vzdálenosti těžišť jednotlivých částí od těžiště celého profilu:

$y_{c1} = y_1 - y_c = 0{,}018\ m$ $\qquad$ $z_{c1} = z_1 - z_c = -0{,}013\ m$

$y_{c2} = 0{,}018\ m$ $\qquad$ $z_{c2} = 0{,}137\ m$

$y_{c3} = 0{,}151\ m$ $\qquad$ $z_{c3} = -0{,}263\ m$

• momenty setrvačnosti k těžišti celého profilu (Steinerova věta):

$$I_{yc} = \sum \left(I_{y_i} + A_i \cdot z_{c_i}^2 \right) = \underbrace{I_{y_1} + A_1 \cdot z_{c1}^2}_{I_{yc1}} - \underbrace{\left(I_{y_2} + A_2 \cdot z_{c_2}^2 \right)}_{I_{yc2}} - \underbrace{\left(I_{y_3} + A_3 \cdot z_{c_3}^2 \right)}_{I_{yc3}} = 85{,}952 \cdot 10^{-4}\ m^4$$

$$I_{zc} = \sum \left(I_{z_i} + A_i \cdot y_{c_i}^2 \right) = 29{,}848 \cdot 10^{-4}\ m^4$$

$$D_{y_cz_c}=\sum\left(D_{y_iz_i}+A_i\cdot y_{ci}\cdot z_{ci}\right)=\underbrace{\cancelto{0}{D_{y_1z_1}}+\cancelto{<0}{A_1\cdot y_{c1}\cdot z_{c1}}}_{D_{y_{c1}z_{c1}}}-\underbrace{\left(\cancelto{0}{D_{y_2z_2}}+\cancelto{>0}{A_2\cdot y_{c2}\cdot z_{c2}}\right)}_{D_{y_{c2}z_{c2}}}-\underbrace{\left(D_{y_3z_3}+\cancelto{<0}{A_3\cdot y_{c3}\cdot z_{c3}}\right)}_{D_{y_{c3}z_{c3}}}=$$

$$=-0{,}655\cdot10^{-4}-0{,}764\cdot10^{-4}-\left(-11{,}414\cdot10^{-4}\right)=9{,}994\cdot10^{-4}\ m^4$$

- úhel natočení hlavních těžišťových os:

$$tg\,2\alpha=\frac{2\cdot D_{y_cz_c}}{I_{z_c}-I_{y_c}}=-0{,}356\longrightarrow 2\alpha=-19{,}61^\circ$$

$$\alpha=-9{,}8^\circ$$

- hlavní (min a max) momenty setrvačnosti:

$$I_y'(\alpha)=I_{y_0}\ (\text{prof. Kabele})=I_{y_c}\cos^2\alpha+I_{z_c}\sin^2\alpha-D_{y_cz_c}\cdot\sin 2\alpha=87{,}679\cdot10^{-4}\ m^4$$

$$I_z'(\alpha)=I_{z_0}=I_{y_c}\cdot\sin^2\alpha+I_{z_c}\cdot\cos^2\alpha+D_{y_cz_c}\cdot\sin 2\alpha=28{,}121\cdot10^{-4}\ m^4$$

