

Je správně?

Podmínky rovnoběžnosti:

$$A + B = 0$$

$$C + D = F$$

$$\sum \mathcal{M}_a: M + 6F - 5A = 0$$

$$\Rightarrow A = \frac{M + 6F}{5} = \frac{50 + 120}{5} = 34 \text{ kN}$$

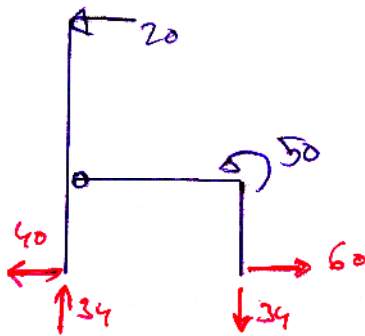
$$B = -34 \text{ kN}$$

$$\sum \mathcal{M}_b: 5B + 20 + M = 0$$

$$\Rightarrow D = \frac{-5B - M}{2} = \frac{170 - 50}{2} = 60 \text{ kN}$$

$$C = -40 \text{ kN}$$

Reakce:

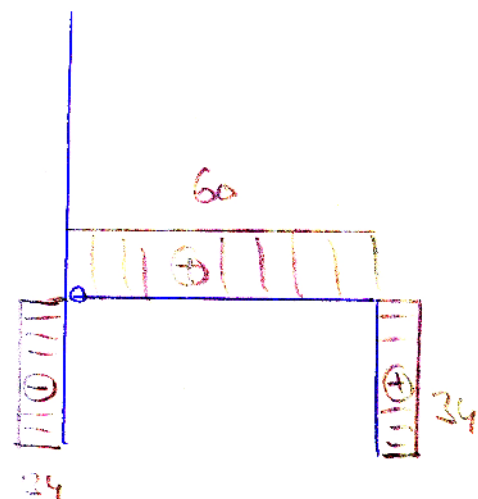
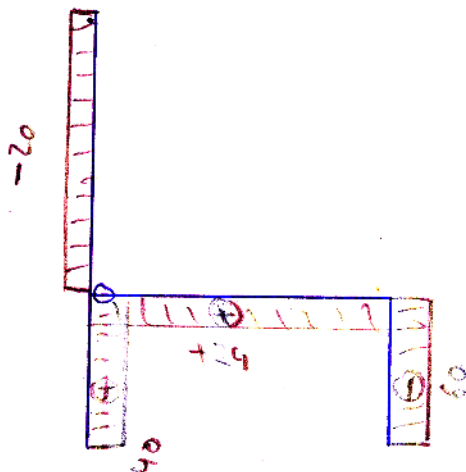
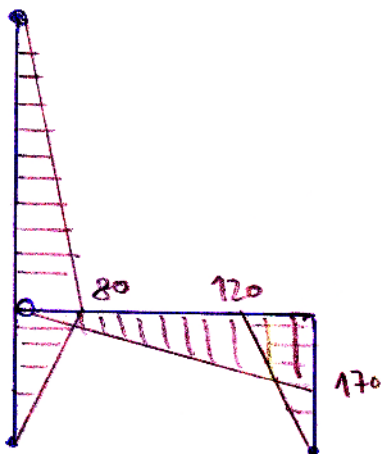


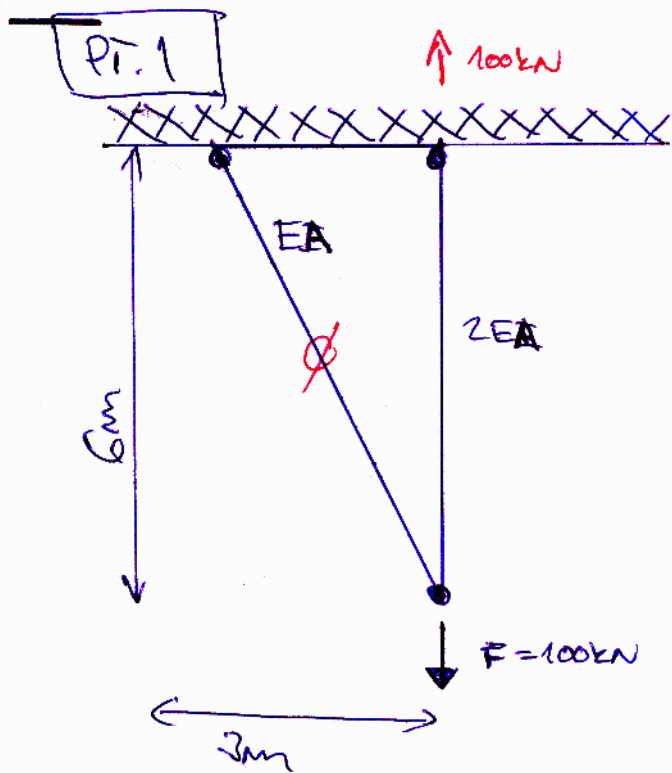
VYKRESLENÍ!

