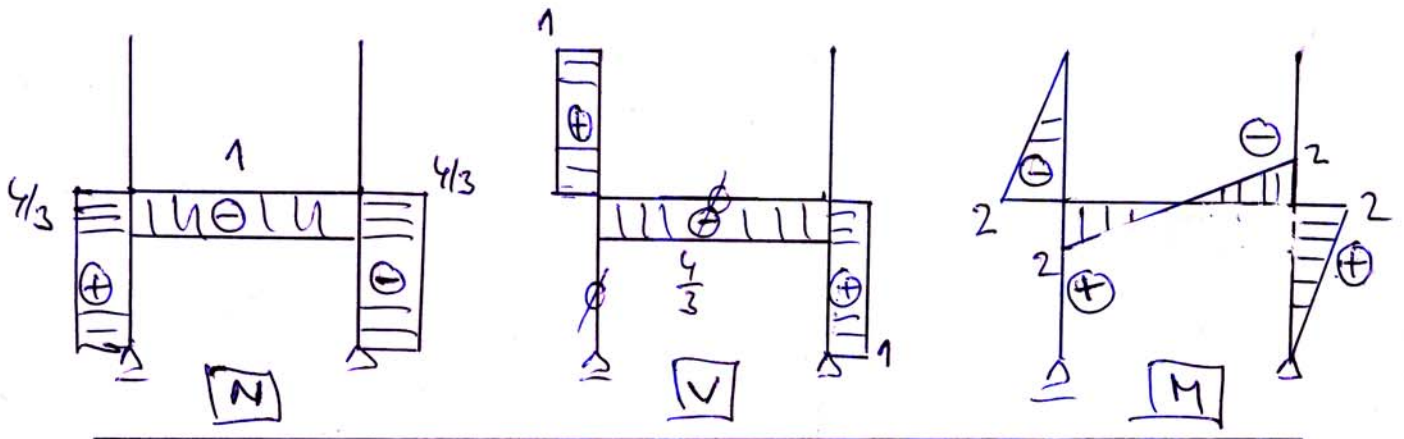
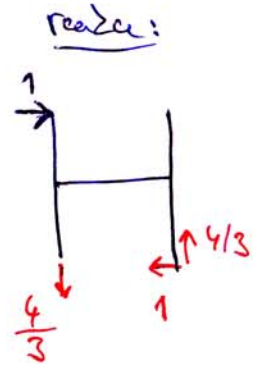
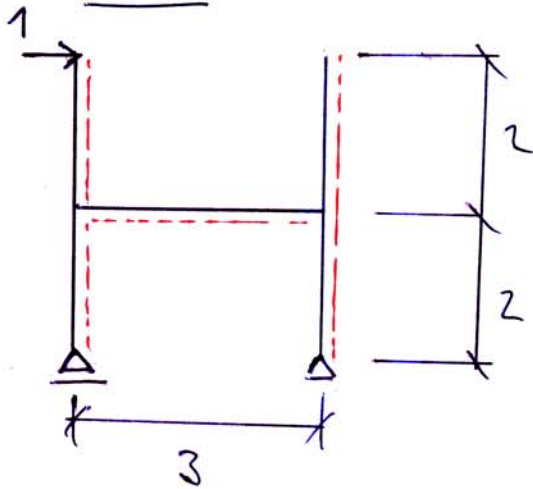
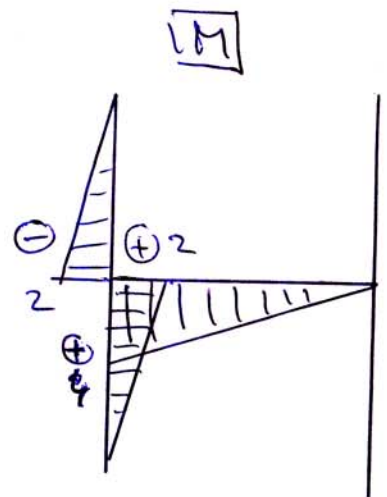
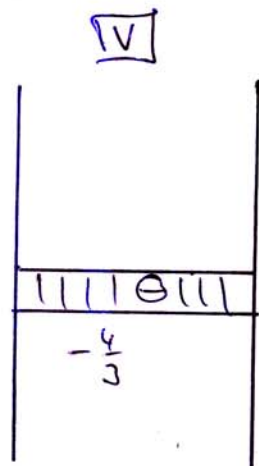
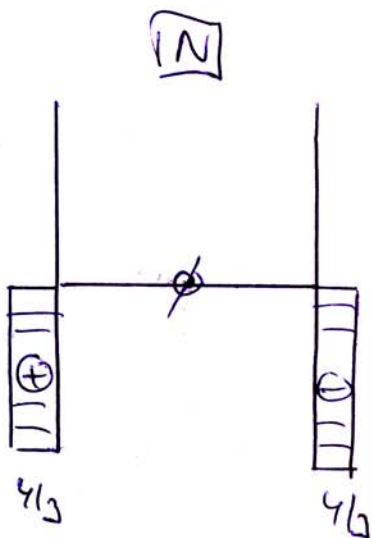
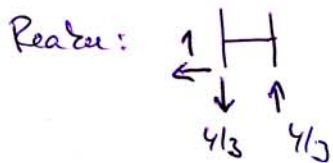


# Předpříklad

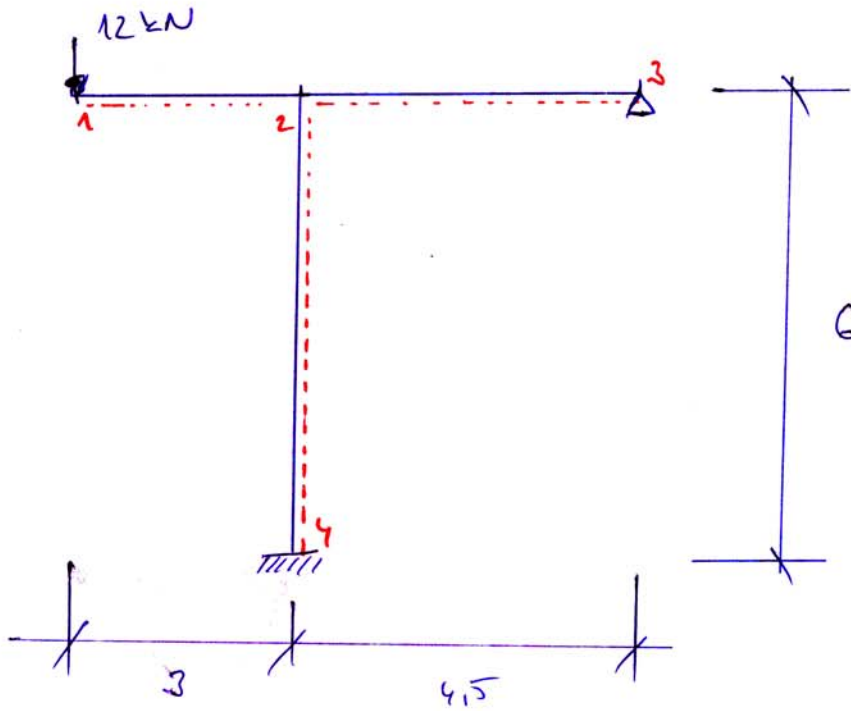
Zadání:



Modifikace - co se stane, když je průběhím posuvnou a neposuvnou podporou?



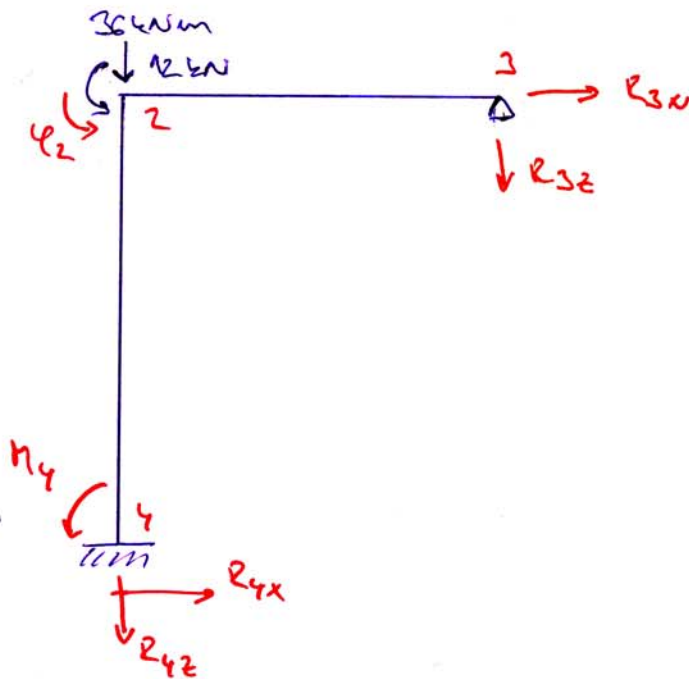
Zadání: Určete reakce a průběhy vnitřních sil.  
Výsledky uveďte.

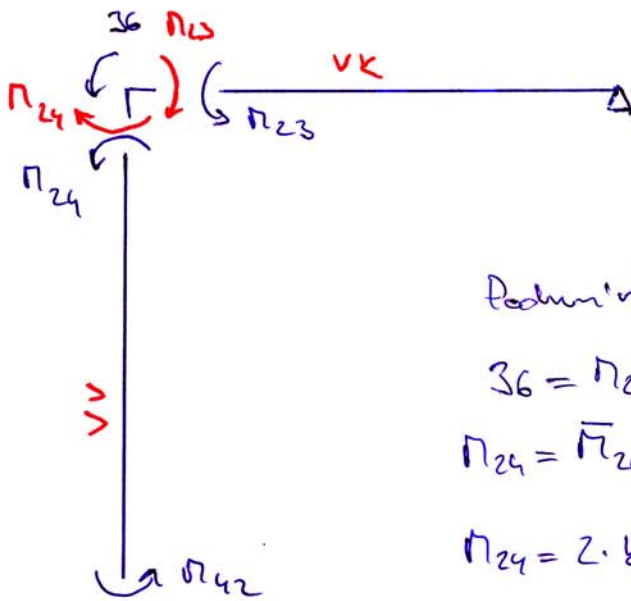


tuhost  $EI =$   
 $= 9000 \text{ kNm}^2 =$   
 $= 9 \text{ MNm}^2$

konstruovat zjednodušeně:

zavědeme nezápornou, zavědeme reakce





Podmínka rovnováhy:

$$36 = M_{24} + M_{23}$$

$$M_{24} = \bar{M}_{24} + 2 \left( \varphi_2 + \varphi_4 + 3 \frac{u_2 - u_4}{L} \right)$$

$$M_{24} = 2 \cdot k \cdot \varphi_2 = \frac{4EI\varphi_2}{L} = \frac{4EI\varphi_2}{6}$$

$$M_{23} = \bar{M}_{23} + \frac{3Z}{2} \left( \varphi_2 + \frac{u_3 - u_2}{L} \right) = \text{prot } V_k$$

$$M_{23} = \frac{3k}{2} \varphi_2 = \frac{3EI\varphi_2}{L} = \frac{3EI\varphi_2}{4.5}$$

Výčet rovnice:  $36 = \frac{4 \cdot 9 \cdot \varphi_2}{6} + \frac{3 \cdot 9 \cdot \varphi_2}{4.5}$

$$36 = 6\varphi_2 + 6\varphi_2$$

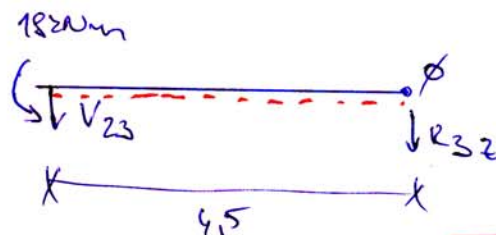
depočet koncové momenty:  $\varphi_2 = 3 \text{ mrad}$

$$M_{24} = 6\varphi_2 = 18 \text{ kNm}$$

$$M_{23} = 6\varphi_2 = 18 \text{ kNm}$$

$$M_{42} = k \cdot \varphi_2 = \frac{2EI}{L} \cdot \varphi_2 = \frac{2 \cdot 9}{6} \cdot 3 = 9 \text{ kNm} = M_4$$

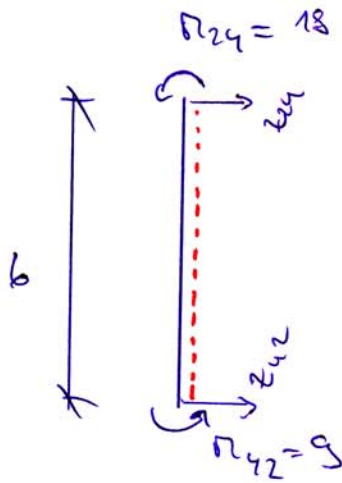
Účet rovnováhy:



$$\sum F_z: 18 - R_{32} \cdot 4.5 = 0 \Rightarrow R_{32} = 4 \text{ kN}$$

$$V_{23} = -4 \text{ kN}$$

$$N_{23} + \frac{18}{4.5} = +4 \text{ kN}$$



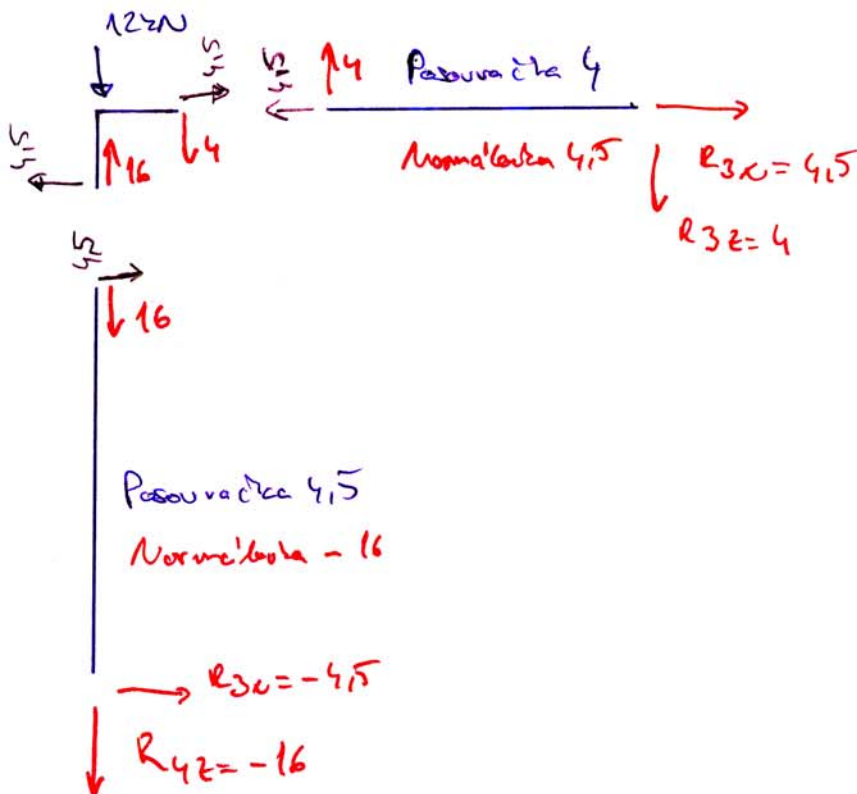
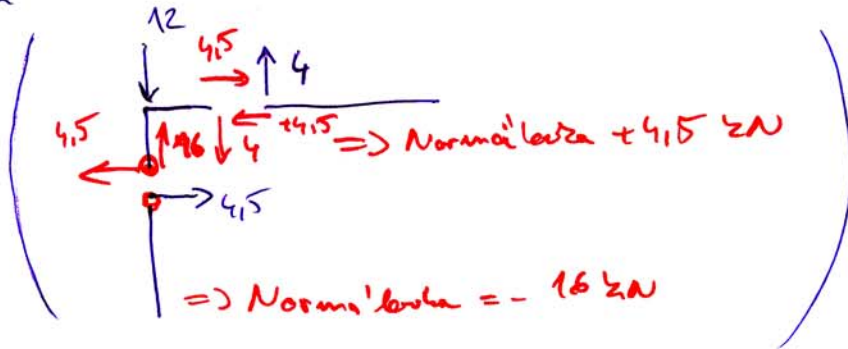
$$\sum \mathcal{M}_z : 18 + 9 + 6 \cdot z_{42} = 0$$

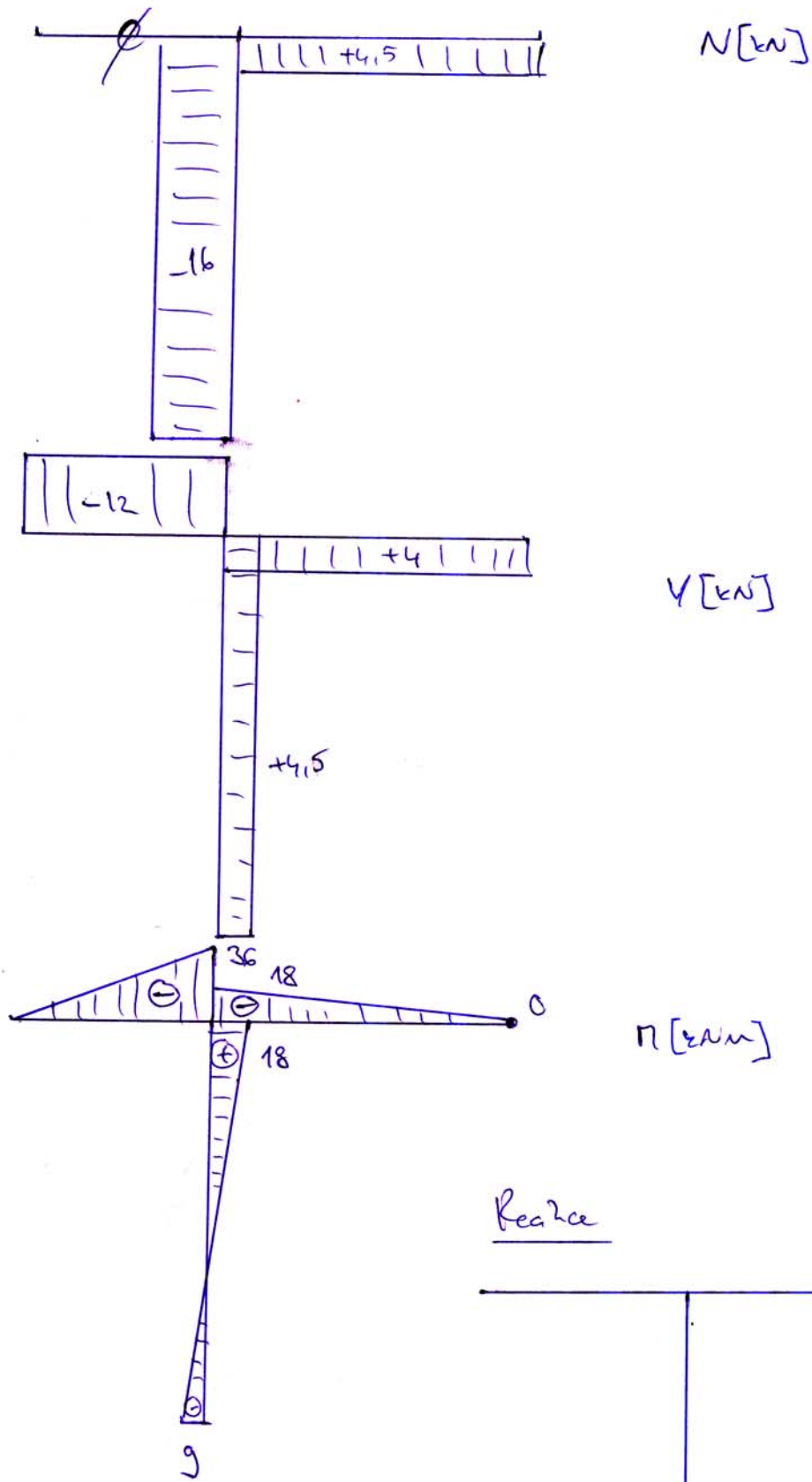
$$z_{42} = \frac{27}{6} = 4,5 \text{ kN}$$

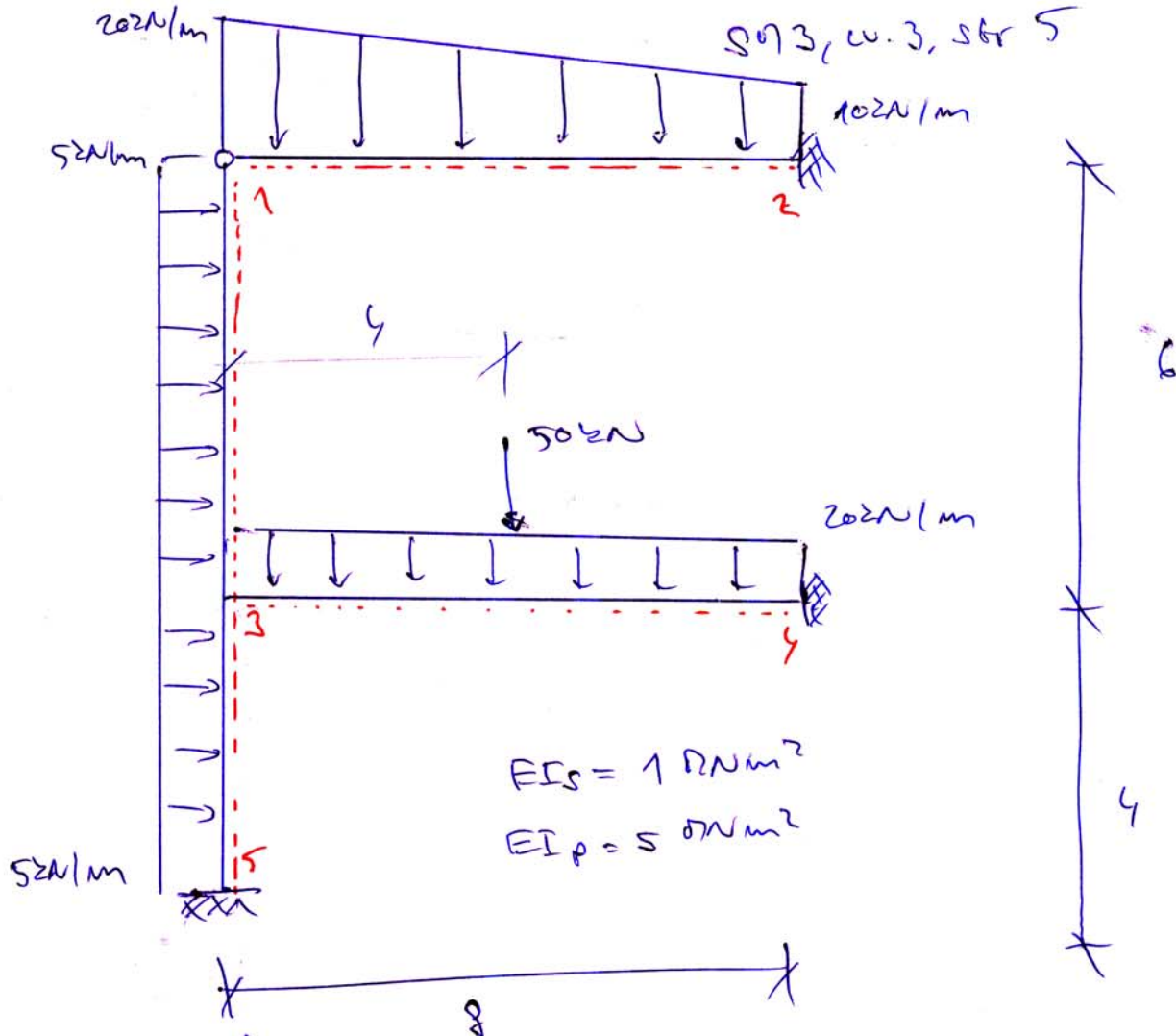
$$z_{24} = 4,5 \text{ kN}$$

$$\text{Neb\u016f} \frac{9+18}{6} = 4,5$$

Vodorovn\u00e1 podmi\u0148ka rovn\u00e1\u017ee se s\u016f\u017e\u00edm\u00e1v:  
+s\u016f\u017e\u00edm\u00e1v







Koncové momenty od síly:

$$\bar{M}_{12} = 0 \quad \bar{M}_{21} = -\frac{1}{8} \cdot 10 \cdot 8^2 - \frac{7 \cdot 10 \cdot 8^2}{120} = -\frac{352}{3}$$

$$\bar{M}_{13} = 0 \quad \bar{M}_{31} = \frac{5 \cdot 6^2}{8} = 22,5$$

$$\bar{M}_{35} = -\frac{1}{12} \cdot 5 \cdot 4^2 = -\frac{20}{3} \quad \bar{M}_{53} = \frac{20}{3}$$

$$\bar{M}_{34} = \frac{1}{12} \cdot 20 \cdot 8^2 + \frac{50 \cdot 8}{8} = \frac{470}{3} \quad \bar{M}_{43} = -\frac{470}{3}$$

Podmínka rovnováhy pro sfčm'č 3-3:

$$M_{31} + M_{34} + M_{35} = 0$$

$$\underbrace{22,5 + \frac{3 \cdot 1}{6} \cdot \varphi_3}_{M_{31}} + \underbrace{\left( +\frac{470}{3} \right) + \frac{4 \cdot 5}{8} \cdot \varphi_3}_{M_{34}} - \underbrace{\frac{20}{3} + \frac{4 \cdot 1}{4} \cdot \varphi_3}_{M_{35}} = 0$$

$$22,5 + \frac{1}{2} \varphi_3 + \frac{470}{3} + \frac{5}{2} \varphi_3 - \frac{20}{3} + \varphi_3 = 0$$

$$\varphi_3 = \frac{-172,5}{4} = -43,125 \text{ mrad}$$

$$M_{12} = 0$$

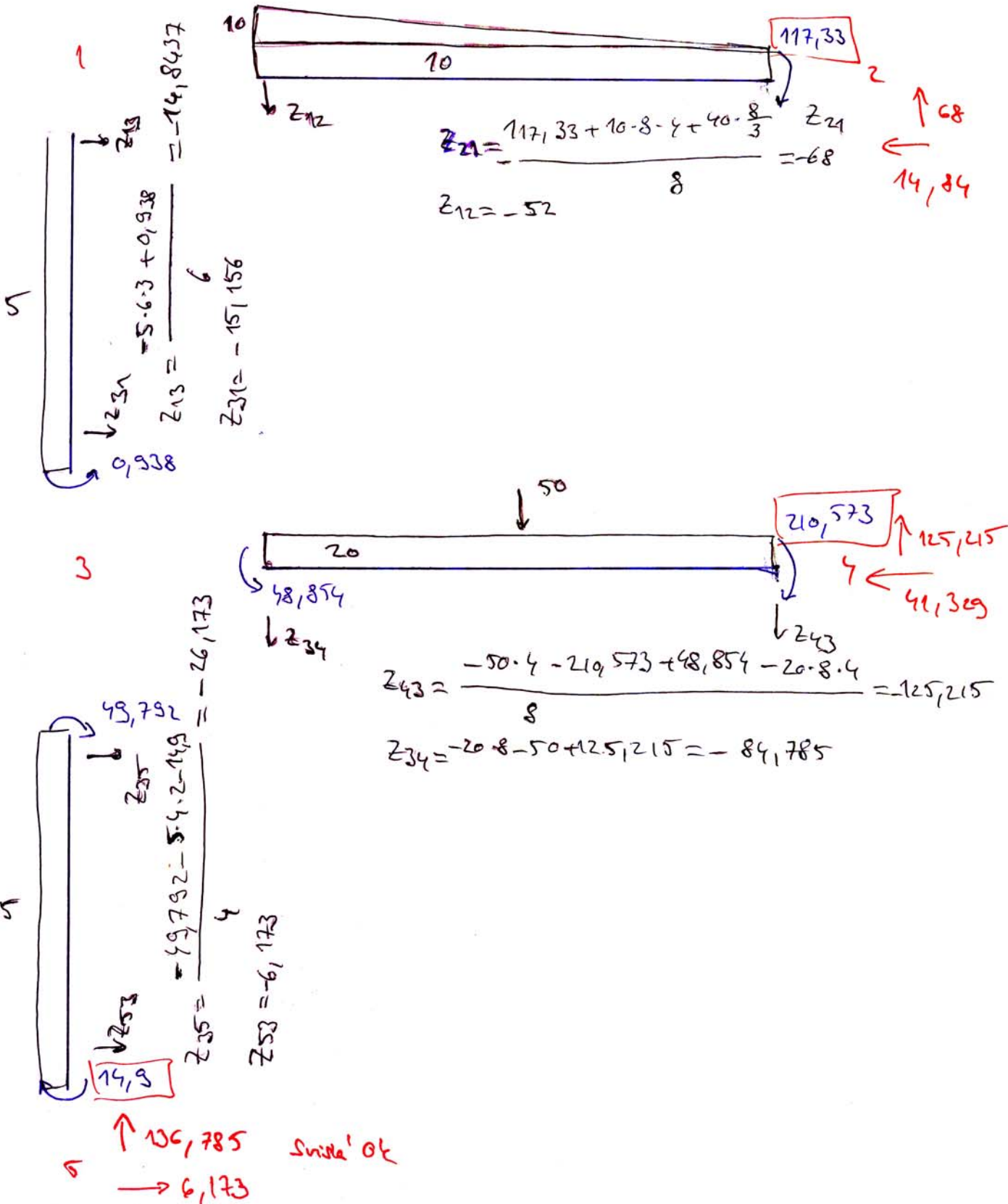
$$M_{21} = -\frac{352}{3} = -117,33$$

$$\bar{M}_{13} = 0$$

$$M_{31} = 22,5 + \frac{1}{2}(-43,125) = 0,9375$$

$$M_{35} = -\frac{20}{3} - 43,125 = -49,792 \quad M_{53} = \frac{20}{3} + \frac{2 \cdot (-43,125)}{4} = -14,9$$

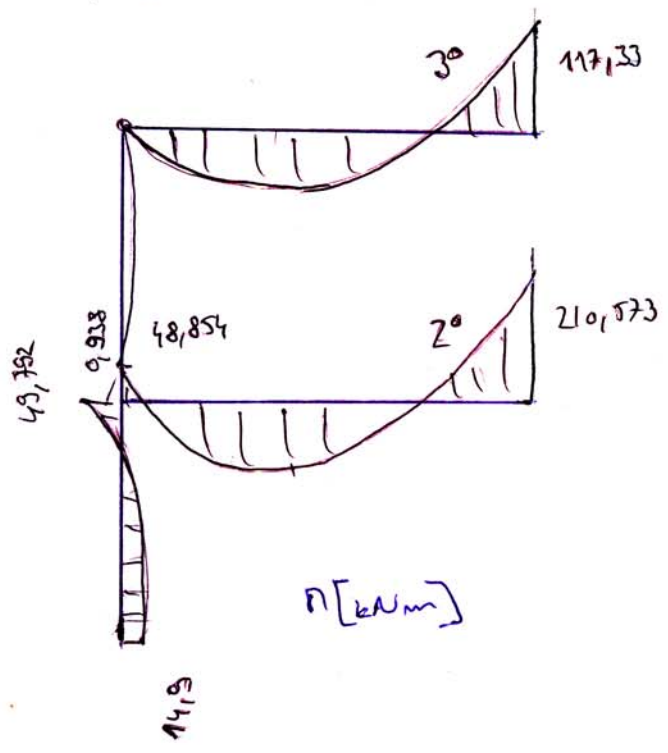
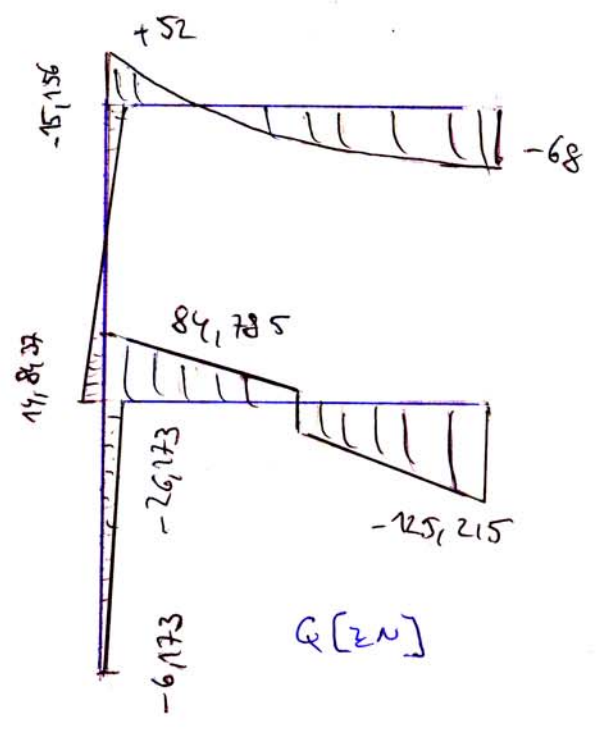
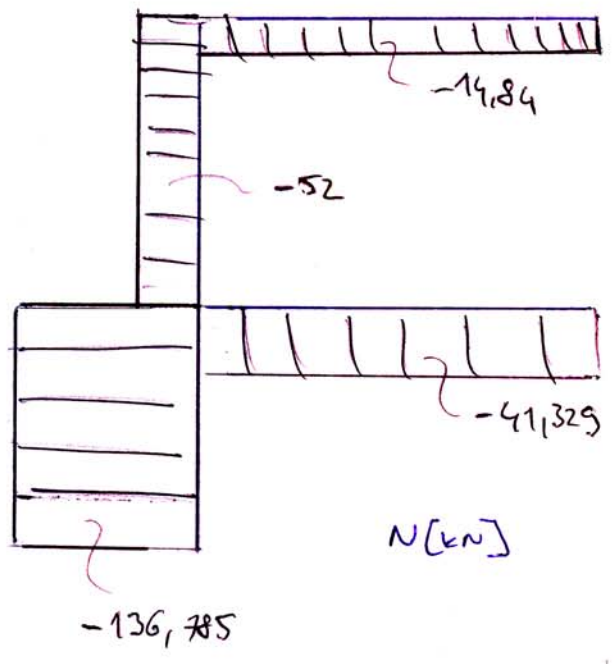
$$M_{34} = \frac{470}{3} + \frac{5}{2} \cdot (-43,125) = 48,8542 \quad M_{43} = -\frac{470}{3} + \frac{2 \cdot 5}{8} \cdot (-43,125) = -210,573$$



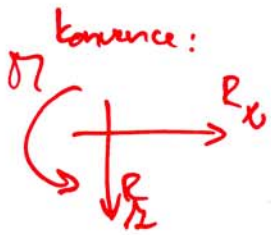
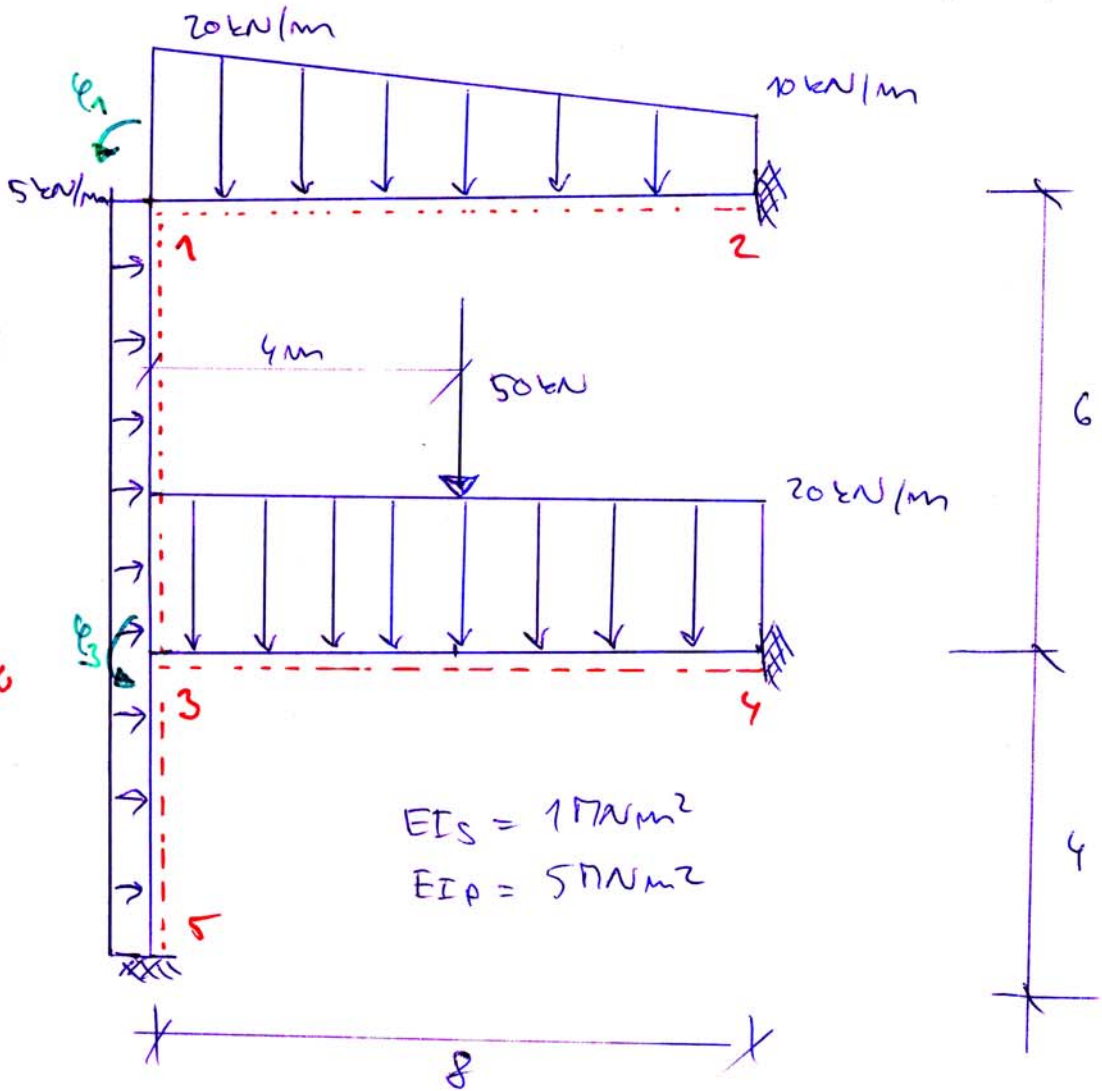
Momentele' pedinute

$$G_2 = -117,33 - 210,573 - 14,9 - 136,785 \cdot 8 + 6,173 \cdot 10 - 41,329 \cdot 6 + 10 \cdot 5 \cdot 5 + 50 \cdot 4 + 20 \cdot 8 \cdot 4 + 10 \cdot 8 \cdot 4 + 40 \cdot \frac{2}{3} \cdot 8 = 0,00633 \quad OK$$

$$G_5 = -117,33 - 210,573 - 14,9 + 68 \cdot 8 + 14,84 \cdot 10 + 41,329 \cdot 4 + 125,215 \cdot 8 - 50 \cdot 4 - 30 \cdot 8 \cdot 4 - 40 \cdot \frac{8}{3} - 10 \cdot 5 \cdot 5 = -0,0337 \quad OK$$







$EIS = 1 \text{ MNm}^2$   
 $EIP = 5 \text{ MNm}^2$

Konvergenzmomente:

$$\bar{M}_{12}^{\square} = \frac{1}{12} \cdot 10 \cdot 8^2 = 53,333 \text{ kNm} \quad \bar{M}_{12}^{\Delta} = \frac{1}{20} \cdot 10 \cdot 8^2 = 32 \text{ kNm}$$

$$\bar{M}_{12} = \bar{M}_{12}^{\square} + \bar{M}_{12}^{\Delta} = 85,333 \text{ kNm}$$

Prot 1,2

$$M_{12} = 85,333 + \frac{4 \cdot 5}{8} \cdot \varphi_1 = 85,333 + 2,5 \varphi_1$$

$$\bar{M}_{21}^{\square} = -\frac{1}{12} \cdot 10 \cdot 8^2 = -53,333 \text{ kNm} \quad \bar{M}_{21}^{\Delta} = -\frac{1}{30} \cdot 10 \cdot 8^2 = -21,333 \text{ kNm}$$

$$\bar{M}_{21} = -74,666 \text{ kNm}$$

$$M_{21} = -74,666 + \frac{2 \cdot 5}{8} \cdot \varphi_1 = -74,666 + 1,25 \varphi_1$$

Prot 1,3

$$\bar{M}_{31} = \frac{1}{12} \cdot 5 \cdot 6^2 = 15 \text{ kNm}$$

$$M_{31} = 15 + \frac{4 \cdot 1}{6} \cdot \varphi_3 + \frac{2 \cdot 1}{6} \cdot \varphi_1 = 15 + 0,666 \varphi_3 + 0,333 \varphi_1$$

$$\bar{M}_{13} = -\frac{1}{12} \cdot 5 \cdot 6^2 = -15 \text{ kNm}$$

$$M_{13} = -15 + \frac{4 \cdot 1}{6} \cdot \varphi_1 + \frac{2 \cdot 1}{6} \cdot \varphi_3 = -15 + 0,666 \varphi_1 + 0,333 \varphi_3$$

$$\text{Prat } 34 \quad \bar{M}_{34} = \frac{1}{12} \cdot 20 \cdot 8^2 + \frac{1}{8} 50 \cdot 8 = 156,667 \text{ kNm}$$

$$M_{34} = 156,667 + \frac{4 \cdot 5}{8} \cdot \varphi_3 = 156,667 + 2,5 \varphi_3$$

$$\bar{M}_{43} = -156,667 \text{ kNm}$$

$$M_{43} = -156,667 + \frac{2 \cdot 5}{8} \varphi_3 = -156,667 + 1,25 \varphi_3$$

$$\text{Prat } 53 \quad \bar{M}_{53} = \frac{1}{12} \cdot 5 \cdot 4^2 = 6,667 \text{ kNm}$$

$$M_{53} = 6,667 + \frac{2 \cdot 1}{4} \varphi_3 = 6,667 + 0,5 \varphi_3$$

$$\bar{M}_{35} = -6,667 \text{ kNm}$$

$$M_{35} = -6,667 + \frac{4 \cdot 1}{4} \varphi_3 = -6,667 + \varphi_3$$

Podmínky rovnováhy:

$$M_{12} + M_{13} = 0$$

$$M_{31} + M_{34} + M_{35} = 0$$

$$85,333 + 2,5 \varphi_1 - 15 + 0,666 \varphi_1 + 0,333 \varphi_3 = 0$$

$$15,0 + 0,667 \varphi_3 + 0,333 \varphi_1 + 156,667 + 2,5 \varphi_3 + (-6,667) + \varphi_3 = 0$$

$$70,333 + 3,166 \varphi_1 + 0,333 \varphi_3 = 0 \quad \Rightarrow \varphi_1 = \frac{-0,333 \varphi_3 - 70,333}{3,166}$$

$$165 + 0,333 \varphi_1 + 4,167 \varphi_3 = 0$$

$$\Rightarrow \varphi_1 = \frac{-4,167 \varphi_3 - 165}{0,333}$$

$$0,333(+0,333 \varphi_3 + 70,333) = 3,166(4,167 \varphi_3 + 165)$$

$$0,11089 \varphi_3 + 23,421 = 13,1927 \varphi_3 + 522,39$$

$$\varphi_3 = \frac{23,421 - 522,39}{13,1927 - 0,11089} = \underline{\underline{-38,1422 \text{ mrad}}}$$

$$\varphi_1 = \underline{\underline{-18,2033 \text{ mrad}}}$$

Novi<sup>4</sup> koncové momenty

$$M_{12} = 85,033 + 2,5 \cdot \varphi_1 = 39,8248 \text{ kNm}$$

$$M_{21} = -74,666 + 1,25 \varphi_1 = -97,4261 \text{ kNm}$$

$$M_{13} = -15 + 0,666 \varphi_1 + 0,033 \varphi_3 = -39,8248 \text{ kNm}$$

$$M_{31} = 15 + 0,666 \varphi_3 + 0,333 \varphi_1 = -16,4644 \text{ kNm}$$

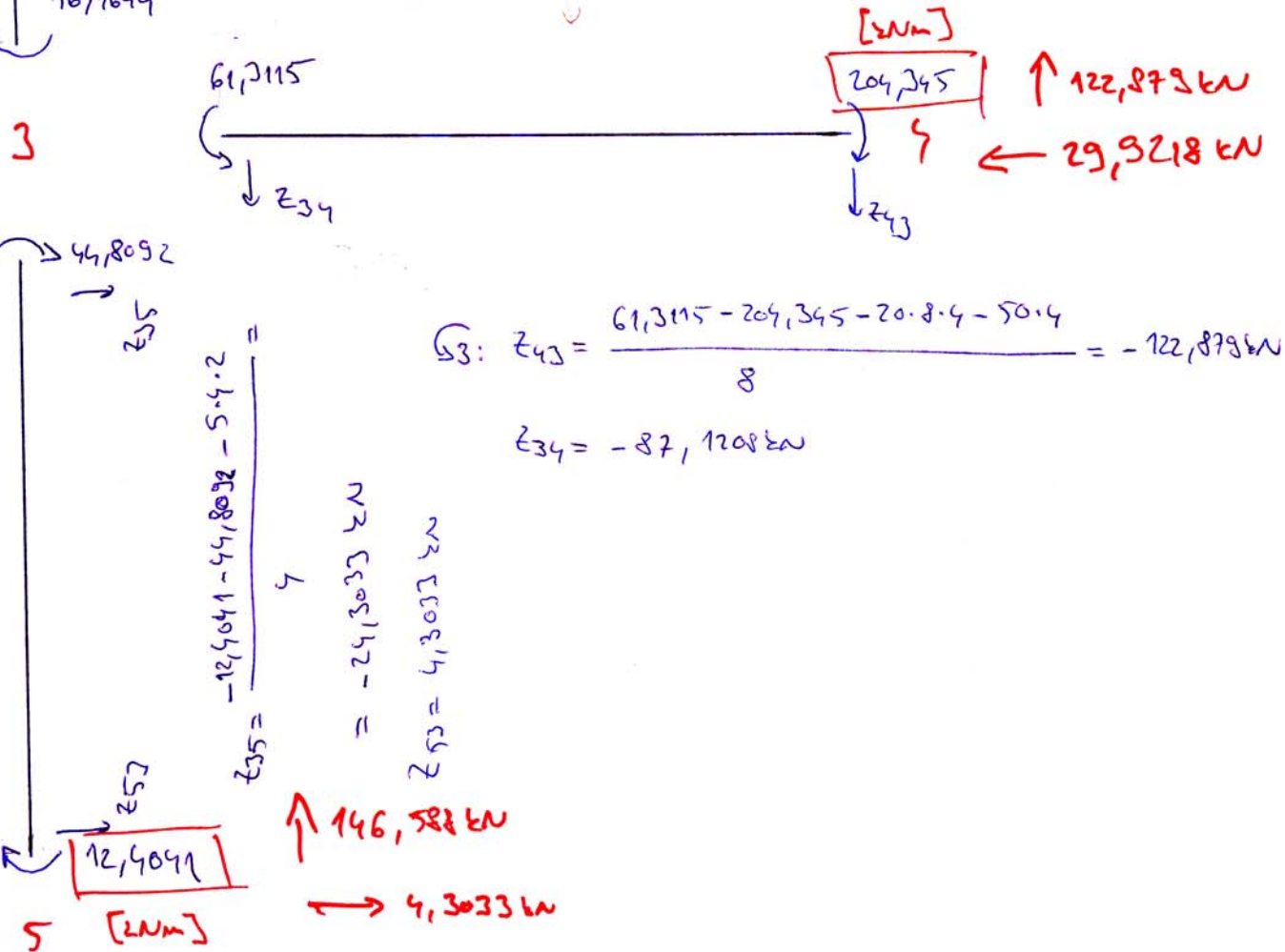
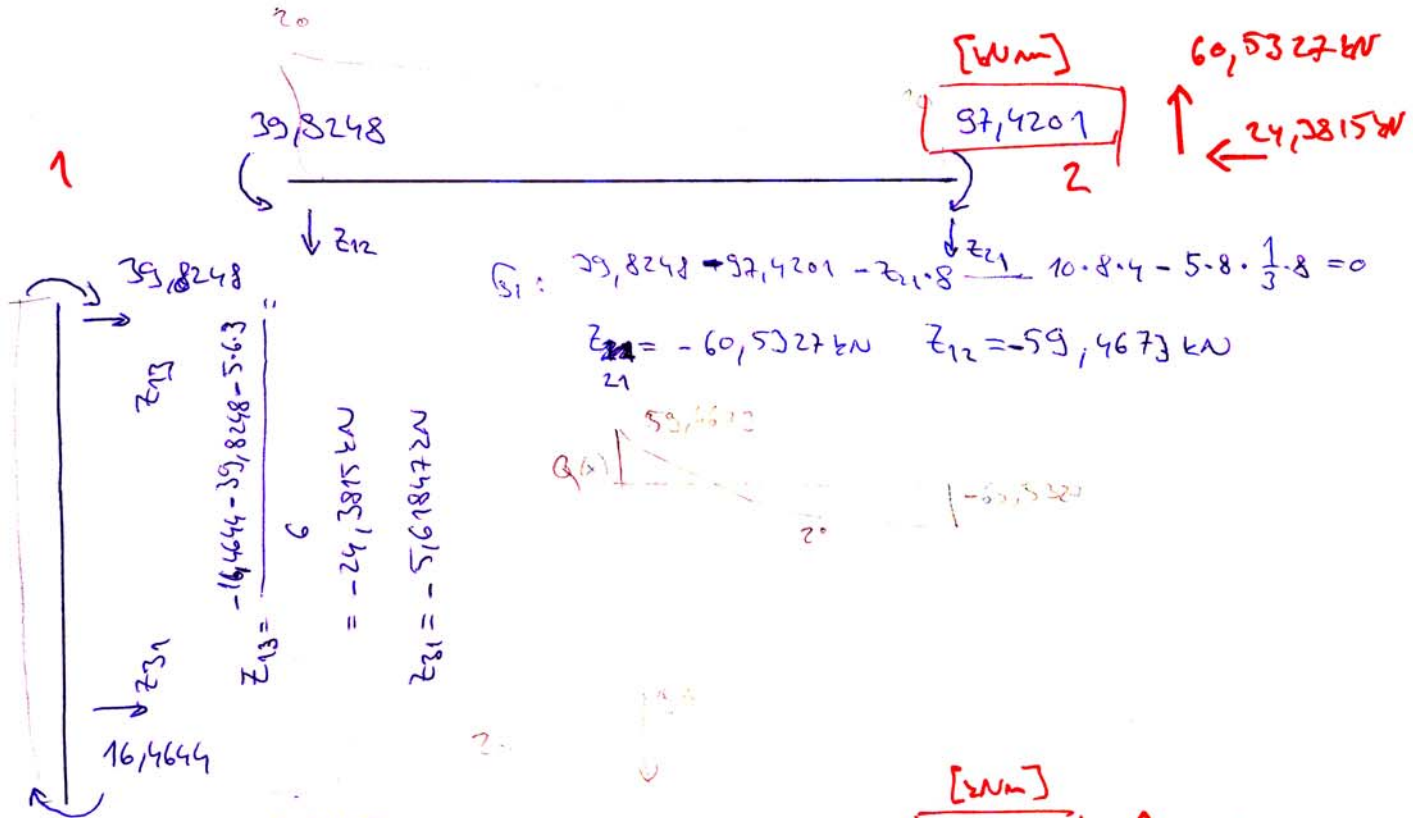
$$M_{34} = 156,667 + 2,5 \varphi_3 = 61,3115 \text{ kNm}$$

$$M_{43} = -156,667 + 1,25 \varphi_3 = -204,345 \text{ kNm}$$

$$M_{35} = -6,667 + \varphi_3 = -44,8092 \text{ kNm}$$

$$M_{53} = 6,667 + 0,5 \cdot \varphi_3 = -12,4091 \text{ kNm}$$

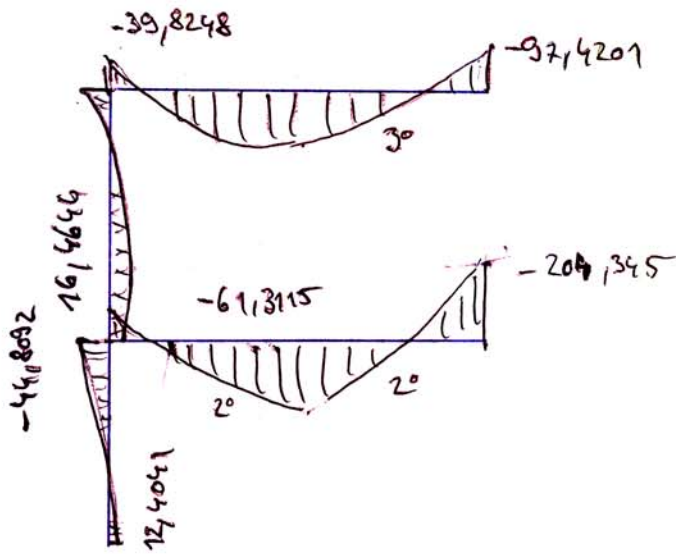
→ KONTROLA



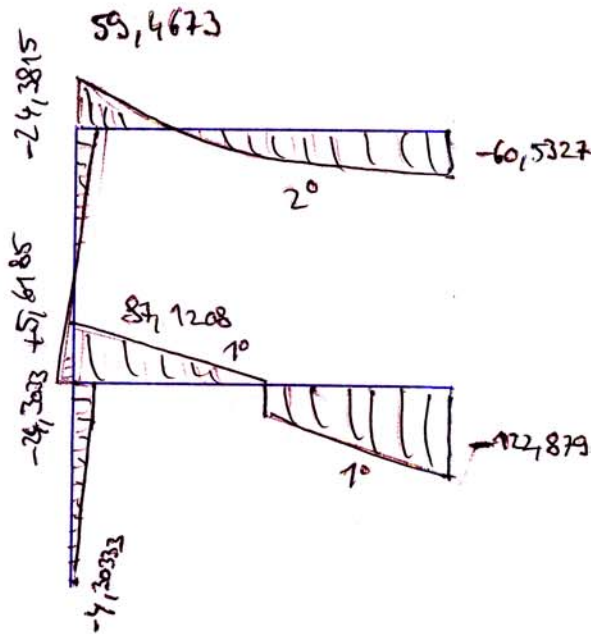
Moment. podmínky  $\sum M:$

$$- 97,4201 - 204,345 - 12,4041 - 146,588 \cdot 8 + 4,3033 \cdot 10 - 29,9218 \cdot 6 + 50 \cdot 5 + 50 \cdot 4 + 20 \cdot 8 \cdot 4 + 10 \cdot 8 \cdot 4 + 5 \cdot 8 \cdot \frac{2}{3} \cdot 8 = -0,0327 \text{ ok}$$

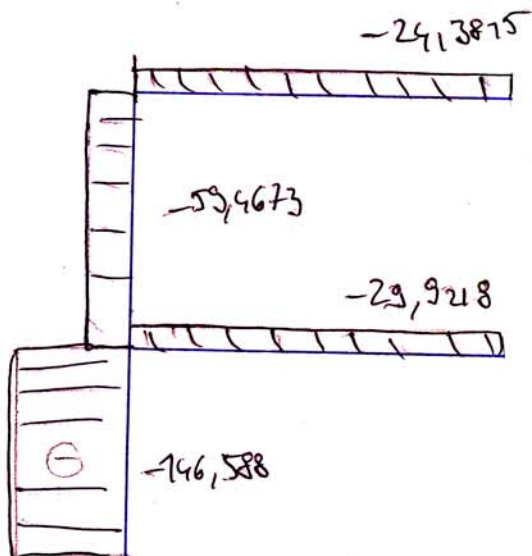
$$\sum M: - 97,4201 - 204,345 - 12,4041 + 24,3815 \cdot 10 + 29,9218 \cdot 4 + (22,879 + 60,5327) \cdot 8 - 30 \cdot 8 \cdot 4 - 5 \cdot 8 \cdot \frac{1}{3} \cdot 8 - 50 \cdot 4 - 50 \cdot 5 = -0,0400 \text{ ok}$$



M [kNm]



Q [kN]



N [kN]