$$D_{y'z'}(\alpha)=D_{y_cz_c}=0\ m^4$$

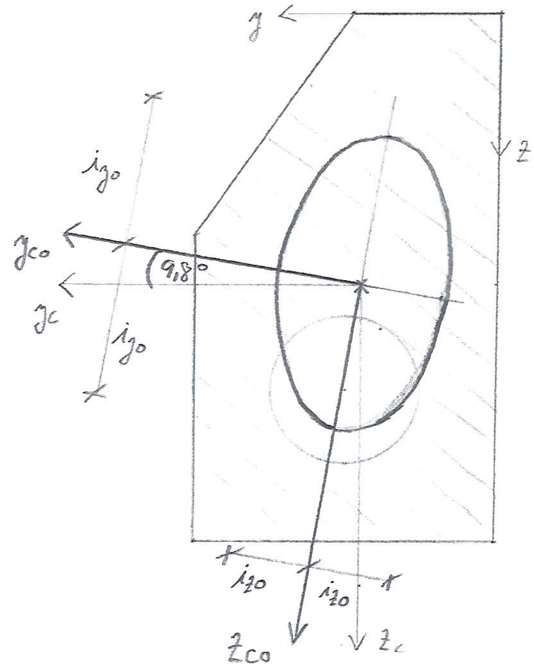
$$\hookrightarrow\ I_{y_0}=I_{max}$$

$$I_{z_0}=I_{min}$$

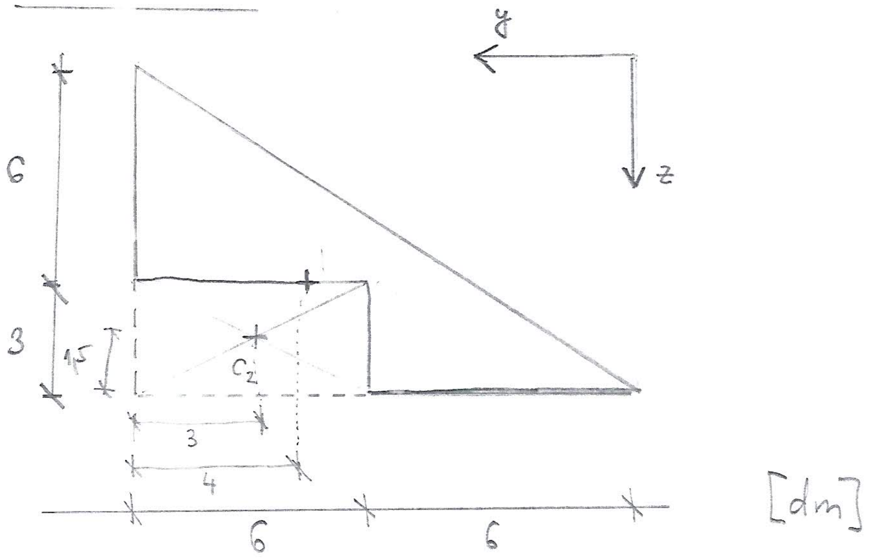
- poloměry setrvačnosti:

$$i_{y_0}=i_{max}=\sqrt{\frac{I_{y_0}}{A}}=0{,}20\ m$$

$$i_{z_0}=i_{min}=\sqrt{\frac{I_{z_0}}{A}}=0{,}113\ m$$



PŘ: Stanovte hlavní centrální momenty setrvačnosti složeného rovinného obrazce a vykreslete elipsu setrvačnosti



a) plochy: $A_1 = \frac{1}{2} \cdot 12 \cdot 9 = 54 \ dm^2$

$A_2 = 6 \cdot 3 = 18 dm^2 \qquad \Rightarrow \qquad A = 54 - 18 = 36 \ dm^2$

b) těžiště:

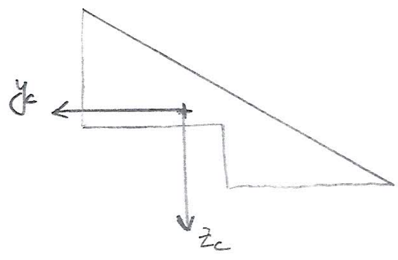
rameno · plocha = stat. moment

$C_1 [8; 6]$

$C_2 [9; 7{,}5] \qquad \Rightarrow$

$$y_c = \frac{\sum S_y}{\sum A} = \frac{54 \cdot 8 - 18 \cdot 9}{54 - 18} = 7{,}5 \ dm$$

$$z_c = \frac{\sum S_z}{\sum A} = \frac{54 \cdot 6 - 18 \cdot 7{,}5}{54 - 18} = 5{,}25 \ dm$$



c) těžiště v centrálním souřadném systému:

$C_1 [y_1 - y_c \ ; \ z_1 - z_c] \ \rightarrow \ C_1 [8 - 7{,}5 \ ; \ 6 - 5{,}25]$

$C_1 [0{,}5 \ ; \ 0{,}75]$

$C_2 [1{,}5 \ ; \ 2{,}25]$

d) <u>momenty setrvačnosti (+ deviační moment)</u>

- $$I_{yc} = \underbrace{\frac{1}{36}\cdot 12\cdot 9^3}_{\text{z tabulky}} + \underbrace{54}_{A_1}\cdot \underbrace{0{,}75^2}_{y_{c_1}^2} - \left(\frac{1}{12}\cdot 6\cdot 3^3 + 18\cdot 2{,}25^2\right) = 168{,}75 \text{ dm}^4$$

- $$I_{zc} = \frac{1}{36}\cdot 9\cdot 12^3 + 54\cdot 0{,}5^2 - \left(\frac{1}{12}\cdot 3\cdot 6^3 + 18\cdot 1{,}5^2\right) = 351{,}0 \text{ dm}^4$$

- $$D_{y_c z_c} = \underbrace{-\frac{1}{72}\cdot 12^2\cdot 9^2}_{\text{z tabulky}} + \underbrace{54}_{A_1}\cdot \underbrace{0{,}75}_{y_{c_1}}\cdot \underbrace{0{,}5}_{z_{c1}} - \left(\emptyset + 18\cdot 2{,}25\cdot 1{,}5\right) = -202{,}5 \text{ dm}^4$$

e) <u>I_{MAX}, I_{MIN}, α_0, i_{max}, i_{min}</u>

- $$I_{1,2} = \frac{I_{y_c} + I_{z_c}}{2} \pm \frac{1}{2}\sqrt{\left(I_{y_c} - I_{z_c}\right)^2 + 4\cdot D_{y_c z_c}^2}$$

- $$I_{1,2} = \frac{168{,}75 + 351{,}0}{2} \pm \frac{1}{2}\sqrt{\left(168{,}75 - 351{,}0\right)^2 + 4\cdot(-202{,}5)^2} =$$

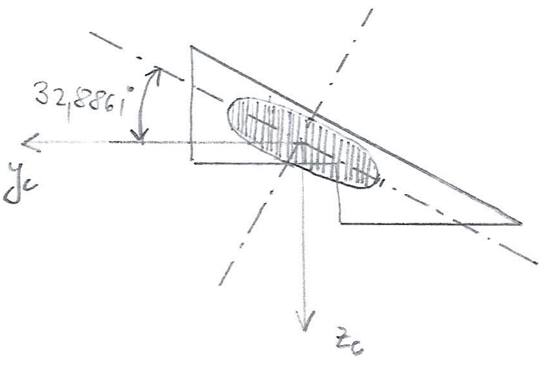
$$= \begin{cases} I_1 \equiv I_{MAX} = 481{,}059 \text{ dm}^4 = I_{z_0} \\ I_2 \equiv I_{MIN} = 37{,}816 \text{ dm}^4 = I_{y_0} \end{cases}$$

