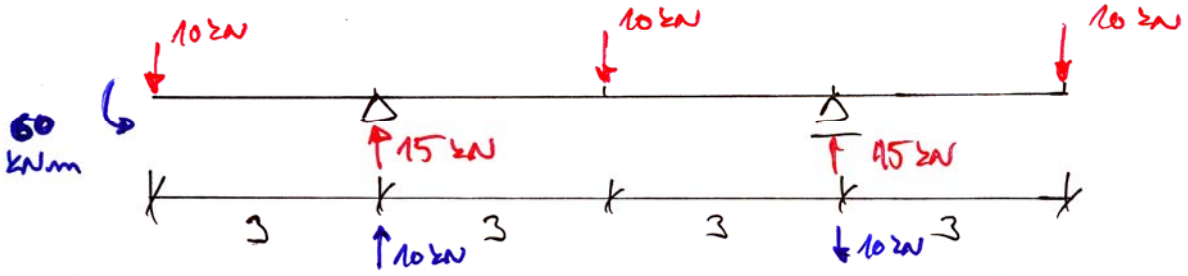
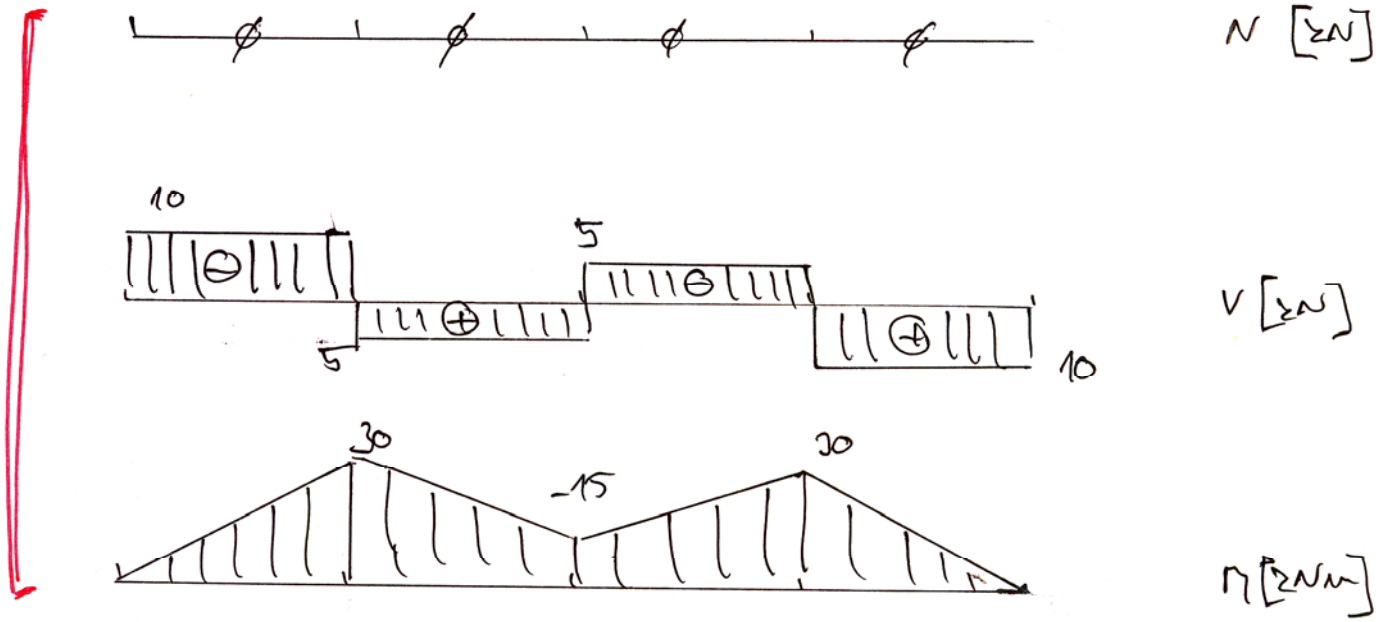
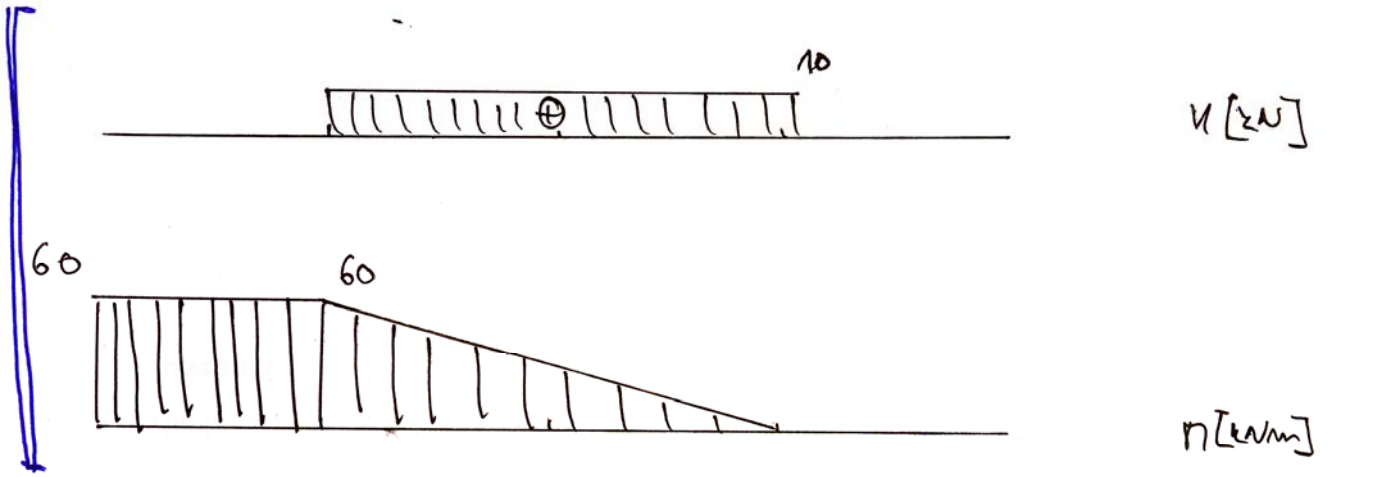
Na rozchrá'th:

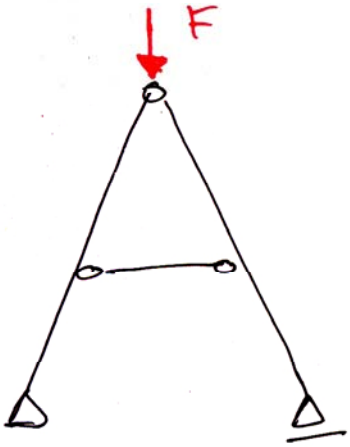
Varianta 1
Varianta 2



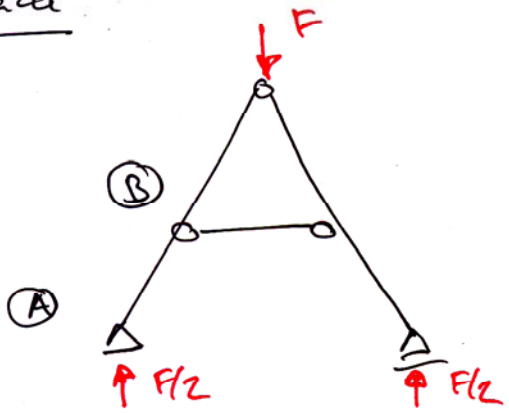


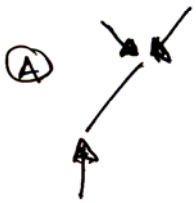


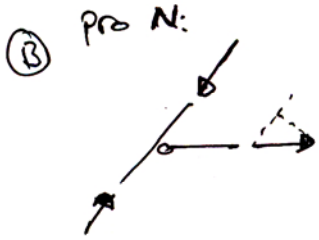
Příklady na stat. určité konstrukce:



