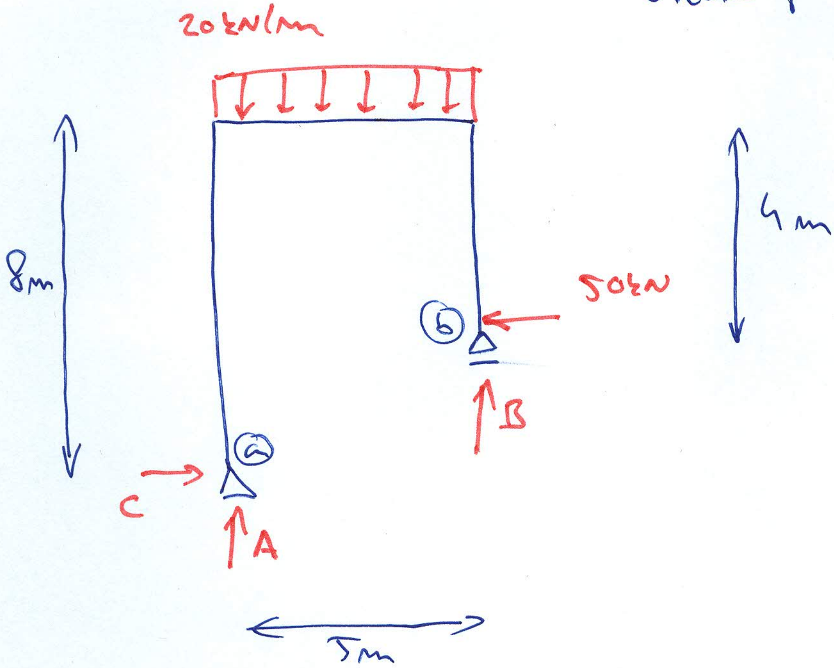
Příklad na rozehřátí:

Určete průběhy M, N, V po konstrukci!



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1) REAKCE

C = 50 kN

$A + B = 5m \cdot 20 kN/m = 100 kN$

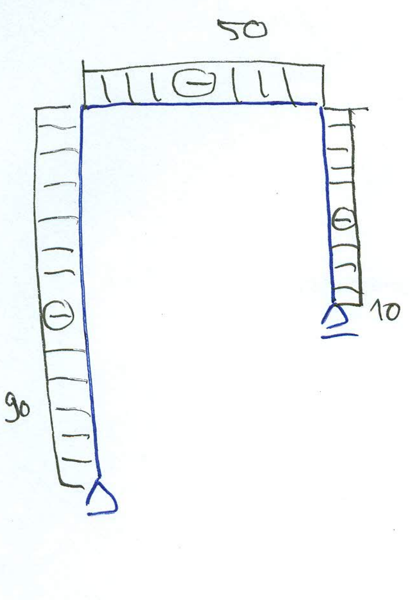
$\Sigma a: \; 5 \cdot B + 50 \cdot 4 - 100 \cdot 2{,}5 = 0 \;\Rightarrow\; B = \dfrac{250 - 200}{5} = 10 \text{ kN}$