- $$i_{MAX} = \sqrt{\frac{I_{MAX}}{A}} = \sqrt{\frac{481{,}059}{36}} = 3{,}656 \text{ dm} = i_{z_0}$$

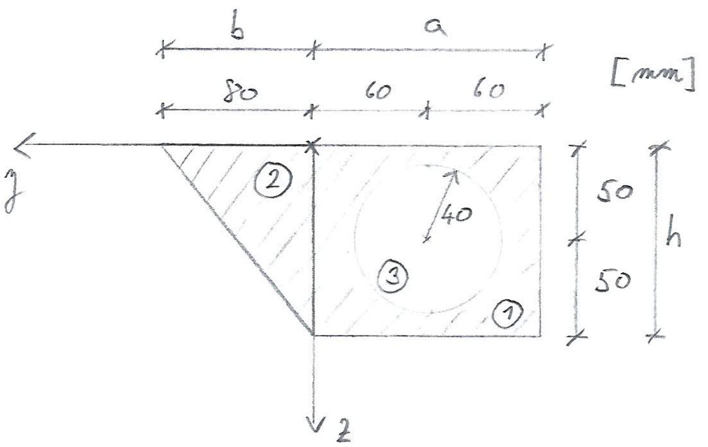
- $$i_{MIN} = \sqrt{\frac{I_{MIN}}{A}} = \sqrt{\frac{37{,}816}{36}} = 1{,}025 \text{ dm} = i_{y_0}$$

- $$tg\, 2\alpha = \frac{2\cdot D_{y_c z_c}}{I_{z_c} - I_{y_c}} = \frac{2\cdot(-202{,}5)}{351{,}0 - 168{,}75} = -2{,}2222$$

$$\Rightarrow 2\alpha = -65{,}7723^\circ$$
$$\alpha = -32{,}8861^\circ$$



• nakreslete elipsu setrvačnosti k původním osám (ne k těžišťovým)



①

$A_1 = 100 \cdot 120 = 12\,000 \text{ mm}^2$

$y_1 = -60 \text{ mm}$
$z_1 = 50 \text{ mm}$ } vzdálenost těžiště od počátku souřadnic

$S_{z1} = A_1 \cdot y_1 = -720\,000 \text{ mm}^3$

$S_{y1} = A_1 \cdot z_1 = 600\,000 \text{ mm}^3$

$I_{y1} = \frac{1}{12} a h^3 = \frac{1}{12} 120 \cdot 100^3 = 10 \cdot 10^6 \text{ mm}^4$
$I_{z1} = \frac{1}{12} a^3 h = 14{,}4 \cdot 10^6 \text{ mm}^4$
$D_{y1z1} = 0 \text{ mm}^4$
} z tabulky (I_{yc}, I_{zc}, D_{yczc})

②

$A_2 = 100 \cdot 80 \cdot \frac{1}{2} = 4000 \text{ mm}^2$

$y_2 = \frac{80}{3} = 26{,}667 \text{ mm}$

$z_2 = \frac{100}{3} = 33{,}333 \text{ mm}$

$S_{z2} = 4000 \cdot 26{,}667 = 106\,666{,}667 \text{ mm}^3$

$S_{y2} = 4000 \cdot 33{,}333 = 133\,333{,}333 \text{ mm}^3$

$I_{y2} = \frac{1}{36} b h^3 = \frac{1}{36} \cdot 80 \cdot 100^3 = 2{,}222 \cdot 10^6 \text{ mm}^4$

$I_{z2} = \frac{1}{36} b^3 h = 1{,}422 \cdot 10^6 \text{ mm}^4$

$D_{y2z2} = -\frac{1}{72} b^2 h^2 = -\frac{1}{72} \cdot 80^2 \cdot 100^2 = -8{,}889 \cdot 10^5 \text{ mm}^4$

③ 

$A_3 = \pi r^2 = 5026{,}548\ mm^2$

$y_3 = -60\ mm$

$z_3 = 50\ mm$

$S_{z3} = 5026{,}548 \cdot (-60) = -301\,592{,}895\ mm^3$

$S_{y3} = 5026{,}548 \cdot 50 = 251\,327{,}412\ mm^3$

$I_{y3} = I_{z3} = \frac{1}{4}\pi r^4 = 2{,}011 \cdot 10^6\ mm^4$

$D_{y_3 z_3} = 0\ mm^4$

- těžiště:

• je to díra → odečítáme

$$y_c = \frac{\sum S_{zi}}{\sum A_i} = \frac{-720\,000 + 106\,666{,}667 - (-301\,592{,}895)}{12\,000 + 4000 - 5026{,}548} = \frac{-311\,740{,}438}{10\,973{,}452} =$$

$$= -28{,}41\ mm$$

$$z_c = \frac{\sum S_{yi}}{\sum A_i} = \frac{482\,005{,}921}{10\,973{,}452} = 43{,}93\ mm$$

pozn.: v tomto příkladu nevztahujeme momenty k těžišti, ale k počátku → výpočet těžiště byl jenom k procvičení, nevyužijeme ho