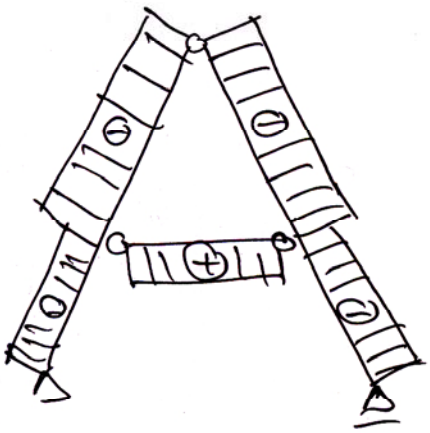
Řešení



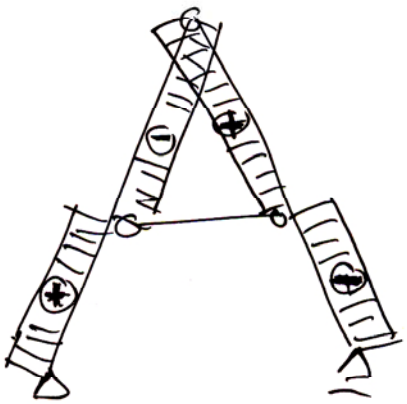




N

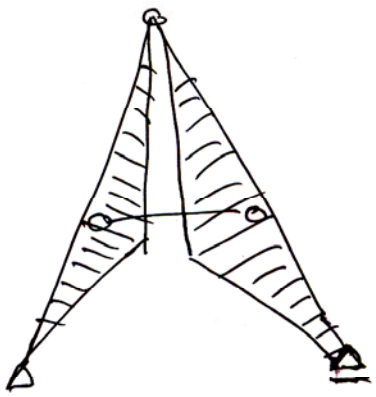


V



antisymetrický průběh posouvajících sil

M

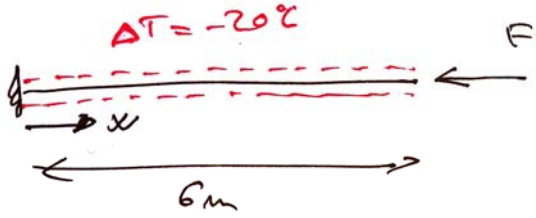


Symetrický průběh momentů

Vysvětlit: Tah v tyčce stejné u u posouvajících sil

Symetrie

Určit posuvy, napětí, deformace



$E = 30\ GPa$

$A = 0{,}2\ m^2$

$\alpha_T = 12 \cdot 10^{-6}\ K^{-1}$

→ vyjdeme z diferenciální rovnice

$$\frac{d}{dx}\left[EA\left(\frac{du}{dx} - \alpha_T \Delta T\right)\right] = -t_x = 0$$

ε_{tot}

$$EA\left(\frac{du}{dx} - \alpha_T \Delta T\right) = C_1$$

$$\frac{du(x)}{dx} = \frac{C_1}{EA} + \alpha_T \Delta T \quad (B)$$

$$u(x) = \frac{C_1 x}{EA} + \alpha_T \Delta T x + C_2 \quad (A)$$

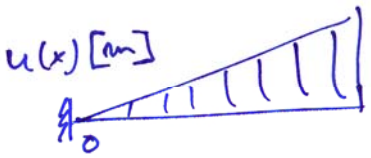
OKRAJOVÉ PODMÍNKY

- (1) $u(0) = 0$ ... kinematická okrajová podmínka
- (2) $N(L) = -F$ ... statická okrajová podmínka

$$A \cdot \sigma(L) = -F$$

$$EA \cdot \left(\frac{du(L)}{dx} - \alpha_T \Delta T\right) = -F \Rightarrow \frac{du(L)}{dx} = -\frac{F}{EA} + \alpha_T \Delta T$$

$-0{,}00164$



- (1)+(A) $\Rightarrow C_2 = 0$
- (2)+(B) $\Rightarrow \frac{C_1}{EA} + \alpha_T \Delta T = -\frac{F}{EA} + \alpha_T \Delta T \Rightarrow C_1 = -F$

$$\Rightarrow u(x) = \frac{-F \cdot x}{EA} + \alpha_T \Delta T \cdot x \; ; \; u(L) = \frac{-200 \cdot 6}{30 \cdot 10^9 \cdot 0{,}2} + 12 \cdot 10^{-6} \cdot (-20) \cdot 6 = -0{,}00164\ m = -1{,}64\ mm$$

($\cdot 10^3$)

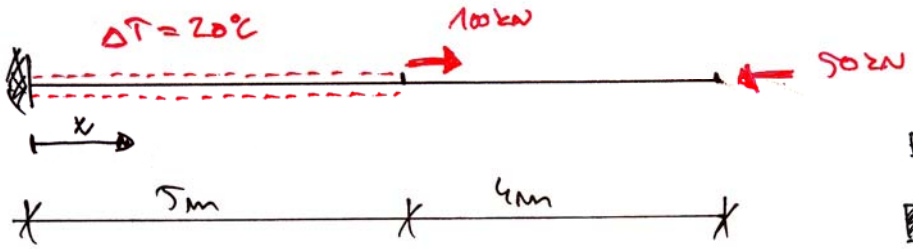
$$\varepsilon(x) = \frac{-F}{EA} + \alpha_T \Delta T = konst = \frac{-200}{30 \cdot 10^6 \cdot 0{,}2} + 12 \cdot 10^{-6} \cdot (-20) = -3{,}33 \cdot 10^{-5} + (-24 \cdot 10^{-4}) = -0{,}00027$$

