TRESCOVA PODMÍNKA PLASTICITY

MISESOVA PODMÍNKA PLASTICITY

TRESCOVA PODMÍNKA PLASTICITY

$\tau_{max}(\sigma) - \tau_0 = 0$

$\hookrightarrow \tau_{max} = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

rovinná napjatost: $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_1 > \sigma_2$

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MISESOVA PODMÍNKA PLASTICITY

$$\sqrt{J_2} - \tau_0 = 0$$

$$\hookrightarrow J_2 = \frac{s_x^2 + s_y^2 + s_z^2}{2} + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$J_2 = \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

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$$\sigma_1 > \sigma_2$$

PŘ: jednoosý tah $\sigma_x > 0$; $\sigma_y = 0$; $\tau_{xy} = 0$

$\sigma_1 = \sigma_x \quad \sigma_2 = 0$

$\tau_{max} = \frac{\sigma_x - 0}{2} - \tau_0 = 0 \rightarrow \sigma_x = 2\tau_0$

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PT : jednoosý tah $\sigma_x > 0$; $\sigma_y = 0$; $\tau_{xy} = 0$

$\sigma_1 = \sigma_x \quad \sigma_2 = 0$

$$\tau_{max} = \frac{\sigma_x - 0}{2} - \tau_0 = 0 \quad \rightarrow \sigma_x = 2\tau_0$$

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$$J_2 = \frac{1}{6}\left[(\sigma_x - 0)^2 + (\sigma_x - 0)^2 + (0-0)^2\right] + 0^2 + 0^2 + 0^2 = \frac{\sigma_x^2}{3}$$

$$\sqrt{\sigma_x^2/3} - \tau_0 = 0$$

TRESCOVA PODMÍNKA PLASTICITY

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$$\tau_{max} = \frac{\sigma_x - 0}{2} - \tau_0 = 0 \rightarrow \sigma_x = 2\tau_0$$

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$$\sqrt{\sigma_x^2/3} - \tau_0 = 0 \quad \rightarrow \sigma_x = \sqrt{3}\,\tau_0 = \sigma_0$$

TRESCOVA PODMÍNKA PLASTICITY

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Př: jednoosý tah $\sigma_x > 0;\ \sigma_y = 0;\ \tau_{xy} = 0$

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MISESOVA PODMÍNKA PLASTICITY

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$$J_2 = \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

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$$\sqrt{\sigma_x^2/3} - \tau_0 = 0 \longrightarrow \sigma_x = \sqrt{3}\,\tau_0 = \sigma_0$$

$\rightarrow$ alternativní zápis $\underbrace{\sqrt{3J_2}}_{\text{efektivní napětí}} - \sigma_0 = 0$

TRESCOVA PODMÍNKA PLASTICITY

$$\tau_{max}(\sigma) - \tau_0 = 0$$

↳ $\tau_{max} = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

rovinná napjatost: $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

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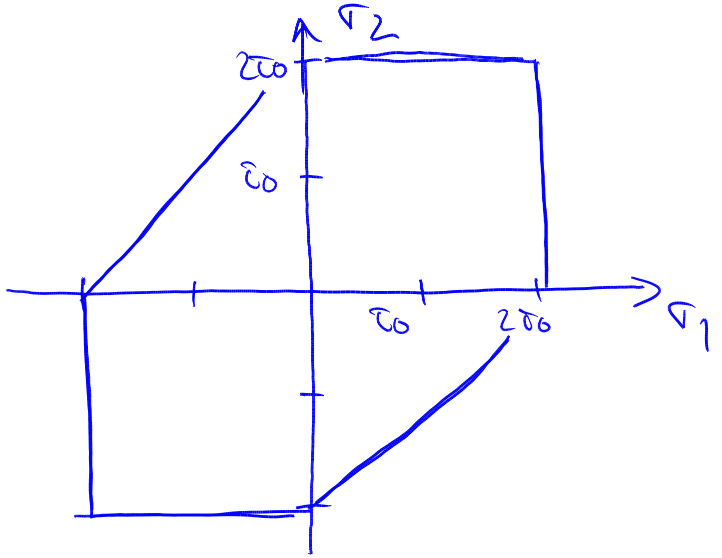
$$\tau_{max} = \frac{\sigma_x - 0}{2} - \tau_0 = 0 \rightarrow \sigma_x = 2\tau_0$$

Plocha plasticity pro rovinnou napjatost



MISESOVA PODMÍNKA PLASTICITY

$$\sqrt{J_2} - \tau_0 = 0$$

↳ $J_2 = \dfrac{s_x^2 + s_y^2 + s_z^2}{2} + \tau_{xy} + \tau_{xz}^2 + \tau_{yz}^2$

$$J_2 = \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$J_2 = \frac{1}{6}\left[(\sigma_x - 0)^2 + (\sigma_x - 0)^2 + (0 - 0)^2\right] + 0^2 + 0^2 + 0^2 = \frac{\sigma_x^2}{3}$$

$$\sqrt{\sigma_x^2/3} - \tau_0 = 0 \rightarrow \sigma_x = \sqrt{3}\,\tau_0 = \sigma_0$$

→ alternativní zápis $\underbrace{\sqrt{3J_2}}_{\text{efektivní napětí}} - \sigma_0 = 0$

TRESCOVA PODMÍNKA PLASTICITY

$$\tau_{max}(\sigma) - \tau_0 = 0$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

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$$\sigma_1 > \sigma_2$$

Př: jednoosý tah $\sigma_x > 0;\ \sigma_y = 0;\ \tau_{xy} = 0$

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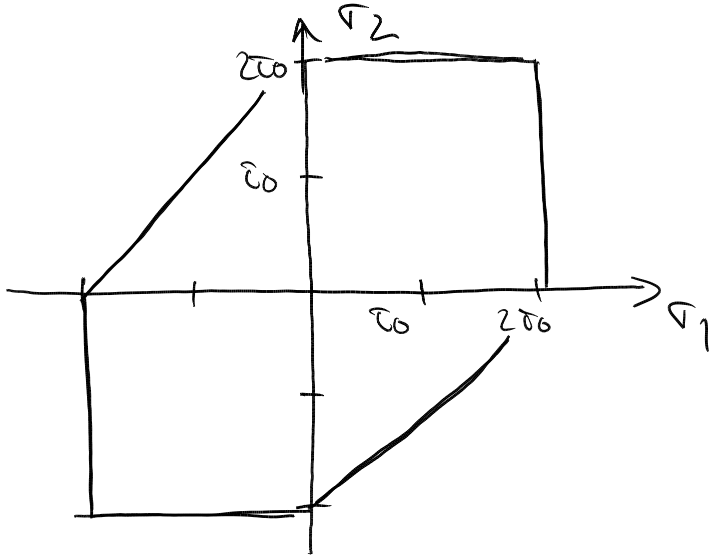


MISESOVA PODMÍNKA PLASTICITY

$$\sqrt{J_2} - \tau_0 = 0$$

$$J_2 = \frac{s_x^2 + s_y^2 + s_z^2}{2} + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

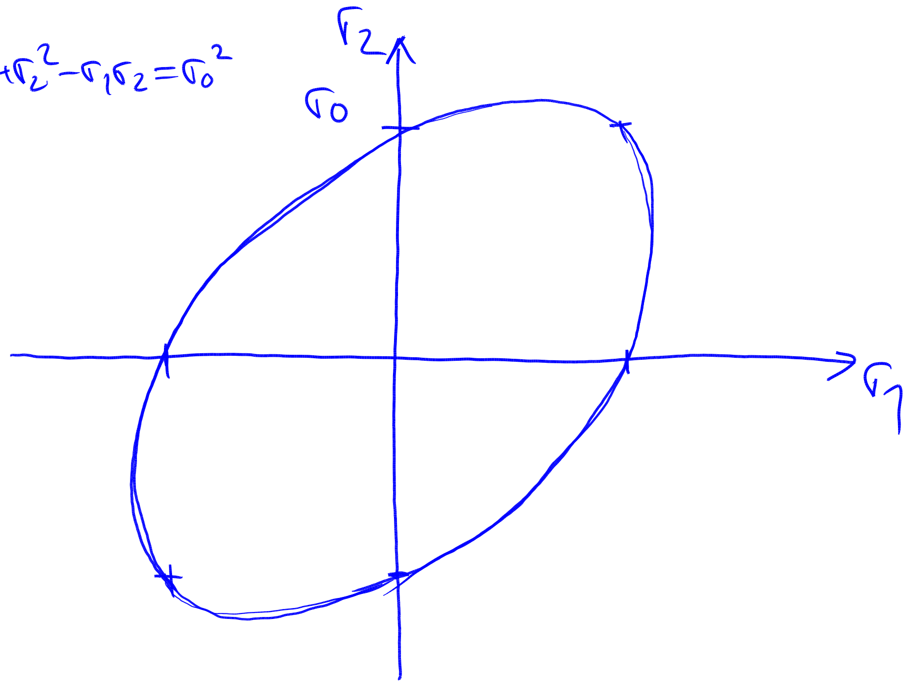
$$J_2 = \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$J_2 = \frac{1}{6}\left[(\sigma_x - 0)^2 + (\sigma_x - 0)^2 + (0-0)^2\right] + 0^2 + 0^2 + 0^2 = \frac{\sigma_x^2}{3}$$