M [kNm]

V [kN]

N [kN]



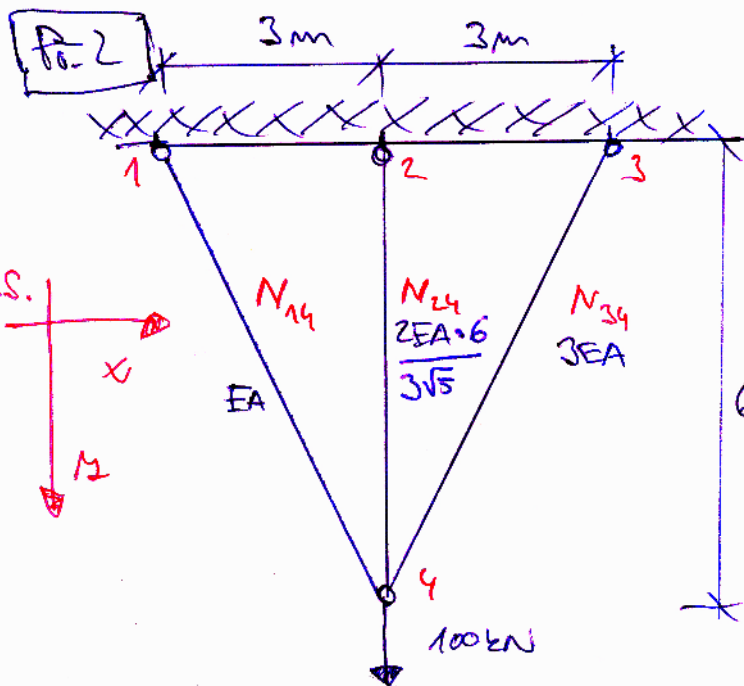


$EA = 1 \text{ GN}$

Statická učitelnost?

Vliv teploty?

Vliv posunu podpory?



$EA = 1 \text{ GN}$



$\alpha = \arctan\left(\frac{3}{6}\right) = 26,57^\circ$

$m_{14} = \frac{EA}{L_{14}} = \frac{1}{3\sqrt{5}} \text{ GN/m}$

$m_{24} = \frac{2EA}{L_{24}} = \frac{2}{3\sqrt{5}} \text{ GN/m}$

$m_{34} = \frac{3EA}{L_{34}} = \frac{1}{\sqrt{5}} \text{ GN/m}$

Počítání rovnováhy:

• rovnováhy svazky a vodorovný posun svazky 4



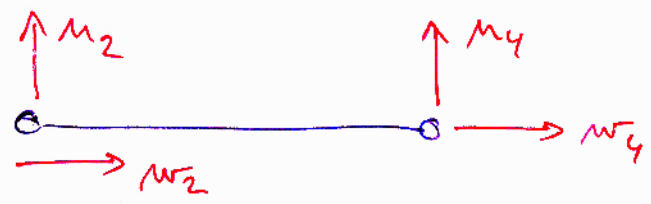
$N_{14} \cdot \cos \alpha + N_{24} + N_{34} \cdot \cos \alpha - 100 = 0$   
 (0,8934)

$\rightarrow: -N_{14} \sin \alpha + N_{34} \cdot \sin \alpha = 0$   
 (0,447)

• Osově síly  $N_{24}$  se vyjádří formou lineární kombinace nezávislých posunů  $u_4, w_4$ .

Prut 2-4

$N_{24}$ :

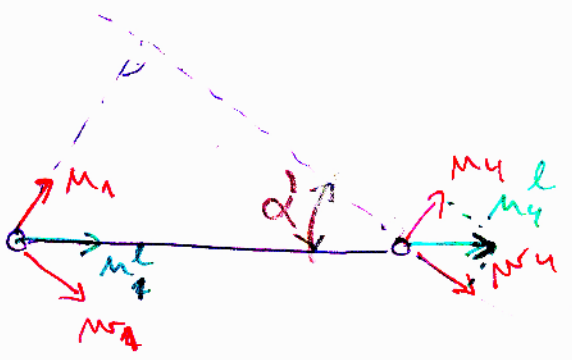


→ vyjádříme formou GLOBÁLNÍCH souřadnic / posunů

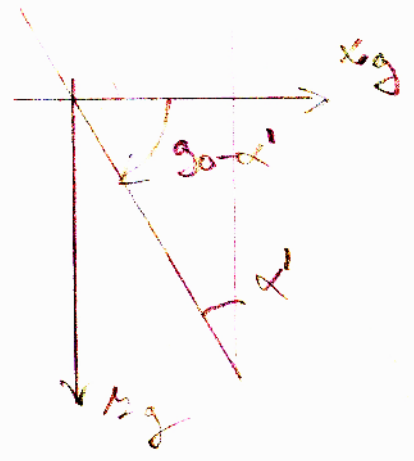
$$N_{24} = M_{24} (w_4 - w_2) = w_4 \cdot \frac{2}{3\sqrt{5}} \text{ GN/m}$$

Prut 1-4

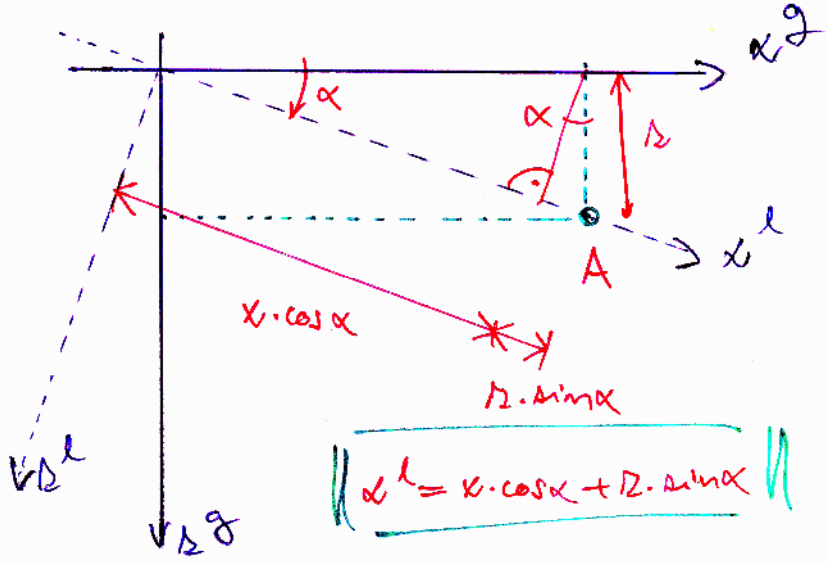
$N_{14}$ :



$$N_{14} = M_{14} (u_4^l - u_1^l)$$



Odbočka:



$$l^l = x \cdot \cos \alpha + l \cdot \sin \alpha$$

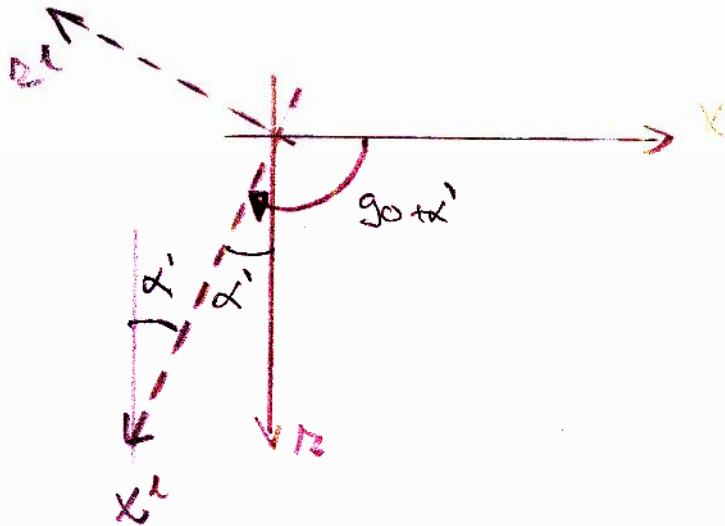
$$M_4^l = M_4^g (\cos(90 - \alpha')) + w_4^g (\sin(90 - \alpha'))$$

$$N_{14} = M_{14} (u_4^l - u_1^l) = \frac{1}{3\sqrt{5}} \text{ GN/m} \left[ \underbrace{w_4 \cos(90 - \alpha')}_{0,447} + \underbrace{w_4 \sin(90 - \alpha')}_{0,894} \right]$$

Prat 3-4:

CVS/str.3

$$N_{34} = m_{34} (u_4^l - u_3^l)$$



$$N_{34} = \frac{1}{\sqrt{5}} \frac{GN}{m} \left[ u_4 \underbrace{\cos(90 + \alpha')}_{-0,447} + w_4 \underbrace{\sin(90 + \alpha')}_{0,894} \right]$$

SOUSTAVA:

$$\uparrow: N_{14} \cos \alpha' + N_{23} + N_{34} \cos \alpha' - 100 \text{ kN} = 0$$

$$\rightarrow: -N_{14} \sin \alpha' + N_{34} \sin \alpha' = 0 \rightarrow N_{34} - N_{14} = 0$$

$$\uparrow: \frac{1}{\sqrt{5}} \frac{GN}{m} [0,447 u_4 + 0,894 w_4] \cdot 0,894 + w_4 \frac{2}{\sqrt{5}} \frac{GN}{m} + \frac{1}{\sqrt{5}} \frac{GN}{m} [-0,447 u_4 + 0,894 w_4] = 0$$

$$\bullet 0,894 = 100 \text{ kN}$$

$$\rightarrow: \frac{1}{\sqrt{5}} \frac{GN}{m} [u_4(-0,447) + 0,894 w_4] - \frac{1}{\sqrt{5}} [0,447 u_4 + 0,894 w_4] = 0$$

$$\uparrow: u_4 \left( \frac{0,447 \cdot 0,894}{\sqrt{5}} - \frac{0,447}{\sqrt{5}} \right) + w_4 \left( \frac{0,894^2}{\sqrt{5}} + \frac{2}{\sqrt{5}} + \frac{0,894^2}{\sqrt{5}} \right) \frac{GN}{m} = 100 \text{ kN}$$

$$\rightarrow: u_4 \left( -0,447 - \frac{0,447}{3} \right) \frac{GN}{m} + w_4 \left( 0,894 - \frac{1}{3} \cdot 0,894 \right) \frac{GN}{m} = 0$$

$$\Rightarrow u_4 = w_4$$

$$\uparrow u_4 = \frac{100 \text{ kN}}{\left[ \frac{0,447 \cdot 0,894 + 0,894^2}{\sqrt{5}} + 2 + \frac{-0,447 \cdot 0,894 + 0,894^2}{\sqrt{5}} \right] \frac{GN}{m}} = \frac{100 \text{ kN} \cdot m}{0,6556 \text{ GN}} = \underline{\underline{0,1525 \text{ mm}}}$$

Zpitve' dora zem'

$$N_{24} = N_{34} \frac{z}{305} \text{ GN/m} = 0,1525 \text{ mm} \cdot \frac{z}{305} \text{ GN/m} = 0,04547 \text{ MN} = 45,47 \text{ kN}$$

$$N_{14} = \frac{1}{305} \text{ GN/m} \left[ (0,447 + 0,894) \cdot 0,1525 \text{ mm} \right] = 0,030985 \text{ MN} = 30,985 \text{ kN}$$

$$N_{34} = \frac{1}{15} \text{ GN/m} \left[ (0,894 - 0,447) \cdot 0,1525 \text{ mm} \right] = 0,030985 \text{ MN} = 30,985 \text{ kN}$$

