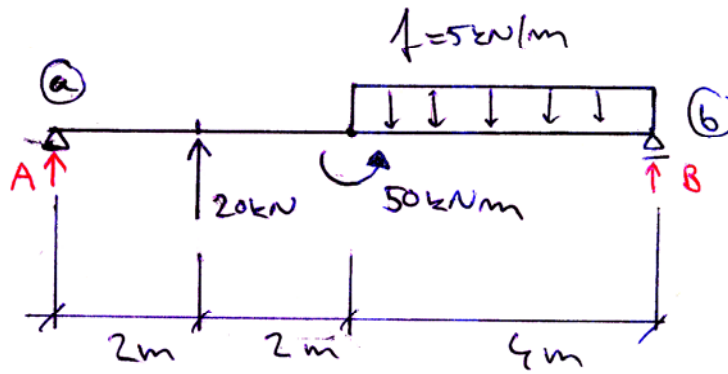


Přípravné cvičení



Ověřte reakce, přičemž vn. síl

$$\sum \mathcal{M}_A: 20 \cdot 2 + 50 - 5 \cdot 4 \cdot 6 + 8 \cdot B = 0$$

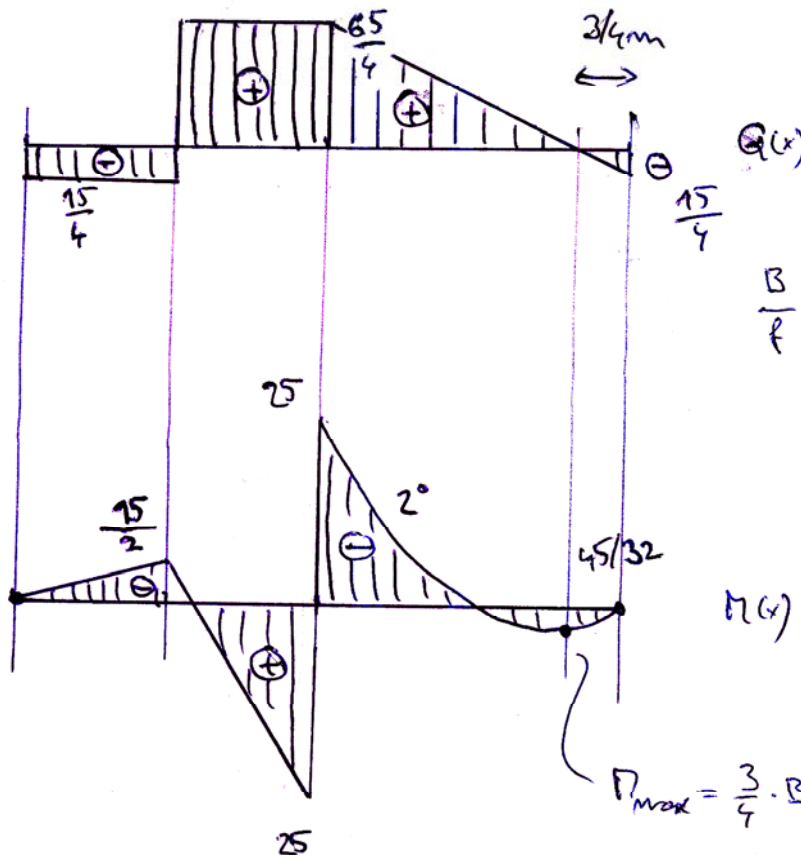
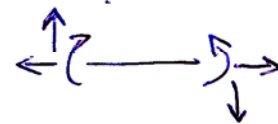
$$B = \frac{15}{4} \text{ kN}$$

$$\sum \mathcal{M}_B: -A \cdot 8 - 20 \cdot 6 + 50 + 5 \cdot 4 \cdot 2 = 0$$

$$A = -\frac{15}{4} \text{ kN}$$

$$\uparrow: 20 + A - 5 \cdot 4 + B = 0 \quad \checkmark$$

Konvence kladných vn. síl

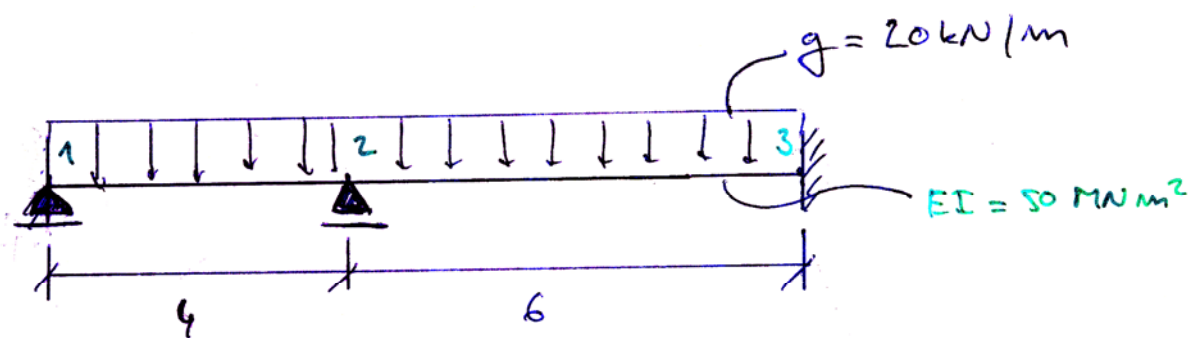


$$\frac{B}{f} = \frac{15/4}{5} = \frac{3}{4} \text{ m}$$

$$M_{\max} = \frac{3}{4} \cdot B - \frac{3}{4} \cdot 5 \cdot \frac{3}{8} =$$

$$= \frac{45}{816} - \frac{45}{32} = \frac{45}{32} \text{ kNm}$$

Pr. 1

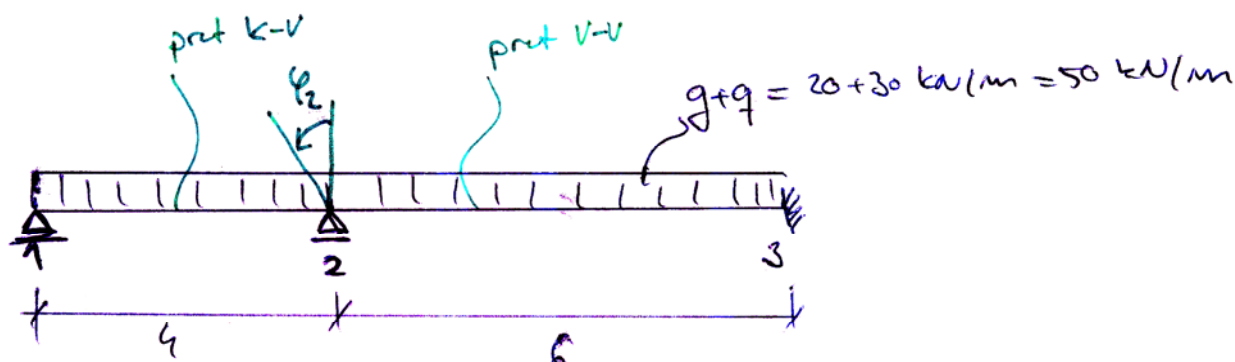


- určiť
- 1) extrémni moment nad podporou (stredmi)
 - 2) maximálny ohybový moment v pravom poli

- kromě stáleho zátěže $q = 20 \text{ kN/m}$ navíc uvažujte
nahodilou zátěž $q = 30 \text{ kN/m}$

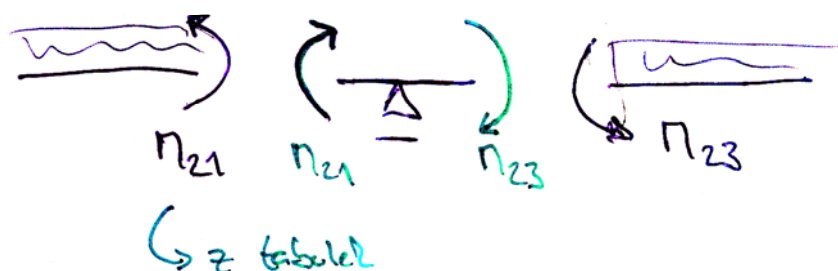
①

rovnice konstrukce:

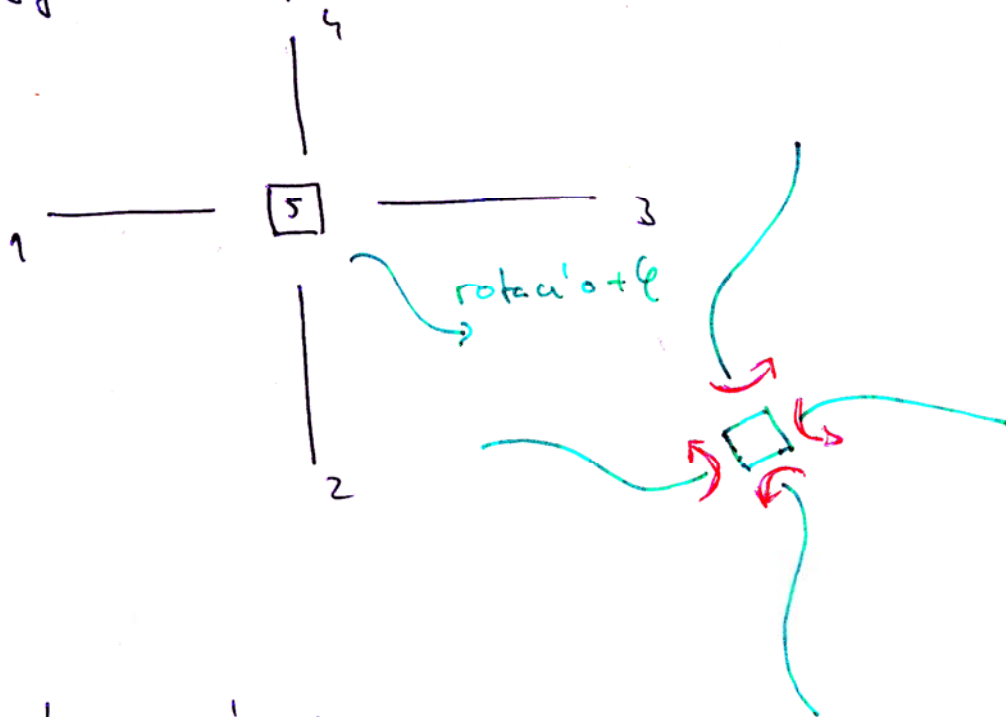


- 1) určit proud
- 2) — — — momenty
- 3) základní rovnice - momentové podmínky rovnováhy

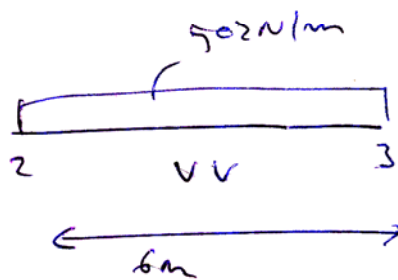
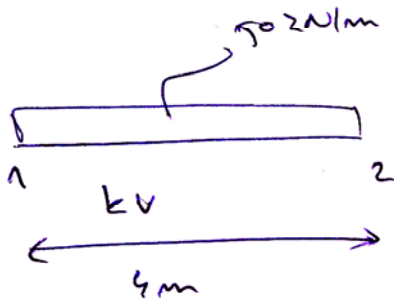
ⓐ koncové momenty



Koncové momenty - ekvální tehy, požad působí
sféricky na prut v každém směru



SÍLOVÉ KONCOVÉ MOMENTY



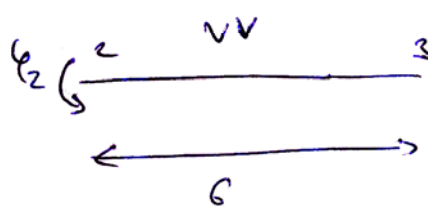
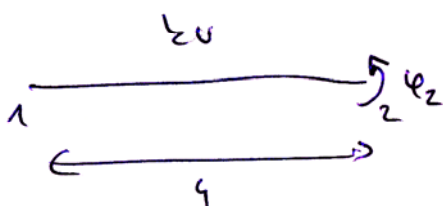
$$\bar{M}_{21} = -\frac{kL^2}{8}$$

$$\bar{M}_{23} = \frac{kL^2}{12} = \frac{50 \cdot 6^2}{12} = 150 \text{ kNm}$$

$$\bar{M}_{21} = -\frac{50 \cdot 4^2}{8} = -100 \text{ kNm}$$

KONCOVÉ ROTENTY SOUVISEJÍCÍ S PROTOČENÍM STŘEŠNÍKŮ

$$k = \frac{2EI}{L}$$



$$M_{21}^{\varphi} = \frac{3k\varphi_2}{2} = \frac{3EI\varphi_2}{L} = \frac{3 \cdot 50 \cdot \varphi_2}{4} = 37,5\varphi_2$$

$$M_{23}^{\varphi} = k2\varphi_2 = \frac{4EI\varphi_2}{L} = \frac{4 \cdot 50 \cdot \varphi_2}{6} = 33,33\varphi_2$$

Global coordinate moments

$$\bar{M}_{21} = \bar{M}_{21} + M_{21}^f = (-100 + 37,5\varphi_2) \text{ kNm}$$

$\uparrow$
kNm

$$\bar{M}_{23} = 150 + 33,333\varphi_2 \text{ (kNm)}$$

Rotational compatibility

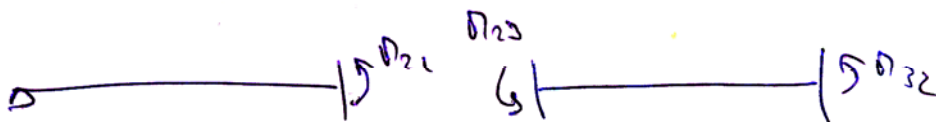
$$\bar{M}_{21} + \bar{M}_{23} = 0$$

$$-100 + 37,5\varphi_2 + 150 + 33,333\varphi_2 = 0$$

$$-50 = 70,833\varphi_2$$

$$\varphi_2 = -0,7059 \text{ m rad}$$

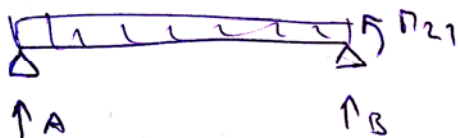
Local coordinate moments



$$M_{21} = -100 + 37,5\varphi_2 = -100 + 37,5 \cdot (-0,7059) = -126,470 \text{ kNm}$$

$$M_{23} = 150 + 33,333 \cdot (-0,7059) = 126,47 \text{ kNm}$$

$$M_{32} = -150 + \frac{2EI\varphi}{L} = \frac{-150 + 2 \cdot 50 \cdot (-0,7059)}{6} = -161,75 \text{ kNm}$$



$$\sum F_y: 4B + M_{21} = 50 \cdot 4 \cdot 2$$

$$B = 131,618 \text{ kN}$$

$$\sum M_A: A = \frac{126,47 + 50 \cdot 4 \cdot 2}{4} = 68,3825 \text{ kN}$$



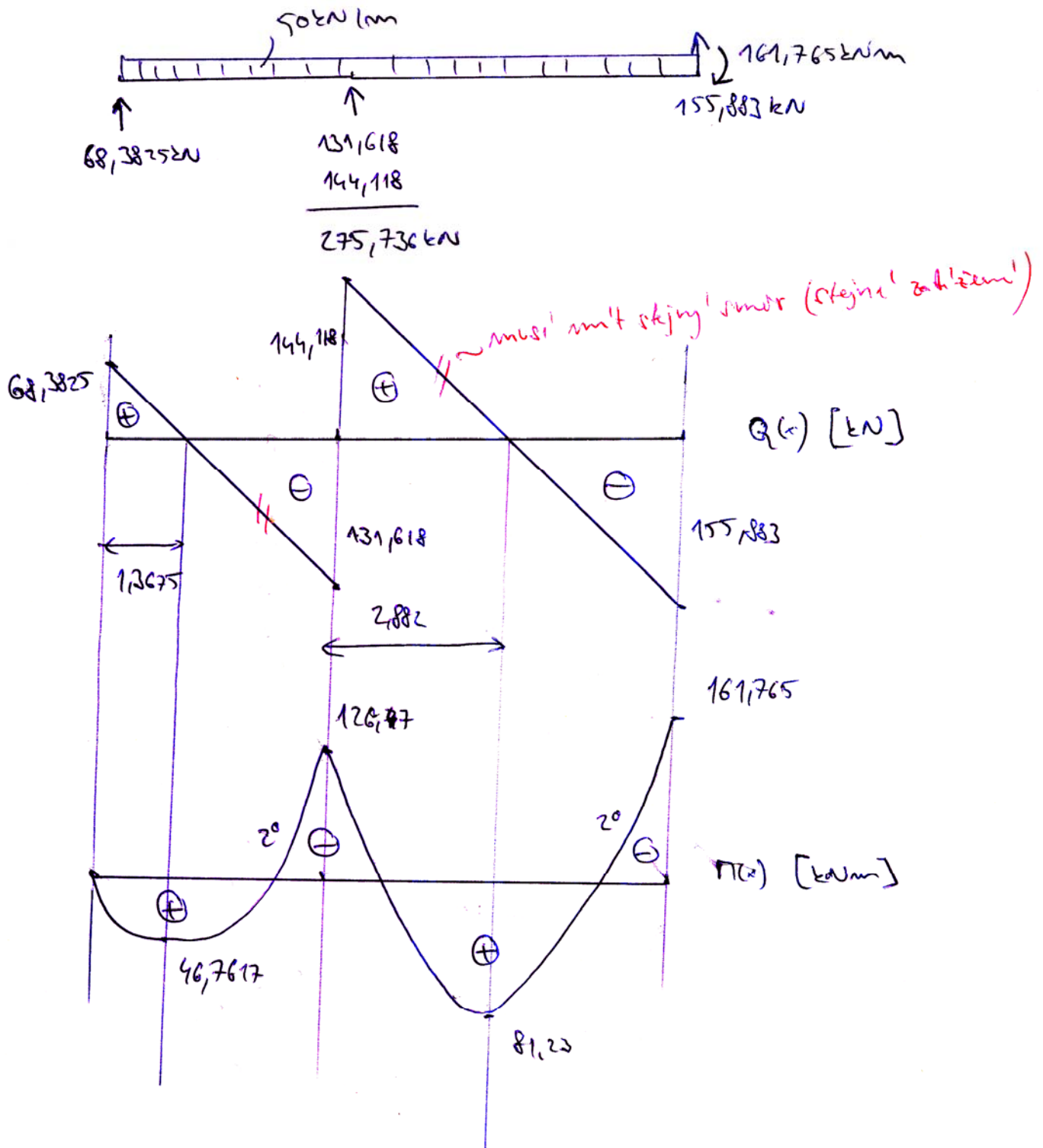
$$\sum F_y: M_{23} + M_{32} + B \cdot 6 - 50 \cdot 6 \cdot 3 = 0$$

$$B = \frac{-M_{23} - M_{32} + 50 \cdot 6 \cdot 3}{6}$$

$$B = 155,883 \text{ kN}$$

$$A = \frac{M_{32} + M_{23} + 50 \cdot 6 \cdot 3}{6} = 144,118 \text{ kN}$$

Kontrola $\Sigma F_z: -68,3825 \cdot 10 - 275,736 \cdot 6 - 161,765 + 50 \cdot 10 \cdot 5 = -0,006$ OK J43, cv. 2, str. 5



extrém v levém poli: $x = 68,3825 / 50 = 1,36765 \text{ m}$

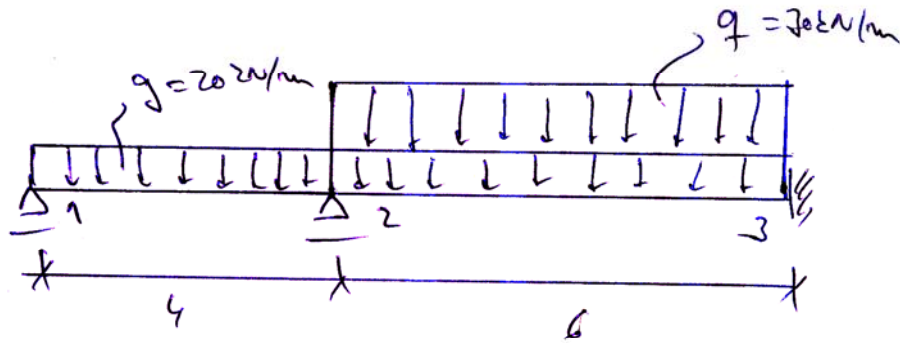
$$M_{\max} = 68,3825 \cdot 1,36765 - 50 \cdot \frac{1,36765^2}{2} = 46,7617 \text{ kNm}$$

extrém v pravém poli: $x = \frac{144,118}{50} = 2,882 \text{ m}$

$$M_{\max} = 144,118 \cdot 2,882 - 50 \cdot \frac{2,882^2}{2} = \text{~~126,47~~} 81,23 \text{ kNm}$$