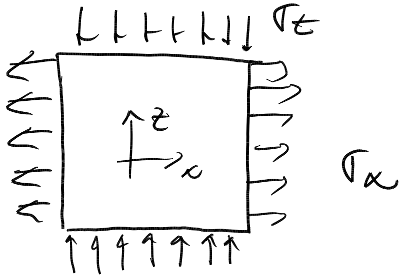
$$\sqrt{\sigma_x^2/3} - \tau_0 = 0 \longrightarrow \sigma_x = \sqrt{3}\,\tau_0 = \sigma_0$$

$\rightarrow$ alternativní zápis $\underbrace{\sqrt{3J_2}}_{\text{efektivní napětí}} - \sigma_0 = 0$

$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_0^2$



Proporcionální zatěžování

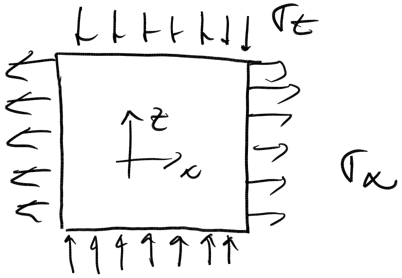


$\sigma_x : \sigma_z = 2 : (-1)$

TRESCA

MISES

Proporcionální zatěžování



$$\sigma_x : \sigma_z = 2 : (-1)$$

TRESCA

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{2\sigma - (-\sigma)}{2} = \frac{3}{2}\sigma$$

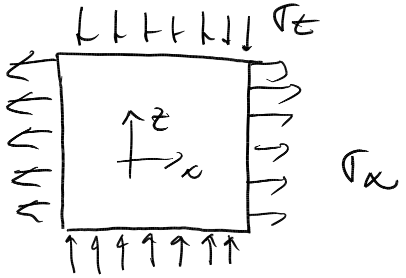
$$\tau_{max}(\sigma) - \tau_0 = 0$$

$$\frac{3}{2}\sigma - \tau_0 = 0 \quad \rightarrow \quad \sigma = \frac{2}{3}\tau_0 ; \quad \sigma_x = \frac{4}{3}\tau_0$$

$$\sigma_z = -\frac{2}{3}\tau_0$$

MISES

Proporcionální zatěžování



$\sigma_x : \sigma_z = 2 : (-1)$

TRESCA

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$$\tau_{max}(\sigma) - \tau_0 = 0$$

$$\frac{3}{2}\sigma - \tau_0 = 0 \quad \to \quad \sigma = \frac{2}{3}\tau_0; \quad \sigma_x = \frac{4}{3}\tau_0$$

$$\sigma_z = -\frac{2}{3}\tau_0$$

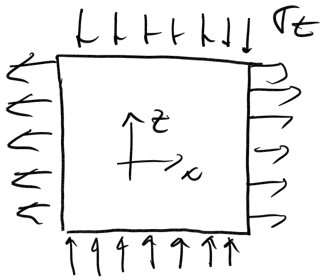
MISES

$\sigma_1 = 2\sigma$; $\sigma_2 = 0$; $\sigma_3 = -\sigma$

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right] =$$

$$= \frac{1}{6}\left[(2\sigma)^2 + (2\sigma - (-\sigma))^2 + (\sigma)^2\right] = \frac{14}{6}\sigma^2 = \frac{7}{3}\sigma^2$$

Proporcionální zatěžování



$\sigma_x : \sigma_z = 2 : (-1)$

TRESCA

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{2\sigma - (-\sigma)}{2} = \frac{3}{2}\sigma$$

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MISES $\quad \sigma_1 = 2\sigma ; \quad \sigma_2 = 0 ; \quad \sigma_3 = -\sigma$

$$J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2\right] =$$

$$= \frac{1}{6}\left[(2\sigma)^2 + (2\sigma-(-\sigma))^2 + (\sigma)^2\right] = \frac{14}{6}\sigma^2 = \frac{7}{3}\sigma^2$$

$$\sqrt{J_2} - \tau_0 = 0$$

$$\sqrt{\frac{7}{3}\sigma^2} - \tau_0 = 0 \longrightarrow \sigma = \frac{\tau_0}{\sqrt{7/3}}$$

$$\sigma_x = \frac{2}{\sqrt{7/3}}\tau_0 \doteq 1{,}309\,\tau_0$$

$$\sigma_z = -\frac{\tau_0}{\sqrt{7/3}} \doteq -0{,}655\,\tau_0$$

→ Určit mez kluzu ve smyku τ_0. Plastického stavu je dosaženo při $\sigma_x = 100$ MPa, $\sigma_z = 200$ MPa

$\tau_{xz} = -100$ MPa (rovinná napjatost)

TRESCA

MISES

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$\tau_{xz} = -100$ MPa (rovinná napjatost)

TRESCA

MISES

$$\sigma_{1,2} = \frac{100+200}{2} \pm \sqrt{\left(\frac{100-200}{2}\right)^2 + (-100)^2} =$$

$$= 150 \pm \sqrt{50^2 + 100^2} = 150 + 111{,}8 = 261{,}8 \text{ MPa}$$

$$= 150 - 111{,}8 = 38{,}2 \text{ MPa}$$

$\sigma_1 = 261{,}8$ MPa

$\sigma_3 = 0$

→ $\tau_{max} = \frac{261{,}8 - 0}{2} = \underline{\underline{130{,}9 \text{ MPa}}} = \tau_0$

→ Určit mez kluzu ve smyku τ_0. Plastického stavu je dosaženo při $\sigma_x = 100$ MPa, $\sigma_z = 200$ MPa

$\tau_{xz} = -100$ MPa (rovinná napjatost)

TRESCA

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$$= 150 - 111{,}8 = 38{,}2 \text{ MPa}$$

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$\sigma_3 = 0$

$$\rightarrow \tau_{max} = \frac{261{,}8 - 0}{2} = \underline{\underline{130{,}9 \text{ MPa}}} = \tau_0$$

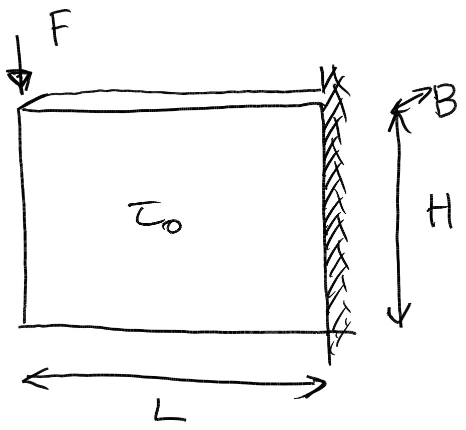
MISES

$$J_2 = \frac{1}{6}\left[(100-0)^2 + (100-200)^2 + (-200)^2\right] + (-100)^2 + 0 + 0 =$$

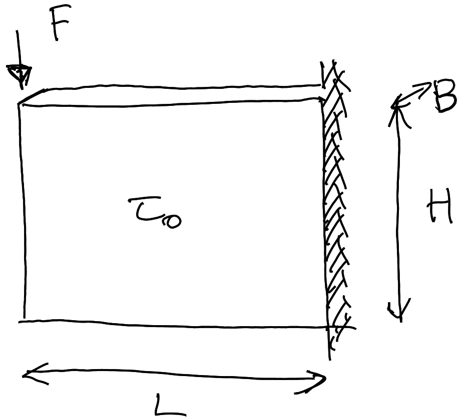
$$= \frac{60\,000}{6} + 10\,000 = 20\,000 \text{ MPa}^2$$

$$\sqrt{J_2} - \tau_0 = 0$$

$$\rightarrow \tau_0 = \sqrt{20\,000} \doteq \underline{\underline{141{,}42 \text{ MPa}}}$$