$$\sigma(x) = E \cdot (\varepsilon - \alpha \Delta T) = -\frac{F}{A} = 1\ MPa$$

→ vyznam integračních konstant

Určit posuny, napětí, deformace

PŘ 1



$E = 30\ GPa$

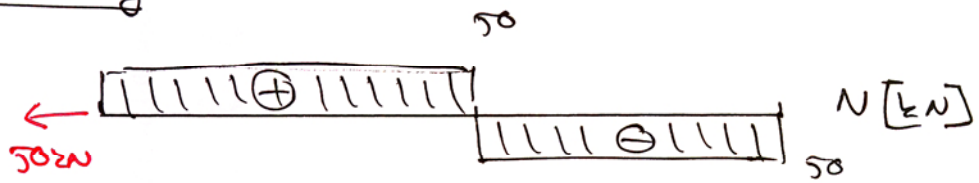
$b = 0{,}4\ m$

$h = 0{,}6\ m$

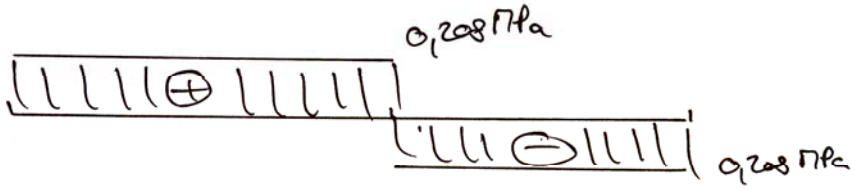
$A = 0{,}24\ m^2$

$\alpha = 12 \cdot 10^{-6}\ K^{-1}$

Vnitřní síly:

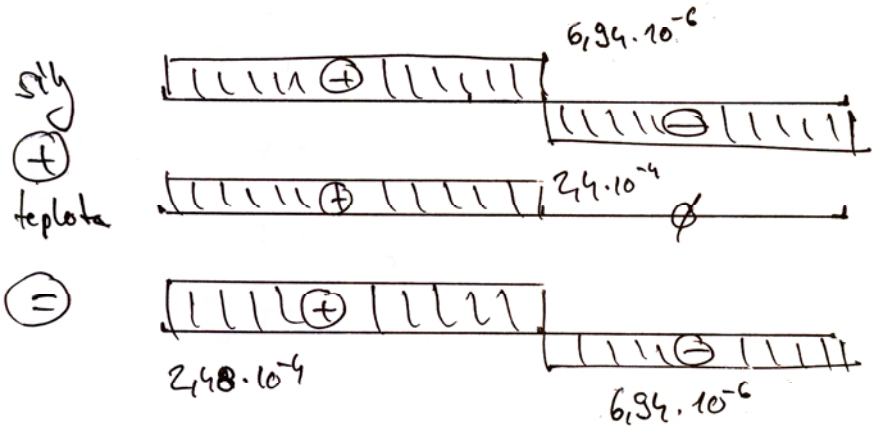


napětí: ... statické rovnice $\sigma = \frac{F}{A} = \frac{50\ kN}{0{,}24\ m^2} = 208{,}333\ kPa \doteq 0{,}208\ MPa$



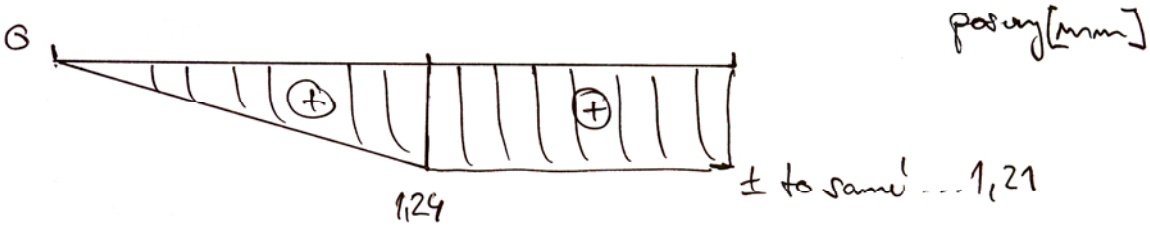
deformace: ... konstitutivní rovnice

$\varepsilon^t = \frac{\sigma}{E} = \frac{0{,}208\ MPa}{30\ GPa} = 6{,}94 \cdot 10^{-6}$



