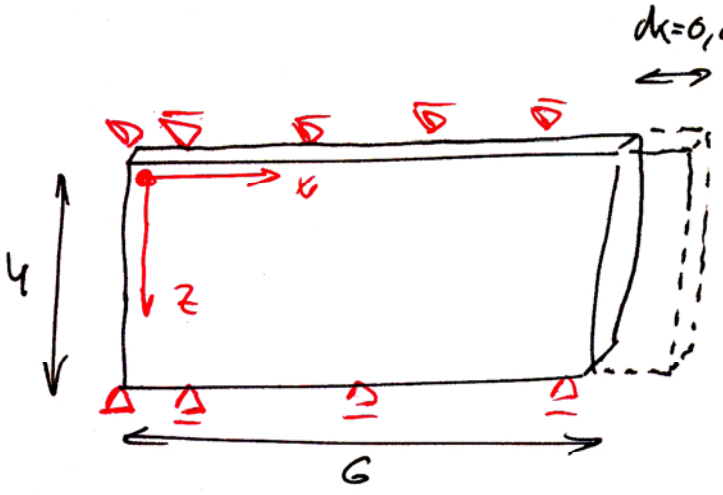
Př1

Vypočtěte $\sigma_x, \sigma_z, \tau_{max}, \sigma_1, \sigma_2, \tau_{xy}$
a úhel α, o který jsou skloněny osy hlavních napětí



$E = 210$ GPa

$\nu = 0,3$

$u(x) = \frac{0,005}{6} \cdot x$

$w(x) = 0$

$\varepsilon_x = \frac{du}{dx} = \frac{0,005}{6} = 8,33 \cdot 10^{-4}$ m $\quad \varepsilon_z = \frac{dw}{dz} = \frac{0}{4} = 0$

$\gamma_{xz} = \frac{du}{dz} + \frac{dw}{dx} = 0$

$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu \varepsilon_z) = \frac{210 \cdot 10^3}{1-0,3^2} \cdot 8,33 \cdot 10^{-4} = 192,31$ MPa

$\sigma_z = \frac{E}{1-\nu^2}(\varepsilon_z + \nu \varepsilon_x) = \frac{210 \cdot 10^3}{1-0,3^2} \cdot 0,3 \cdot 8,33 \cdot 10^{-4} = 57,69$ MPa

$\sigma_{max} = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \frac{192,31 + 57,69}{2} + \sqrt{\left(\frac{192,31 - 57,69}{2}\right)^2} =$

$= 192,31$ MPa

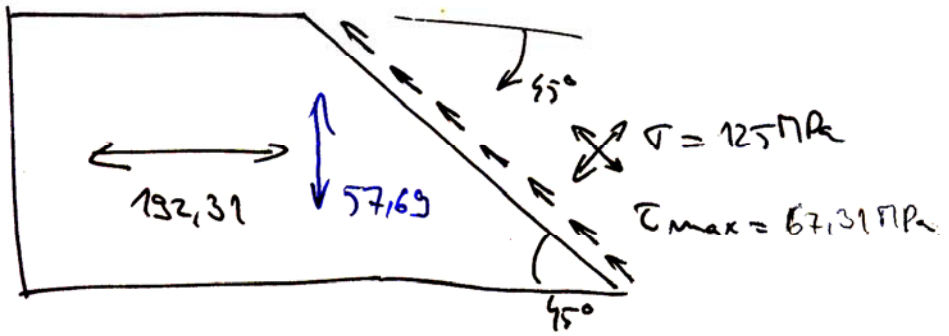
$\sigma_{min} =$ -"- $\ominus$ ——— " ——— $= 57,69$ MPa

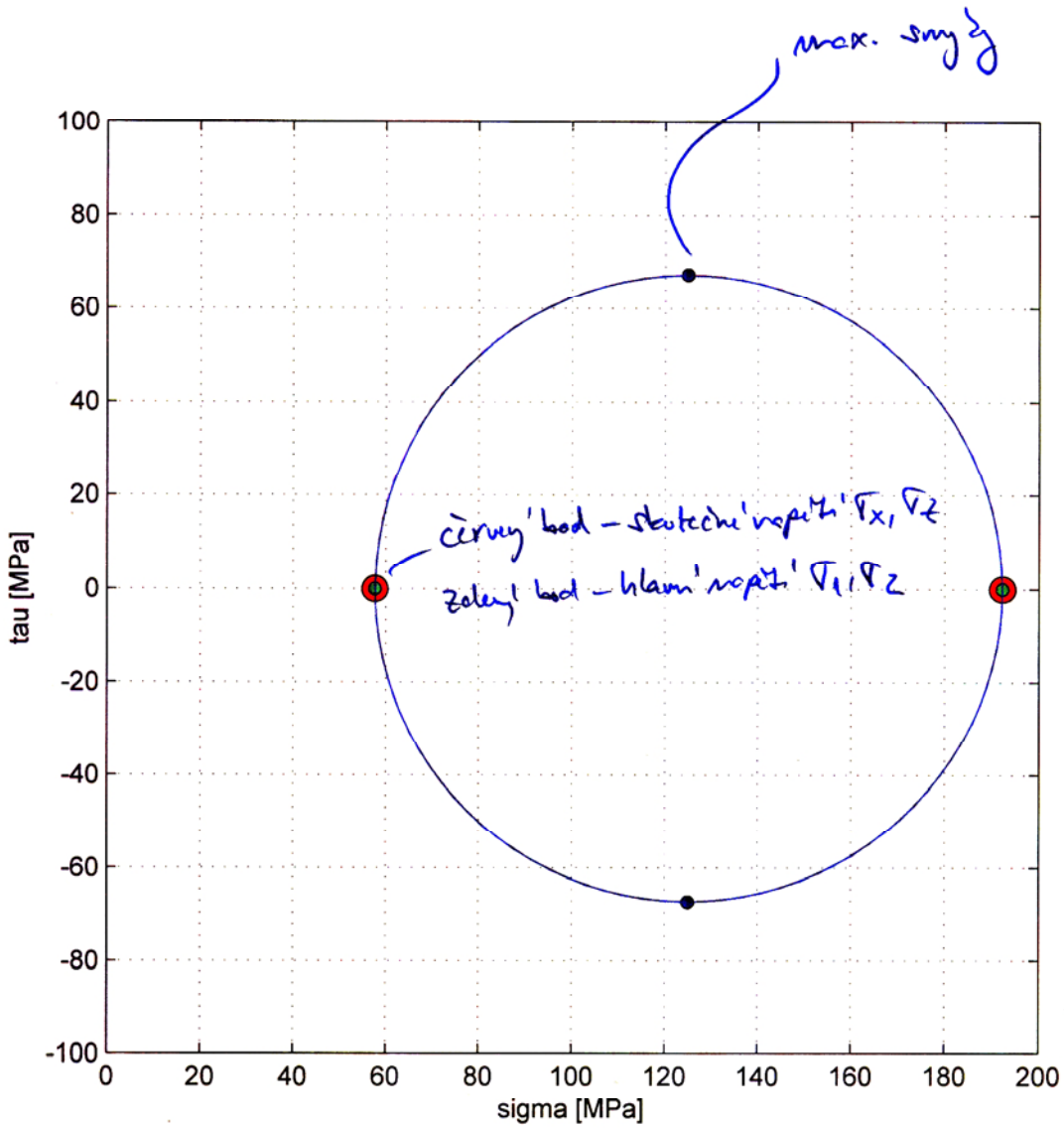
$\alpha = 0,5 \cdot \operatorname{atan}\left(\frac{2\tau_{xz}}{\sigma_x - \sigma_z}\right) = 0$ (hlavní napětí)

maximální smyk $\beta = \alpha + 45° = 45°$

$\sigma_{mean} = \frac{\sigma_1 + \sigma_2}{2} = \frac{192,31 + 57,69}{2} = 125$ MPa

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{\left(\frac{192,31 - 57,69}{2}\right)^2} = 67,31$ MPa



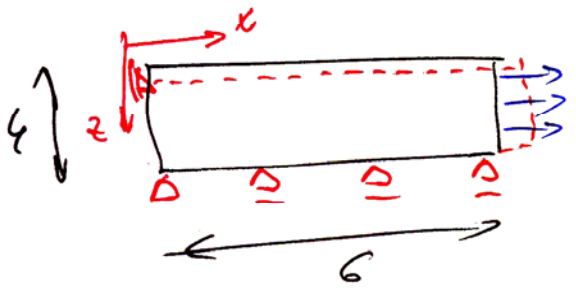


$$\sigma_x' = \sigma_x \cos^2\alpha + \sigma_z \sin^2\alpha + \tau_{xz} \sin 2\alpha$$

$$\sigma_z' = \sigma_x \sin^2\alpha + \sigma_z \cos^2\alpha - \tau_{xz} \sin 2\alpha$$

$$\tau_{xz}' = \frac{\sigma_z - \sigma_x}{2} \sin 2\alpha + \tau_{xz} \cos 2\alpha$$

1



Kromě dalšího určit posun horního pravého rohu!

$\sigma_x = 150$ MPa $\quad \sigma_z = 0$ MPa $\quad \tau_{xz} = 0$

$$\sigma_{max} = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \frac{150}{2} + \frac{150}{2} = 150 \text{ MPa}$$

$$\sigma_{min} = \frac{\sigma_x + \sigma_z}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = 0 \text{ MPa}$$

$\alpha = 0$

$\beta = 45°$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \frac{150}{2} = 75 \text{ MPa}$$

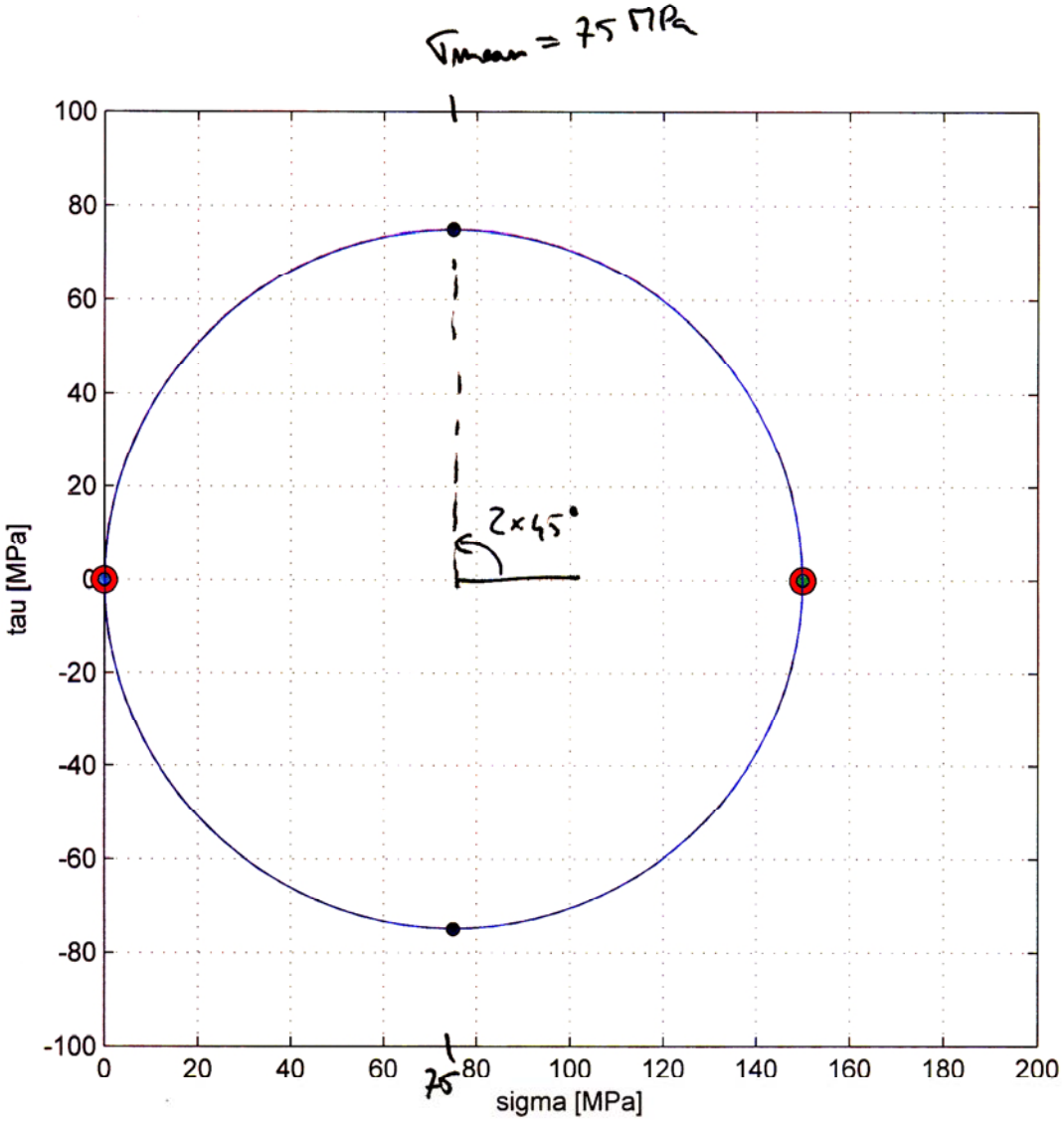
$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu \sigma_z\right] = \frac{150}{210 \cdot 10^3} = 7{,}14 \cdot 10^{-4}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \sigma_x \cdot \nu\right] = -\frac{150 \cdot 0{,}3}{210 \cdot 10^3} = -2{,}14 \cdot 10^{-4}$$

$\gamma_{xz} = 0$

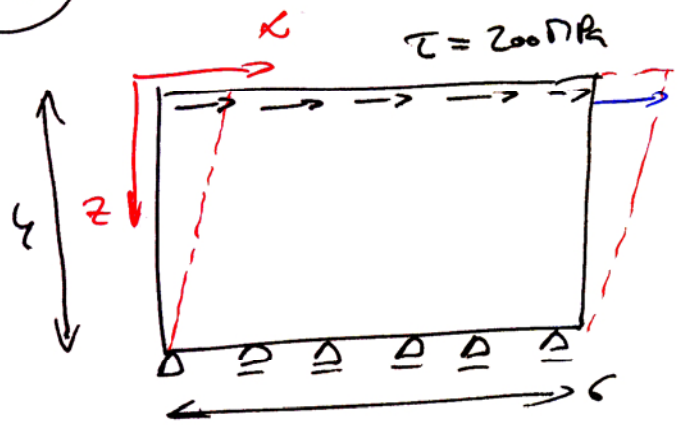
Posun na konci:

- ve směru osy x ... $u(6,0) = \varepsilon_x \cdot 6 = 7{,}14 \cdot 10^{-4} \cdot 6 = 0{,}0043$ m
- " " z ... $w(6,0) = -\varepsilon_z \cdot 4 = 2{,}14 \cdot 10^{-4} \cdot 4 = 0{,}000856$ m


σmean = 75 MPa
2×45°
tau [MPa]
sigma [MPa]
100
80
60
40
20
0
-20
-40
-60
-80
-100
0
20
40
60
75
80
100
120
140
160
180
200

P6)

Prpa, cv. 9, str. 5



$$G = \frac{E}{2(1+\nu)} = \frac{210 \cdot 10^9}{2(1+0{,}3)} = 80{,}769\,GPa$$

$\tau_{xz} = -200\,MPa \quad \sigma_x = 0 \quad \sigma_z = 0$

$$\gamma_{xz} = \frac{\tau_{xz}}{G} = \frac{-200}{80{,}769 \cdot 10^3} = -0{,}0025$$

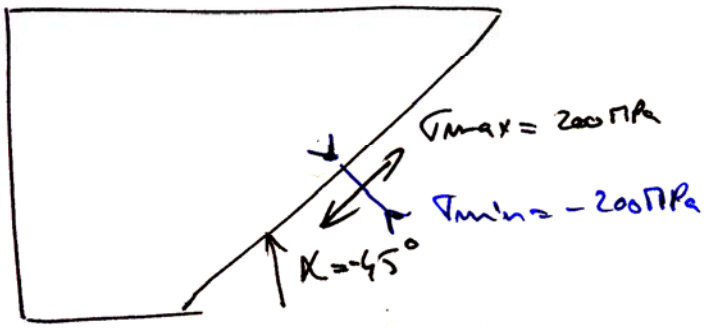
$\varepsilon_x = \varepsilon_z = 0$

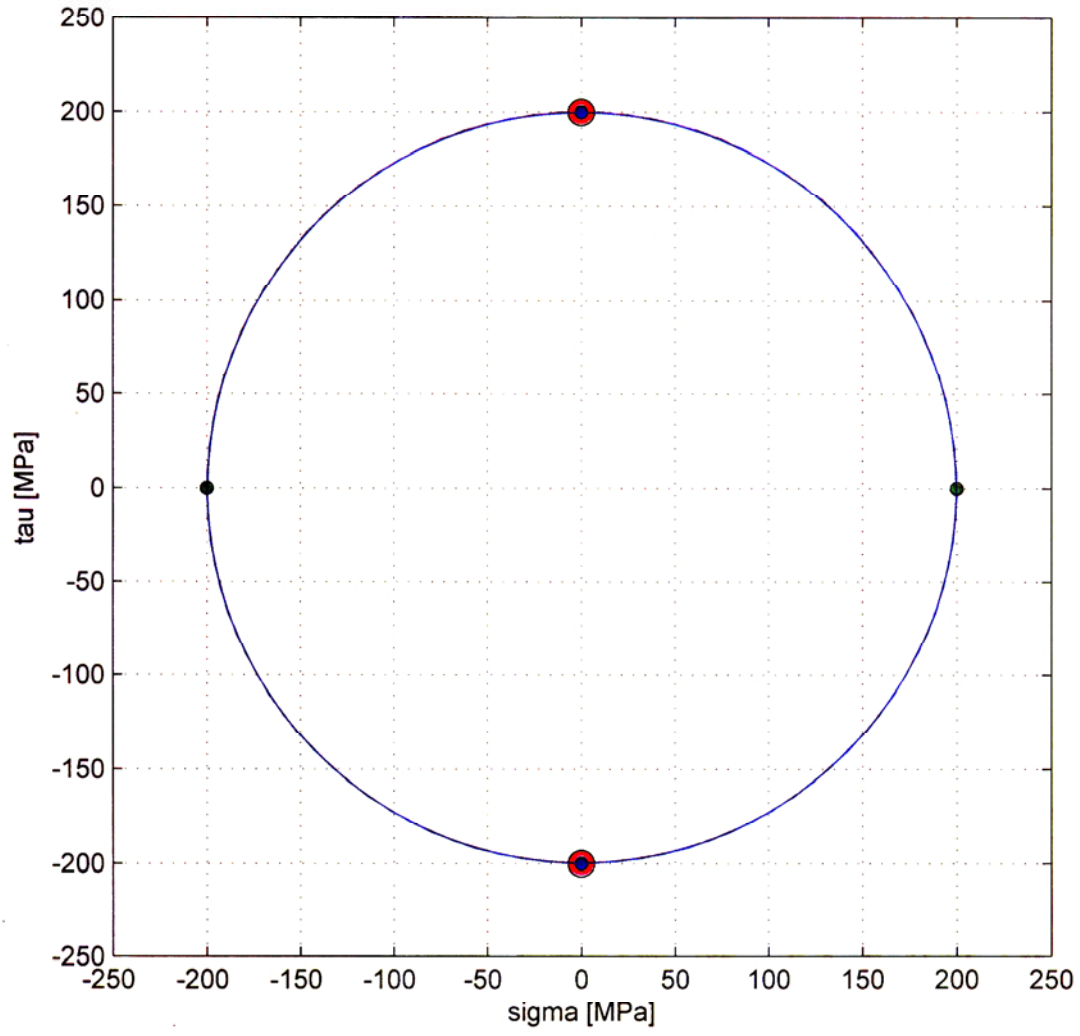
$$\sigma_{max} = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{yz}^2} = 200$$