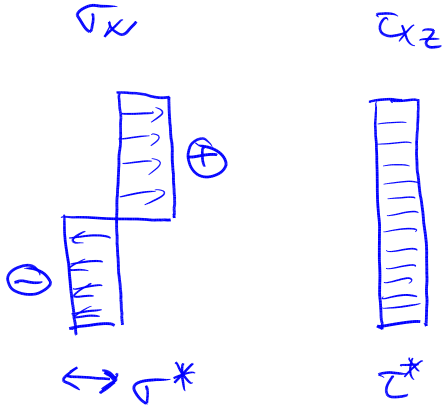


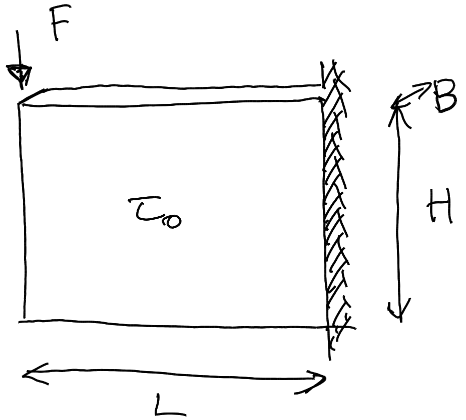
F
B
τ_0
H
L



$M = F \cdot L$

$V = F$





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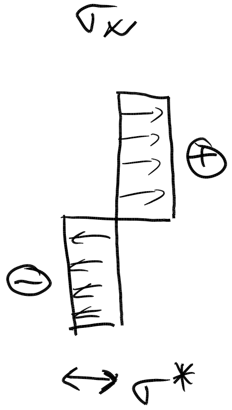
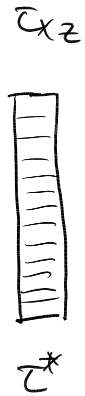
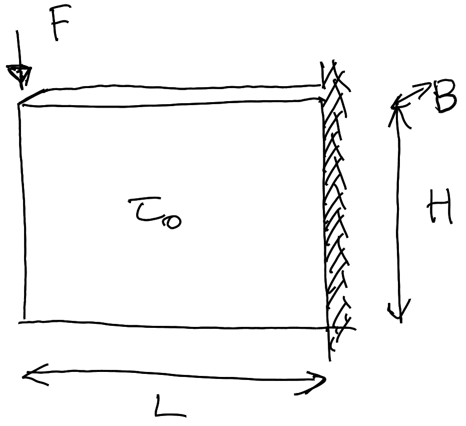


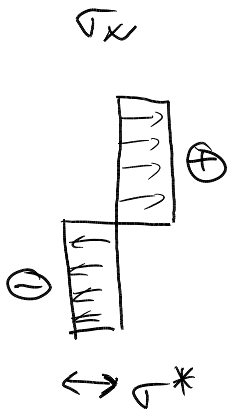


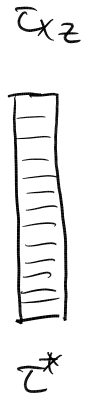
$M = \sigma^* \cdot B \cdot \frac{H}{2} \cdot \frac{H}{2} = \sigma^* B H^2 / 4$



$M = F \cdot L$

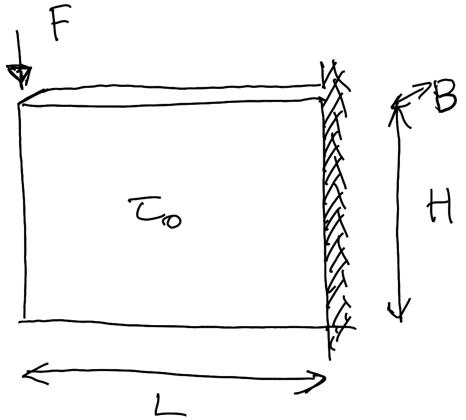
$V = F$





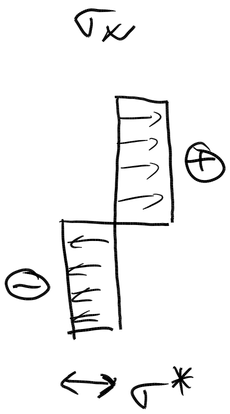
$M = \sigma^* \cdot B \cdot \frac{H}{2} \cdot \frac{H}{2} = \sigma^* B H^2 / 4$

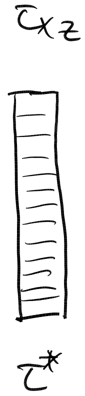
$V = \tau^* B H$



$M = F \cdot L$

$V = F$





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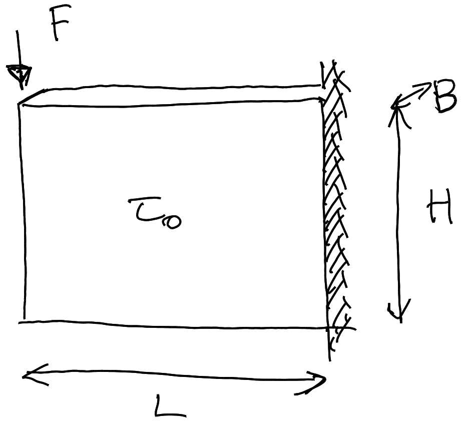

$$M = F \cdot L$$
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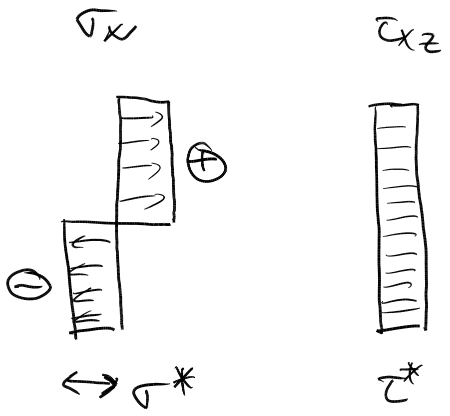
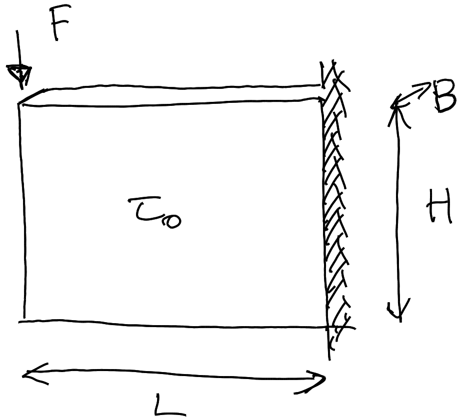


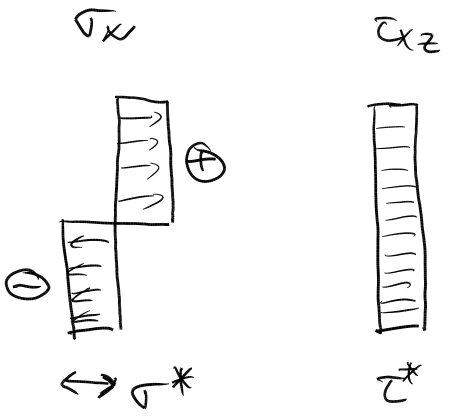
$$M = \sigma^* \cdot B \cdot \frac{H}{2} \cdot \frac{H}{2} = \sigma^* B H^2 / 4$$
$$V = \tau^* B H$$

$$\rightarrow FL = \sigma^* B H^2/4 \quad \rightarrow \sigma^* = \frac{4FL}{BH^2}$$
$$\rightarrow F = \tau^* BH \quad \rightarrow \tau^* = \frac{F}{BH}$$



$$M = F \cdot L$$

$$V = F$$



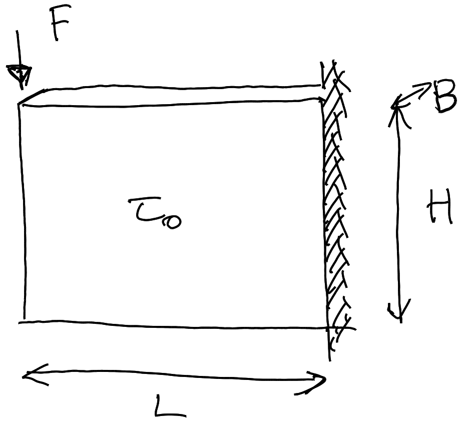
$$M = \sigma^* \cdot B \cdot \frac{H}{2} \cdot \frac{H}{2} = \sigma^* B H^2 / 4$$

$$V = \tau^* B H$$

$$\rightarrow FL = \sigma^* B H^2 / 4 \quad \rightarrow \sigma^* = \frac{4FL}{BH^2}$$

$$\rightarrow F = \tau^* BH \quad \rightarrow \tau^* = \frac{F}{BH}$$

rovinná napjatost, $\sigma_x = \sigma^*$, $\sigma_z = 0$, $\tau_{xz} = \tau^*$



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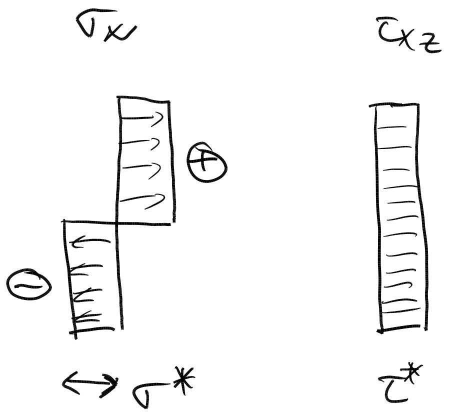

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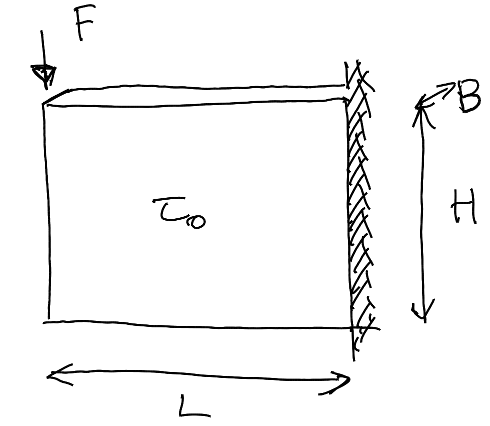
Trescova podmínka plasticity $\rightarrow \sigma_{1,2} = \frac{\sigma^* + 0}{2} \pm \sqrt{\left(\frac{\sigma^* - 0}{2}\right)^2 + \tau^{*2}} = \frac{2FL}{BH^2} \pm \sqrt{\left(\frac{2FL}{BH^2}\right)^2 + \left(\frac{F}{BH}\right)^2} = \frac{2FL}{BH^2} \pm \sqrt{\frac{4F^2L^2}{B^2H^4} + \frac{F^2}{B^2H^2}}$



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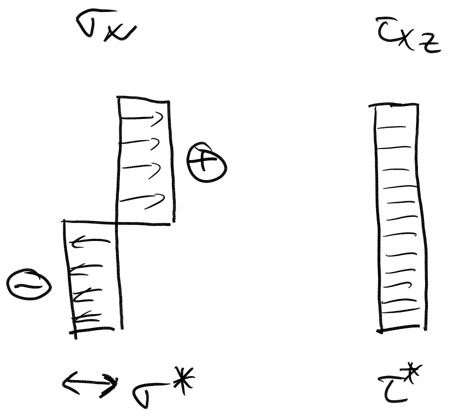

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$\tau_{max} = \tau_0$

$\tau_{max} = \frac{2\sqrt{4F^2L^2/B^2H^4 + F^2/B^2H^2}}{2} = \tau_0$

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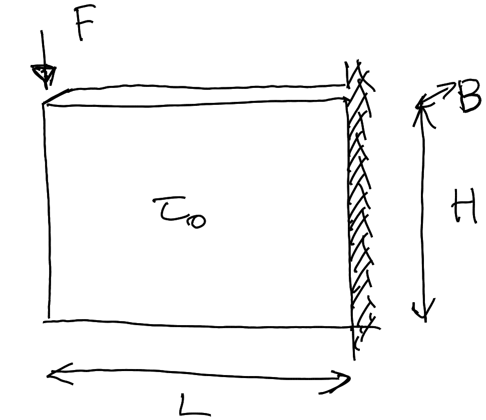

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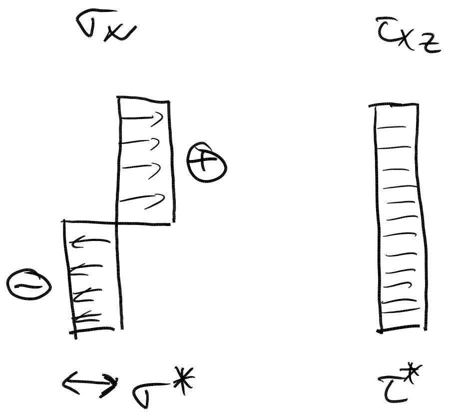

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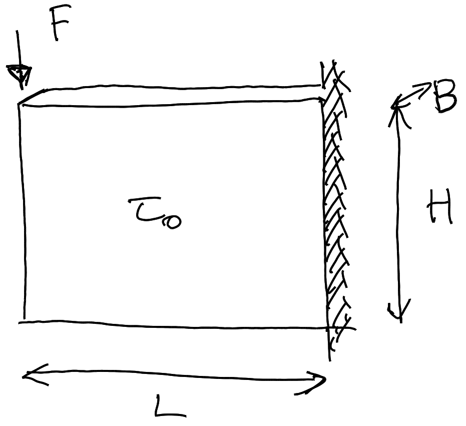

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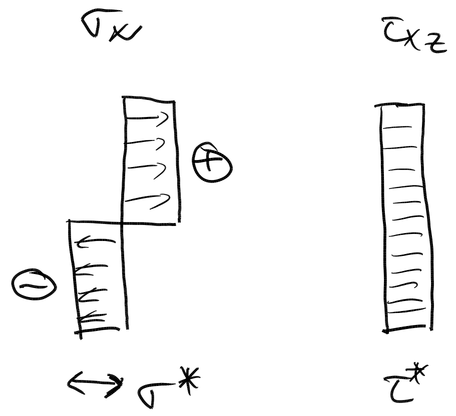

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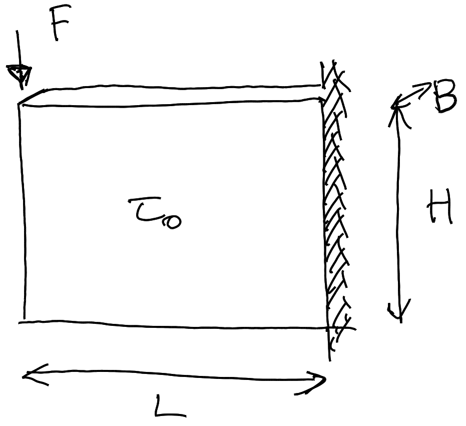

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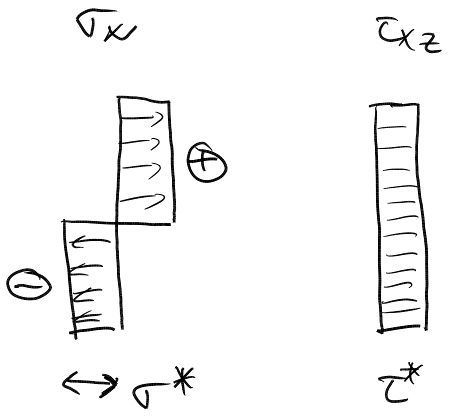

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rovinná napjatost, $\sigma_x = \sigma^*$, $\sigma_z = 0$, $\tau_{xz} = \tau^*$

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$L \ll H \rightarrow F = \tau_0 BH$

$L \gg H \rightarrow F = \tau_0 BH \cdot \frac{H}{2L}$

Misesova podmínka plasticity

$$\sqrt{J_2} - \tau_0 = 0$$

$$J_2 = \frac{1}{6}\left[(\sigma_x - \cancel{\sigma_y})^2 + (\sigma_x - \cancel{\sigma_z})^2 + \cancel{(\sigma_y - \sigma_z)^2}\right] + \cancel{\tau_{xy}^2} + \tau_{xz}^2 + \cancel{\tau_{yz}^2} = \frac{1}{6} 2\sigma_x^2 + \tau_{xz}^2 = \frac{\sigma_x^2}{3} + \tau_{xz}^2$$

$$\sqrt{\left(\frac{4FL}{BH^2}\right)^2 \cdot \frac{1}{3} + \left(\frac{F}{BH}\right)^2} - \tau_0 = 0$$

Misesova podmínka plasticity

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$$\sqrt{\left(\frac{4FL}{BH^2}\right)^2 \cdot \frac{1}{3} + \left(\frac{F}{BH}\right)^2} - \tau_0 = 0$$

$$\sqrt{\frac{16F^2L^2}{B^2H^4} \cdot \frac{1}{3} + \frac{F^2}{B^2H^2}} = \tau_0$$

$$\frac{F}{BH}\sqrt{\frac{16}{3}\frac{L^2}{H^2} + 1} = \tau_0$$

$$\rightarrow F = \frac{\tau_0 BH}{\sqrt{\frac{16}{3}\frac{L^2}{H^2} + 1}}$$

Misesova podmínka plasticity

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$$\rightarrow F = \frac{\tau_0 BH}{\sqrt{\frac{16}{3}\frac{L^2}{H^2} + 1}}$$

pouze ohyb: $\sigma^* = \frac{4FL}{BH^2} \rightarrow F = \frac{\tau_0\sqrt{3}BH^2}{4L}$