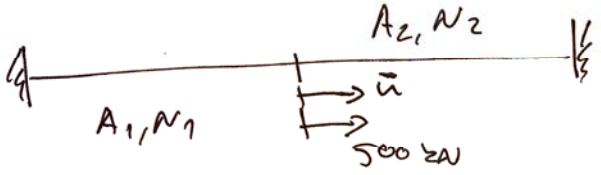
$\varepsilon^t = \alpha_T \cdot \Delta T = 12 \cdot 10^{-6} \cdot 20 = 0{,}000\,24 = 2{,}4 \cdot 10^{-4}$

posuny $u(x) = \int \varepsilon(x)\,dx = \varepsilon(x) \cdot x + C$



! $-6{,}94 \cdot 10^{-6} \cdot 4\ m = -0{,}0278\ mm$

Př 2



$E = 30\,GPa$

$A_1 = 0{,}2\,m^2$

$A_2 = 0{,}1\,m^2$

→ určit posunutí, napětí, deformaci

→ z diferenciální rovnice

$$\frac{d}{dx}\left(EA\frac{du}{dx}\right) = -f_x = 0$$

$$EA\frac{du}{dx} = C_1$$

$$u = \frac{C_1 x}{EA} + C_2$$

- 1. interval:

OKRAJOVÉ PODMÍNKY: $u(0) = 0$ $\quad u(4) = \bar{u}$

$\downarrow$

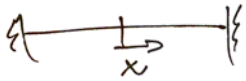
$C_2 = 0\,m$

$\downarrow \bar{u} = \frac{C_1 \cdot 4}{EA_2} \Rightarrow C_1 = \frac{\bar{u}\,EA_1}{4} = 330\,kN$

$$\rightarrow u(x) = \frac{\bar{u}x}{4};\quad \varepsilon(x) = \frac{\bar{u}}{4};\quad \sigma(x) = \frac{E\bar{u}}{4};\quad N_1 = \frac{EA_1\bar{u}}{4}$$

- 2. interval

Zavedu lokální souřadný systém



(1) $u(0) = \bar{u}$

(2) $u(4) = 0$

$$u(x) = \frac{C_3 x}{EA_2} + C_4$$

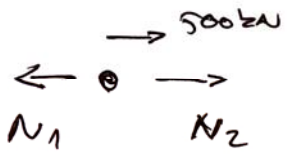
z (1) $\Rightarrow C_4 = \bar{u}\,[m] = 0{,}222\,mm$

z (2) $\Rightarrow 0 = \frac{4 \cdot C_3}{EA_2} + \bar{u} \Rightarrow C_3 = -\frac{\bar{u}\,EA_2}{4} = -166{,}5\,kN$

$$\rightarrow u(x) = -\frac{\bar{u}x}{4} + \bar{u}$$

$$\varepsilon(x) = -\frac{\bar{u}}{4};\quad \sigma(x) = \frac{-E\bar{u}}{4};\quad N_2 = \frac{-EA_2\bar{u}}{4}$$

- mutná rovnováha ve styčníku



$\rightarrow: \quad 500 + N_2 = N_1$

$$500 + \frac{-EA_2\,\bar{u}}{4} = \frac{EA_1\,\bar{u}}{4}$$

$$2000 = \bar{u}(EA_1 + EA_2) \rightarrow \bar{u} = \frac{2000}{EA_1 + EA_2} = \frac{2000\ kN}{30000 \cdot 0{,}2 + 30000 \cdot 0{,}1} =$$

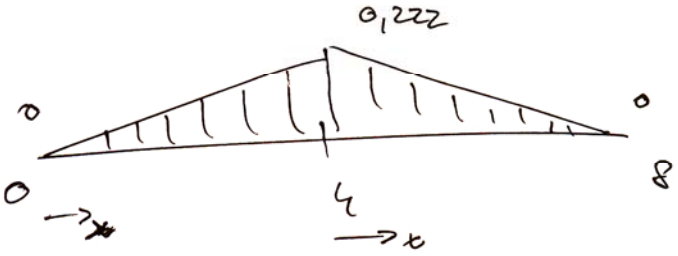
$$= 0{,}222\ mm$$

- 1. interval

POSUNY: $u_1(x) = \frac{0{,}222 \cdot x}{4}$ [mm]