$\sigma_{min} = -200\,MPa$

$\alpha = -45°$

$$u(6,0) = -y \cdot \gamma_{xz} = -4 \cdot -0{,}0025 = 9{,}9691 \cdot 10^{-3}\,m$$


σmax = 200 MPa
σmin = −200 MPa
α = −45°


250
200
150
100
50
0
-50
-100
-150
-200
-250
tau [MPa]
-250
-200
-150
-100
-50
0
50
100
150
200
250
sigma [MPa]

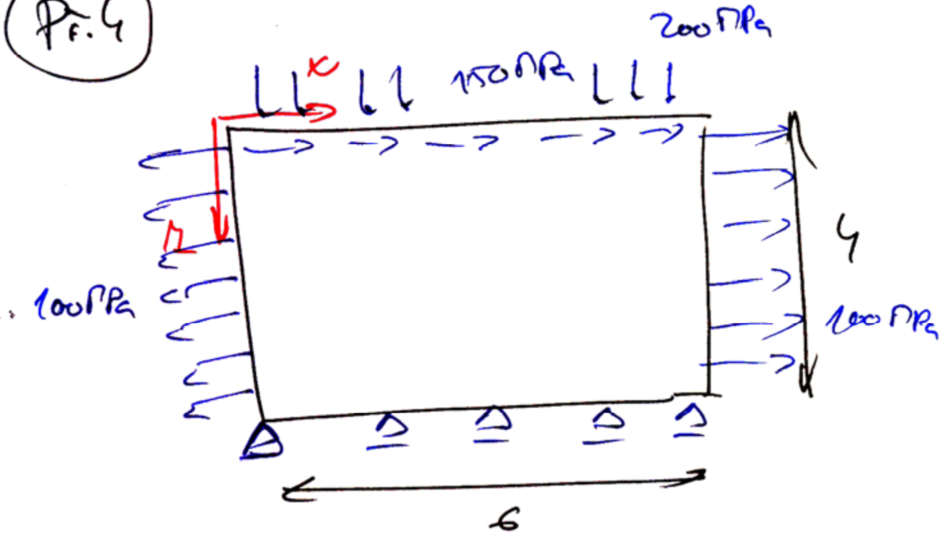
3 mod

Př. 4

Prpe, cv. 9, str. 7

Posun horního pravého rohu ?

+ další věci ...



$\tau_{xz} = -150\,\text{MPa} \quad \sigma_x = 100\,\text{MPa} \quad \sigma_z = -200\,\text{MPa}$

$$\gamma_{xy} = \frac{\tau_{xz}}{G} = \frac{-150 \cdot 10^6}{80\,769 \cdot 10^9} = -0{,}0019$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu \sigma_z\right] = \left[100 \cdot 10^6 - 0{,}3 \cdot (-200 \cdot 10^6)\right] / 210 \cdot 10^9 = 7{,}619 \cdot 10^{-4}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu \sigma_x\right] = \left[-200 \cdot 10^6 - 0{,}3 \cdot 100 \cdot 10^6\right] : 210 \cdot 10^9 = -0{,}0011$$

$$\sigma_{max} = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \frac{100 - 200}{2} + \sqrt{\left(\frac{300}{2}\right)^2 + 150^2} = 162{,}13\,\text{MPa}$$

$$\sigma_{min} = \quad \ominus \quad = \quad \ominus \quad = -262{,}13\,\text{MPa}$$

$$\alpha = \frac{1}{2} \cdot \operatorname{atan}\left(\frac{2\tau_{xz}}{\sigma_x - \sigma_z}\right) = \frac{1}{2} \cdot \operatorname{atan}\left(\frac{2 \cdot (-150)}{100 + 200}\right) = -22{,}5^\circ$$

$$\beta = \alpha + 45^\circ = 22{,}5$$

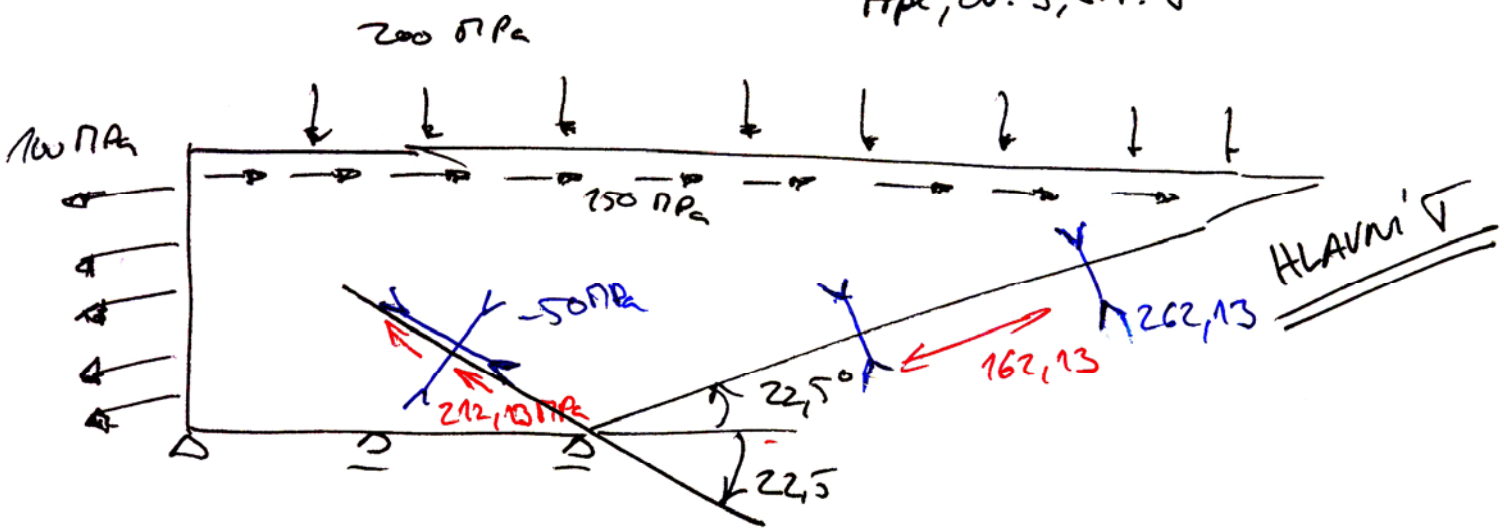
$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = -50\,\text{MPa}$$

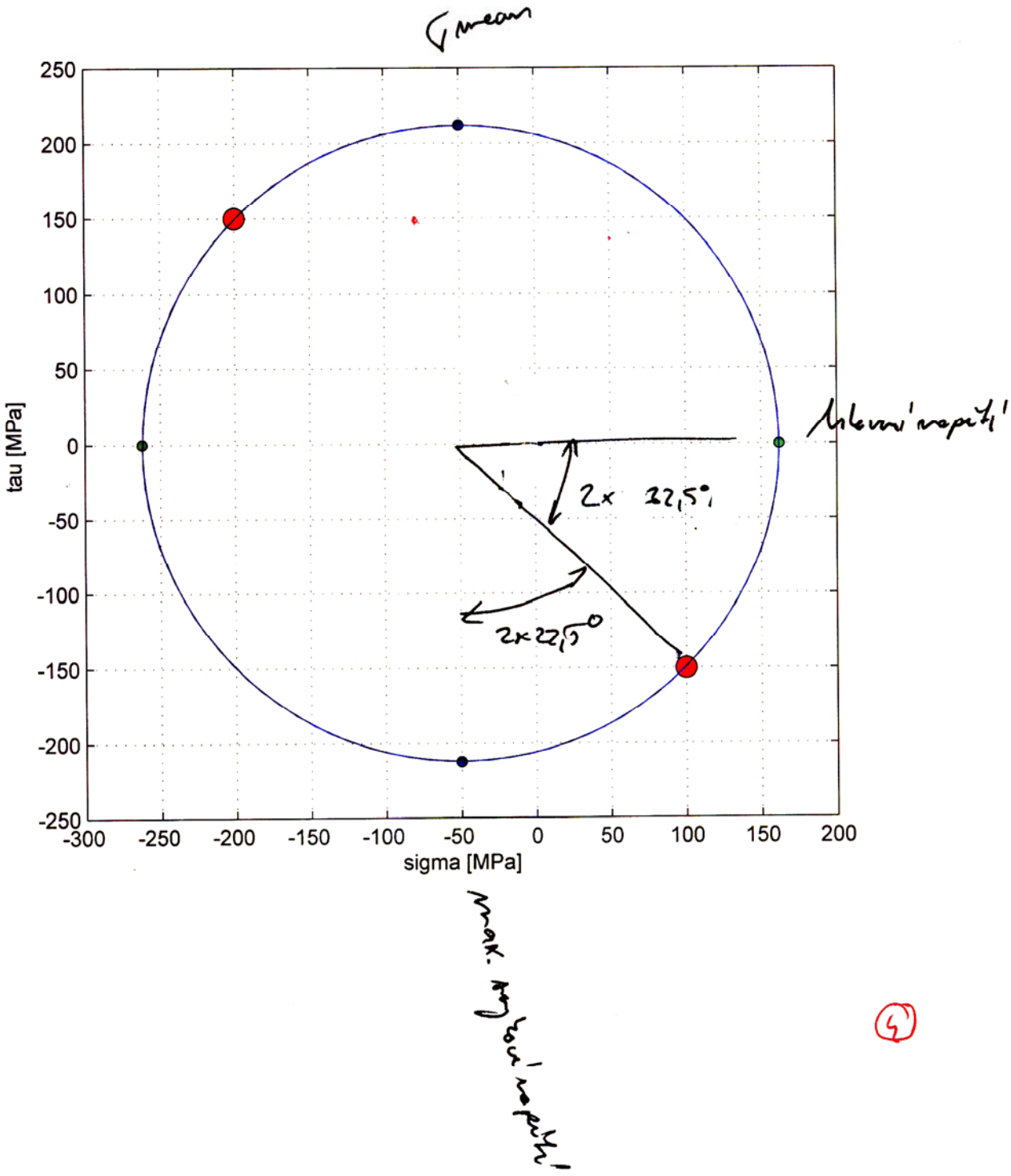
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{\left(\frac{100 + 200}{2}\right)^2 + 150^2} = 212{,}13 \cdot \text{MPa}$$

$$u(6,0) = \varepsilon_x \cdot 6 - 4\gamma_{xz} = 0{,}012\,\text{m}$$

$$w(6,0) = -4 \cdot \varepsilon_z = 0{,}0044\,\text{m}$$

Příp. cv. 9, str. 8





hlavní napětí

max. smyková napětí