$\sigma^* \rightarrow \tau_0\sqrt{3}$

pouze smyk: $\tau^* = \tau_0 = \frac{F}{BH} \rightarrow F = \tau_0 BH$

Misesova podmínka plasticity

$$\sqrt{J_2} - \tau_0 = 0$$

$$J_2 = \frac{1}{6}\left[(\sigma_x - \cancel{\sigma_y})^2 + (\sigma_x - \cancel{\sigma_z})^2 + \cancel{(\sigma_y - \sigma_z)^2}\right] + \cancel{\tau_{xy}^2} + \tau_{xz}^2 + \cancel{\tau_{yz}^2} = \frac{1}{6}\, 2\sigma_x^2 + \tau_{xz}^2 = \frac{\sigma_x^2}{3} + \tau_{xz}^2$$

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$$\frac{F}{BH}\sqrt{\frac{16}{3}\frac{L^2}{H^2} + 1} = \tau_0$$

$$\rightarrow F = \frac{\tau_0 BH}{\sqrt{\frac{16}{3}\frac{L^2}{H^2} + 1}} = \frac{\tau_0 BH}{\sqrt{\frac{3H^2 + 16L^2}{3H^2}}}$$

$L \gg H$

$$\frac{\tau_0 BH}{\sqrt{\frac{16L^2}{3H^2}}} = \frac{\tau_0 BH^2\sqrt{3}}{4L}$$

pouze ohyb: $\sigma^* = \frac{4FL}{BH^2} \rightarrow F = \frac{\tau_0\sqrt{3}\,BH^2}{4L}$ ($\sigma^* \rightarrow \tau_0\sqrt{3}$)

pouze smyk: $\tau^* = \tau_0 = \frac{F}{BH} \rightarrow F = \tau_0 BH$

Misesova podmínka plasticity

$$\sqrt{J_2} - \tau_0 = 0$$

$$J_2 = \frac{1}{6}\left[(\sigma_x - \cancel{\sigma_y})^2 + (\sigma_x - \cancel{\sigma_z})^2 + \cancel{(\sigma_y - \sigma_z)^2}\right] + \cancel{\tau_{xy}^2} + \tau_{xz}^2 + \cancel{\tau_{yz}^2} = \frac{1}{6} 2\sigma_x^2 + \tau_{xz}^2 = \frac{\sigma_x^2}{3} + \tau_{xz}^2$$

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$$\frac{F}{BH}\sqrt{\frac{16}{3}\frac{L^2}{H^2} + 1} = \tau_0$$

$$\rightarrow F = \frac{\tau_0 BH}{\sqrt{\frac{16}{3}\frac{L^2}{H^2} + 1}} = \frac{\tau_0 BH}{\sqrt{\frac{3H^2 + 16L^2}{3H^2}}}$$

$L >> H$

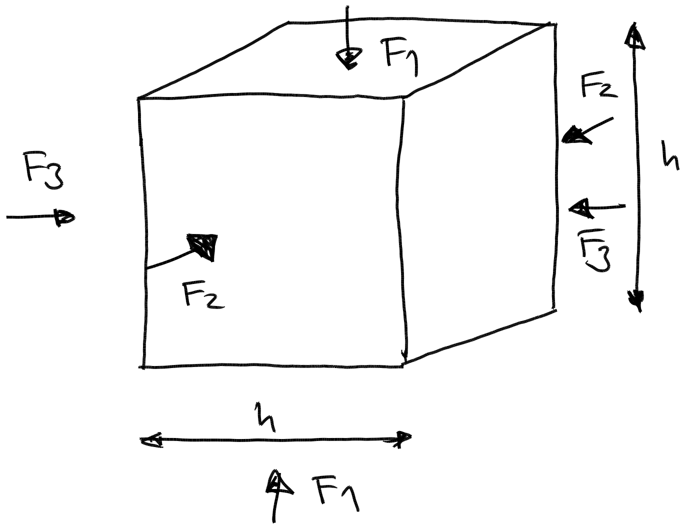
$$\rightarrow \frac{\tau_0 BH}{\sqrt{\frac{16L^2}{3H^2}}} = \frac{\tau_0 BH^2\sqrt{3}}{4L}$$

pouze ohyb: $\sigma^* = \frac{4FL}{BH^2} \rightarrow F = \frac{\tau_0\sqrt{3}\,BH^2}{4L}$, kde $\sigma^* \to \tau_0\sqrt{3}$

pouze smyk: $\tau^* = \tau_0 = \frac{F}{BH} \rightarrow F = \tau_0 BH$

$H >> L$

$$\frac{\tau_0 BH}{1}$$

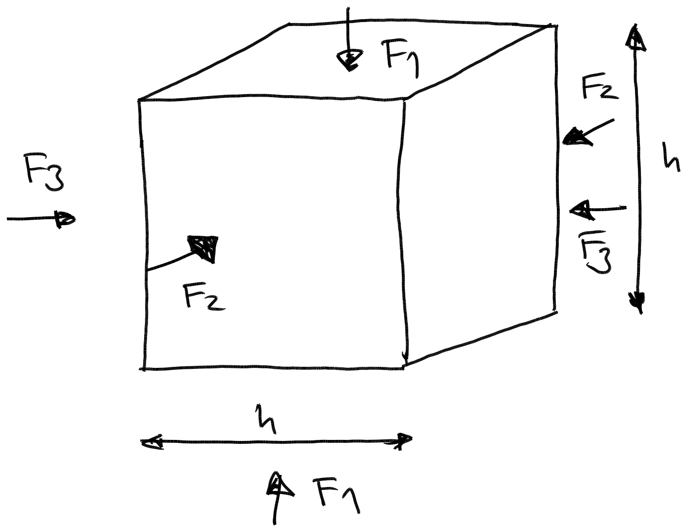


$h = 200$ mm

Pružnoplastický model s Trescovou podmínkou plasticity

1) $E, \nu = ?$

2) $\tau_0 = ?$



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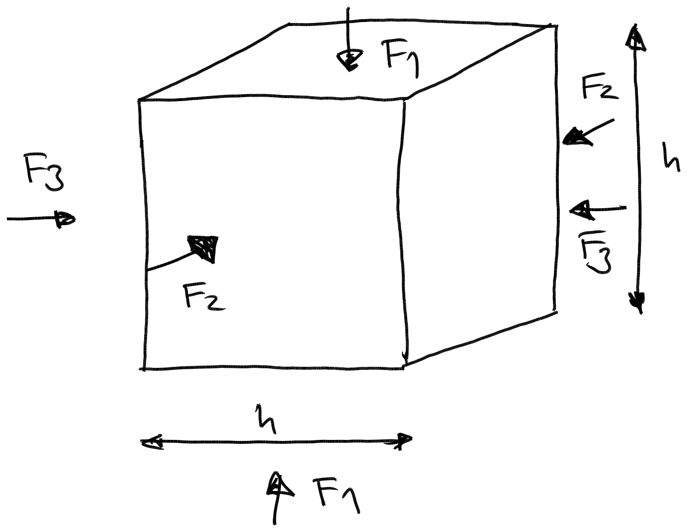
Pružnoplastický model s Trescovou podmínkou plasticity

1) $E, \nu = ?$

2) $\tau_0 = ?$

2 experiment $\longrightarrow$ pouze vodorovné síly $F_2 = F_3$, $\sigma_m = -3$ MPa, pružné chování, $\varepsilon_V = -0{,}06$, výška se zvětšila o 3 mm



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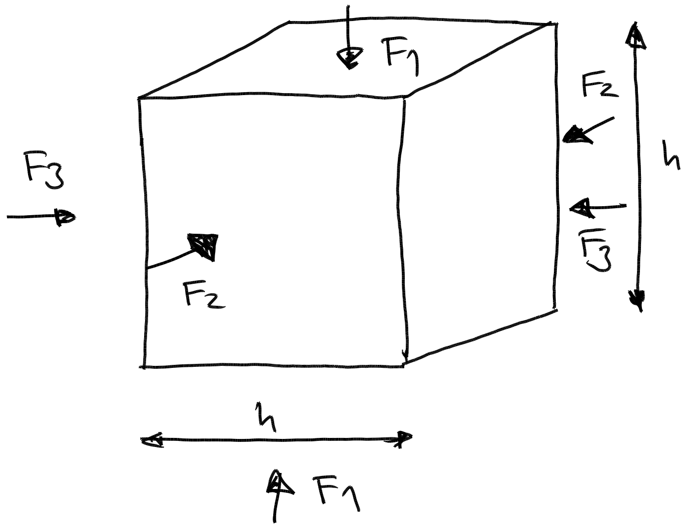
→ trojosá napjatost $F_1 = 100\ kN$, $F_2 = 200\ kN$, $F_3 = 300\ kN$ plastický stav



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Pružnoplastický model s Trescovou podmínkou plasticity

1) $E, \nu = ?$

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$\rightarrow$ trojosá napjatost $F_1 = 100$ kN, $F_2 = 200$ kN, $F_3 = 300$ kN plastický stav

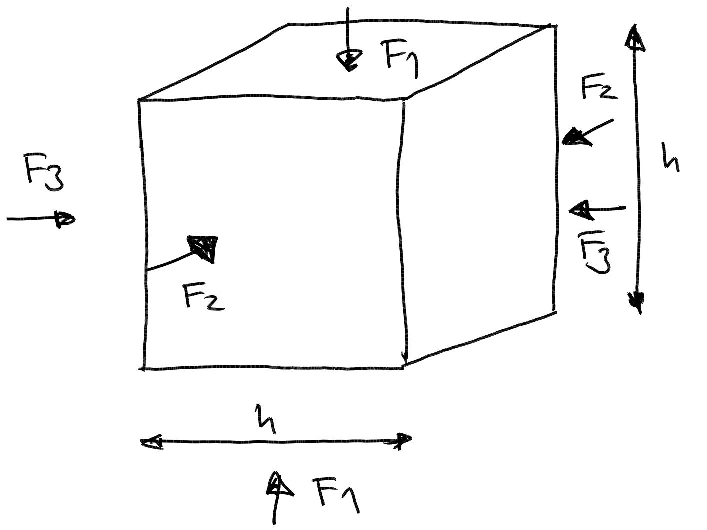
---

σ_m = střední napětí $= \dfrac{\sigma_x + \sigma_y + \sigma_z}{3}$

volba souřadnicového systému

$\rightarrow \sigma_z = 0, \; \sigma_x = \sigma_y$

$$\sigma_m = \frac{2\sigma_x}{3} \rightarrow \sigma_x = \frac{3\sigma_m}{2} = \frac{3(-3)}{2} = -4{,}5 \text{ MPa}$$



$h = 200\ mm$

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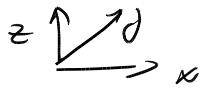
volba souřadnicového systému



$\rightarrow \sigma_z = 0,\ \sigma_x = \sigma_y$

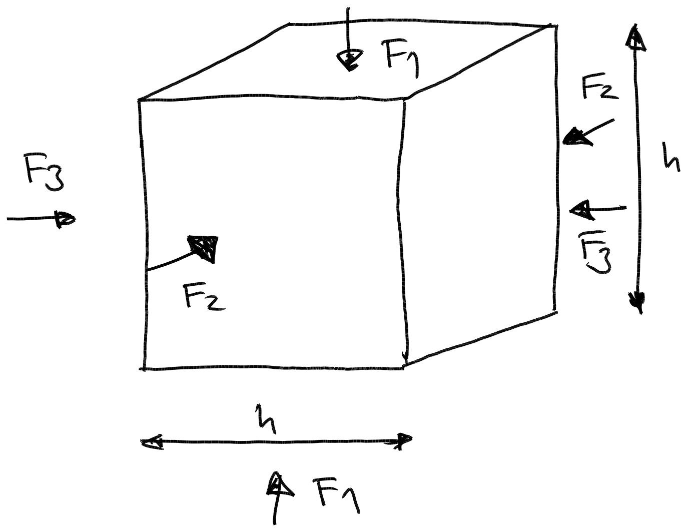
$$\sigma_m = \frac{2\sigma_x}{3} \rightarrow \sigma_x = \frac{3\sigma_m}{2} = \frac{3(-3)}{2} = -4{,}5\ MPa$$

ε_V = objemová deformace $= \varepsilon_x + \varepsilon_y + \varepsilon_z$

$\varepsilon_x = \varepsilon_y$

$\varepsilon_z = \dfrac{3}{200} = 0{,}015$

$$\varepsilon_V = 2\varepsilon_x + \varepsilon_z \rightarrow \varepsilon_x = \frac{\varepsilon_V - \varepsilon_z}{2} = \frac{-0{,}06 - 0{,}015}{2} = -0{,}0375$$



$h = 200$ mm

Pružnoplastický model s Trescovou podmínkou plasticity

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2) $\tau_0 = ?$

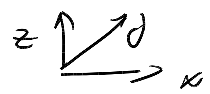
2 experimenty → pouze vodorovné síly $F_2 = F_3$, $\sigma_m = -3$ MPa, pružné chování, $\varepsilon_V = -0{,}06$, výška se zvětšila o 3 mm

→ trojosá napjatost $F_1 = 100$ kN, $F_2 = 200$ kN, $F_3 = 300$ kN plastický stav

---

$$\sigma_m = \text{střední napětí} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

volba souřadnicového systému



$\rightarrow \sigma_z = 0, \ \sigma_x = \sigma_y$

$$\sigma_m = \frac{2\sigma_x}{3} \rightarrow \sigma_x = \frac{3\sigma_m}{2} = \frac{3(-3)}{2} = -4{,}5 \text{ MPa}$$

$$\varepsilon_V = \text{objemová deformace} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

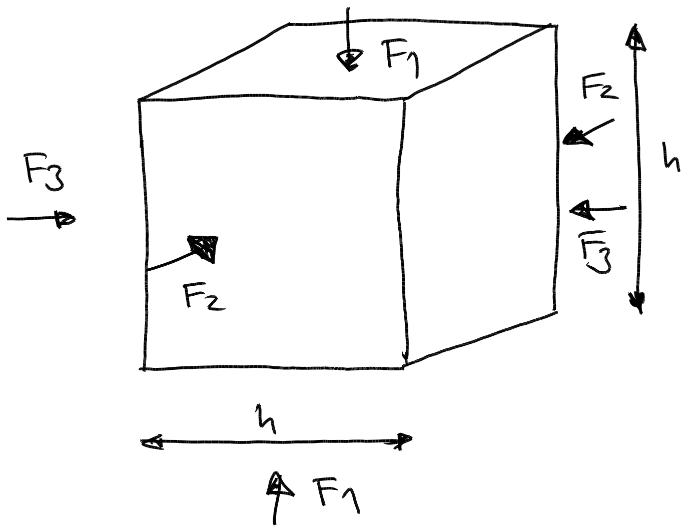
$\varepsilon_x = \varepsilon_y$

$$\varepsilon_z = \frac{3}{200} = 0{,}015$$

$$\varepsilon_V = 2\varepsilon_x + \varepsilon_z \rightarrow \varepsilon_x = \frac{\varepsilon_V - \varepsilon_z}{2} = \frac{-0{,}06 - 0{,}015}{2} = -0{,}0375$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu \overset{\sigma_x}{\cancel{\sigma_y}} - \nu \overset{0}{\cancel{\sigma_z}}\right) \qquad -0{,}0375 = \frac{1}{E}\left(-4{,}5(1-\nu)\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\underset{0}{\cancel{\sigma_z}} - \nu\sigma_x - \nu \underset{\sigma_x}{\cancel{\sigma_y}}\right) \qquad 0{,}015 = \frac{-2\nu(-4{,}5)}{E}$$



$h = 200\ \text{mm}$

Pružnoplastický model s Trescovou podmínkou plasticity

1) $E, \nu = ?$

2) $\tau_0 = ?$

2 experimenty → pouze vodorovné síly $F_2 = F_3$, $\sigma_m = -3\ \text{MPa}$, pružné chování, $\varepsilon_V = -0{,}06$, výška se zvětšila o 3 mm

→ trojosá napjatost $F_1 = 100\ \text{kN}$, $F_2 = 200\ \text{kN}$, $F_3 = 300\ \text{kN}$ plastický stav

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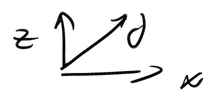
σ_m = střední napětí $= \dfrac{\sigma_x + \sigma_y + \sigma_z}{3}$

volba souřadnicového systému



$\to \sigma_z = 0,\ \sigma_x = \sigma_y$

$$\sigma_m = \frac{2\sigma_x}{3} \to \sigma_x = \frac{3\sigma_m}{2} = \frac{3(-3)}{2} = -4{,}5\ \text{MPa}$$

ε_V = objemová deformace $= \varepsilon_x + \varepsilon_y + \varepsilon_z$

$\varepsilon_x = \varepsilon_y$

$\varepsilon_z = \dfrac{3}{200} = 0{,}015$

$$\varepsilon_V = 2\varepsilon_x + \varepsilon_z \to \varepsilon_x = \frac{\varepsilon_V - \varepsilon_z}{2} = \frac{-0{,}06 - 0{,}015}{2} = -0{,}0375$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\cancel{\sigma_y}^{\,\sigma_x} - \nu\cancel{\sigma_z}^{\,0}\right) \qquad -0{,}0375 = \frac{1}{E}\left(-4{,}5(1-\nu)\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\cancel{\sigma_z}_{\,0} - \nu\sigma_x - \nu\cancel{\sigma_y}_{\,\sigma_x}\right) \qquad 0{,}015 = \frac{-2\nu(-4{,}5)}{E}$$

$$\left.\right\} \quad E = 100\ \text{MPa}, \quad \nu = \frac{1}{6}$$

Určení parametru Trescovy podmínky plasticity

$$\sigma_x = \frac{F_3}{A} = \frac{-0{,}3\ MN}{0{,}04\ m^2} = -7{,}5\ MPa$$

$$\sigma_y = \frac{F_2}{A} = \frac{-0{,}2}{0{,}04} = -5\ MPa$$

$$\sigma_z = \frac{F_1}{A} = \frac{-0{,}1}{0{,}04} = -2{,}5\ MPa$$

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Podmínka plasticity: $\tau_{max}(\sigma) - \overset{\text{parametr}}{\tau_0} = 0$

τ_{max} smykové napětí $= \dfrac{\sigma_{max} - \sigma_{min}}{2}$

$$\rightarrow \tau_{max} = \frac{-2{,}5 - (-7{,}5)}{2} = 2{,}5\ \mathrm{MPa} = \tau_0$$

Určení parametru Trescovy podmínky plasticity

$$\sigma_x = \frac{F_3}{A} = \frac{-0,3\ MN}{0,04\ m^2} = -7,5\ MPa$$

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parametr

Podmínka plasticity: $\tau_{max}(\sigma) - \tau_0 = 0$

τ_{max} smykové napětí $= \dfrac{\sigma_{max} - \sigma_{min}}{2}$

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3) ověření pružného chování při 1. zkoušce

$$\tau_{max} = \frac{0 - (-4,5)}{2} = 2,25\ MPa$$

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$\tau_{max} < \tau_0$ → materiál se chová pružně

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4) kolikrát se musí zvýšit zatížení 1. zkoušky, aby bylo dosaženo plast. stavu?

Určení parametru Trescovy podmínky plasticity

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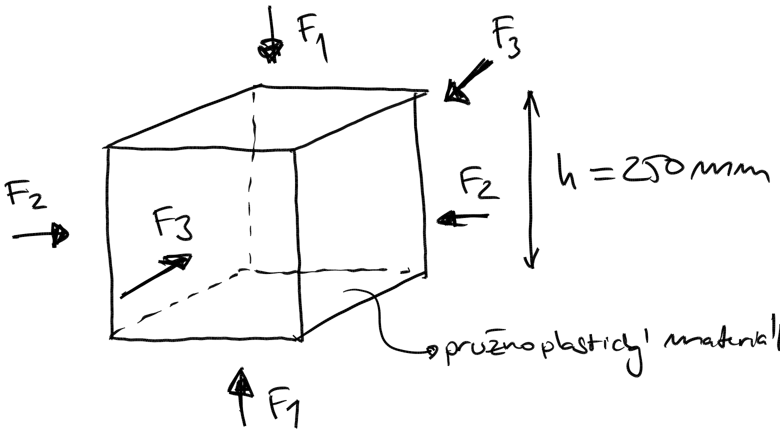
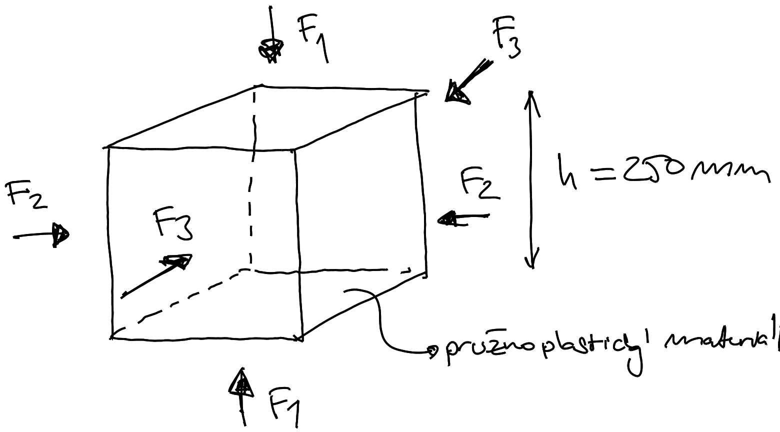
$$\frac{2{,}5}{2{,}25} = 1{,}111$$



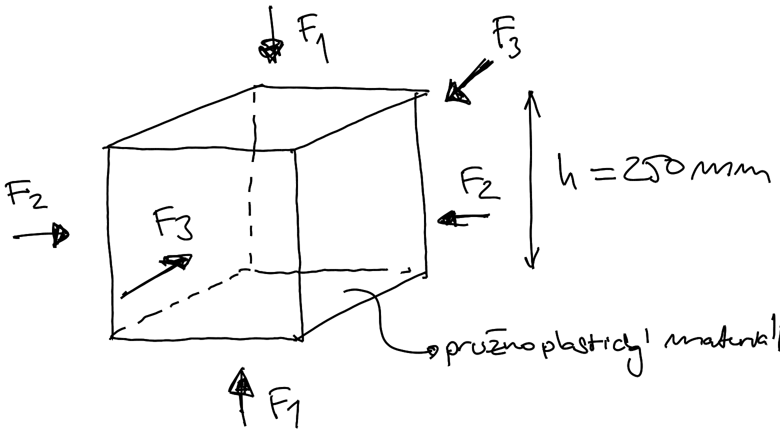
pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 20MPa$, $\nu = 0{,}1$



1 vzorek, zatěžování: • $F_1 = F_2 = F_3 = 100\ \mathrm{kN} \rightarrow \varepsilon_V = -0{,}00768$

• $F_1 = \text{konst}$, $F_2 \neq F_3$ rostou rychlostí 5 kN/s … F_2, 10 kN/s … F_3 až do dosažení plastického stavu.

pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 20\ \mathrm{MPa}$, $\nu = 0{,}1$



1 vzorek, zatěžování: • $F_1 = F_2 = F_3 = 100$ kN $\rightarrow$ $\varepsilon_V = -0{,}00768$

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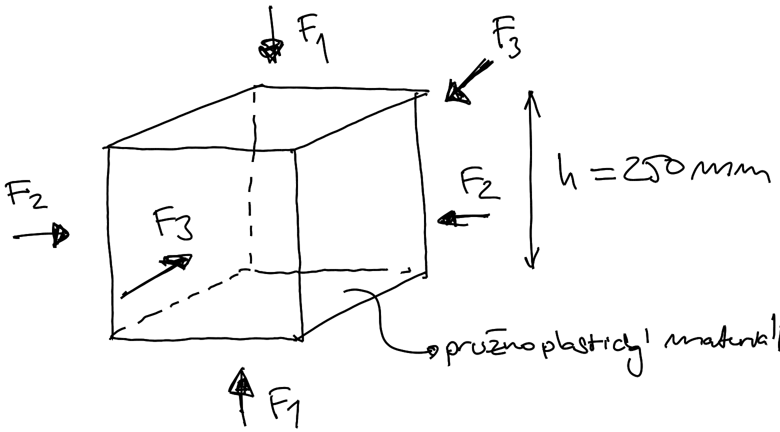
10 kN/s ... F_3

až do dosažení plastického stavu.

• pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 20$ MPa, $\nu = 0{,}1$

1) Určení modulu pružnosti, E



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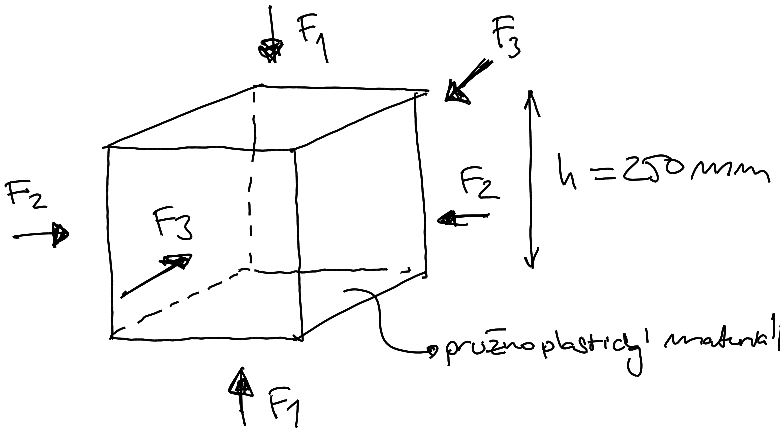
• pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 20$ MPa, $\nu = 0{,}1$

1) Určení modulu pružnosti, E

$$\sigma_x = \sigma_y = \sigma_z = \frac{-100 \cdot 10^{-3}}{0{,}25^2} = -1{,}6 \text{ MPa}$$



1 vzorek, zatěžování: • $F_1 = F_2 = F_3 = 100$ kN $\rightarrow$ $\varepsilon_V = -0{,}00768$

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až do dosažení plastického stavu.

pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 20$ MPa, $\nu = 0{,}1$

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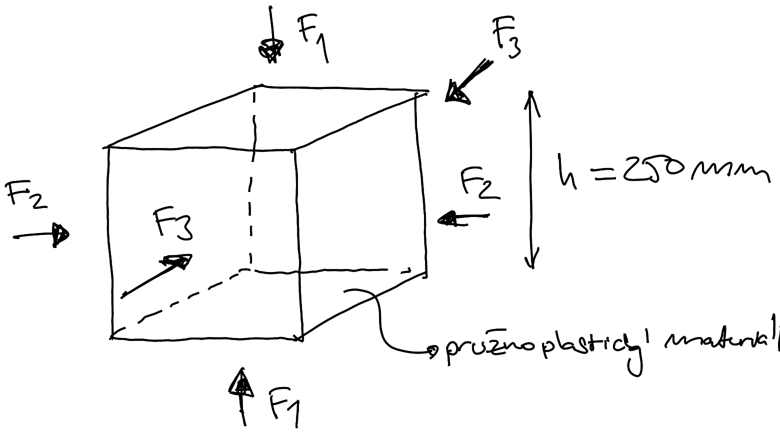
$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z = 3\varepsilon_x = 3\frac{\sigma}{E}(1-2\nu)$$



1 vzorek, zatěžování: • $F_1 = F_2 = F_3 = 100\,kN \rightarrow \varepsilon_V = -0{,}00768$

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až do dosažení plastického stavu.

pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 20\,MPa$, $\nu = 0{,}1$

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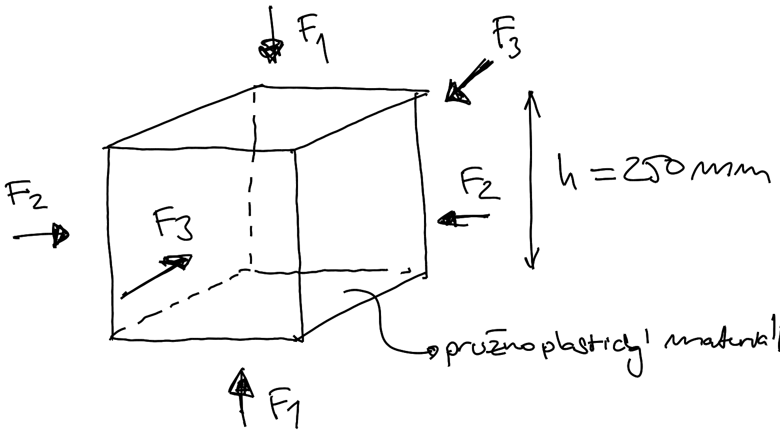
$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z = 3\varepsilon_x = 3\frac{\sigma}{E}(1-2\nu) \rightarrow E = \frac{3\sigma(1-2\nu)}{\varepsilon_V} = \frac{3\cdot(-1{,}6)\cdot(1-2\cdot 0{,}1)}{-0{,}00768} = \underline{500\ MPa}$$



1 vzorek, zatěžování: • $F_1 = F_2 = F_3 = 100\,kN \rightarrow \varepsilon_V = -0{,}00768$

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až do dosažení plastického stavu.

pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 2\,MPa$, $\nu = 0{,}1$

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2) velikost sil při dosažení plastického stavu

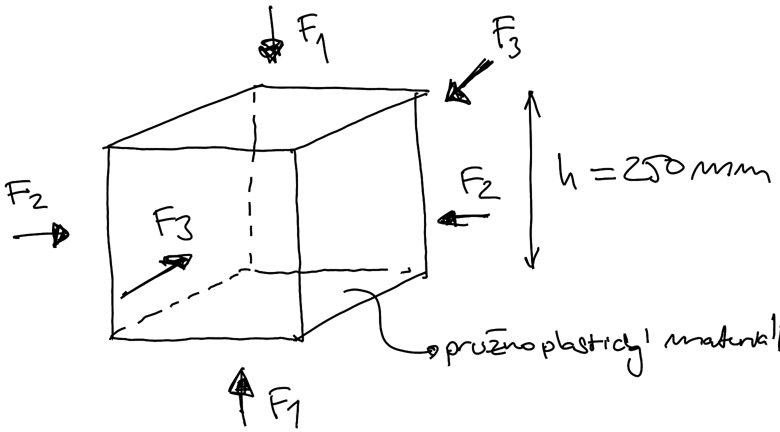
Misesova podmínka plasticity: $f(\sigma) = \sqrt{J_2} - \tau_0$

$$J_2 = \frac{s_1^2 + s_2^2 + s_3^2}{2}$$

$$J_2 = \frac{1}{2}(s_x^2 + s_y^2 + s_z^2) + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$J_2 = \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right]$$



1 vzorek, zatěžování: • $F_1 = F_2 = F_3 = 100\,kN \rightarrow \varepsilon_V = -0{,}00768$

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pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 20\,MPa$, $\nu = 0{,}1$

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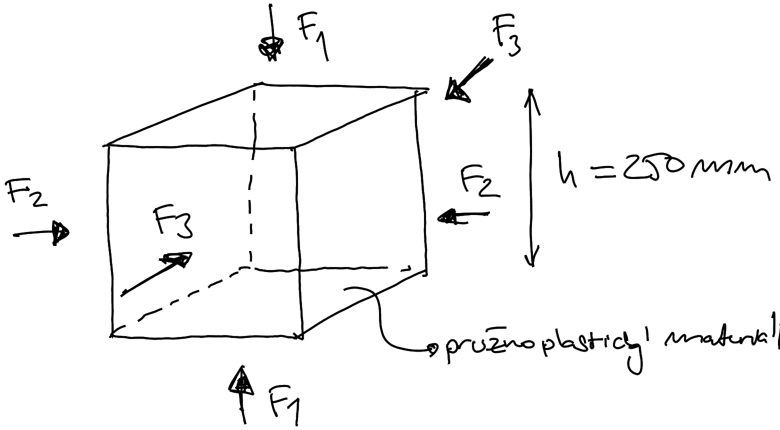
$$J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2\right]$$

$$\sigma_1 > \sigma_2 > \sigma_3$$

$\hookrightarrow \sigma \qquad \hookrightarrow \sigma - \Delta\sigma_2 \qquad \hookrightarrow \sigma - 2\Delta\sigma_2$



pružnoplastický materiál s Misesovou podmínkou plasticity, $\tau_0 = 20\,MPa$, $\nu = 0{,}1$

1 vzorek, zatěžování:

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až do dosažení plastického stavu.

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$$J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2\right]$$

$$\sigma_1 > \sigma_2 > \sigma_3$$

$$\sigma \qquad \sigma - \Delta\sigma_2 \qquad \sigma - 2\Delta\sigma_2$$

$$J_2 = \frac{1}{6}\left[(\Delta\sigma_2)^2 + (2\Delta\sigma_2)^2 + (\Delta\sigma_2)^2\right] = \Delta\sigma_2^2$$

$$\rightarrow \sqrt{\Delta\sigma_2^2} = \tau_0 = 20\,MPa$$

$$\Delta\sigma_2 = \pm 20\,MPa$$

$$\Delta F_2 = 20 \cdot 0{,}25^2 = 1{,}25\,MN$$

$$t = \frac{1{,}25}{5 \cdot 10^{-3}} = 250\,s$$

$$F_1 = 100\,kN$$

$$F_2 = 100 + 1250 = 1350\,kN$$

$$F_3 = 100 + 2500 = 2600\,kN$$

3) Varianta pro Trescovu podmínku plasticity s $\tau_0 = 20\ MPa$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_0$$

$$\Delta\sigma_3 = -2\tau_0 = -2 \cdot 20\ MPa = -40\ MPa$$

$$\Delta F_3 = 40\ MPa \cdot 0{,}25^2 = 2{,}5\ MN$$

$$\Delta F_2 = 1{,}25\ MN$$

$\rightarrow$ $F_1 = 0{,}1\ MN$

$F_2 = 1{,}35\ MN$

$F_3 = 2{,}6\ MN$