$\Rightarrow A = 90 \text{ kN}$

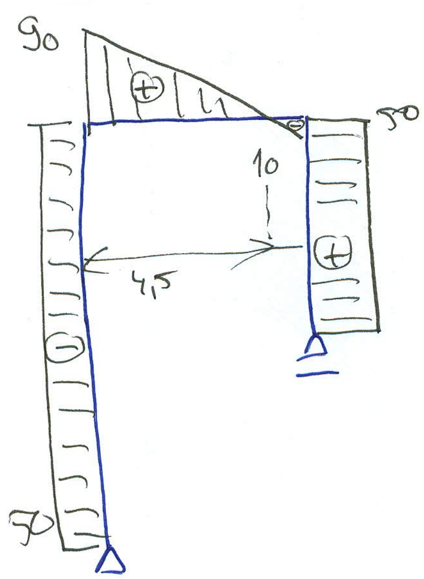
- vyzkoušet pomocí superpozice

$-400 + 90 \cdot 4{,}5 / 2 = -197{,}5 \text{ kNm}$

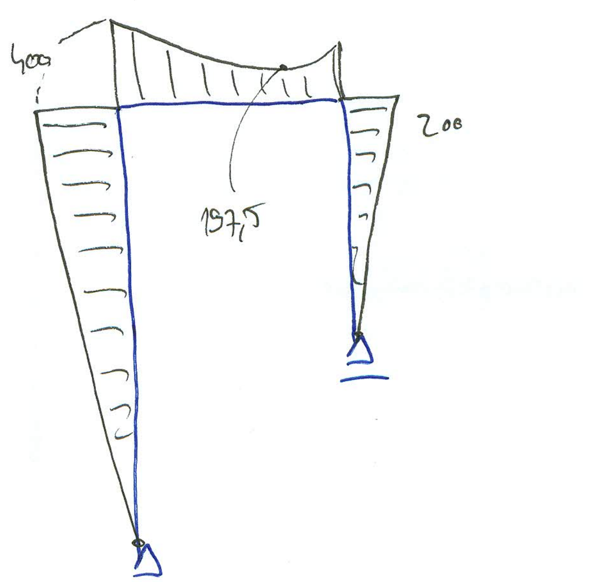
2) PRŮBĚHY VNITŘNÍCH SIL



N[kN]

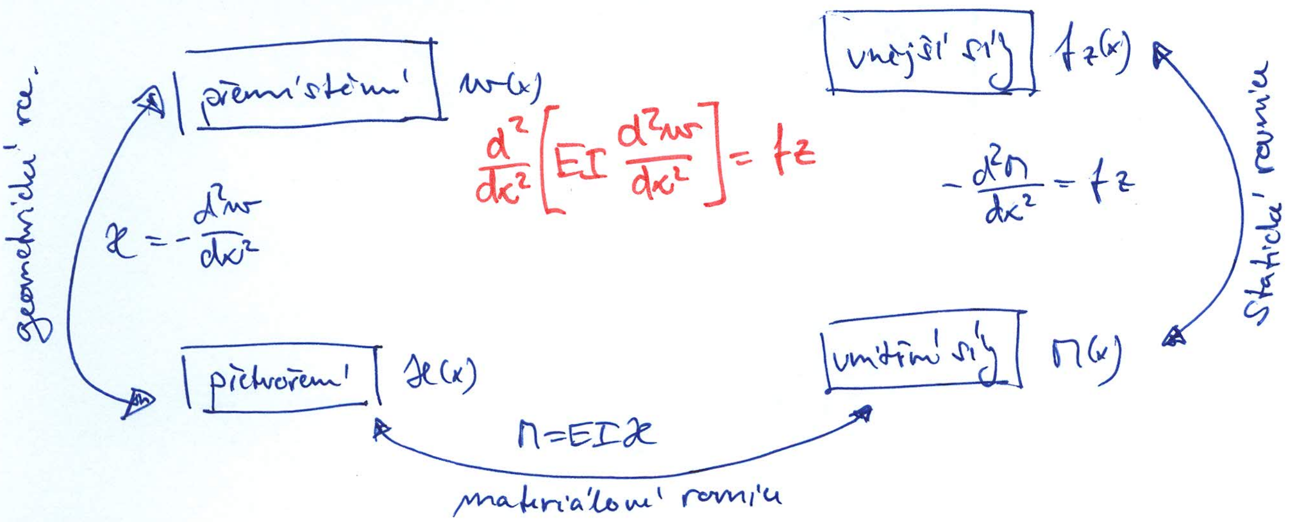


V[kN]



M[kNm]

Základní vztahy a rovnice pro prut namáhaný ohybem

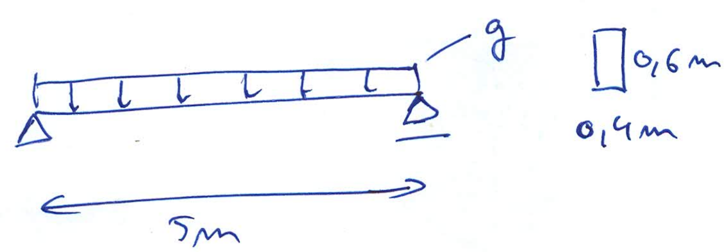


...dále platí $\kappa(x) = \frac{d\varphi(x)}{dx}$ $\qquad \varphi(x) = -\frac{dw}{dx}$!

+Schwedlerovy vztahy $\frac{dQ}{dx} + f_z(x) = 0$

$$\frac{dM}{dx} - Q(x) = 0$$

Příklad: Vypočtěte maximální průhyb nosníku zatíženého vlastní tíhou a vypočtěte maximální normálová napětí. Načrtněte funkce $w(x)$; $w'(x)$; $w''(x)$; $w'''(x)$



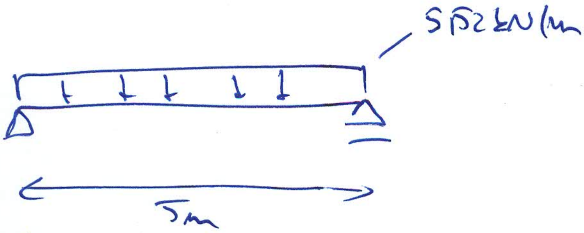
$E = 30\,GPa$

$\rho = 23\,kN/m^3$

Průřezové charakteristiky: $A = 0{,}4\,m \cdot 0{,}6\,m = 0{,}24\,m^2$

$$I = \frac{1}{12} \cdot 0{,}4 \cdot 0{,}6^3 = 0{,}0072\,m^4$$

Liniové zatížení $\qquad g = A \cdot \rho = 0{,}24 \cdot 23 = 5{,}52\,kN/m$



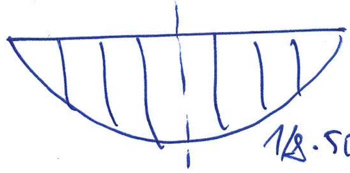
VN. SÍLY

V[kN]

$\boxed{M(x) =}\ 13{,}8 \cdot x - \frac{5{,}52 \cdot x^2}{2} = \boxed{13{,}8x - 2{,}76x^2}$

... v kNm

M[kNm]



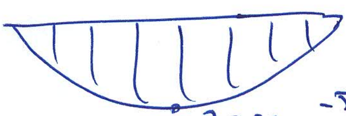
$1/8 \cdot 5{,}52 \cdot 5^2 = 17{,}25$ kNm

KŘIVOST

$M = EI\varkappa \Rightarrow \varkappa = \frac{M}{EI} = \frac{M(x)}{\underbrace{30 \cdot 10^9 \cdot 0{,}0072}_{2{,}16 \cdot 10^8\ Nm^2}}$

$\varkappa = \frac{13{,}8 \cdot x - 2{,}76x^2}{2{,}16 \cdot 10^5} = \boxed{6{,}389 \cdot 10^{-5} x - 1{,}278 \cdot 10^{-5} x^2}$

$\varkappa$[1/m]



$7{,}986 \cdot 10^{-5}$

NATOČENÍ

$\varkappa = \frac{d\varphi(x)}{dx} \Rightarrow \varphi = \int \varkappa(x)\,dx = -\frac{1{,}278 \cdot 10^{-5} x^3}{3} + \frac{6{,}389 \cdot 10^{-5} x^2}{2} + C_1$

konstrukce je symetrická $\Rightarrow \varphi(2{,}5) = 0$

$\Rightarrow 0 = \frac{-1{,}278 \cdot 10^{-5} \cdot 2{,}5^3}{3} + \frac{6{,}389 \cdot 10^{-5} \cdot 2{,}5^2}{2} + C_1$

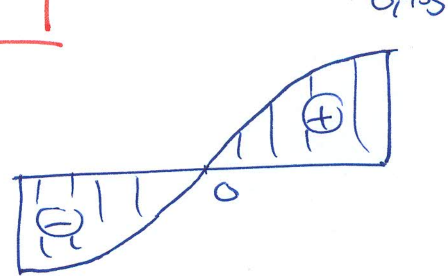
$\Rightarrow C_1 = -1{,}3309 \cdot 10^{-4}$

$\Rightarrow \boxed{\varphi = -\frac{1{,}278 \cdot 10^{-5} x^3}{3} + \frac{6{,}389 \cdot 10^{-5} x^2}{2} - 1{,}3309 \cdot 10^{-4}}$

$\varphi(0) = -1{,}3309 \cdot 10^{-4}$ rad $= -0{,}13309$ mrad

$\varphi(5) = 0{,}000133$ rad $= 0{,}133$ mrad

$\varphi(x)$ [mrad]

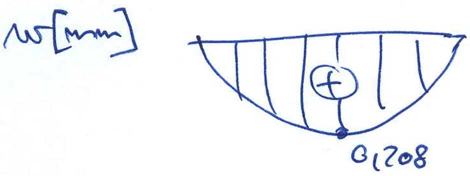


PRŮHYB

$$\boxed{w(x) = -\int \varphi(x)\,dx = +\frac{+1{,}278\cdot10^{-5}x^4}{12} - \frac{6{,}385\cdot10^{-5}x^3}{6} + 1{,}3309\cdot10^{-4}\cdot x + \cancel{C_2}^{\,0}}$$

$w(2{,}5) = 2{,}079\cdot10^{-4}\ m = 0{,}208\ mm$

kontrola: $w(5) = 3{,}33\cdot10^{-8}\ m \cong 0$

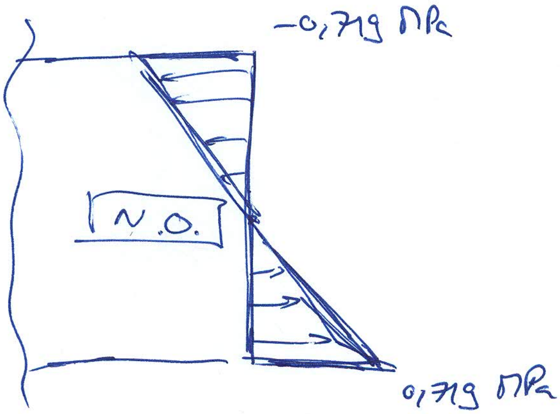


→ změnit 2. možnost pro 2 integrační konstanty a okrajové podmínky $w(0)=0$ + $w(L)=0$

pokud by se počítalo obecně ... $w\left(\frac{L}{2}\right) = \frac{5}{384}\,\frac{qL^4}{EI}$

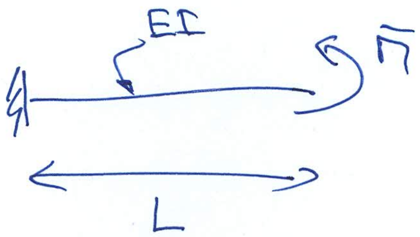
NORMÁLOVÉ NAPĚTÍ

$$\sigma_x(x) = \frac{M}{I}\cdot z = \frac{17{,}25\cdot10^3}{0{,}0072} \begin{cases} \cdot(-0{,}3) = -0{,}719\ MPa \\ \cdot(0{,}3) = +0{,}719\ MPa \end{cases}$$



Prpe, cv. 3, str. 4,5

Vypočtěte maximální natočení a průhyb!



$EI = 300\ MNm^2$

$L = 8\ m$

$M = 150\ kNm$

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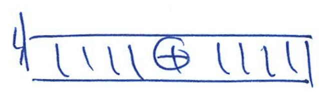
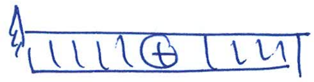
průběh momentů:



$M(x) = \bar{M}$

křivost:

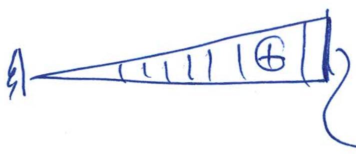


$\varkappa(x) = \dfrac{\bar{M}}{EI}$

natočení

$\varphi(0) = 0$

$\varkappa = \dfrac{d\varphi}{dx}$

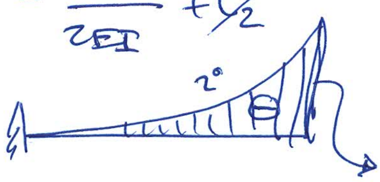


$\dfrac{\bar{M}}{EI} \cdot L$

$\varphi(x) = \dfrac{\bar{M}}{EI} \cdot x$

průhyb

$$w = -\int \varphi(x)\,dx = -\frac{\bar{M}x^2}{2EI} + \cancel{C_2} \to 0$$

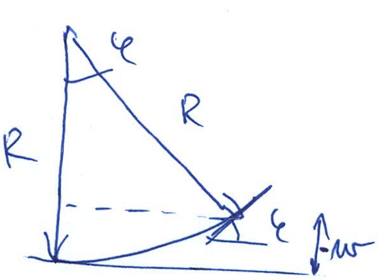


$-\dfrac{\bar{M}L^2}{2EI}$

$$\max.\ \varphi = \frac{\bar{M}}{EI} \cdot L = \frac{150 \cdot 10^3}{300 \cdot 10^6} \cdot 8 = 0{,}004\ rad = 4\ mrad$$

$$\max\ w = -\frac{\bar{M}L^2}{2EI} = \frac{-150 \cdot 10^3 \cdot 8^2}{2 \cdot 300 \cdot 10^6} = -0{,}016\ m = -16\ mm$$

