Vzorce z přednášky:

- Cauchyho podmínky rovnováhy: $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \bar{X} = 0$
  - další cyklickou záměnou indexů

$\tau_{xy} = \tau_{yx}$ $\quad \tau_{xz} = \tau_{zx}$ $\quad \tau_{yz} = \tau_{zy}$ – vzájemnost smykových napětí

- Deformace: $\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right)$

$$\vdots$$

- Napětí: $\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_x + \nu\varepsilon_z\right]$

$\sigma_m = k \cdot \varepsilon_v$. k – objemový modul $= \frac{E}{3(1-2\nu)}$

σ_m – střední napětí

ε_v – volumetrická část deformace

změna objemu

$s_x = \sigma_x - \sigma_m$

$e_x = \varepsilon_x - \varepsilon_v/3$

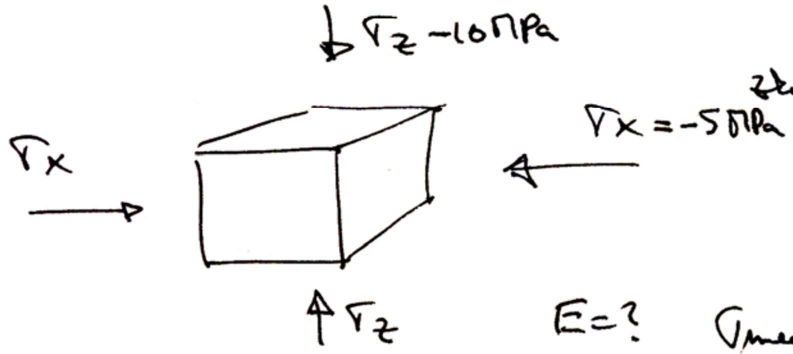
deviatorické napětí a deformace

změna tvaru

Pr. 1

Prpe, cv. 10, str. 2

Určete materiálové charakteristiky vzorku



stlačení $du = -0{,}01\,mm$

$dw = -0{,}03\,mm$

$dv = ?$

$E = ?$ $\sigma_{mean} = ?$ $s_i = ?$

$\nu = ?$ $\varepsilon_v = ?$ $K = ?$ $e_i = ?$

hrana $a = 10\,cm$

$$\varepsilon_x = \frac{\partial u}{\partial x} = \ldots = \frac{du}{a} = \frac{-0{,}01\cdot 10^{-3}}{0{,}1} = -1\cdot 10^{-4}$$

$$\varepsilon_z = \frac{dw}{a} = \frac{-0{,}03\cdot 10^{-3}}{0{,}1} = -3\cdot 10^{-4}$$

$$(1)\quad \varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu \cancelto{0}{\sigma_y} - \nu\sigma_z\right) \qquad \varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_z$$

$$(2)\quad \varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_x - \nu\sigma_y\right) \qquad \varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}\sigma_x$$

rovnost pro E:

$$\frac{\sigma_x - \nu\sigma_z}{\varepsilon_x} = \frac{\sigma_z - \nu\sigma_x}{\varepsilon_z} \Rightarrow \nu = \left(\frac{\sigma_z}{\varepsilon_z} - \frac{\sigma_x}{\varepsilon_x}\right) : \left(\frac{\sigma_x}{\varepsilon_z} - \frac{\sigma_z}{\varepsilon_x}\right)$$

$$\nu = \frac{\left(\frac{-10\cdot 10^{6}}{-3\cdot 10^{-4}} - \frac{-5\cdot 10^{6}}{-1\cdot 10^{-4}}\right)}{\left(\frac{-5\cdot 10^{6}}{-3\cdot 10^{-4}} - \frac{-10\cdot 10^{6}}{-1\cdot 10^{-4}}\right)} = \underline{\underline{0{,}2}}$$

$$E = \frac{\sigma_x - \nu\sigma_z}{\varepsilon_x} = \frac{-5\cdot 10^{6} - 0{,}2\cdot(-10\cdot 10^{6})}{-1\cdot 10^{-4}} = $$

$$= 3\cdot 10^{10}\,Pa = \underline{\underline{30\,GPa}}$$

→ Beton

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x - \nu\sigma_z\right) = \frac{1}{30\cdot 10^{9}}\left(0 - 0{,}2\cdot(-5\cdot 10^{6}) - 0{,}2\cdot(-10\cdot 10^{6})\right) = 1\cdot 10^{-4}$$

$$dv = a\cdot\varepsilon_y = 0{,}1\cdot 1\cdot 10^{-4} = 1\cdot 10^{-5}\,m = 0{,}01\,mm$$

Relativní změna objemu

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z = -1\cdot 10^{-4} + 1\cdot 10^{-4} - 3\cdot 10^{-4} = \underline{\underline{-3\cdot 10^{-4}}}$$

Střední napětí

$$\sigma_{mean} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{-5+0-10}{3} = \underline{\underline{-5\,MPa}}$$

$$\sigma_{mean} = K\cdot\varepsilon_V \longrightarrow K = \frac{\sigma_{mean}}{\varepsilon_V} = \frac{-5\cdot 10^{6}}{-3\cdot 10^{-4}} = 1{,}667\cdot 10^{10}\,Pa = \underline{\underline{16{,}67\,GPa}}$$

Jinak:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z)$$

$$\sum \quad \varepsilon_V = \frac{1}{E}\left[\sigma_x + \sigma_y + \sigma_z\right]\cdot\left[1-2\nu\right]$$

$$K = \frac{\sigma_{mean}}{\varepsilon_V} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}\cdot\frac{E}{(\sigma_x+\sigma_y+\sigma_z)(1-2\nu)} = \frac{E}{3(1-2\nu)}$$

kontrola $K = \dfrac{30\cdot 10^{9}}{3(1-2\cdot 0{,}2)} = \underline{16{,}667\,GPa}$

Prpe cv. 10, str. 4

Deviatorická napětí a deformace

$s_x = \sigma_x - \sigma_{mean} = -5\,MPa - -5\,MPa = 0\,MPa$

$s_y = \sigma_y - \sigma_{mean} = 5\,MPa$

$s_z = \sigma_z - \sigma_{mean} = -10 - -5\,MPa = -5\,MPa$

$\Sigma\ 0\,MPa$ ... ok

$e_x = \varepsilon_x - \varepsilon_v/3 \qquad \varepsilon_v/3 = -1\cdot 10^{-4}$

$e_x = -1\cdot 10^{-4} - -1\cdot 10^{-4} = 0$

$e_y = \varepsilon_y - \frac{\varepsilon_v}{3} = 1\cdot 10^{-4} - -1\cdot 10^{-4} = 2\cdot 10^{-4}$

$e_z = \varepsilon_z - \frac{\varepsilon_v}{3} = -3\cdot 10^{-4} - -1\cdot 10^{-4} = -2\cdot 10^{-4}$

$\Sigma\ 0$ ... ok

platí $\boxed{s_x = 2Ge_x}$

Př 2

Prpe, cv. 10, str. 5

Vypočítejte složky napětí $\sigma_x, \sigma_y, \sigma_z, \sigma_{mean}$

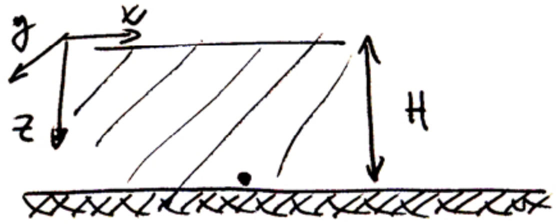
velikost deformací $\varepsilon_x, \varepsilon_y, \varepsilon_z, \varepsilon_V$

$\nu = 0{,}4$

$E = 3\ GPa$

$H = 10\ m$

$\rho = 20\ kN/m^3$



Řešení! předpoklad $\varepsilon_z \neq 0 \quad \varepsilon_x, \varepsilon_y = 0 \quad \Rightarrow \sigma_x, \sigma_y \neq 0;\ \sigma_x = \sigma_y$

$$\sigma_z = H \cdot \rho = 10 \cdot 20 \frac{kN}{m^2} = -200\ kPa$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) \qquad 0 = \frac{1}{E}(\sigma_x - \nu\sigma_x - \nu\sigma_z)$$

$$\Rightarrow \sigma_x = \frac{\nu\sigma_z}{(1-\nu)} = \frac{0{,}4(-200)}{(1-0{,}4)} = -133{,}333\ kPa$$

$$\sigma_y = -133{,}333\ kPa$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = (-200 - 0{,}4 \cdot 2 \cdot (-133{,}333)) : 3 \cdot 10^6 = -0{,}000031$$

$$\sigma_{mean} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{2 \cdot (-133{,}33) - 200}{3} = \underline{-155{,}555\ kPa}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = 0 + 0 + (-0{,}000031) = \underline{-31 \cdot 10^{-6}}$$

obecně:

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - 2\nu\frac{\nu\sigma_z}{(1-\nu)}\right) = \frac{1}{E}\left(\sigma_z \underbrace{\left[1 - \frac{2\nu^2}{(1-\nu)}\right]}\right)$$

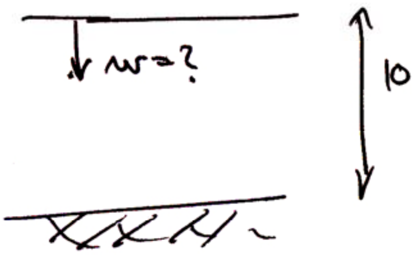
$$\frac{1-\nu-2\nu^2}{1-\nu} = \frac{(1+\nu)(1-2\nu)}{1-\nu}$$

$$\sigma_z = \frac{\overbrace{E(1-\nu)}^{E_{oed} = \text{edometrický modul pružnosti}}}{(1+\nu)(1-2\nu)} \cdot \varepsilon_z$$

Př3

Prpe, cv. 10, str. 6

<u>Vypočtěte velikost sednutí na povrchu</u> (úvomek od vl. tíhy)



$\rho = 20\,kN/m^3$ $\quad\quad \nu = 0{,}4$

$w = \int \varepsilon_z \, dz$ $\quad\quad E = 3\,GPa$

$\varepsilon_z = \dfrac{\sigma_z}{E_{oed}}$; $\quad\quad \sigma_z = \rho \cdot z$

$$w = \int \frac{\sigma_z(z)}{E_{oed}} dz = \frac{1}{E_{oed}} \int \rho z \, dz = \frac{\rho}{E_{oed}} \left[\frac{z^2}{2} \right]_0^{10}$$

$$E_{oed} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = \frac{3(1-0{,}4)}{(1+0{,}4)(1-2\cdot 0{,}4)} = 6{,}42857\,GPa$$

$$w = \frac{\rho}{E_{oed}} \cdot \frac{z^2}{2} = \frac{20\cdot 10^3}{6{,}429\cdot 10^9} \cdot \frac{10^2}{2} = 0{,}000156\,m = \underline{\underline{0{,}156\,mm}}$$