Modeling of concrete creep based on microprestress-solidification theory

Petr Havlásek & Milan Jirásek

Department of Mechanics,
Faculty of Civil Engineering, Czech Technical University in Prague,
Thákurova 7, 16629 Prague 6 - Dejvice, Czech Republic

Abstract

Creep of concrete is strongly affected by the evolution of pore humidity and temperature, which in turn depend on the environmental conditions and on the size and shape of the concrete member. Current codes of practice take that into account only approximately, in a very simplified way. More realistic description can be achieved by advanced models, such as model B3 and its improved version that uses the concept of microprestress. The value of microprestress is influenced by the evolution of pore humidity and temperature. In this paper, values of parameters used by the microprestress-solidification theory (MPS) are recommended and their influence on the creep compliance function is evaluated and checked against experimental data from the literature. Certain deficiencies of MPS are pointed out and its modified version is proposed and validated.

1 Introduction

In contrast to metals, concrete exhibits creep already at room temperature. This phenomenon results into a gradual but considerable increase of deformation at sustained loads and needs to be taken into account in design and analysis of concrete structures. The present paper examines an advanced concrete creep model, which extends the original B3 model [1] and uses the concepts of solidification [5, 6] and microprestress [2, 3, 4]. The main objective of the paper is to clarify the role of non-traditional model parameters and provide hints on their identification. The creep tests performed by Fahmi, Polivka and Bresler [7], covering creep of both sealed and drying specimens under elevated and variable temperatures, are used as a source of experimental data and are compared with the results of numerical simulations.

All numerical computations have been performed using the finite element package OOFEM [8, 9, 10] developed mainly at the CTU in Prague by Bořek Patzák.

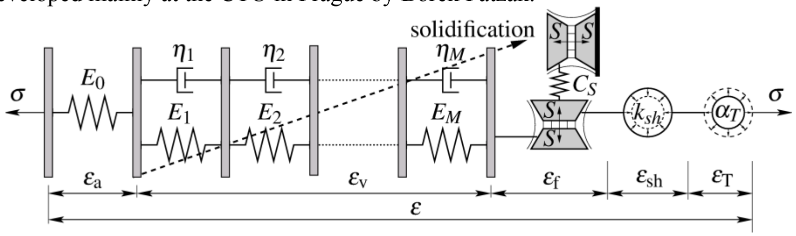

Fig. 1 Rheological scheme of the complete hygro-thermo-mechanical model

2 Description of the material model

The complete constitutive model for creep and shrinkage of concrete can be represented by the rheological scheme shown in Fig. 1. It consists of (i) a non-aging elastic spring, representing instantaneous elastic deformation, (ii) a solidifying Kelvin chain, representing short-term creep, (iii) an aging dashpot with viscosity dependent on the microprestress, *S*, representing long-term creep, (iv) a shrinkage unit, representing volume changes due to drying, and (v) a unit representing thermal expansion. In the experiments, shrinkage and thermal strains were measured separately on load-free specimens and subtracted from the strain of the loaded specimen under the same environmental conditions. It should be noted that even after subtraction of shrinkage and thermal strain, the evolution of mechanical strain is affected by humidity and temperature. Dry concrete creeps less than wet one, but the process of drying accelerates creep. Elevated temperature leads to faster cement

Proc. of the 9th *fib* International PhD Symposium in Civil Engineering
July 22 to 25, 2012, Karlsruhe Institute of Technology (KIT), Germany

hydration and thus to faster reduction of compliance due to aging, but it also accelerates the viscous processes that are at the origin of creep and the process of microprestress relaxation.

The microprestress is understood as the stress in the microstructure generated due to large localized volume changes during the hydration process. It builds up at very early stages of microstructure formation and then is gradually reduced by relaxation processes. Additional microprestress is generated by changes of internal relative humidity and temperature. This is described by the non-linear differential equation [2].

$$\frac{\mathrm{d}S}{\mathrm{d}t} + \psi_S(T,h)c_0S^2 = k_1\left|\frac{\mathrm{d}(T\ln h)}{\mathrm{d}t}\right| \tag{1}$$

in which T denotes the absolute temperature, h is the relative pore humidity (partial pressure of water vapor divided by the saturation pressure), c_0 and k_1 are constant parameters, and ψ_S is a variable factor that reflects the acceleration of microprestress relaxation at higher temperature and its deceleration at lower humidity (compared to the standard conditions). Owing to the presence of the absolute value operator on the right-hand side of (1), additional microprestress is generated by both drying and wetting, and by both heating and cooling, as suggested in [2].

3 Numerical simulations

In this section, experimental data of Fahmi, Polivka and Bresler are compared to results obtained with the MPS theory, which reduces to the standard B3 model in the special case of basic creep. All examples concerning drying and thermally induced creep have been run as a staggered problem, with the heat and moisture transport analyses preceding the mechanical one. The available experimental data contained the mechanical strains (due to elasticity and creep), with the thermal and shrinkage strains subtracted.

The four parameters of the B3 model describing the basic creep, q_1, q_2, q_3 and q_4, were determined from the composition of the concrete mixture and from the compressive strength using empirical formulae according to [1]. The result of this prediction exceeded the expectations; only minor adjustments were necessary to get the optimal fit (see the first part of the strain evolution in Fig. 2 (left). The following values were used: q_1 = 19.5, q_2 = 160, q_3 = 5.25 and q_4 = 12.5 (all in 10^{-6}/MPa). The MPS theory uses three additional parameters, c_0, k_1 and c, but parameter c should be equal to $c_0\ q_4$. It has been found that the remaining parameters c_0 and k_1 are not independent. What matters for creep is only their product. In practical computations, k_1 can be set to a fixed value (e.g. 1 MPa/K), and c_0 can be varied until the best fit with experimental data is obtained; in all the following figures c_0 is specified in MPa^{-1}day^{-1}. All other parameters were used according to standard recommendations.

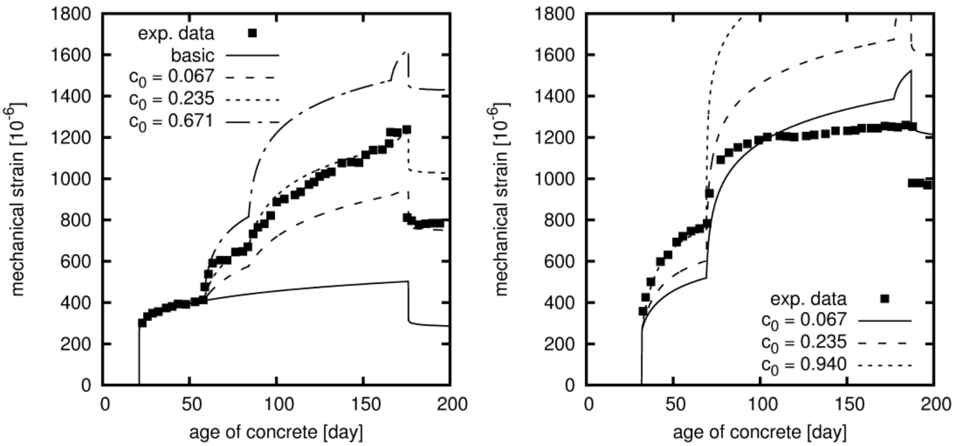

Fig. 2 Mechanical strain evolution for sealed specimens with relative pore humidity assumed to be 98%, loaded at time t' = 21 days (left), and for drying specimens at 50% relative environmental humidity, loaded at time t' = 32 days (right); both specimens are loaded by compressive stress 6.27 MPa

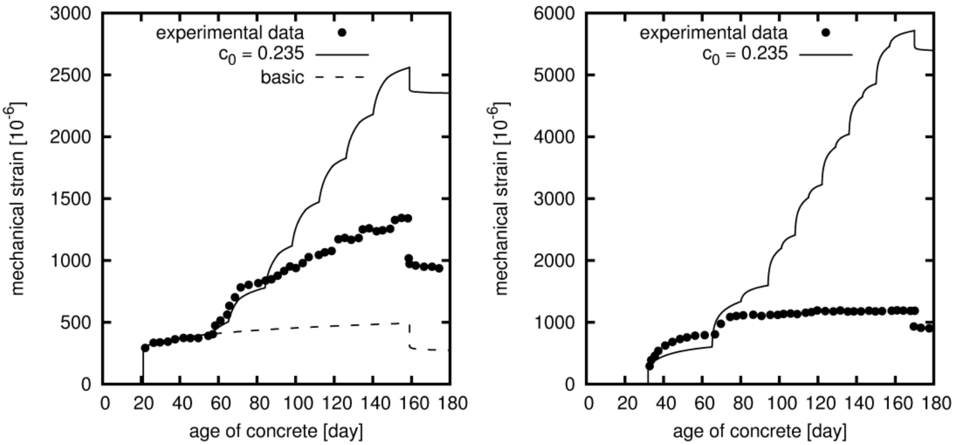

Fig. 3 Mechanical strain evolution for sealed specimens with relative pore humidity assumed to be 98%, loaded at time t' = 21 days (left), and for drying specimens at 50% relative environmental humidity, loaded at time t' = 32 days (right); both specimens are loaded by compressive stress 6.27 MPa and subjected to cyclic variations of temperature

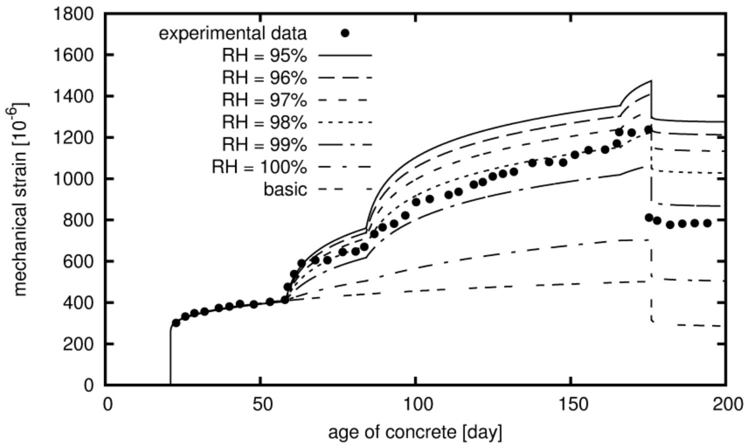

Fig. 4 Mechanical strain evolution for sealed specimens, loaded by compressive stress 6.27 MPa from age 21 days, with the assumed relative humidity of pores varied from 95% to 100%; parameters of MPS theory: k_1 = 1 MPa/K, c_0 = 0.235 MPa^{-1}day^{-1}

A really good fit of the first experimental data set (98% RH, i.e., h = 0.98) was obtained for c_0 = 0.235 MPa^{-1}day^{-1}; see Fig. 2 (left). The agreement is satisfactory except for the last interval, which corresponds to unloading. It is worth noting that the thermally induced part of creep accounts for more than a half of the total creep (compare experimental data with solid curve labeled *basic* in Fig. 2 (left). Unfortunately, with default values of the other parameters, the same value of c_0 could not be used to fit the second experimental data set, because it would have led to overestimation of creep; see the dashed curve in Fig. 2 (right). In the first loading interval of 37 days, creep takes place at room temperature and the best agreement would be obtained with parameter c_0 set to 0.940 MPa^{-1}day^{-1}; see the dash-dotted curve in Fig. 2 (right). However, at the later stage when the temperature rises to 60°C, the creep would be grossly overestimated. A reasonable agreement during this stage of loading is obtained with c_0 reduced to 0.067 MPa^{-1}day^{-1} (solid curve in Fig. 2 (right)), but then the creep is underestimated in the first interval in Fig. 2 (left).

For the last two testing programs with cyclic variations of temperature, the agreement between experimental and computed data is reasonable only until the end of the second heating cycle (solid curves in Fig. 3). For the sealed specimen, the final predicted compliance exceeds the measured value almost twice, for the drying specimen almost five times. In order to obtain a better agreement, parameter c_0 would have to be reduced, but this would result into an underestimation of creep in the first two testing programs.

Another deficiency of the model is illustrated by the graphs in Fig. 4. They refer to the first set of experiments. As documented by the solid curve in Fig. 2 (left), a good fit was obtained by setting parameter c_0 = 0.235 MPa^{-1}day^{-1}, assuming that the relative pore humidity is 98%. The pores are initially completely filled with water; however, even if the specimen is perfectly sealed, the relative humidity slightly decreases due to the water deficiency caused by the hydration reaction. The problem is that the exact value of pore humidity in a sealed specimen and its evolution in time are difficult to determine. In simple engineering calculations, a constant value of 98% is often used; unfortunately, the model response is quite sensitive to this choice (see Fig. 4). The source of such a strong sensitivity is in the assumption that the instantaneously generated microprestress is proportional to the absolute value of the change of $T\ln(h)$; see the right-hand side of (1). Rewriting (1) as

$$\frac{\mathrm{d}S}{\mathrm{d}t} + \psi_S(T,h)c_0S^2 = k_1\left|\ln h\frac{\mathrm{d}T}{\mathrm{d}t} + \frac{\mathrm{T}}{\mathrm{h}}\frac{\mathrm{d}h}{\mathrm{d}t}\right| \tag{2}$$

we can see that at (almost) constant humidity close to 100%, the right-hand side is proportional to the magnitude of temperature rate, with proportionality factor $k_1|\ln(h)| \approx k_1(1-h)$.

4 Improved material model and its validation

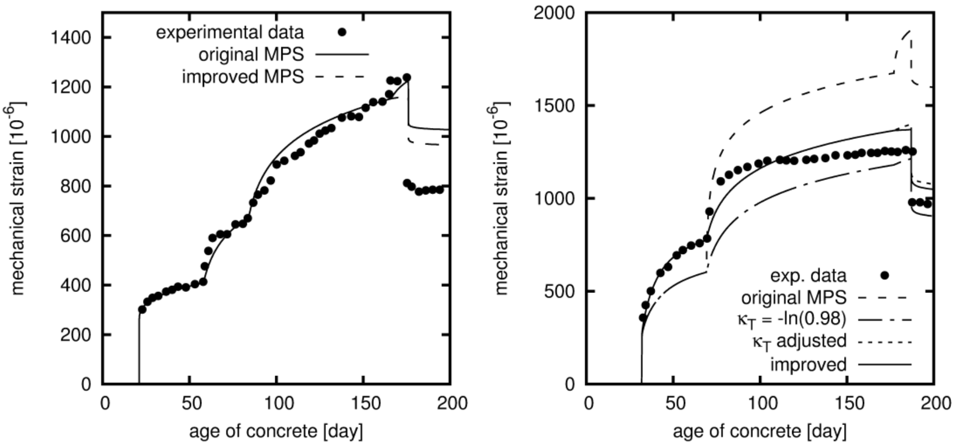

Fig. 5 Mechanical strain evolution for sealed specimens loaded at time t' = 21 days (left), and for drying specimens loaded at time t' = 32 days (right); both specimens are loaded by compressive stress 6.27 MPa and subjected to cyclic variations of temperature

As a simple remedy to overcome these problems, the microprestress relaxation equation (1) is replaced by

$$\frac{\mathrm{d}S}{\mathrm{d}t} + \psi_S(T,h)c_0S^2 = k_1\left|\frac{\mathrm{T}}{\mathrm{h}}\frac{\mathrm{d}h}{\mathrm{d}t} - \kappa_T k_T(T)\frac{\mathrm{d}T}{\mathrm{d}t}\right| \tag{3}$$

$$\text{with} \quad k_T(T) = \mathrm{e}^{-c_T(T_{max}-T)} \tag{4}$$

in which κ_T and c_T are new parameters and T_{max} is the maximum previously reached temperature. With $\kappa_T = 0.02$, the creep curves in Fig. 4 plotted for different assumed pore humidities would be almost identical with the solid curve that nicely fits experimental results. Introduction of a new parameter provides more flexibility, which is needed to improve the fit of the second testing program in Fig. 2 (right), with combined effects of drying and temperature variation. For sealed specimens and monotonous thermal loading, only the product $c_0 k_1 \kappa_T$ matters, and so the good fit in Fig. 2 (right) could be obtained with different combinations of κ_T and c_0.

The results are shown in Fig. 5 for sustained thermal loading and in Fig. 6 for cyclic thermal loading. In these plots, the curves labeled *original MPS* show results obtained with standard MPS. Data series κ_T = -ln(0.98) were obtained with c_0 = 0.235 MPa^{-1}day^{-1}, k_1 = 1 MPa/K, κ_T = 0.020203 and c_T = 0. Data series κ_T *adjusted* correspond to parameters c_0 = 0.235 MPa^{-1}day^{-1}, k_1 = 4 MPa/K, κ_T = 0.005051 and c_T = 0. Note that in the case of constant relative humidity (Fig. 5 (left) and Fig. 6 (left)) these series coincide with data series *original MPS*. The best agreement with experimental data is obtained with c_0 = 0.235 MPa^{-1}day^{-1}, k_1 = 4 MPa/K, κ_T = 0.005051 and c_T = 0.3 K^{-1}; these series are labeled *improved*. In Fig. 5 (left), only a small change can be observed compared to data series *original MPS*; these differences arise when the temperature ceases to be monotonous. For the sealed specimen Fig. 5 (left), this change is detrimental, but the deterioration is negligible compared to a substantial improvement in the case of cyclic thermal loading (Fig. 6).

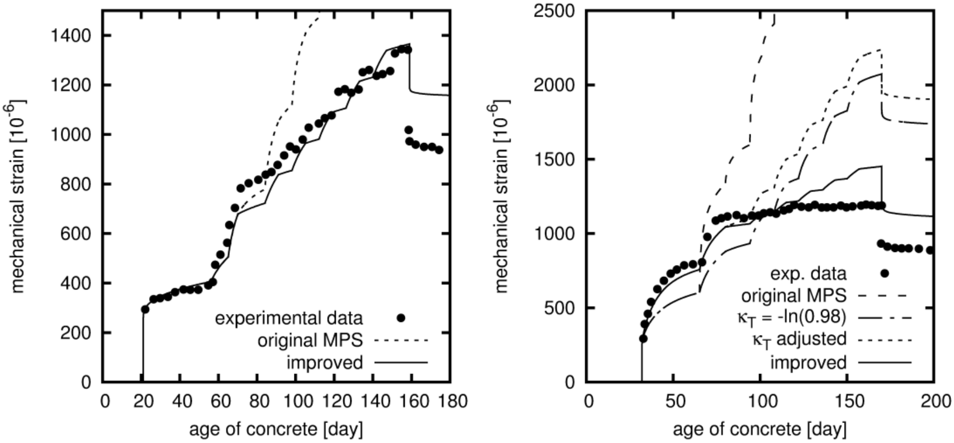

Fig. 6 Mechanical strain evolution for sealed specimens loaded at time t' = 21 days (left), and for drying specimens loaded at time t' = 32 days (right); both specimens are loaded by compressive stress 6.27 MPa and subjected to cyclic variations of temperature

5 Conclusions and further work

The material model based on the MPS theory has been successfully implemented into the finite element package OOFEM and has been used in simulations of concrete creep at variable temperature and humidity.

For sealed specimens subjected to variable temperature, the results predicted by the MPS theory are very sensitive to the assumed value of relative pore humidity (which is slightly below 100% due to self-desiccation). In order to overcome this deficiency, a modified version of the model has been proposed and successfully validated. Excessive sensitivity to the specific choice of relative humidity has been eliminated. Also, it has become easier to calibrate the model because thermal and moisture effects on creep are partially separated.

The original MPS theory grossly overestimates creep when the specimen is subjected to cyclic temperature. A new variable k_T has been introduced in order to reduce the influence of subsequent thermal cycles on creep. This modification does not affect creep tests in which the evolution of temperature is monotonous.

Even though the existing material model has been improved, it is not yet applicable to a general experimental setup. Comparing the results of experimental measurements of Pickett [11] and the data obtained from FE simulations (see Fig. 7), it is apparent that the experiment is well fitted in the case of basic creep (first part of series B) and monotonous drying (series C). Even the effect of curing before loading and drying is captured satisfactorily (series E and F). On the other hand the influence of drying and wetting cycles on creep is not described correctly (series D), e.g. at the end of the experiment (t = 140 days) the computed total deflection exceeds measured data more than two times. Further adjustments will need to be addressed in future work.

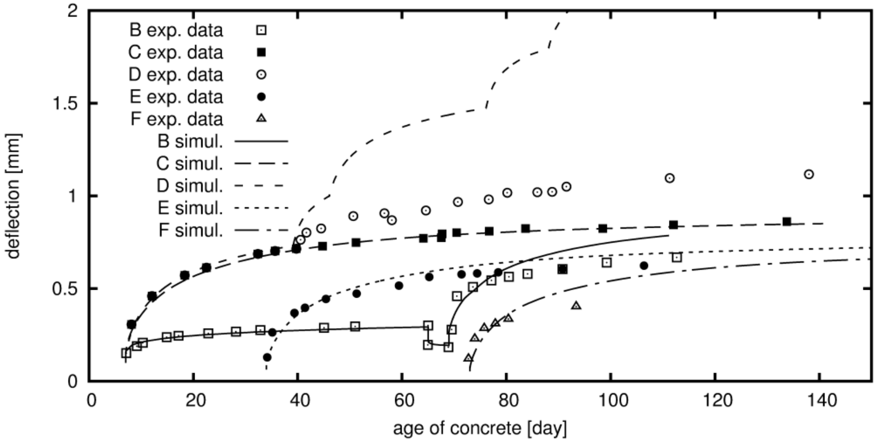

Fig. 7 Time evolution of deflection of prismatic beams subjected to three and four point bending (points refer experimental data, lines to data obtained from FE simulations; B = sealed conditions until reloading, C, E, F = cured until loading, then drying, D = drying and wetting cycles)

Acknowledgements: Financial support for this work was provided by projects SGS "Numerické modelování v mechanice konstrukcí a materiálů", reg. number OHK1-009/12 and 103/09/H078. The financial support is gratefully acknowledged.

References

[1] Bažant, Z.P. and Baweja, S.: Creep and shrinkage prediction model for analysis and design of concrete structures: Model B3. In: Adam Neville Symposium: Creep and Shrinkage – Structural Design Effects (2000)

[2] Bažant, Z.P., Cedolin, L. and Cusatis, G.: Temperature effect on concrete creep modeled by microprestress-solidification theory. In: Journal of Engineering Mechanics, 130 (2004) No. 6, pp. 691-699

[3] Bažant, Z.P., Hauggaard, A.B. and Ulm, F.: Microprestress-solidification theory for concrete creep I: Aging and drying effects. In: Journal of Engineering Mechanics, 123 (1997) No. 11, pp. 1188-1194

[4] Bažant, Z.P., Hauggaard, A.B. and Ulm, F.: Microprestress-solidification theory for concrete creep II: Algorithm and verification. In: Journal of Engineering Mechanics, 123 (1997) No. 11, pp. 1195-1201

[5] Bažant, Z.P., and Prasannan, S.: Solidification theory for concrete creep. I: Formulation. In: Journal of Engineering Mechanics, 115 (1989) No. 8, pp. 1691-1703

[6] Bažant, Z.P., and Prasannan, S.: Solidification theory for concrete creep. II: Verification and application. In: Journal of Engineering Mechanics, 115 (1989) No. 8, pp. 1704-1725

[7] Fahmi, H.M., Polivka, M. and Bresler, B.: Effects of sustained and cyclic elevated temperature on creep of concrete. In: Cement and Concrete Research, (1972) No. 2, pp. 591-606

[8] Patzák, B.: OOFEM home page. http://www.oofem.org, (2000)

[9] Patzák, B., and Bittnar, Z.: Design of object oriented finite element code. In: Advances in Engineering Software, 32 (2001) No. 10-11, pp. 759-767

[10] Patzák, B., Rypl, D. and Bittnar, Z.: Parallel explicit finite element dynamics with nonlocal constitutive models. In: Computers Structures, 79 (2001) No. 26-28, pp. 2287-2297

[11] Pickett, G.: The effect of change in moisture-content on the creep of concrete under a sustained load. In: Journal of the ACI, 13 (1942) No. 4, pp. 333-355