Jinak: geometrická interpretace

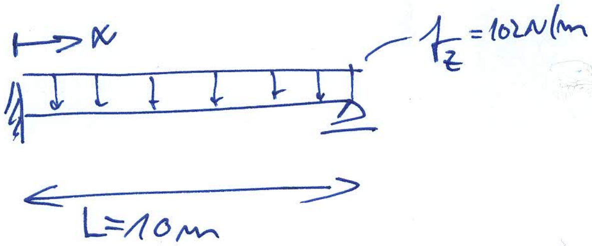


$R = \dfrac{1}{\varkappa} = 2000\ m \qquad \varphi = \dfrac{360}{2\pi r} \cdot L = 0{,}229183° = 0{,}004\ rad \left(\dfrac{L}{r}\right)$

$-w = R(1 - \cos\varphi) = 0{,}016\ m$

Příklad 2:

Na dané konstrukci určete maximální průhyb, natočení, křivost, moment, posouvající sílu



$E = 30$ GPa

$I = 0{,}01$ m^4

Okrajové podmínky:

$w(0) = 0$ 1) $\quad w(L) = 0$ 3) — kinematické o.p.

$w'(0) = 0$ 2) $\quad \boxed{M(L) = 0}$ — statické o.p.

Dif. rovnice 4. řádu → 4 o.p.

$$\frac{d^2}{dx^2}\left[EI\,\frac{d^2 w}{dx^2}\right] = f_z$$

(protože je konstrukce S.N., musíme vyjít z diferenciální rovnice ohybové čáry)

!

$$-M = EI\frac{d^2 w}{dx^2} = \frac{f x^2}{2} + C_1 x + C_2$$

$$M(L) = 0 = \frac{fL^2}{2} + C_1 L + C_2 \rightarrow C_2 = -\frac{fL^2}{2} - C_1 L$$

$$\Rightarrow EI\frac{d^2 w}{dx^2} = \frac{fx^2}{2} + C_1 x + \left(-\frac{fL^2}{2} - C_1 L\right)$$ .... o 1 int. konstantu méně !

NATOČENÍ

$$\frac{dw(x)}{dx} = \frac{1}{EI}\left[\frac{fx^3}{6} + C_1\frac{x^2}{2} - \frac{fL^2 x}{2} - C_1 L x + C_3\right]$$

ad 2) $$-\varphi(0) = \frac{dw(0)}{dx} = 0 = \frac{1}{EI}\left[\cancel{\frac{f0^3}{6}} + \cancel{C_1\frac{0^2}{2}} - \cancel{\frac{fL^2\cdot 0}{2}} - \cancel{C_1 L\cdot 0} + C_3\right]$$

$C_3 = 0$

$$\frac{dw}{dx} = \frac{1}{EI}\left[\frac{fx^3}{6} + \frac{C_1 x^2}{2} - \frac{fL^2 x}{2} - C_1 L x\right]$$

PRŮHYB

$$w(x) = \frac{1}{EI}\left[\frac{f x^4}{24} + \frac{C_1 x^3}{6} - \frac{f L^2 x^2}{4} - C_1 L \frac{x^2}{2} + C_4\right]$$

ad 1) $w(0)=0 \rightarrow C_4 = 0$

ad 3) $w(L)=0 \rightarrow \frac{fL^4}{24} + \frac{C_1 L^3}{6} - \frac{fL^4}{4} - C_1\frac{L^3}{2} = 0$

$$\Rightarrow C_1 = \frac{\frac{fL^4}{4} - \frac{fL^4}{24}}{\frac{L^3}{6} - \frac{L^3}{2}} = \frac{\frac{5}{24} fL^4}{-\frac{1}{3}L^3} = -\frac{5}{8} fL$$

... po dosazení C_1

→ průběh momentů

$$M(x) = -\frac{fx^2}{2} + \frac{5}{8} fLx + \frac{fL^2}{2} - \frac{5}{8} fL^2 = -\frac{fx^2}{2} + \frac{5}{8} fLx - \frac{1}{8} fL^2$$

$M(0) = -\frac{1}{8} fL^2 \qquad M(L) = 0 \ldots$ ok

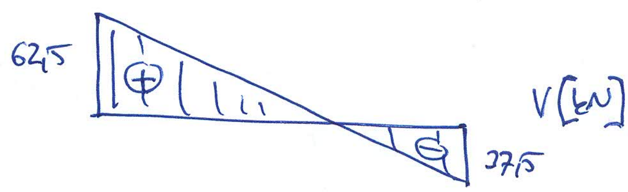
$M_{max} \ldots$ pro $V(x) = 0 = -fx + \frac{5}{8} fL = 0 \Rightarrow x = \frac{5}{8} L$

$$M_{max}\left(\frac{5}{8}L\right) = -f \cdot \frac{25}{2 \cdot 64} \cdot L^2 + \frac{5}{8} \cdot fL^2 \cdot \frac{5}{8} - \frac{1}{8} fL^2 =$$

$$= fL^2\left(-\frac{25}{128} + \frac{25}{64} - \frac{1}{8}\right) = +\frac{9}{128} fL^2$$