$-126,47$

② schematische Konstruktion



$$\Pi_{21} = \bar{\Pi}_{21} + k \cdot \frac{3}{2} \varphi_2 = -\frac{1}{8} q L^2 + \frac{3 \varphi_2 E I}{L} = -\frac{20 \cdot 4^2}{8} + 37,5 \varphi_2 = -40 + 37,5 \varphi_2$$

$$\Pi_{23} = \bar{\Pi}_{23} + k \cdot 2 \varphi_2 = \frac{1}{12} q L^2 + \frac{4 E I \varphi_2}{L} = \frac{1}{12} \cdot 20 \cdot 6^2 + \frac{4 \cdot 20}{6} \varphi_2 = 150 + 33,333 \varphi_2$$

$$\Pi_{21} + \Pi_{23} = 0 \Rightarrow -40 + 37,5 \varphi_2 + 150 + 33,333 \varphi_2 = 0$$

$$\varphi_2 \cdot 70,833 = -110$$

$$\varphi_2 = -1,5529 \text{ mrad}$$

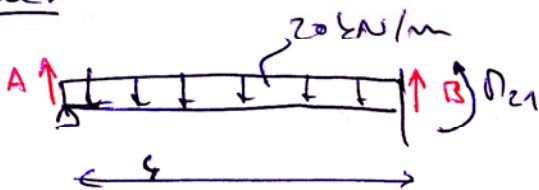
Konkrete Momente:

$$M_{21} = -40 + 37,5 \varphi_2 = -98,235 \text{ kNm}$$

$$\Pi_{23} = 150 + 33,333 \varphi_2 = 98,2353 \text{ kNm}$$

$$\Pi_{32} = \bar{M}_{32} + k \varphi_2 = -\frac{1}{12} q L^2 + \frac{2 E I \varphi_2}{L} = -150 - 25,8824 = -175,882 \text{ kNm}$$

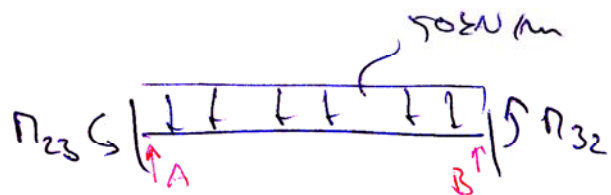
Reaktionen:



$$\sum F_y = 0: 4B + \Pi_{21} - 20 \cdot 4 \cdot 2 = 0$$

$$B = \frac{20 \cdot 4 \cdot 2 - \Pi_{21}}{4} = 64,5588 \text{ kN}$$

$$A = 80 - 64,5588 = 15,4413 \text{ kN}$$

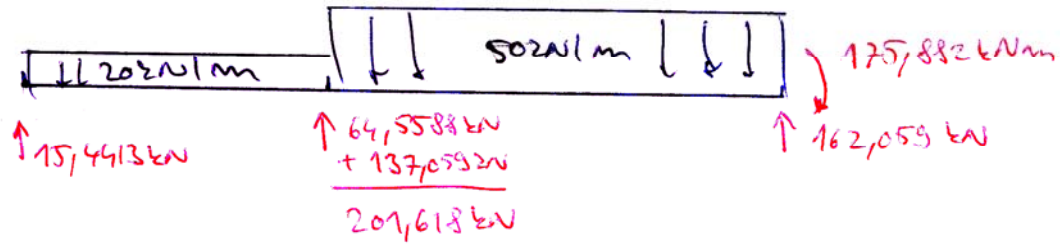


$$\sum F_y = 0: A = \frac{\Pi_{32} + \Pi_{23} + 20 \cdot 6 \cdot 3}{6}$$

$$= \frac{98,2353 - 175,882 + 0}{6} = -13,877 \text{ kN}$$

$$B = 162,059 \text{ kN}$$

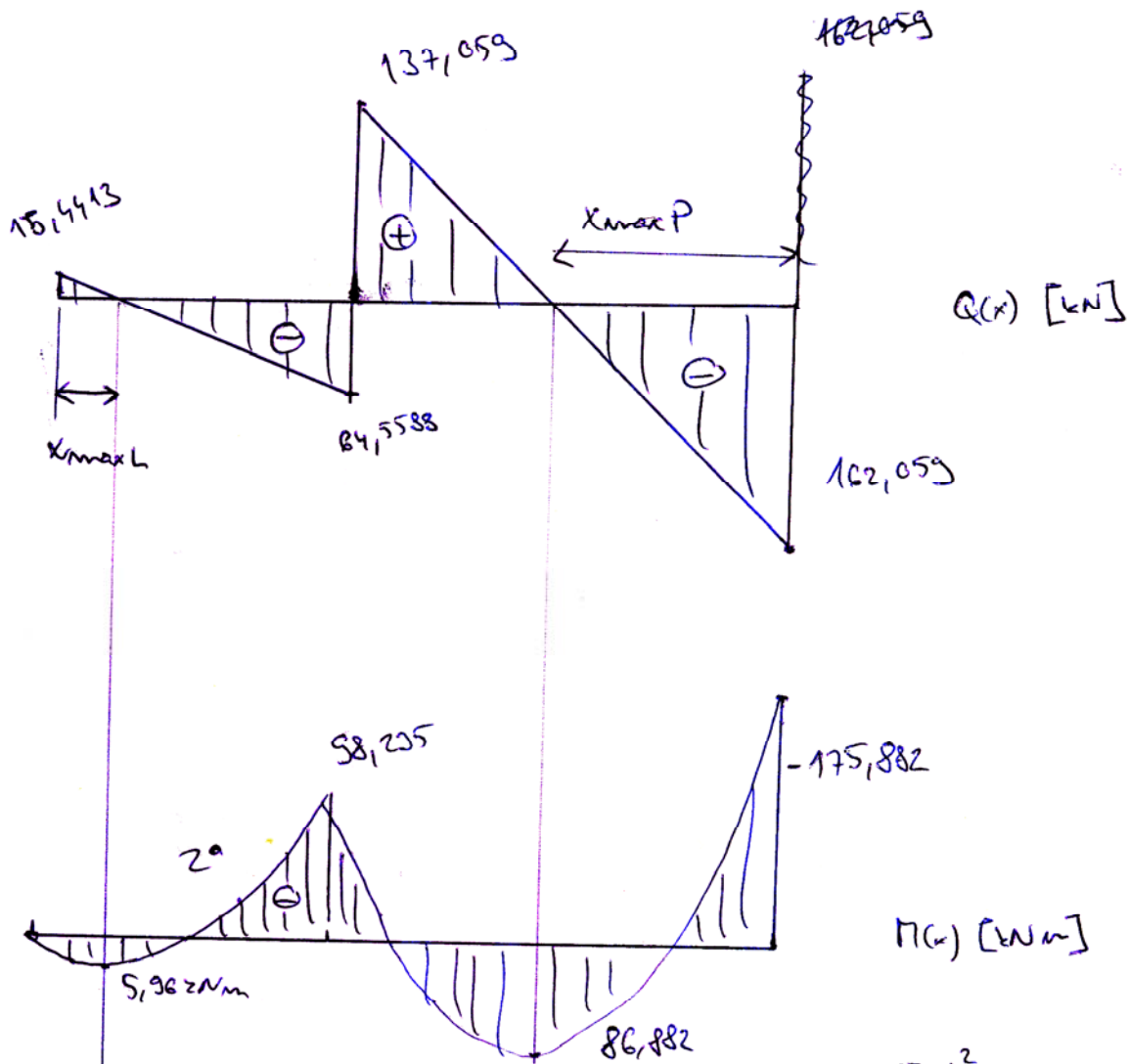
Reakce v podporačích



Kontrola - moment

$$-10 \cdot 15,4413 - 201,618 \cdot 6 + 20 \cdot 4 \cdot 8 + 50 \cdot 6 \cdot 3 - 175,882 \doteq -0,003 \quad \text{OK}$$

Výkresy, extrém



$$x_{\max L} = 15,4413 / 20 = 0,7721 \text{ m}$$

$$M_{\max L} = 15,4413 \cdot 0,7721 - 20 \cdot \frac{0,7721^2}{2} = 5,9612 \text{ kNm}$$

$$x_{\max P} = 162,059 / 50 = 3,2412 \text{ m}$$

$$M_{\max P} = 162,059 \cdot 3,2412 - 50 \cdot \frac{3,2412^2}{2} = -175,882 = 86,882$$