- momenty setrvačnosti k počátku (Steinerova věta):

$$I_{yP1} = I_{y1} + A_1 \cdot z_1^2 = \frac{1}{12} ah^3 + a \cdot h \cdot \left(\frac{h}{2}\right)^2 = \frac{1}{12} ah^3 + \frac{1}{4} ah^3 = \frac{4}{12} ah^3 = \frac{1}{3} ah^3 \text{ (jako v tabulce)}$$

Počátek

$$= \frac{1}{3}\, 120 \cdot 100^3 = 40 \cdot 10^6\ mm^4$$

$$I_{yP2} = I_{y2} + A_2 \cdot z_2^2 \rightarrow \text{tabulka} \rightarrow = \frac{1}{12} bh^3 = 6{,}667 \cdot 10^6\ mm^4$$

$$I_{yP3} = I_{y3} + A_3 \cdot z_3^2 \rightarrow \text{nejde využít tabulka!} \rightarrow = \frac{\pi r^4}{4} + \left(\frac{h}{2}\right)^2 \cdot \pi r^2 = 14{,}577 \cdot 10^6\ mm^4$$

$$I_{yP} = I_{yP1} + I_{yP2} - I_{yP3} = 32{,}09 \cdot 10^6\ mm^4$$

díra

$I_{zp1} \rightarrow$ tabulka $\rightarrow = \frac{1}{3} a^3 h = 57{,}6 \cdot 10^6 \text{ mm}^4$

$I_{zp2} \rightarrow$ tabulka $\rightarrow = \frac{1}{12} b^3 h = 4{,}267 \cdot 10^6 \text{ mm}^4$

$I_{zp3} \rightarrow$ NE tabulka $\rightarrow = \frac{\pi r^4}{4} + \left(\frac{a}{2}\right)^2 \cdot \pi r^2 = 20{,}106 \cdot 10^6 \text{ mm}^4$

$$I_{zp} = I_{zp_1} + I_{zp_2} - I_{zp_3} = 41{,}761 \cdot 10^6 \text{ mm}^4$$

$$D_{y_{p1} z_{p1}} = D_{y_1 z_1} + y_1 \cdot z_1 \cdot A = 0 + \left(-\frac{a}{2}\right)\cdot\left(\frac{h}{2}\right)\cdot a \cdot h = -\frac{a^2 h^2}{4} \quad \times \quad \oplus \frac{a^2 h^2}{4} \text{ v tabulce}$$

$$= 0 - 60 \cdot 50 \cdot 120 \cdot 100 = -36{,}0 \cdot 10^6 \text{ mm}^4$$

$\Downarrow$

- u D_{yz} pozor na kvadranty, když jsme v II. kvadrantu $\rightarrow$ z je $\oplus$, ale y je $\ominus$ $\rightarrow$ $\ominus$ (opačné znaménko než v tabulce)

$$D_{y_{p2} z_{p2}} = +\frac{1}{24} b^2 h^2 = 2{,}667 \cdot 10^6 \text{ mm}^4$$ (I. kvadrant $\rightarrow$ znaménko jako v tabulce)

$$D_{y_{p3} z_{p3}} = D_{y_3 z_3} + y_3 \cdot z_3 \cdot A = 0 + \left(-\frac{a}{2}\right)\left(\frac{h}{2}\right)\pi r^2 = -15{,}08 \cdot 10^6 \text{ mm}^4$$

$$D_{y_p z_p} = D_{y_{p1} z_{p1}} + D_{y_{p2} z_{p2}} - D_{y_{p3} z_{p3}} = -36{,}0 \cdot 10^6 + 2{,}667 \cdot 10^6 - (-15{,}08 \cdot 10^6) = -18{,}254 \cdot 10^6 \text{ mm}^4$$

- úhel natočení hlavních os:

$$tg\, 2\alpha = \frac{2 \cdot D_{y_p z_p}}{I_{zp} - I_{yp}} = -3{,}775 \rightarrow 2\alpha = -75{,}163^\circ$$

$$\alpha = -37{,}6^\circ$$

- hlavní (min a max) momenty setrvačnosti:

$$I_{y'}(\alpha) = I_{y0} (\text{prof. Kabele}) = I_{yp} \cos^2\alpha + I_{zp} \sin^2\alpha - D_{y_p z_p} \cdot \sin 2\alpha = 18{,}042 \cdot 10^6 \text{ mm}^4 = I_{min}$$