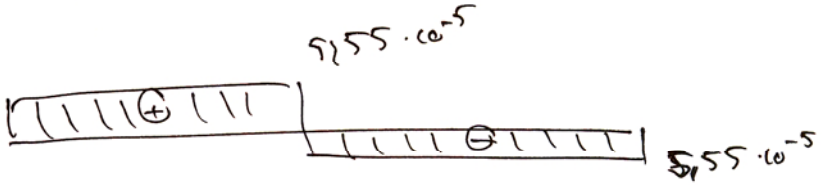
- 2. interval

$u_2(x) = 0{,}222 - \frac{0{,}222 \cdot x}{4}$



DEFORMACE

$$\varepsilon_1(x) = \frac{\bar{u}}{4} = \frac{0{,}222 \cdot 10^{-3}}{4} = 5{,}55 \cdot 10^{-5} \qquad \varepsilon_2 = -\frac{\bar{u}}{4} = -5{,}55 \cdot 10^{-5}$$



NAPĚTÍ

$$\sigma_1(x) = \frac{E\bar{u}}{4} = \frac{30\,000 \cdot 0{,}222 \cdot 10^{-3}}{4} = 1{,}665\ MPa \qquad \sigma_2 = \frac{-E\bar{u}}{4} = -1{,}665\ MPa$$

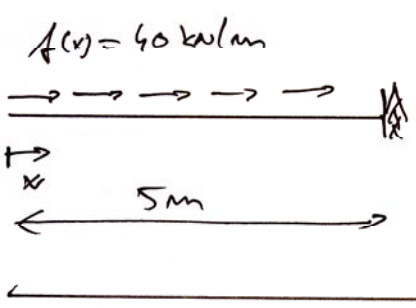
SÍLY

$$N_1 = A_1\sigma_1 = 0{,}2 \cdot 1{,}665 \cdot 10^3\ kN = 333\ kN \qquad N_2 = A_2\sigma_2 = 0{,}1 \cdot \left(-1{,}665 \cdot 10^{+3}\right) kN =$$

$$= -166{,}5\ kN$$

kontrola:

Nosník zatížený spojitým osovým zatížením



OKRAJOVÉ PODMÍNKY

(1) $u(5) = 0$ m (kinematická) (2) $N(0) = 0$ kN (statická o.p.)

→ diferenciální rovnice

$$\frac{d}{dx}\left[EA\left(\frac{du}{dx}\right)\right] = -f_x$$

$$EA\frac{du(x)}{dx} = -f_x(x)\cdot x + C_1$$

$$u(x) = -\frac{f_x(x)x^2}{EA\cdot 2} + C_1 x/EA + C_2$$

→ z (1)

$$0 = \frac{-f_x(x)5^2}{EA\cdot 2} + \frac{5C_1}{EA} + C_2$$

→ z (2)

$$EA\frac{du(0)}{dx} = 0$$

$$0 = 0\cdot f_x(x) + C_1 \Rightarrow C_1 = 0 \text{ kN}$$

$$0 = \frac{-f_x(x)5^2}{EA\cdot 2} + \frac{5C_1}{EA} + C_2 \Rightarrow C_2 = \frac{f_x(x)\cdot 5^2}{2EA} = \frac{40\cdot 10^3\cdot 5^2}{2\cdot 30\cdot 10^9\cdot 0{,}4}\frac{N\cdot m}{N} =$$

$$= 0{,}0000417\text{ m} = 0{,}0417\text{ mm}$$

$$u(x) = \frac{-40\cdot 10^3 x^2}{2\cdot 30\cdot 10^9\cdot 0{,}4} + 0{,}0417\cdot 10^{-3}\ [m]$$

$$u(x) = -1{,}667\cdot 10^{-6}x^2 + 4{,}167\cdot 10^{-5}\ [m]$$

$$\varepsilon(x) = \frac{-f_x(x)\cdot x}{EA} = \frac{-40\cdot 10^3\cdot x}{30\cdot 10^9\cdot 0{,}4} = -3{,}33\cdot 10^{-6}x$$

$$\sigma(x) = -\frac{f_x(x)\cdot x}{A} = \frac{-40\cdot 10^3\cdot x}{A} = 100000\cdot x\ [Pa] = 0{,}1x\ [MPa]$$

$$N = -f_x(x)\cdot x = -40\cdot 10^3 x\ [N] = -40x\ [kN]$$

Vyřešeno!: