Podmínky plasticity pro materiály s vnitřním třením

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MOHR – COULOMB

DRUCKER – PRAGER

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$$f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0\cos\varphi$$

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c_0 – koheze

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$$f(\sigma) = 3\alpha_\varphi\sigma_m + \sqrt{J_2} - \tau_0 = 0$$

α_φ – součinitel vnitřního tření

τ_0 – mez kluzu ve smyku

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c_0 – koheze

→ speciální volba $\varphi = 0$

$$f(\sigma) = \frac{\sigma_{max}}{2} - \frac{\sigma_{min}}{2} - c_0 = \frac{\sigma_{max} - \sigma_{min}}{2} - c_0$$

$$= \tau_{max} - c_0$$ = TRESCOVA PODMÍNKA PLASTICITY

$(c_0 = \tau_0)$

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α_φ – součinitel vnitřního tření

τ_0 – mez kluzu ve smyku

→ speciální volba $\alpha_\varphi = 0$

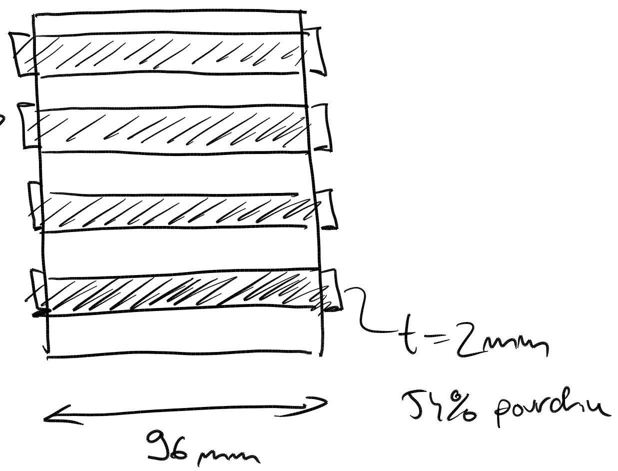
$f(\sigma) = 0 + \sqrt{J_2} - \tau_0 = 0$ = MISESOVA PODMÍNKA PLASTICITY

Hliník $E = 70\ GPa$

$\sigma_0 = 215\ MPa$

Beton $f_c = 60\ MPa$

$E = 40\ GPa$



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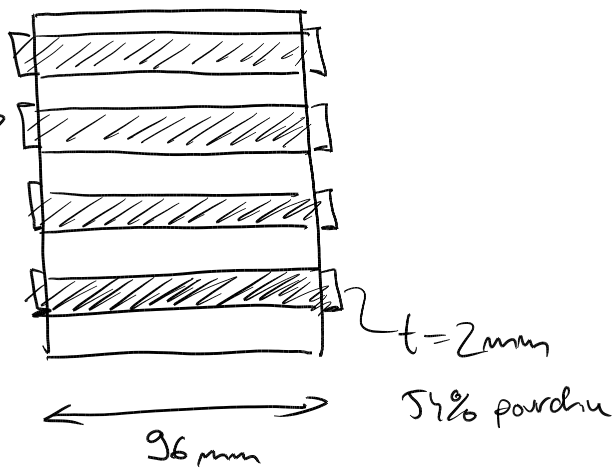

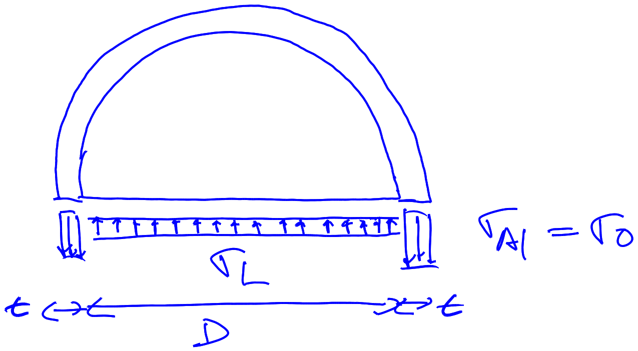

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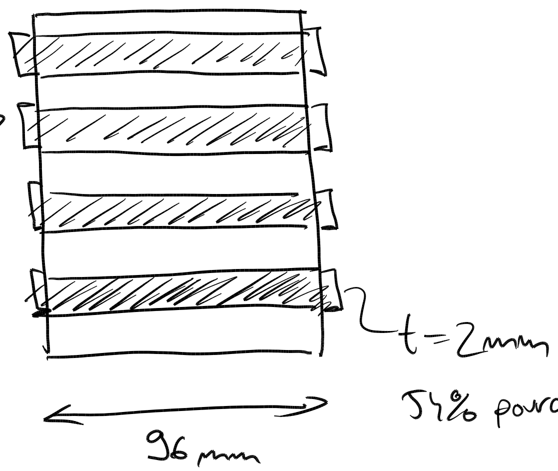

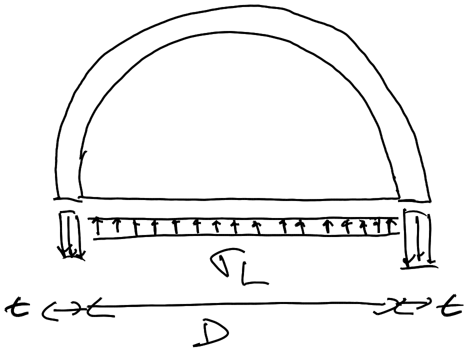

$\sigma_{Al} = \sigma_0$

podmínka rovnováhy: $2t\sigma_0 = \sigma_L D$

$$\rightarrow \sigma_L = \frac{2t\sigma_0}{D}$$

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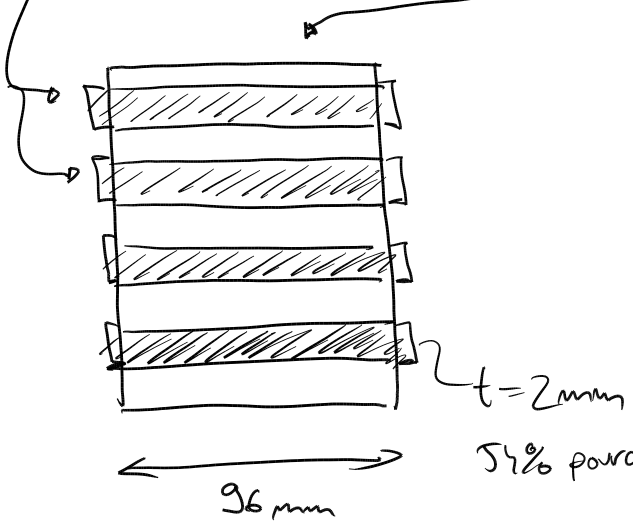
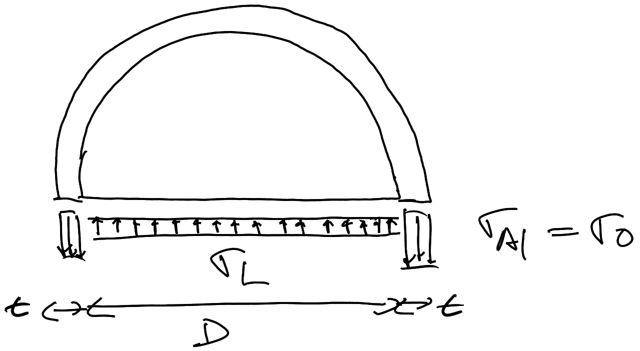


54% povrchu



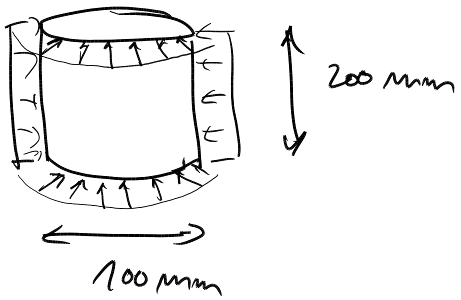
podmínka rovnováhy: $2t\sigma_0 = \sigma_L D$

$$\rightarrow \sigma_L = \frac{2t\sigma_0}{D}$$

pro 54% povrchu $\sigma_L = \dfrac{0{,}54 \cdot 2 \cdot 2 \cdot 215}{96} = 4{,}8375\ MPa$

Příklad

$\sigma_L = 5\,MPa$



$f_c = 60\,MPa$

$f_t = 6\,MPa$

$E = 40\,GPa$

$\nu = 0{,}2$

Příklad

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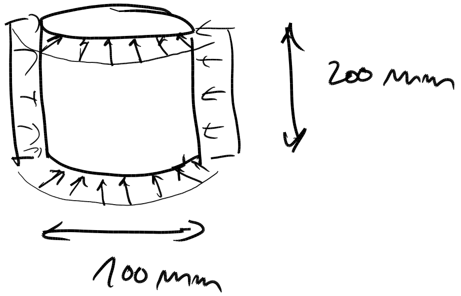

$f_c = 60\,MPa$

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1) Určit prodloužení výšky vzorku při zatížení příčným tlakem $\sigma_L = -5\,MPa$

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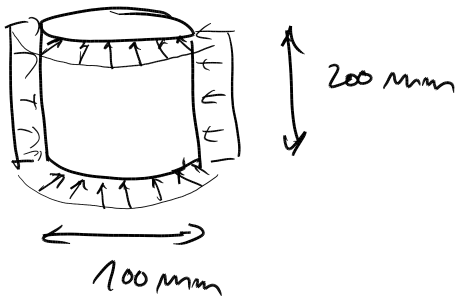

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svislý směr – A (axial)

vodorovný směr – L (lateral)

$$\varepsilon_A = \frac{1}{E}\left(\sigma_A - \nu\sigma_L - \nu\sigma_L\right) = \frac{1}{40\cdot 10^3}\left[0 - 0{,}2\,(-5) - 0{,}2\,(-5)\right] = 0{,}05\cdot 10^{-3}$$

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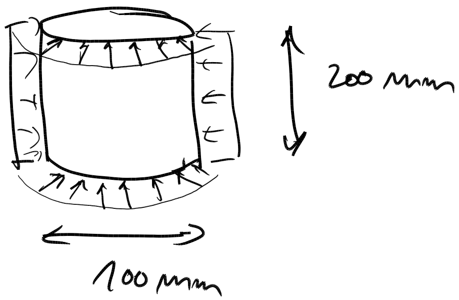

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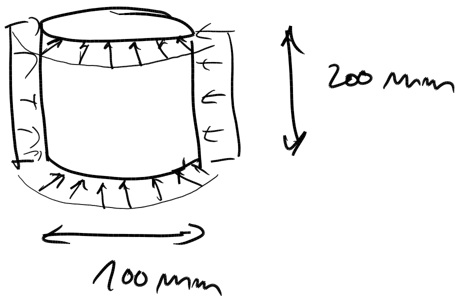

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2) Určit objemovou deformaci v tomto stavu

$$\varepsilon_V = \varepsilon_A + \varepsilon_L + \varepsilon_L$$

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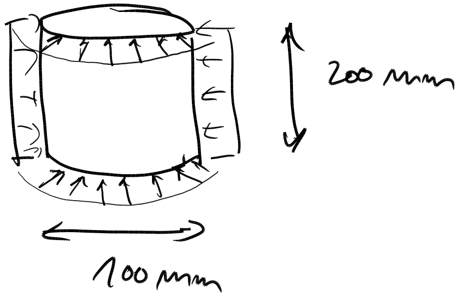

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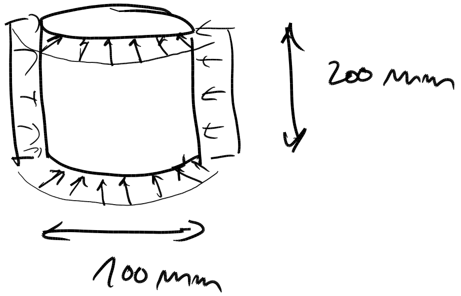

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$$\varepsilon_V = (0{,}05 - 0{,}1 - 0{,}1)\cdot 10^{-3} = \underline{-0{,}15\cdot 10^{-3}}$$

Výpočet parametrů M-C a D-P podmínky plasticity

1) MOHR-COULOMB

$$f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0 \cos\varphi$$

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$$\rightarrow 0 = \frac{1+\sin\varphi}{2}\cdot 6 - \frac{1-\sin\varphi}{2}\cdot 0 - c_0\cdot\cos\varphi \qquad \text{II}$$

I − II : $0 = -\frac{1+\sin\varphi}{2}\cdot 6 - \frac{1-\sin\varphi}{2}(-60)$

$$(1+\sin\varphi)\cdot 6 = (1-\sin\varphi)\,60$$

$$1+\sin\varphi = 10 - 10\sin\varphi$$

$$11\sin\varphi = 9$$

$$\sin\varphi = \frac{9}{11}$$

$$\varphi = 54{,}903^\circ$$

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$$\text{I}-\text{II}:\quad 0 = -\frac{1+\sin\varphi}{2}\cdot 6 - \frac{1-\sin\varphi}{2}(-60)$$

$$\text{I}:\quad c_0 = -\frac{1-\sin\varphi}{2\cdot\cos\varphi}(-60) = 30\,\frac{1-\sin\varphi}{\cos\varphi} = \underline{9{,}487\ MPa}$$

$$(1+\sin\varphi)\cdot 6 = (1-\sin\varphi)\,60$$

$$1+\sin\varphi = 10 - 10\sin\varphi$$

$$11\sin\varphi = 9$$

$$\sin\varphi = \frac{9}{11}$$

$$\underline{\varphi = 54{,}903^\circ}$$

2) DRUCKER - PRAGER $\qquad f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0$

$f_c = 60\ MPa \qquad (\sigma_1 = 0,\ \sigma_2 = 0,\ \sigma_3 = -60\ MPa)$

$f_t = 6\ MPa \qquad (\sigma_1 = 6\ MPa,\ \sigma_2 = 0,\ \sigma_3 = 0)$

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$f_t = 6$ MPa $\qquad (\sigma_1 = 6$ MPa$,\ \sigma_2 = 0,\ \sigma_3 = 0)$

$\rightarrow\ \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0 = 0$

$I_1 \hookrightarrow (0+0+(-60))$

$J_2 \rightarrow \frac{1}{6}\left[0^2 + 60^2 + 60^2\right] = 1200\ (\text{MPa}^2)$

$-60\alpha_\varphi + \sqrt{1200} - \tau_0 = 0$

$\rightarrow\ \alpha_\varphi(6+0+0) + \sqrt{\frac{1}{6}\left[6^2+6^2+0\right]} - \tau_0 = 0$

$6\alpha_\varphi + \sqrt{12} - \tau_0 = 0$

2) DRUCKER – PRAGER $\quad f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0$

$f_c = 60\ \text{MPa} \quad (\sigma_1 = 0,\ \sigma_2 = 0,\ \sigma_3 = -60\ \text{MPa})$

$f_t = 6\ \text{MPa} \quad (\sigma_1 = 6\ \text{MPa},\ \sigma_2 = 0,\ \sigma_3 = 0)$

$\rightarrow \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0 = 0$

$I_1 = (0+0+(-60)) \qquad J_2 = \frac{1}{6}\left[0^2 + 60^2 + 60^2\right] = 1200\ (\text{MPa}^2)$

$-60\alpha_\varphi + \sqrt{1200} - \tau_0 = 0 \quad (\text{I})$

$\rightarrow \alpha_\varphi(6+0+0) + \sqrt{\frac{1}{6}\left[6^2+6^2+0\right]} - \tau_0 = 0$

$6\alpha_\varphi + \sqrt{12} - \tau_0 = 0 \quad (\text{II})$

$(\text{II})-(\text{I}):\ 66\alpha_\varphi + \sqrt{12} - \sqrt{1200} = 0$

$$\alpha_\varphi = \frac{\sqrt{1200} - \sqrt{12}}{66} = \underline{\underline{0{,}4724}}$$

případně obecně:

$$\alpha_\varphi = \frac{f_c - f_t}{\sqrt{3}\,(f_t + f_c)} \ ;\ f_c > 0\ (!!!)$$

2) DRUCKER – PRAGER $\quad f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0$

$f_c = 60$ MPa $\quad (\sigma_1 = 0,\ \sigma_2 = 0,\ \sigma_3 = -60\ \text{MPa})$

$f_t = 6$ MPa $\quad (\sigma_1 = 6\ \text{MPa},\ \sigma_2 = 0,\ \sigma_3 = 0)$

$\rightarrow \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0 = 0$

$\quad I_1 = (0+0+(-60))$

$\quad J_2 = \frac{1}{6}\left[0^2 + 60^2 + 60^2\right] = 1200\ (\text{MPa}^2)$

$-60\alpha_\varphi + \sqrt{1200} - \tau_0 = 0 \quad (\text{I})$

$\rightarrow \alpha_\varphi(6+0+0) + \sqrt{\frac{1}{6}\left[6^2+6^2+0\right]} - \tau_0 = 0$

$6\alpha_\varphi + \sqrt{12} - \tau_0 = 0 \quad (\text{II})$

$(\text{II})-(\text{I})$: $66\alpha_\varphi + \sqrt{12} - \sqrt{1200} = 0$

$$\alpha_\varphi = \frac{\sqrt{1200} - \sqrt{12}}{66} = \underline{\underline{0{,}4724}}$$

případně obecně:

$$\alpha_\varphi = \frac{f_c - f_t}{\sqrt{3}(f_t + f_c)} \quad ; f_c > 0\ (!!!)$$

Dopočet τ_0 z (I):

$\tau_0 = \sqrt{1200} - 60 \cdot \alpha_\varphi = \sqrt{1200} - 60 \cdot 0{,}4724 =$

$= \underline{\underline{6{,}298\ \text{MPa}}}$

2) DRUCKER – PRAGER $\quad f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0$

$f_c = 60\ \mathrm{MPa} \quad (\sigma_1 = 0,\ \sigma_2 = 0,\ \sigma_3 = -60\ \mathrm{MPa})$

$f_t = 6\ \mathrm{MPa} \quad (\sigma_1 = 6\ \mathrm{MPa},\ \sigma_2 = 0,\ \sigma_3 = 0)$

$\rightarrow \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0 = 0$

$\hookrightarrow (0+0+(-60)) \qquad \rightarrow \frac{1}{6}\left[0^2 + 60^2 + 60^2\right] = 1200\ (\mathrm{MPa}^2)$

$-60\alpha_\varphi + \sqrt{1200} - \tau_0 = 0 \quad (\mathrm{I})$

$\rightarrow \alpha_\varphi(6+0+0) + \sqrt{\frac{1}{6}\left[6^2 + 6^2 + 0\right]} - \tau_0 = 0$

$6\alpha_\varphi + \sqrt{12} - \tau_0 = 0 \quad (\mathrm{II})$

$(\mathrm{II}) - (\mathrm{I}):\quad 66\alpha_\varphi + \sqrt{12} - \sqrt{1200} = 0$

$\alpha_\varphi = \frac{\sqrt{1200} - \sqrt{12}}{66} = \underline{\underline{0{,}4724}}$

připadně obecně:

$\alpha_\varphi = \frac{f_c - f_t}{\sqrt{3}(f_t + f_c)}\ ;\ f_c > 0\ (!!!)$

Dopočet τ_0 z (I):

$\tau_0 = \sqrt{1200} - 60 \cdot \alpha_\varphi = \sqrt{1200} - 60 \cdot 0{,}4724 =$

$= \underline{6{,}298\ \mathrm{MPa}}$

$\tau_0 = f_t\left(\alpha_\varphi + \frac{1}{\sqrt{3}}\right)$

Určení stavu pro D-P a M-C pro zadanou napjatost!

$\sigma_x = 5\,MPa$

$\sigma_y = -5\,MPa$

$\sigma_z = -10\,MPa$

Určení stavu pro D-P a M-C pro zadanou napjatost:

$\sigma_x = 5\,MPa$

$\sigma_y = -5\,MPa$

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$f_{MC}(\sigma) = 0 \rightarrow$ plastický stav

Určení stavu pro D-P a M-C pro zadanou napjatost:

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$f_{MC}(\sigma) = 0$ → plastický stav

$f_{DP}(\sigma) = -3{,}38$ → pružný stav

Určení stavu pro D-P a M-C pro zadanou napjatost:

$\sigma_x = 5\,MPa$

$\sigma_y = -5\,MPa$

$\sigma_z = -10\,MPa$

$\sigma_x = 3\,MPa$

$\sigma_y = \sigma_x$

$\sigma_z = -16{,}5\,MPa$

$f_{MC}(\sigma) = 0 \rightarrow$ plastický stav

$f_{DP}(\sigma) = -3{,}38 \rightarrow$ pružný stav

Určení stavu pro D-P a M-C pro zadanou napjatost:

$\sigma_x = 5\,MPa$

$\sigma_y = -5\,MPa$

$\sigma_z = -10\,MPa$

$\sigma_x = 3\,MPa$

$\sigma_y = \sigma_x$

$\sigma_z = -16{,}5\,MPa$

$f_{MC}(\sigma) = 0 \rightarrow$ plastický stav

$f_{DP}(\sigma) = -3{,}38 \rightarrow$ pružný stav

$f_{MC}(\sigma) = -1{,}23 \rightarrow$ pružný stav

Určení stavu pro D-P a M-C pro zadanou napjatost!

$\sigma_x = 5\,MPa$

$\sigma_y = -5\,MPa$

$\sigma_z = -10\,MPa$

$\sigma_x = 3\,MPa$

$\sigma_y = \sigma_x$

$\sigma_z = -16{,}5\,MPa$

$f_{MC}(\sigma) = 0 \rightarrow$ plastický stav

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$f_{MC}(\sigma) = -1{,}23 \rightarrow$ pružný stav

$f_{DP}(\sigma) = 0 \rightarrow$ plastický stav

Výpočet svislého napětí, při kterém je dosaženo plastického stavu:

1) MOHR–COULOMB $\quad f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0\cos\varphi$; $\varphi = 54{,}903^\circ$, $c_0 = 9{,}487$ MPa

Výpočet svislého napětí, při kterém je dosaženo plastického stavu:

1) MOHR–COULOMB

$$f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0\cos\varphi \;;\; \varphi = 54{,}903^\circ,\; c_0 = 9{,}487\ \text{MPa}$$

$\sigma_{max} = -5\ \text{MPa}$

$\sigma_{min} = ?$

$$\sigma_{min} = \frac{-c_0 \cdot \cos\varphi + \frac{1+\sin\varphi}{2}\sigma_{max}}{\frac{1-\sin\varphi}{2}} = -110\ \text{MPa}$$

Výpočet svislého napětí, při kterém je dosaženo plastického stavu:

1) MOHR–COULOMB

$$f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0\cos\varphi \; ; \quad \varphi = 54{,}903^{\circ}, \quad c_0 = 9{,}487 \text{ MPa}$$

$\sigma_{max} = -5$ MPa

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2) DRUCKER-PRAGER

$$f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0 \; ; \quad \alpha_\varphi = 0{,}4724, \quad \tau_0 = 6{,}298 \text{ MPa}$$

Výpočet svislého napětí, při kterém je dosaženo plastického stavu:

1) Mohr–Coulomb $$f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0\cos\varphi \quad ; \quad \varphi = 54{,}903^\circ, \; c_0 = 9{,}487\ \text{MPa}$$

$\sigma_{max} = -5\ \text{MPa}$

$\sigma_{min} = ?$

$$\sigma_{min} = \frac{-c_0 \cdot \cos\varphi + \frac{1+\sin\varphi}{2}\sigma_{max}}{\frac{1-\sin\varphi}{2}} = -110\ \text{MPa}$$

2) Drucker–Prager $$f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0 \quad ; \quad \alpha_\varphi = 0{,}4724, \; \tau_0 = 6{,}298\ \text{MPa}$$

$I_1 = 2\sigma_L + \sigma_A$

$$J_2 = \frac{1}{6}\left[(\sigma_A - \sigma_L)^2 + (\sigma_A - \sigma_L)^2 + \cancel{(\sigma_L - \sigma_L)^2}\right] = \frac{(\sigma_A - \sigma_L)^2}{3}$$

Výpočet svislého napětí, při kterém je dosaženo plastického stavu:

1) MOHR–COULOMB $\quad f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0\cos\varphi$; $\varphi = 54{,}903°$, $c_0 = 9{,}487$ MPa

$\sigma_{max} = -5$ MPa

$\sigma_{min} = ?$ $\quad \sigma_{min} = \dfrac{-c_0\cdot\cos\varphi + \frac{1+\sin\varphi}{2}\sigma_{max}}{\frac{1-\sin\varphi}{2}} = -110$ MPa

2) DRUCKER–PRAGER $\quad f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0$; $\alpha_\varphi = 0{,}4724$, $\tau_0 = 6{,}298$ MPa

$I_1 = 2\sigma_L + \sigma_A$

$J_2 = \frac{1}{6}\left[(\sigma_A-\sigma_L)^2 + (\sigma_A-\sigma_L)^2 + \cancel{(\sigma_L-\sigma_L)^2}\right] = \frac{(\sigma_A-\sigma_L)^2}{3}$

$\alpha_\varphi(2\sigma_L+\sigma_A) \pm (\sigma_A-\sigma_L)\cdot\frac{1}{\sqrt{3}} - \tau_0 = 0$

$\rightarrow 2\sigma_L\alpha_\varphi - \frac{\sigma_L}{\sqrt{3}} + \sigma_A\alpha_\varphi + \frac{\sigma_A}{\sqrt{3}} - \tau_0 = 0 \rightarrow \sigma_{A_1} = \dfrac{\sigma_L\left(\frac{1}{\sqrt{3}} - 2\alpha_\varphi\right) + \tau_0}{\frac{1}{\sqrt{3}} + \alpha_\varphi}$

Výpočet svislého napětí, při kterém je dosaženo plastického stavu:

1) MOHR–COULOMB $f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0\cos\varphi$; $\varphi = 54{,}903°$, $c_0 = 9{,}487$ MPa

$\sigma_{max} = -5$ MPa

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2) DRUCKER–PRAGER $f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0$; $\alpha_\varphi = 0{,}4724$, $\tau_0 = 6{,}298$ MPa

$I_1 = 2\sigma_L + \sigma_A$

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$\alpha_\varphi(2\sigma_L + \sigma_A) \pm (\sigma_A - \sigma_L) \cdot \frac{1}{\sqrt{3}} - \tau_0 = 0$

$\rightarrow 2\sigma_L\alpha_\varphi - \frac{\sigma_L}{\sqrt{3}} + \sigma_A\alpha_\varphi + \frac{\sigma_A}{\sqrt{3}} - \tau_0 = 0 \rightarrow \sigma_{A_1} = \dfrac{\sigma_L\left(\frac{1}{\sqrt{3}} - 2\alpha_\varphi\right) + \tau_0}{\frac{1}{\sqrt{3}} + \alpha_\varphi}$

$\rightarrow 2\sigma_L\alpha_\varphi + \frac{\sigma_L}{\sqrt{3}} + \sigma_A\left(\alpha_\varphi - \frac{1}{\sqrt{3}}\right) - \tau_0 = 0 \rightarrow \sigma_{A_2} = \dfrac{\sigma_L\left(-\frac{1}{\sqrt{3}} - 2\alpha_\varphi\right) + \tau_0}{\alpha_\varphi - \frac{1}{\sqrt{3}}}$

Výpočet svislého napětí, při kterém je dosaženo plastického stavu:

1) MOHR–COULOMB

$$f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0\cos\varphi \;;\; \varphi = 54{,}903^\circ ,\; c_0 = 9{,}487\ \mathrm{MPa}$$

$\sigma_{max} = -5\ \mathrm{MPa}$

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$$\sigma_{min} = \frac{-c_0\cdot\cos\varphi + \frac{1+\sin\varphi}{2}\sigma_{max}}{\frac{1-\sin\varphi}{2}} = -110\ \mathrm{MPa}$$

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$$f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0 \;;\; \alpha_\varphi = 0{,}4724 ,\; \tau_0 = 6{,}298\ \mathrm{MPa}$$

$I_1 = 2\sigma_L + \sigma_A$

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$$\alpha_\varphi(2\sigma_L+\sigma_A) \pm (\sigma_A-\sigma_L)\cdot\frac{1}{\sqrt{3}} - \tau_0 = 0$$

$$\rightarrow 2\sigma_L\alpha_\varphi - \frac{\sigma_L}{\sqrt{3}} + \sigma_A\alpha_\varphi + \frac{\sigma_A}{\sqrt{3}} - \tau_0 = 0 \rightarrow \sigma_{A_1} = \frac{\sigma_L\left(\frac{1}{\sqrt{3}} - 2\alpha_\varphi\right) + \tau_0}{\frac{1}{\sqrt{3}} + \alpha_\varphi} = 7{,}75\ \mathrm{MPa}$$

$$\rightarrow 2\sigma_L\alpha_\varphi + \frac{\sigma_L}{\sqrt{3}} + \sigma_A\left(\alpha_\varphi - \frac{1}{\sqrt{3}}\right) - \tau_0 = 0 \rightarrow \sigma_{A_2} = \frac{\sigma_L\left(-\frac{1}{\sqrt{3}} - 2\alpha_\varphi\right) + \tau_0}{\alpha_\varphi - \frac{1}{\sqrt{3}}} = \underline{\underline{-132{,}5\ \mathrm{MPa}}}$$

Výpočet svislého napětí, při kterém je dosaženo plastického stavu:

1) **MOHR–COULOMB** $\quad f(\sigma) = \frac{1+\sin\varphi}{2}\sigma_{max} - \frac{1-\sin\varphi}{2}\sigma_{min} - c_0 \cos\varphi$; $\varphi = 54{,}903^\circ$, $c_0 = 9{,}487$ MPa

$\sigma_{max} = -5$ MPa

$\sigma_{min} = ?$

$$\sigma_{min} = \frac{-c_0 \cdot \cos\varphi + \frac{1+\sin\varphi}{2}\sigma_{max}}{\frac{1-\sin\varphi}{2}} = -110 \text{ MPa}$$

2) **DRUCKER–PRAGER** $\quad f(\sigma) = \alpha_\varphi I_1 + \sqrt{J_2} - \tau_0$; $\alpha_\varphi = 0{,}4724$, $\tau_0 = 6{,}298$ MPa

$I_1 = 2\sigma_L + \sigma_A$

$$J_2 = \frac{1}{6}\left[(\sigma_A - \sigma_L)^2 + (\sigma_A - \sigma_L)^2 + \cancel{(\sigma_L - \sigma_L)^2}\right] = \frac{(\sigma_A - \sigma_L)^2}{3}$$

$$\alpha_\varphi(2\sigma_L + \sigma_A) \pm (\sigma_A - \sigma_L)\cdot\frac{1}{\sqrt{3}} - \tau_0 = 0$$

$$\rightarrow 2\sigma_L\alpha_\varphi - \frac{\sigma_L}{\sqrt{3}} + \sigma_A\alpha_\varphi + \frac{\sigma_A}{\sqrt{3}} - \tau_0 = 0 \rightarrow \sigma_{A1} = \frac{\sigma_L\left(\frac{1}{\sqrt{3}} - 2\alpha_\varphi\right) + \tau_0}{\frac{1}{\sqrt{3}} + \alpha_\varphi} = 7{,}75 \text{ MPa}$$

$$\rightarrow 2\sigma_L\alpha_\varphi + \frac{\sigma_L}{\sqrt{3}} + \sigma_A\left(\alpha_\varphi - \frac{1}{\sqrt{3}}\right) - \tau_0 = 0 \rightarrow \sigma_{A2} = \frac{\sigma_L\left(-\frac{1}{\sqrt{3}} - 2\alpha_\varphi\right) + \tau_0}{\alpha_\varphi - \frac{1}{\sqrt{3}}} = \underline{\underline{-132{,}5 \text{ MPa}}}$$

Pro okamžik dosažení plastického stavu (D-P) dopočítat pružnou deformaci

$\sigma_L = -5\,MPa$ $\nu = 0{,}2$

$\sigma_A = -132{,}5\,MPa$ $E = 40\,GPa$

Pro okamžik dosažení plastického stavu (D-P) dopočítat pružnou deformaci

$\sigma_L = -5\,MPa$ $\qquad$ $\nu = 0{,}2$

$\sigma_A = -132{,}5\,MPa$ $\qquad$ $E = 40\,GPa$

$$\varepsilon_A = \frac{1}{E}\left[\sigma_A - 2\nu\sigma_L\right] = \frac{1}{40\cdot 10^3}\left[-132{,}5 - 2\nu(-5)\right] = -3{,}2625\cdot 10^{-3}$$

Pro okamžik dosažení plastického stavu (D-P) dopočítat pružnou deformaci

$\sigma_L = -5\,MPa$ $\quad \nu = 0{,}2$

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$$\varepsilon_L = \frac{1}{E}\left[\sigma_L - \nu\sigma_L - \nu\sigma_A\right] = \frac{1}{40\cdot 10^3}\left[-5 - 0{,}2\cdot(-5) - 0{,}2\cdot(-132{,}5)\right] = 0{,}5625\cdot 10^{-3}$$

Pro okamžik dosažení plastického stavu (D-P) dopočítat pružnou deformaci

$\sigma_L = -5\,\text{MPa}$ $\quad\quad \nu = 0{,}2$

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objemová deformace

$$\varepsilon_V = \varepsilon_A + 2\varepsilon_L = -2{,}1375\cdot 10^{-3}$$

Pro okamžik dosažení plastického stavu (D-P) dopočítat pružnou deformaci

$\sigma_L = -5\,MPa$ $\nu = 0{,}2$

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Objemová deformace

$$\varepsilon_V = \varepsilon_A + 2\varepsilon_L = -2{,}1375\cdot 10^{-3}$$

Zatížení pokračuje, poměr složek napětí je konstantní

→ vypočítat plastickou deformaci pro okamžik, kdy $\varepsilon_A = -1\,\%$

Pro okamžik dosažení plastického stavu (D-P) dopočítat pružnou deformaci

$\sigma_L = -5\,MPa$ $\nu = 0{,}2$

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pro sdružený zákon plastického přetváření

$$\dot{\varepsilon}_p = \dot{\lambda}\,\frac{\partial f(\sigma)}{\partial \sigma}$$

Pro okamžik dosažení plastického stavu (D-P) dopočítat pružnou deformaci

$\sigma_L = -5\,MPa$ $\quad\nu = 0{,}2$

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$$\varepsilon_L = \frac{1}{E}\left[\sigma_L - \nu\sigma_L - \nu\sigma_A\right] = \frac{1}{40\cdot 10^3}\left[-5 - 0{,}2\cdot(-5) - 0{,}2\cdot(-132{,}5)\right] = 0{,}5625\cdot 10^{-3}$$

objemová deformace

$$\varepsilon_V = \varepsilon_A + 2\varepsilon_L = -2{,}1375\cdot 10^{-3}$$

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NESDRUŽENÝ ZÁKON

$$\dot{\varepsilon}_p = \dot{\lambda}\,\frac{\partial g(\sigma)}{\partial \sigma}$$

Pro okamžik dosažení plastického stavu (D-P) dopočítat pružnou deformaci

$\sigma_L = -5\,MPa$ $\quad\quad\nu = 0{,}2$

$\sigma_A = -132{,}5\,MPa$ $\quad\quad E = 40\,GPa$

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Objemová deformace

$$\varepsilon_V = \varepsilon_A + 2\varepsilon_L = -2{,}1375\cdot 10^{-3}$$

Zatížení pokračuje, poměr složek napětí je konstantní

$\rightarrow$ vypočítat plastickou deformaci pro okamžik, kdy $\varepsilon_A = -1\,\%$

pro sdružený zákon plastického přetváření

$$\dot{\varepsilon}_p = \dot{\lambda}\,\frac{\partial f(\sigma)}{\partial \sigma}$$

NESDRUŽENÝ ZÁKON

$$\dot{\varepsilon}_p = \dot{\lambda}\,\frac{\partial g(\sigma)}{\partial \sigma}$$

$\rightarrow$ pro D-P zákon $\alpha_\phi \rightarrow \alpha_\psi$

$\hookrightarrow$ součinitel dilatace

$$\frac{\partial g(\sigma)}{\partial \sigma_\alpha} = \frac{\partial I_1}{\partial \sigma_\alpha} \alpha_\psi + \frac{\partial \sqrt{J_2}}{\partial \sigma_\alpha}$$

$$\frac{\partial g(\sigma)}{\partial \sigma_x} = \frac{\partial I_1}{\partial \sigma_x} \alpha_\varphi + \frac{\partial \sqrt{J_2}}{\partial \sigma_x}$$

$$\frac{\partial(\sigma_x + \sigma_y + \sigma_z)}{\partial \sigma_x} = 1 + 0 + 0$$

$$\frac{\partial g(\sigma)}{\partial \sigma_x} = \frac{\partial I_1}{\partial \sigma_x} \alpha_\psi + \frac{\partial \sqrt{J_2}}{\partial \sigma_x}$$

$$\frac{\partial(\sigma_x + \sigma_y + \sigma_z)}{\partial \sigma_x} = 1+0+0$$

$$\frac{1}{2\sqrt{J_2}} \frac{\partial J_2}{\partial \sigma_x}$$

$$\frac{\partial \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right]}{\partial \sigma_x} = \frac{1}{6}\left[2(\sigma_x - \sigma_y) + 2(\sigma_x - \sigma_z)\right]$$

$$\frac{\partial g(\sigma)}{\partial \sigma_x} = \frac{\partial I_1}{\partial \sigma_x}\alpha_\varphi + \frac{\partial \sqrt{J_2}}{\partial \sigma_x}$$

$$\frac{\partial(\sigma_x+\sigma_y+\sigma_z)}{\partial \sigma_x} = 1+0+0$$

$$\frac{1}{2\sqrt{J_2}}\underbrace{\frac{\partial J_2}{\partial \sigma_x}}$$

$$\frac{\partial \frac{1}{6}\left[(\sigma_x-\sigma_y)^2+(\sigma_x-\sigma_z)^2+(\sigma_y-\sigma_z)^2\right]}{\partial \sigma_x} = \frac{1}{6}\left[2(\sigma_x-\sigma_y)+2(\sigma_x-\sigma_z)\right] =$$

$$= \frac{1}{3}\left[2\sigma_x-\sigma_y-\sigma_z\right]$$

$$\frac{\partial g(\sigma)}{\partial \sigma_x} = \frac{\partial I_1}{\partial \sigma_x} \alpha_\psi + \frac{\partial \sqrt{J_2}}{\partial \sigma_x}$$

$$\frac{\partial (\sigma_x + \sigma_y + \sigma_z)}{\partial \sigma_x} = 1 + 0 + 0$$

$$\frac{1}{2\sqrt{J_2}} \underbrace{\frac{\partial J_2}{\partial \sigma_x}}$$

$$\frac{\partial \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right]}{\partial \sigma_x} = \frac{1}{6}\left[2(\sigma_x - \sigma_y) + 2(\sigma_x - \sigma_z)\right] =$$

$$= \frac{1}{3}\left[2\sigma_x - \sigma_y - \sigma_z\right] =$$

$$= \frac{1}{3}\left[3\sigma_x - 3\sigma_m\right] = s_x$$

$$\frac{\partial g(\sigma)}{\partial \sigma_x} = \frac{\partial I_1}{\partial \sigma_x}\alpha_\varphi + \frac{\partial \sqrt{J_2}}{\partial \sigma_x}$$

$$\frac{\partial(\sigma_x+\sigma_y+\sigma_z)}{\partial \sigma_x} = 1+0+0$$

$$\frac{1}{2\sqrt{J_2}}\underbrace{\frac{\partial J_2}{\partial \sigma_x}}$$

$$\frac{\partial \frac{1}{6}\left[(\sigma_x-\sigma_y)^2+(\sigma_x-\sigma_z)^2+(\sigma_y-\sigma_z)^2\right]}{\partial \sigma_x} = \frac{1}{6}\left[2(\sigma_x-\sigma_y)+2(\sigma_x-\sigma_z)\right] =$$

$$= \frac{1}{3}\left[2\sigma_x - \sigma_y - \sigma_z\right] =$$

$$= \frac{1}{3}\left[3\sigma_x - 3\sigma_m\right] = S_x$$

$$\Rightarrow \frac{\partial g(\sigma)}{\partial \sigma_x} = \alpha_\varphi + \frac{S_x}{2\sqrt{J_2}} \qquad \frac{\partial g(\sigma)}{\partial \sigma_y} = \alpha_\varphi + \frac{S_y}{2\sqrt{J_2}}$$

$$\frac{\partial g(\sigma)}{\partial \sigma_x} = \frac{\partial I_1}{\partial \sigma_x}\alpha_\varphi + \frac{\partial \sqrt{J_2}}{\partial \sigma_x}$$

$$\frac{\partial(\sigma_x+\sigma_y+\sigma_z)}{\partial \sigma_x} = 1+0+0$$

$$\frac{1}{2\sqrt{J_2}}\underbrace{\frac{\partial J_2}{\partial \sigma_x}}$$

$$\frac{\partial \frac{1}{6}\left[(\sigma_x-\sigma_y)^2+(\sigma_x-\sigma_z)^2+(\sigma_y-\sigma_z)^2\right]}{\partial \sigma_x} = \frac{1}{6}\left[2(\sigma_x-\sigma_y)+2(\sigma_x-\sigma_z)\right] =$$

$$= \frac{1}{3}\left[2\sigma_x-\sigma_y-\sigma_z\right] =$$

$$= \frac{1}{3}\left[3\sigma_x-3\sigma_m\right] = s_x$$

$$\Rightarrow \frac{\partial g(\sigma)}{\partial \sigma_x} = \alpha_\varphi + \frac{s_x}{2\sqrt{J_2}} \qquad \frac{\partial g(\sigma)}{\partial \sigma_y} = \alpha_\varphi + \frac{s_y}{2\sqrt{J_2}}$$

pro $f = g$ $\quad \dfrac{\partial g(\sigma)}{\partial \sigma_x} = \alpha_\varphi + \dfrac{s_x}{2\sqrt{J_2}}$

$\longrightarrow$ zpět k příkladu: $\sigma_m = \frac{1}{3}\left(-5 + (-5) + (-132{,}5)\right) = -47{,}5 \text{ MPa}$

$s_x = „s_L" = \sigma_L - \sigma_m = -5 - (-47{,}5) = 42{,}5 \text{ MPa}$

$s_z = „s_A" = \sigma_A - \sigma_m = -85 \text{ MPa}$

$\longrightarrow$ zpět k příkladu: $\sigma_m = \frac{1}{3}\left(-5 + (-5) + (-132{,}5)\right) = -47{,}5\ \text{MPa}$

$$S_x = \text{"}S_L\text{"} = \sigma_L - \sigma_m = -5 - (-47{,}5) = 42{,}5\ \text{MPa}$$

$$S_z = \text{"}S_A\text{"} = \sigma_A - \sigma_m = -85\ \text{MPa}$$

$$J_2 = \frac{1}{6}\left[(\sigma_A - \sigma_L)^2 + (\sigma_A - \sigma_L)^2 + (\sigma_L - \sigma_L)^2\right] = \frac{1}{3}(\sigma_A - \sigma_L)^2$$

$\longrightarrow$ zpět k příkladu: $\sigma_m = \frac{1}{3}\left(-5 + (-5) + (-132{,}5)\right) = -47{,}5 \text{ MPa}$

$$s_x = \text{“}s_L\text{“} = \sigma_L - \sigma_m = -5 - (-47{,}5) = 42{,}5 \text{ MPa}$$

$$s_z = \text{“}s_A\text{“} = \sigma_A - \sigma_m = -85 \text{ MPa}$$

$$J_2 = \frac{1}{6}\left[(\sigma_A - \sigma_L)^2 + (\sigma_A - \sigma_L)^2 + (\sigma_L - \sigma_L)^2\right] = \frac{1}{3}(\sigma_A - \sigma_L)^2 = 5418{,}75 \text{ MPa}^2$$

$$\sqrt{J_2} = 73{,}61 \text{ MPa}$$

$$\frac{\partial f(\sigma)}{\partial \sigma_x} = \underbrace{\alpha_\varphi}_{0{,}472} + \frac{42{,}5}{2 \cdot 73{,}61} = 0{,}761$$

$\longrightarrow$ zpět k příkladu: $\sigma_m = \frac{1}{3}\left(-5+(-5)+(-132{,}5\right) = -47{,}5\ MPa$

$S_x = \text{"}S_L\text{"} = \sigma_L - \sigma_m = -5-(-47{,}5) = 42{,}5\ MPa$

$S_z = \text{"}S_A\text{"} = \sigma_A - \sigma_m = -85\ MPa$

$J_2 = \frac{1}{6}\left[(\sigma_A-\sigma_L)^2+(\sigma_A-\sigma_L)^2+(\sigma_L-\sigma_L)^2\right] = \frac{1}{3}(\sigma_A-\sigma_L)^2 = 5418{,}75\ MPa^2$

$\sqrt{J_2} = 73{,}61\ MPa$

$$\frac{\partial f(\sigma)}{\partial \sigma_x} = \underbrace{\alpha_\varphi}_{0{,}472} + \frac{42{,}5}{2\cdot 73{,}61} = 0{,}761$$

$$\frac{\partial f(\sigma)}{\partial \sigma_z} = 0{,}472 + \frac{-85}{2\cdot 73{,}61} = -0{,}105$$

$\longrightarrow$ zpět k příkladu: $\sigma_m = \frac{1}{3}\left(-5 + (-5) + (-132{,}5)\right) = -47{,}5 \text{ MPa}$

$$S_x = \text{"}S_L\text{"} = \sigma_L - \sigma_m = -5 - (-47{,}5) = 42{,}5 \text{ MPa}$$

$$S_z = \text{"}S_A\text{"} = \sigma_A - \sigma_m = -85 \text{ MPa}$$

$$J_2 = \frac{1}{6}\left[(\sigma_A - \sigma_L)^2 + (\sigma_A - \sigma_L)^2 + (\sigma_L - \sigma_L)^2\right] = \frac{1}{3}(\sigma_A - \sigma_L)^2 = 5418{,}75 \text{ MPa}^2$$

$$\sqrt{J_2} = 73{,}61 \text{ MPa}$$

$$\frac{\partial f(\sigma)}{\partial \sigma_x} = \underbrace{\alpha_\varphi}_{0{,}472} + \frac{42{,}5}{2 \cdot 73{,}61} = 0{,}761$$

$$\frac{\partial f(\sigma)}{\partial \sigma_z} = 0{,}472 + \frac{-85}{2 \cdot 73{,}61} = -0{,}105$$

pokud $\varepsilon_A^{tot} = -0{,}01 = \varepsilon_A^{el} + \varepsilon_A^{pl}$

pak $\lambda = \dfrac{\varepsilon_A^{tot} - \varepsilon_A^{el}}{\partial f / \partial \sigma_A} = \dfrac{-0{,}01 - (-3{,}2625 \cdot 10^{-3})}{-0{,}105} = 0{,}0642$

$\longrightarrow$ zpět k příkladu: $\sigma_m = \frac{1}{3}\left(-5+(-5)+(-132{,}5)\right) = -47{,}5\ \mathrm{MPa}$

$s_x = \text{"}s_L\text{"} = \sigma_L - \sigma_m = -5-(-47{,}5) = 42{,}5\ \mathrm{MPa}$

$s_z = \text{"}s_A\text{"} = \sigma_A - \sigma_m = -85\ \mathrm{MPa}$

$$J_2 = \frac{1}{6}\left[(\sigma_A-\sigma_L)^2+(\sigma_A-\sigma_L)^2+(\sigma_L-\sigma_L)^2\right] = \frac{1}{3}(\sigma_A-\sigma_L)^2 = 5418{,}75\ \mathrm{MPa}^2$$

$\sqrt{J_2} = 73{,}61\ \mathrm{MPa}$

$$\frac{\partial f(\sigma)}{\partial \sigma_x} = \underbrace{\alpha_\varphi}_{0{,}472} + \frac{42{,}5}{2\cdot 73{,}61} = 0{,}761$$

$$\frac{\partial f(\sigma)}{\partial \sigma_z} = 0{,}472 + \frac{-85}{2\cdot 73{,}61} = -0{,}105$$

pokud $\varepsilon_A^{tot} = -0{,}01 = \varepsilon_A^{el} + \varepsilon_A^{pl}$

pak $\lambda = \dfrac{\varepsilon_A^{tot} - \varepsilon_A^{el}}{\partial f/\partial \sigma_A} = \dfrac{-0{,}01-(-3{,}2625\cdot 10^{-3})}{-0{,}105} = 0{,}0642$

$\rightarrow \varepsilon_{pl,L} = \lambda \cdot \dfrac{\partial f(\sigma)}{\partial \sigma_x} = 0{,}0642 \cdot 0{,}761 = 0{,}0488$

$\varepsilon_{pl,A} = -0{,}006738$

kdyby chom mít $g(\sigma) \neq f(\sigma)$

$\alpha_\varphi = \alpha_\varphi / 2 = 0{,}1236$, pak $\dfrac{\partial g}{\partial \sigma_x} = 0{,}5249$

$\dfrac{\partial g}{\partial \sigma_z} = -0{,}3412$

kdybychom měli $g(\sigma) \neq f(\sigma)$

$$\alpha_\varphi = \alpha\varphi/2 = 0{,}236 \,, \quad \text{pak} \quad \frac{\partial g}{\partial \sigma_x} = 0{,}5249$$

$$\frac{\partial g}{\partial \sigma_z} = -0{,}3412$$

$\lambda = 0{,}01375$

$\varepsilon_{pl,A} = -0{,}0067375$

$\varepsilon_{pl,L} = 0{,}0104$

kdybychom měli $g(\sigma) \neq f(\sigma)$

$$\alpha_\psi = \alpha_\varphi / 2 = 0{,}236 \, , \quad \text{pak} \quad \frac{\partial g}{\partial \sigma_x} = 0{,}5249$$

$$\frac{\partial g}{\partial \sigma_z} = -0{,}3412$$

$\lambda = 0{,}01375$

$\varepsilon_{pl,A} = -0{,}0067375$

$\varepsilon_{pl,L} = 0{,}0104$

pro $g = f$

$\varepsilon_V^{pl} = 0{,}091$

pro $g \neq f$

$\varepsilon_V^{pl} = 0{,}014$ → 6,5x snížení rychlosti $\varepsilon_{pl,V}$

Příklad : směr plastické deformace pro jednoosý tlak a jednoosý tah

TLAK : $\sigma_z = -60$ MPa $\rightarrow \sigma_m = -20$ MPa $\rightarrow s_z = -40$ MPa

$\sigma_x = 0$ MPa

$s_x = 20$ MPa

Příklad: směr plastické deformace pro jednoosý tlak a jednoosý tah

TLAK: $\sigma_z = -60\ MPa$ $\rightarrow \sigma_m = -20\ MPa$ $\rightarrow s_z = -40\ MPa$

$\sigma_x = 0\ MPa$ $\qquad s_x = 20\ MPa$

$J_2 = 1200\ MPa^2$

$$\frac{\partial f}{\partial \sigma_z} = -0{,}105$$

$$\frac{\partial f}{\partial \sigma_x} = 0{,}761$$

Příklad : směr plastické deformace pro jednoosý tlak a jednoosý tah

TLAK : $\sigma_z = -60$ MPa $\rightarrow \sigma_m = -20$ MPa $\rightarrow s_z = -40$ MPa

$\sigma_x = 0$ MPa

$s_x = 20$ MPa

$J_2 = 1200$ MPa^2

$$\frac{\partial f}{\partial \sigma_z} = -0{,}105 \qquad \frac{\partial g}{\partial \sigma_z} = -0{,}341$$

$$\frac{\partial f}{\partial \sigma_x} = 0{,}761 \qquad \frac{\partial g}{\partial \sigma_x} = 0{,}525$$

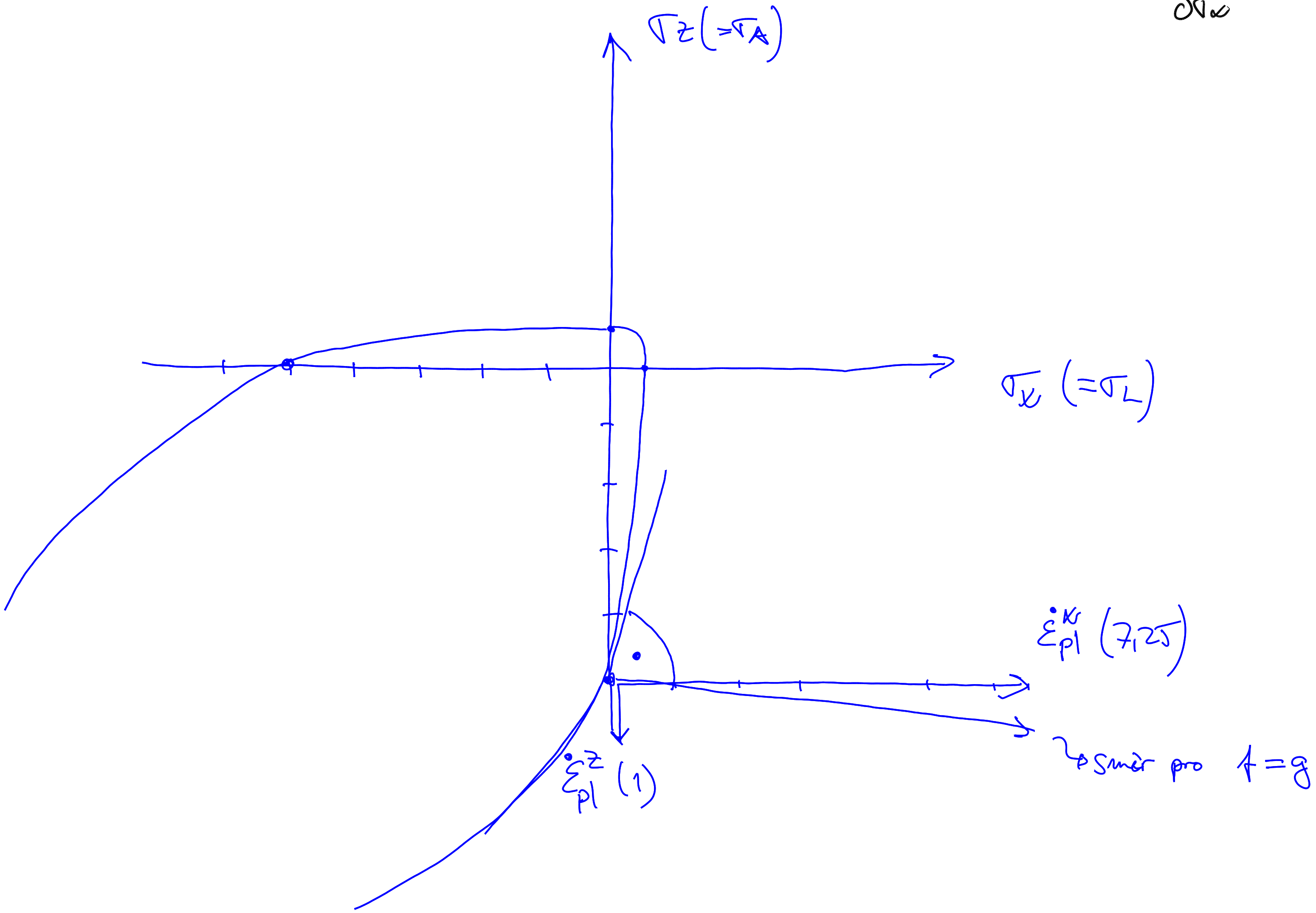
Příklad: směr plastické deformace pro jednoosý tlak a jednoosý tah

TLAK: $\sigma_z = -60\ \mathrm{MPa}$, $\sigma_x = 0\ \mathrm{MPa}$ $\rightarrow \sigma_m = -20\ \mathrm{MPa}$ $\rightarrow s_z = -40\ \mathrm{MPa}$, $s_x = 20\ \mathrm{MPa}$

$J_2 = 1200\ \mathrm{MPa}^2$

$$\frac{\partial f}{\partial \sigma_z} = -0{,}105 \qquad \frac{\partial g}{\partial \sigma_z} = -0{,}341$$

$$\frac{\partial f}{\partial \sigma_x} = 0{,}761 \qquad \frac{\partial g}{\partial \sigma_x} = 0{,}525$$



Příklad: <u>směr</u> plastické deformace pro <u>jednoosý</u> <u>tlak</u> a <u>jednoosý</u> <u>tah</u>

TLAK: $\sigma_z = -60$ MPa $\rightarrow \sigma_m = -20$ MPa $\rightarrow s_z = -40$ MPa

$\sigma_x = 0$ MPa

$s_x = 20$ MPa

$J_2 = 1200$ MPa^2

$$\frac{\partial f}{\partial \sigma_z} = -0{,}105 \qquad \frac{\partial g}{\partial \sigma_z} = -0{,}341$$

$$\frac{\partial f}{\partial \sigma_x} = 0{,}761 \qquad \frac{\partial g}{\partial \sigma_x} = 0{,}525$$