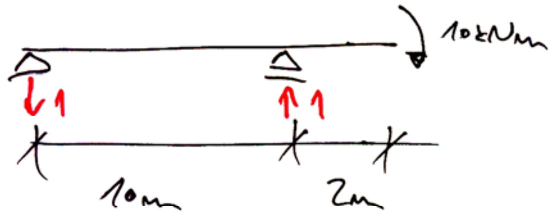
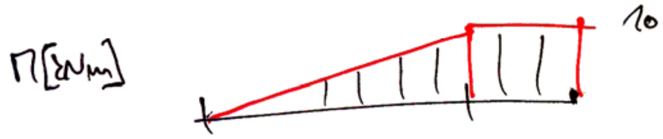
Pro daný nosník vypočtěte natočení nad pravou podporou



$EI = 40\,000\ kNm^2$



$M(x) = -x$ $\qquad$ $M(x) = -10$

$$M = EI\varkappa = EI\cdot(-w'')$$

$$\Rightarrow w'' = -\frac{M}{EI} = \frac{x}{EI}$$

$$w' = \frac{x^2}{2EI} + C_1$$

$$w = \frac{x^3}{6EI} + C_1 x + C_2$$

$$w(0) = 0 \Rightarrow C_2 = 0$$

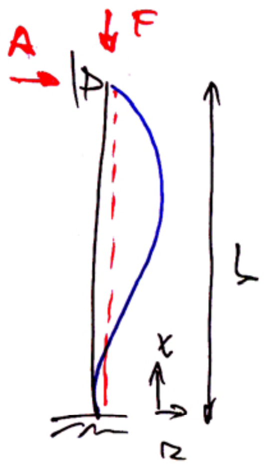
$$w(10) = 0 \Rightarrow 0 = \frac{10^3}{6\cdot 40000} + C_1\cdot 10$$

$$\Rightarrow C_1 = -\frac{1}{2400} \doteq -4{,}1667\cdot 10^{-4}$$

$$\Rightarrow w' = \frac{x^2}{2\cdot 40000} - 4{,}1667\cdot 10^{-4}$$

$$\varphi(10) = -w'(10) = -\left(\frac{10^2}{2\cdot 40000} - 4{,}1667\cdot 10^{-4}\right) = -8{,}333\cdot 10^{-4}\ \text{rad}$$

Geometrickou metodou určete kritické břemeno F_{cr}



$$M(x) = -A(L-x) + F \cdot w = EI\kappa = -EI\,w''$$

$$\rightarrow w'' + \frac{F}{EI} w = \frac{A(L-x)}{EI}$$

$$\alpha^2 = \frac{F}{EI}$$

$$\Rightarrow w'' + \alpha^2 w = \frac{A(L-x)}{EI}$$

- Homogenní část – řešení ve tvaru:

$$w_h = C_1 \sin \alpha x + C_2 \cos \alpha x$$

- Partikulární řešení

$$w_p = C_3 x + C_4$$

$$w_p'' = 0$$

$$0 + \frac{F}{EI}(C_3 x + C_4) = \frac{A(L-x)}{EI}$$

$$\frac{F C_3 x}{EI} + \frac{F C_4}{EI} = \frac{AL}{EI} - \frac{Ax}{EI}$$

$$\Rightarrow C_3 = -\frac{A}{F}$$

$$\Rightarrow C_4 = \frac{AL}{F}$$

$$\Rightarrow w_p = \frac{A}{F}(L-x)$$

$$w = w_h + w_p = C_1 \cos \alpha x + C_2 \sin \alpha x + \frac{A}{F}(L-x)$$

$$w' = -C_1 \alpha \sin \alpha x + C_2 \alpha \cos \alpha x - \frac{A}{F}$$

$$w'' = -C_1 \alpha^2 \cos \alpha x - C_2 \alpha^2 \sin \alpha x$$

Podmínky: $x = 0$ ... $w(0) = 0 = C_1 + \frac{A}{F} \cdot L \Rightarrow C_1 = -\frac{A}{F} \cdot L$

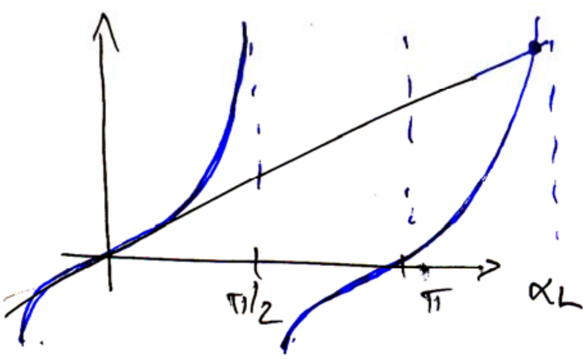
$w'(0) = 0 = C_2 \alpha - \frac{A}{F} \Rightarrow C_2 = \frac{A}{F \cdot \alpha}$