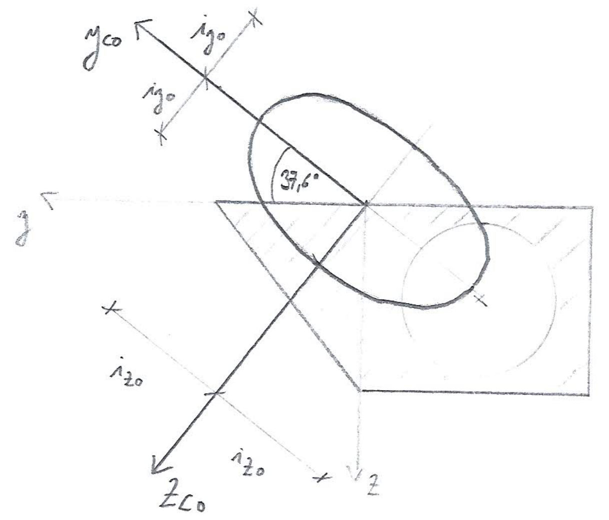
$$I_{z'}(\alpha) = I_{z0} = I_{yp} \cdot \sin^2\alpha + I_{zp} \cdot \cos^2\alpha + D_{y_p z_p} \cdot \sin 2\alpha = 55{,}808 \cdot 10^6 \text{ mm}^4 = I_{max}$$

$$D_{y'z'}(\alpha) = D_{y_p z_p 0} = 0 \text{ mm}^4$$

- poloměry setrvačnosti:

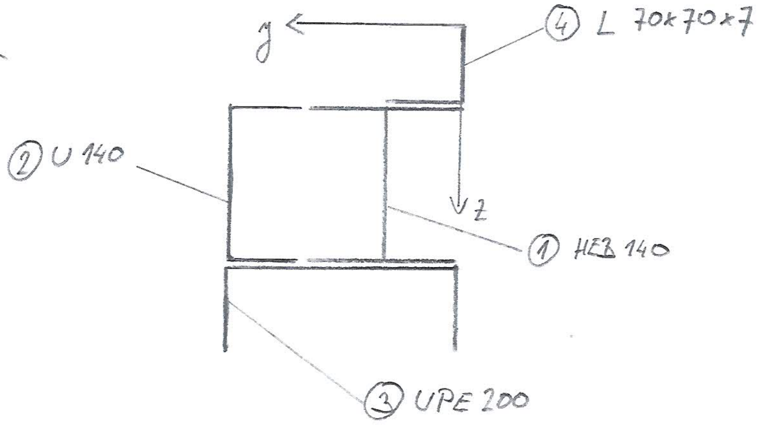
$$i_{y0} = i_{min} = \sqrt{\frac{I_{y0}}{A}} = 40,55 \text{ mm}$$

$$i_{z0} = i_{max} = \sqrt{\frac{I_{z0}}{A}} = 71,37 \text{ mm}$$



Svařenec

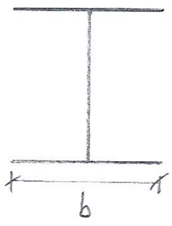
pozn.: výšky válcovaných profilů jsou rovny číslu v názvu



TABULKY

↓

① HEB 140 (str. 14)



$A_1 = 4300\ mm^2$

$I_{y,1} = 15,1 \cdot 10^6\ mm^4$

$I_{z,1} = 5,5 \cdot 10^6\ mm^4$

$b_1 = 140\ mm$

• vzdálenost těžiště od počátku souřadnic

$y_1 = 70\ mm$

$z_1 = 140\ mm$

$S_{z,1} = A_1 \cdot y_1 = 301\ 000\ mm^3$

$S_{y,1} = A_1 \cdot z_1 = 602\ 000\ mm^3$

② U140 (str. 18)

$A_2 = 2040\ mm^2$

$I_{y,2} = 6,05 \cdot 10^6\ mm^4$

$I_{z,2} = 625 \cdot 10^3\ mm^4$

$b_2 = 60\ mm$

$e_2 = 17,6\ mm$

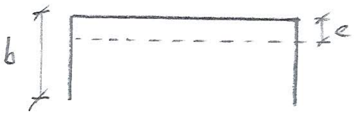
$y_2 = 182,4\ mm$

$z_2 = 140\ mm$

$S_{z,2} = A_2 \cdot y_2 = 372\ 096\ mm^3$

$S_{y,2} = A_2 \cdot z_2 = 285\ 600\ mm^3$

③ UPE200 (str. 20)



$A_3 = 2350\ mm^2$

$I_{y,3} = 1370 \cdot 10^3\ mm^4$

$I_{z,3} = 15,4 \cdot 10^6\ mm^4$

$b_3 = 76\ mm$

$e_3 = 23,3\ mm$

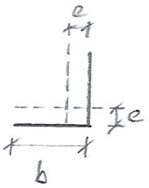
$y_3 = 100\ mm$

$z_3 = 233,3\ mm$

$S_{z,3} = 235\ 000\ mm^3$

$S_{y,3} = 548\ 255\ mm^3$

④ L 70×70×7 (str. 22)



$A_4 = 942\ mm^2$

$I_{y,4} = 423 \cdot 10^3\ mm^4$

$I_{z,4} = 423 \cdot 10^3\ mm^4$

$b_4 = 70\ mm$

$e_4 = 19,6\ mm$

$y_4 = 19,6\ mm$

$z_4 = 50,4\ mm$

$S_{z,4} = 18\ 463,2\ mm^3$

$S_{y,4} = 47\ 476,8\ mm^3$

- těžiště:

$$y_c = \frac{\sum S_{zi}}{\sum A_i} = \frac{S_{z,1}+S_{z,2}+S_{z,3}+S_{z,4}}{A_1+A_2+A_3+A_4} = \frac{926\,559{,}2}{9632} = 96{,}196\ mm$$

$$z_c = \frac{\sum S_{yi}}{\sum A_i} = \frac{1\,483\,331{,}8}{9632} = 154\ mm$$

- vzdálenosti těžišť jednotlivých profilů od těžiště celého svařence:

$y_{c1} = y_1 - y_c = -26{,}196\ mm$ $\quad z_{c1} = z_1 - z_c = -14\ mm$

$y_{c2} = y_2 - y_c = 86{,}204\ mm$ $\quad z_{c2} = z_2 - z_c = -14\ mm$

$y_{c3} = 3{,}804\ mm$ $\quad z_{c3} = 79{,}3\ mm$

$y_{c4} = -76{,}596\ mm$ $\quad z_{c4} = -103{,}6\ mm$

- momenty setrvačnosti k těžišťovým osám svařence (Steinerova věta):

$$I_{yc} = \sum\left(I_{yi} + A_i \cdot z_{ci}^2\right) = \underbrace{I_{y1} + A_1 \cdot z_{c1}^2}_{I_{yc1}} + \underbrace{I_{y,2} + A_2 \cdot z_{c2}^2}_{I_{yc2}} + \underbrace{I_{y,3} + A_3 \cdot z_{c3}^2}_{I_{yc3}} + \underbrace{I_{y,4} + A_4 \cdot z_{c4}^2}_{I_{yc4}} =$$
$$= 49{,}074 \cdot 10^6\ mm^4$$

$$I_{zc} = \sum\left(I_{zi} + A_i \cdot y_{ci}^2\right) = 45{,}619 \cdot 10^6\ mm^4$$

- deviační momenty k těžišťovým osám svařence:

- z tabulek: $D_{y1z1} = 0\ mm^4$
$D_{y2z2} = 0\ mm^4$
$D_{y3z3} = 0\ mm^4$
$D_{y4z4} = 244\,000\ mm^4$

$$D_{yczc} = \sum\left(D_{yizi} + A_i\, y_{ci} \cdot z_{ci}\right) = \underbrace{D_{y1z1} + A_1 \overset{>0}{\cancel{y_{c1} z_{c1}}}}_{D_{yc1zc1}} + \underbrace{D_{y2z2} + A_2 \overset{<0}{\cancel{y_{c2} z_{c2}}}}_{D_{yc2zc2}} + \underbrace{D_{y3z3} + A_3 \overset{>0}{\cancel{y_{c3} z_{c3}}}}_{D_{yc3zc3}} +$$
$$+ \underbrace{D_{y4z4} + A_4 \overset{>0}{\cancel{y_{c4} z_{c4}}}}_{D_{yc4zc4}} = 7{,}543 \cdot 10^6\ mm^4$$

- úhel natočení hlavních těžišťových os:

$$\operatorname{tg} 2\alpha = \frac{2 \cdot D_{y_c z_c}}{I_{z_c} - I_{y_c}} \longrightarrow \alpha = -38,55°$$

- hlavní (min a max) momenty setrvačnosti:

$$I_{y'}(\alpha) = I_{y0}(\text{prof. kabele}) = I_{y_c} \cos^2\alpha + I_{z_c} \sin^2\alpha - D_{y_c z_c} \sin 2\alpha = 55,085 \cdot 10^6 \ mm^4$$

$$I_{z'}(\alpha) = I_{z0} = I_{y_c} \sin^2\alpha + I_{z_c} \cos^2\alpha + D_{y_c z_c} \sin 2\alpha = 39,608 \cdot 10^6 \ mm^4$$

$$\hookrightarrow I_{y0} = I_{max}$$

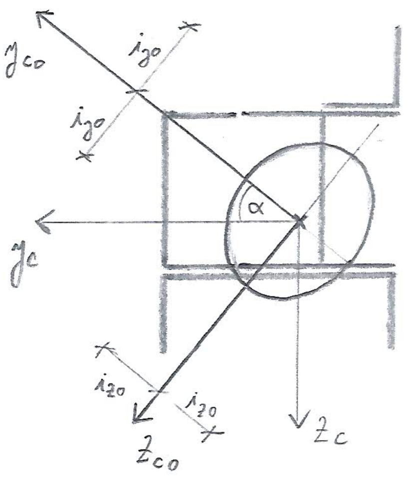
$$I_{z0} = I_{min}$$

$$D_{y'z'}(\alpha) = D_{yz0} = 0 \ mm^4$$

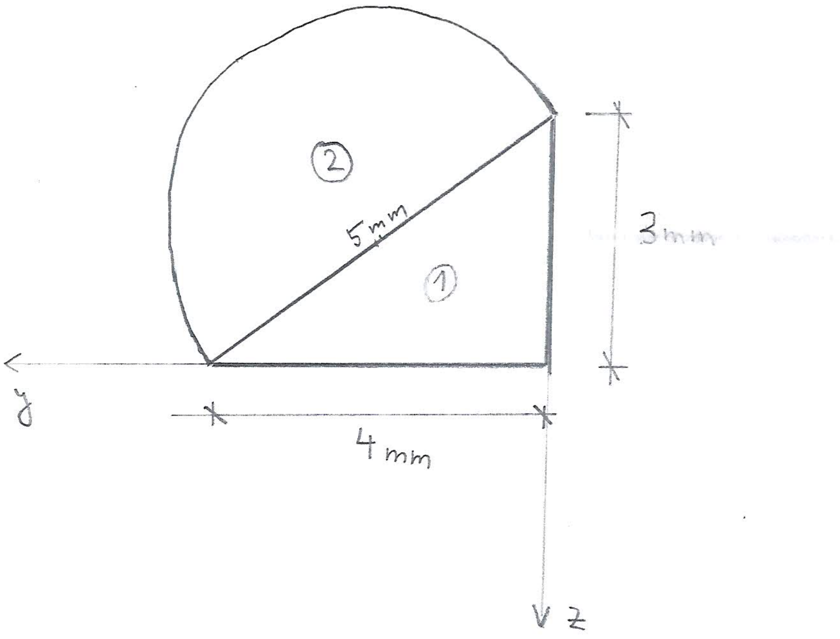
- poloměry setrvačnosti:

$$i_{y0} = i_{max} = \sqrt{\frac{I_{y0}}{A}} = 75,62 \ mm$$

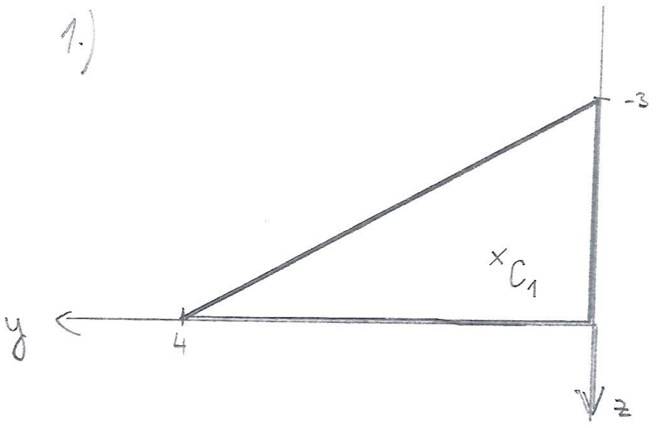
$$i_{z0} = i_{min} = \sqrt{\frac{I_{z0}}{A}} = 64,13 \ mm$$



PŘ: Momenty setrvačnosti, ..., elipsa setrvačnosti



1.)



$y_{c1} = \frac{4}{3} = 1{,}\overline{3333}$ mm

$z_{c1} = -\frac{3}{3} = -1$ mm

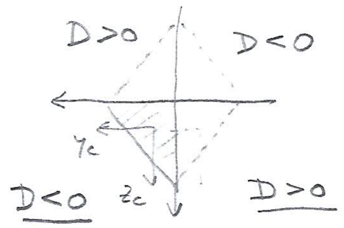
$\Big\} \; C_1\,[1{,}\overline{33}; -1]$

$A_1 = \frac{1}{2} \cdot 4 \cdot 3 = 6\ mm^2$

$I_{yc1} = \frac{1}{36}\, b h^3 = \frac{1}{36} \cdot 4 \cdot 3^3 = 3\ mm^4$

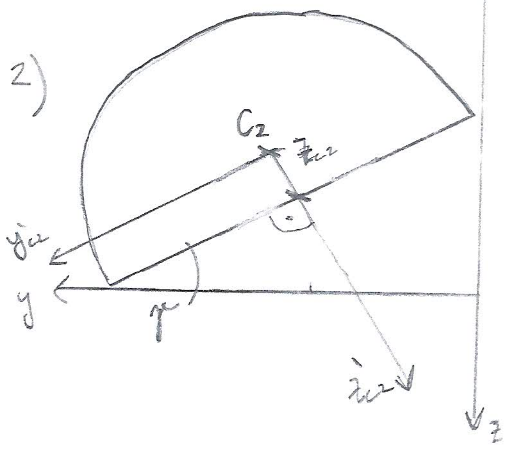
$I_{zc1} = \frac{1}{36}\, h b^3 = \frac{1}{36} \cdot 3 \cdot 4^3 = 5{,}\overline{3333}\ mm^4$

$|D_{y_c z_c}| = \frac{b^2 h^2}{72}$,



$D_{y_{c1} z_{c1}} = \frac{b^2 h^2}{72} = \frac{4^2 3^2}{72} = 2\ mm^4$

2)

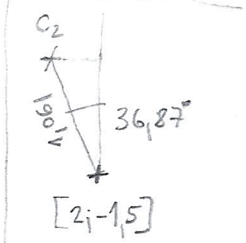


$z_{c2} \doteq 0{,}4244\, r = 0{,}4244 \cdot \frac{5}{2} = 1{,}061$ mm

$tg\, \gamma = \frac{3}{4} \Rightarrow \gamma = 36{,}87^\circ$

$y_{c2} = 2 + 1{,}061 \cdot \sin 36{,}87^\circ = 2{,}64$ mm

$z_{c2} = -1{,}5 - 1{,}061 \cdot \cos 36{,}87^\circ = -2{,}35$ mm



$$A_2 = \frac{\pi r^2}{2} = \frac{\pi \cdot 2{,}5^2}{2} = 9{,}817\ mm^2$$
$$I'_{yc_2} \doteq 0{,}1098\, r^4 = 0{,}1098 \cdot 2{,}5^4 = 4{,}2891\ mm^4$$
$$I'_{zc_2} \doteq 0{,}3928\, r^4 = 0{,}3928 \cdot 2{,}5^4 = 15{,}3438\ mm^4$$
$$D'_{yc_2zc_2} = 0$$

- pro transformaci do (ne)pootočených os lze využít tenzorové transformace :

$$\underline{I}' = \begin{bmatrix} I_{y'} & -D_{y'z'} \\ -D_{y'z'} & I_{z'} \end{bmatrix} , \qquad \underline{I} = \begin{bmatrix} I_y & -D_{yz} \\ -D_{yz} & I_z \end{bmatrix}$$

$$\underline{T} = \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix}$$

$\underline{T}$ je ortogonální, proto $\underline{T}^{-1} = \underline{T}^T = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix}$

$$\underline{I}' = \underline{T}\,\underline{I}\,\underline{T}^T$$

transformační matice

to, kam transformujeme

to, co transformujeme

$$\Rightarrow \boxed{\begin{aligned} I_{y}' &= I_y \cos^2\gamma + I_z \sin^2\gamma - D_{yz} \sin 2\gamma \\ I_z' &= I_y \sin^2\gamma + I_z \cos^2\gamma + D_{yz} \sin 2\gamma \\ D_{y'z'} &= \tfrac{1}{2}(I_y - I_z) \sin 2\gamma + D_{yz} \cos 2\gamma \end{aligned}}$$

- v případě našeho půlkruhu se natáčí do os y a z :

$$I_{yc2} = 4{,}2891 \cdot (\cos 36{,}87°)^2 + 15{,}3438 \cdot (\sin 36{,}87)^2 = 8{,}2688\ mm^4$$
$$I_{zc2} = 4{,}2891 \cdot (\sin 36{,}87°)^2 + 15{,}3438\, (\cos 36{,}87)^2 = 11{,}3641\ mm^4$$
$$D_{yc2zc2} = \frac{1}{2}\left(4{,}2891 - 15{,}3438\right) \cdot \sin(2 \cdot 36{,}87) = -5{,}3063\ mm^4$$

- <u>těžiště celého průřezu:</u>

$$y_c = \frac{\sum S_{zi}}{\sum A_i} = \frac{1{,}\overline{33} \cdot 6 + 2{,}64 \cdot 9{,}817}{6 + 9{,}817} = \frac{33{,}917}{15{,}817} = 2{,}144\ mm$$

$$z_c = \frac{\sum S_{yi}}{\sum A_i} = \frac{-1 \cdot 6 - 2{,}35 \cdot 9{,}817}{6 + 9{,}817} = -1{,}838\ mm$$

- vzdálenosti težišť jednotlivých částí od celkového težiště:

1) $\bar{y}_{c1} = y_{c1} - y_c = 1{,}333 - 2{,}144 = -0{,}811\ mm$
$\bar{z}_{c1} = z_{c1} - z_c = -1 - (-1{,}838) = 0{,}838\ mm$

2) $\bar{y}_{c2} = y_{c2} - y_c = 2{,}64 - 2{,}144 = 0{,}496\ mm$
$\bar{z}_{c2} = z_{c2} - z_c = -2{,}35 - (-1{,}838) = -0{,}512\ mm$

- momenty setrvačnosti vzhledem k osám procházejícím celkovým težištěm:

$$I_{yc} = I_{yc1} + A_1 \cdot \bar{z}_{c1}^2 + I_{yc2} + A_2 \cdot \bar{z}_{c2}^2 = 3 + 6 \cdot 0{,}838^2 + 8{,}2688 + 9{,}817 \cdot (-0{,}512)^2 = \underline{14{,}7224\ mm^4}$$

$$I_{zc} = 5{,}3333 + 6 \cdot (-0{,}811)^2 + 11{,}3641 + 9{,}817 \cdot 0{,}496^2 = \underline{23{,}0589\ mm^4}$$

$$D_{y_c z_c} = 2 + 6 \cdot 0{,}838 \cdot (-0{,}811) + (-5{,}3063) + 9{,}817 \cdot (-0{,}512) \cdot 0{,}496 = \underline{-9{,}8771\ mm^4}$$

- hlavní momenty setrvačnosti, úhel pootočení k hlavním osám, poloměry setrvačnosti:

$$I_{1,2} = \frac{I_{yc} + I_{zc}}{2} \pm \frac{1}{2}\sqrt{(I_{yc} - I_{zc})^2 + 4 D_{y_c z_c}} = \frac{14{,}7224 + 23{,}0589}{2} \pm \frac{1}{2}\sqrt{(14{,}7224 - 23{,}0589)^2 + 4 \cdot (-9{,}8771)}$$

$I_{y0} = 16{,}1525\ mm^4$
$I_{z0} = 21{,}6288\ mm^4$

$\Rightarrow$

$i_{y0} = \sqrt{\frac{I_{y0}}{A}} = \sqrt{\frac{16{,}1525}{15{,}817}} = 1{,}011\ mm$

$i_{z0} = \sqrt{\frac{I_{z0}}{A}} = \sqrt{\frac{21{,}1525}{15{,}817}} = 1{,}156\ mm$

$$tg\, 2\alpha = \frac{2 D_{y_c z_c}}{I_{zc} - I_{yc}} = \frac{2 \cdot (-9{,}8771)}{23{,}0589 - 14{,}7224} = -2{,}3696$$

$\Rightarrow \alpha = -33{,}6°$