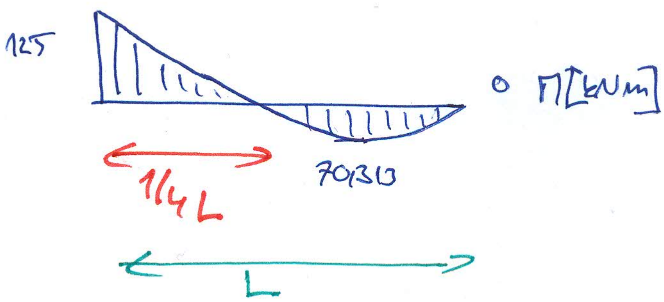
$V(0) = \frac{5}{8} fL = \frac{5}{8} \cdot 10 \cdot 10 = 62{,}5$ kN

$V(L) = -fL + \frac{5}{8} fL = -37{,}5$ kN



$M(0) = -\frac{1}{8} fL^2 = -125$ kNm

$M_{max} = \frac{9}{128} fL^2 \doteq 70{,}313$ kNm



$$M(x) = 0 \equiv -\frac{fx^2}{2} + \frac{5}{8} fLx - \frac{1}{8} fL^2 = 0$$

↳ pro extrémní hodnoty natočení

$$x_{1,2} = \frac{-\frac{5}{8} fL \pm \sqrt{\left(\frac{5}{8} fL\right)^2 - 4 \cdot \frac{f}{2} \cdot \frac{1}{8} fL^2}}{-f} = \frac{-\frac{5}{8} fL \pm \sqrt{\frac{9}{64} f^2 L^2}}{-f} =$$

$$= \frac{\frac{5}{8} fL \pm \frac{3}{8} fL}{f}$$

$x_1 = L$

$x_2 = \frac{1}{4} L$

Průběh natočení a průhybu

$$w(x) = \frac{1}{EI}\left[f\frac{x^4}{24} + \frac{-\frac{5}{8} f L x^3}{6} - \frac{f L^2 x^2}{4} + \frac{5}{8} f L^2 \frac{x^2}{2} \right] =$$

$$= \frac{f x^2}{2EI}\left[\frac{x^2}{12} - \frac{5Lx}{24} + \frac{1}{8}L^2 \right]$$

$$\varphi(x) = -\frac{dw}{dx} = \frac{f}{EI}\left[-\frac{x^3}{6} + \frac{5Lx^2}{16} - \frac{xL^2}{8} \right]$$

Extrémní hodnota natočení φ

$$\varphi_1 = \frac{10\cdot 10^3}{30\cdot 10^9 \cdot 0{,}01} \underbrace{\left[-\frac{L^3}{6} + \frac{5L^3}{16} - \frac{L^3}{8} \right]}_{\frac{1}{48}L^3} = \underline{\underline{0{,}694 \text{ mrad}}} \qquad (x = L)$$

$$\varphi_2 = \frac{10\cdot 10^3}{30\cdot 10^9 \cdot 0{,}01} \cdot \underbrace{\left[-\frac{\left(\frac{1}{4}L\right)^3}{6} + \frac{5L^3\cdot\frac{1}{16}}{16} - \frac{\frac{1}{4}L^3}{8} \right]}_{-\frac{11}{768}L^3} = \underline{\underline{0{,}477 \text{ mrad}}} \qquad \left(x = \frac{L}{4}\right)$$

Extrémní průhyb

$$\varphi(x) = -\frac{f x}{2EI} \underbrace{\left[+\frac{x^2}{3} - \frac{5Lx}{8} + \frac{L^2}{4} \right]}$$

1. kořen $x = 0$ … vetknutí

$$x_{12} = \frac{\frac{5L}{8} \pm \sqrt{\frac{25L^2}{64} - \frac{1}{3}L^2}}{\frac{2}{3}} = \frac{\frac{5L}{8} \pm L\frac{\sqrt{33}}{24}}{\frac{2}{3}}$$

12,9654 m → mimo interval

$\boxed{5{,}78465 \text{ m}}$

$$w(5{,}78465) = 0{,}001805 \text{ m} = \underline{\underline{1{,}805 \text{ mm}}}$$