$x = L$ ... $w(L) = 0 = -\frac{A}{F} L \cos \alpha L + \frac{A}{F\alpha} \sin \alpha L = 0$

$$\rightarrow \sin \alpha L = \cos(\alpha L) \cdot \alpha L$$

$$\operatorname{tg} \alpha L = \alpha L$$

$$\alpha L \doteq 4{,}494$$



$$\alpha^2 = \frac{F_{cr}}{EI} \Rightarrow F_{cr} = \alpha^2 EI = \frac{\alpha^2 L^2}{L^2} EI = \frac{4{,}494^2 EI}{L^2} = \frac{EI\pi^2}{L_{cr}^2}$$

$$\boxed{L_{cr} = \frac{\pi \cdot L}{4{,}494} \doteq 0{,}699 L}$$

$$w = -\frac{A}{F} L \cos\alpha x + \frac{A}{F\alpha}\sin\alpha x + \frac{A}{F}(L-x)$$

$$w = \frac{A}{F}\left(\frac{\sin\alpha x}{\alpha} - L\cos\alpha x + L - x\right)$$, kde $\alpha L = 4{,}49$

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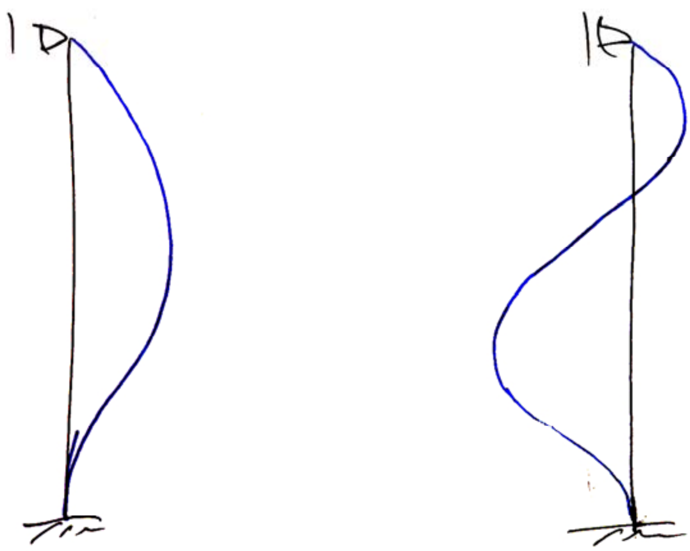
další $\alpha L = 7{,}728$

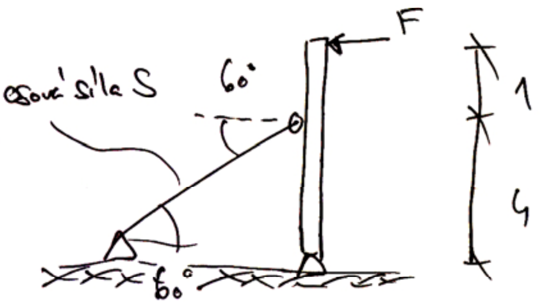


$$\frac{4{,}494^2 EI}{L^2} = \frac{20{,}196\, EI}{L}$$

$$\frac{7{,}728^2 EI}{L^2} = \frac{59{,}722\, EI}{L}$$

… 2,96 × více

Pro danou konstrukci určete kritickou velikost zatížení



Momentová podmínka rovnováhy

$5 \cdot F = 4 \cdot S \cdot \cos 60°$

$\Rightarrow S = 2{,}5 F$

$S_{krit} = EI \frac{\pi^2}{L^2}$

$I = I_{min} = I_z = 11{,}4 \cdot 10^{-6}\, m^4$ $E = 210\ GPa$

$L = \frac{4}{\sin 60} = 4{,}6188\, m$ $A = 0{,}0118\, m^2$

- $S_{krit} = 210 \cdot 10^9 \cdot 11{,}4 \cdot 10^{-6} \cdot \frac{\pi^2}{4{,}6188^2} = 1{,}10756 \cdot 10^6\, N = 1{,}10756\, MN$
- $F_{krit} = S_{krit} / 2{,}5 = \frac{1{,}10756}{2{,}5} = \underline{\underline{0{,}443\, MN}}$
- $\sigma_{krit} = \frac{S_{krit}}{A} = \frac{1{,}10756}{0{,}0118} = 93{,}861\, MPa$
- štíhlost $\lambda = \frac{L}{i}$ $i = \sqrt{\frac{I}{A}} = \sqrt{\frac{11{,}4 \cdot 10^{-6}}{0{,}0118}} = 0{,}03108\, m$

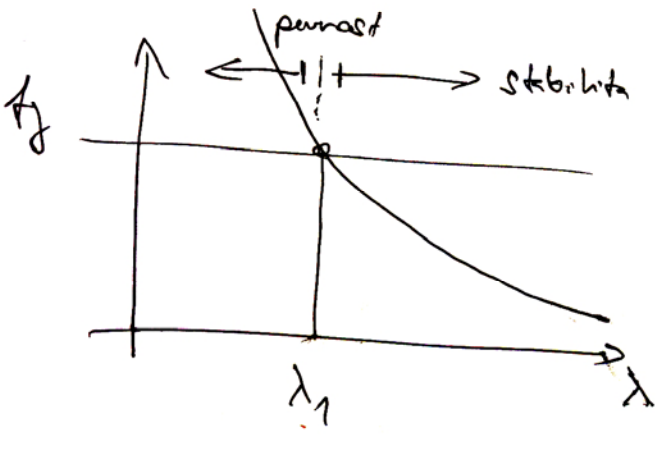
$\lambda = \frac{4{,}6188}{0{,}03108} = 148{,}61$

$\lambda_1 = \pi \sqrt{\frac{210 \cdot 10^3}{235}} = 93{,}913$

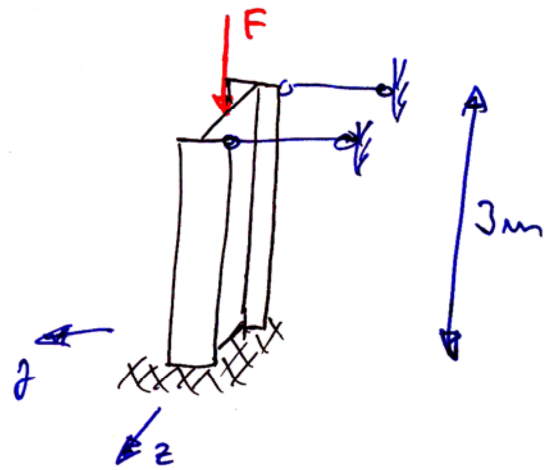
$\lambda > \lambda_1$ ---> rozhoduje stabilita

$\sigma_{krit} = \frac{N_{krit}}{A} = \frac{EI\pi^2}{L^2 \cdot A} =$

$= \frac{Ei^2\pi^2}{L^2} = \frac{E\pi^2}{\lambda^2} \rightarrow \lambda_1 = \pi \sqrt{\frac{E}{\sigma_{krit}}}$



Vypočtěte kritické zatížení ocelového profilu zatíženého podle obrázku.



I 400: $I_y = 291 \cdot 10^{-6}\ m^4$

$I_z = 11{,}4 \cdot 10^{-6}\ m^4$

$A = 11{,}8 \cdot 10^{-3}\ m^2$

$E = 210\ GPa$

$\boxed{S235}$

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1) vybočení ve směru osy y $\qquad L_y = 0{,}7 \cdot 3{,}0 = 2{,}1\ m$

$$F_{krit}^{y} = 210 \cdot 10^{3} \cdot 11{,}4 \cdot 10^{-6} \cdot \frac{\pi^2}{2{,}1^2} = \boxed{5{,}36\ MN}$$

$$i_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{11{,}4 \cdot 10^{-6}}{11{,}8 \cdot 10^{-3}}} = 0{,}0311\ m$$

$$\lambda_z = \frac{L_y}{i_z} = \frac{2{,}1}{0{,}0311} = \boxed{67{,}56}$$

2) vybočení ve směru osy z $\qquad L_z = 2 \cdot 3\ m = 6\ m$

$$F_{krit}^{z} = 210 \cdot 10^{3} \cdot 291 \cdot 10^{-6} \cdot \frac{\pi^2}{6^2} = \boxed{16{,}75\ MN}$$

$$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{291{,}0 \cdot 10^{-6}}{11{,}8 \cdot 10^{-3}}} = 0{,}157\ m$$

$$\lambda_y = \frac{L_z}{i_y} = \frac{6}{0{,}157} = \boxed{38{,}216}$$

3) tlak

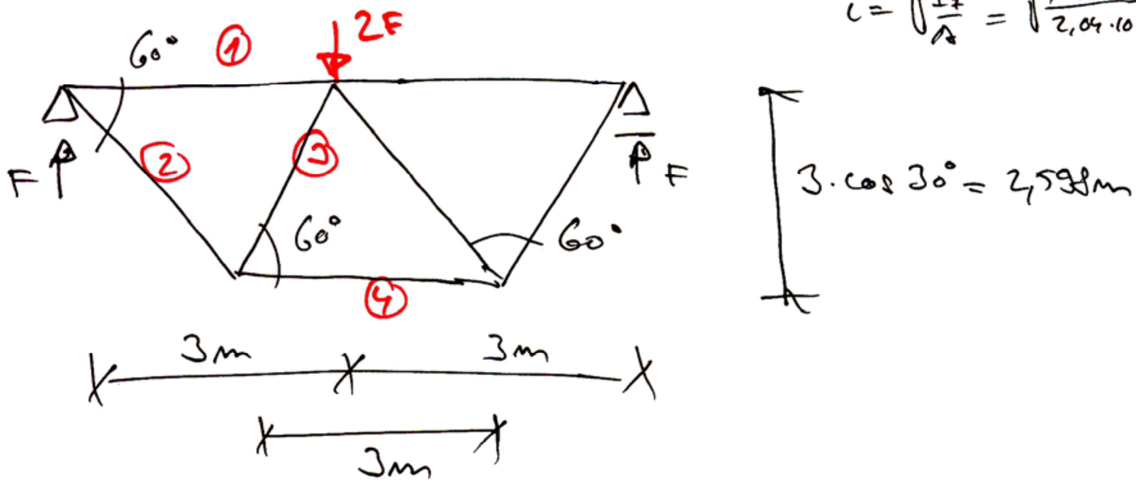
$$F_{max} = A \cdot f_y = 11{,}8 \cdot 10^{-3} \cdot 235 = \boxed{2{,}773\ MN}$$

$$\lambda_1 = \pi \sqrt{\frac{E}{\sigma_{kluz}}} = \pi \sqrt{\frac{210 \cdot 10^{3}}{235}} = 93{,}913$$

Určete kritické zatížení příhradové konstrukce. Pruty jsou stejné;

$[\ 140;\ I_z = 0{,}625 \cdot 10^{-6}\,m^4;\ A = 2{,}04 \cdot 10^{-3}\,m^2;\ E = 210\ GPa$

$i = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{0{,}625}{2{,}04 \cdot 10^{3}}} = 0{,}0175\,m$



Výpočet osových sil:

$\uparrow: \quad F - S_2 \cdot \cos 30° = 0 \quad \Rightarrow \quad S_2 = 1{,}1547\,F$

$\rightarrow: \quad S_1 + S_2 \cdot \cos 60° = 0 \quad \Rightarrow \quad S_1 = -1{,}1547 \cdot F \cdot \cos 60° = -0{,}57735\,F$

$G: \quad -3 \cdot F + 2{,}598 \cdot S_4 = 0 \quad \Rightarrow \quad S_4 = 1{,}1547\,F$

$\uparrow: \quad S_3 \cdot \cos 30° + F = 0 \quad \Rightarrow \quad \boxed{S_3 = -1{,}1547\,F}$ = největší tlak

$$S_3\,krit = EI \cdot \frac{\pi^2}{L^2} = 210 \cdot 10^9 \cdot 0{,}625 \cdot 10^{-6} \cdot \frac{\pi^2}{3^2} = 143\,932\,N = 0{,}143932\,MN$$

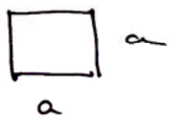
- $\rightarrow F_{krit} = \frac{S_3}{1{,}1547} = \frac{0{,}143932}{1{,}1547} = 0{,}1246\,MN$

max. zatížení ... $2F = 0{,}2492\,MN$

- $\sigma_{krit} = \frac{S_3\,krit}{A} = \frac{0{,}143932}{2{,}04 \cdot 10^{-3}} = 70{,}55\,MPa$

- štíhlost $\lambda = \frac{L}{i} = \frac{3}{0{,}0175} = 171{,}39$

Určete poměr kritických sil dvou prutů, které jsou stejně podepřeny, mají stejnou průřezovou plochu, ale rozdílný tvar průřezu.



$S = a^2$ $\qquad\qquad$ $S = \pi r^2$

$a^2 = \pi r^2$

$I = \frac{1}{12} a^4 = \frac{1}{12} \cdot \pi^2 r^4$ $\qquad\qquad$ $I = \frac{\pi r^4}{4}$

$F_k^{\square} = EI\pi^2/L^2 = \frac{E\pi^4 r^4}{12L^2}$ $\qquad\qquad$ $F_k^{\circ} = EI\pi^2/L^2 = \frac{E\pi^3 r^4}{4L^2}$

$$\frac{F_k^{\square}}{F_k^{\circ}} = \frac{\cancel{E}\cancel{\pi^4}\cancel{r^4} / \cancel{12}\cancel{L^2}}{\cancel{E}\cancel{\pi^3}\cancel{r^4} / \cancel{4}\cancel{L^2}} = \frac{\pi}{3} \doteq 1{,}0472$$