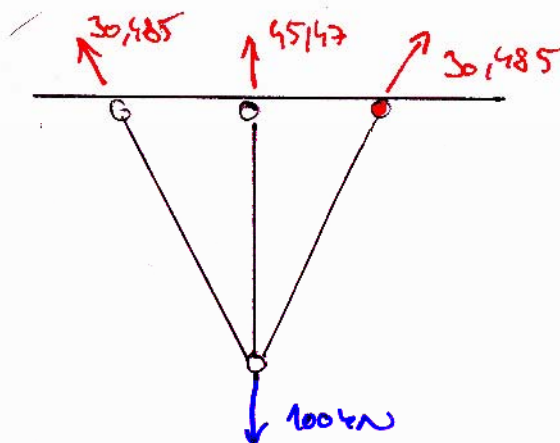
Kontrola: Silove' podum'ny' ravnovaz'ly

$N_{14} = N_{34}$     ok

$N_{14} \cos \alpha' + N_{24} + N_{34} \cos \alpha' - 100 \text{ kN} = 0$

$(2 \cdot 0,894 \cdot 30,985 + 45,47 - 100) \text{ kN} = -0,0228 \text{ kN} \dots \text{ok}$

momentova' podum'nlca

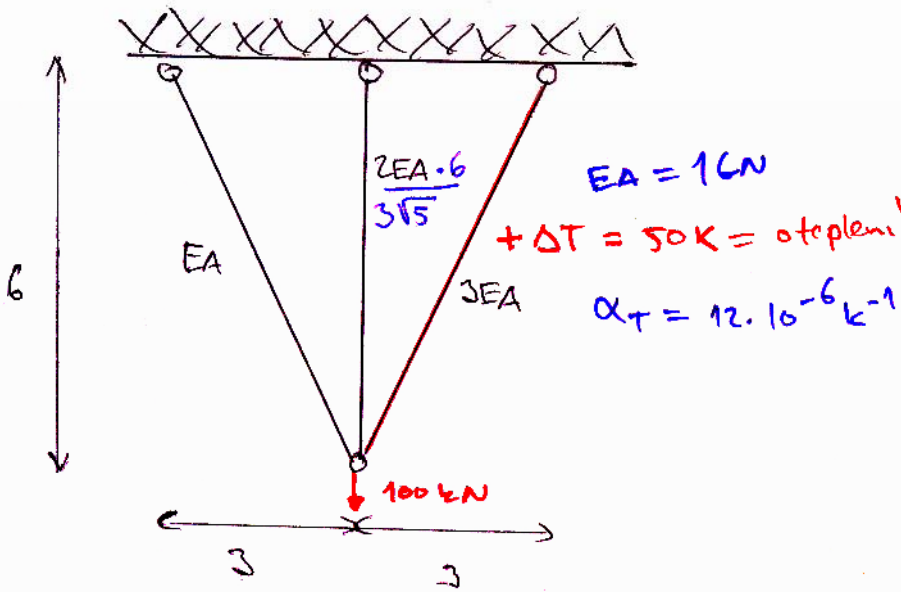


$100 \cdot 3 - 45,47 \cdot 3 - 30,985 \cdot \cos \alpha' \cdot 6 = 0,068 \dots \text{ok}$

↓  
0,894

**Př. 3**

- Stejná konstrukce, jimi z A' z em'



ODBOČKA: VLIV TEPLOTNÍCH ZMĚN

Nemí-li bráněno roztažením  $\epsilon_T = \alpha_T \Delta T$

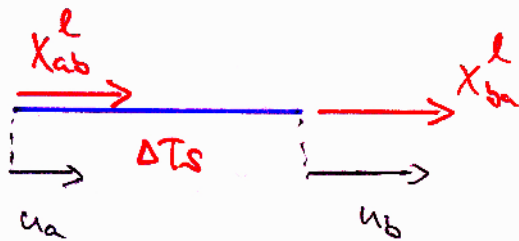
$$\epsilon = \frac{\sigma_x}{E} + \alpha_T \Delta T$$

$$\sigma_x = \frac{N}{A}$$

celková prodloužení =  $\epsilon_x \cdot L$

$$\Delta L = \frac{L \cdot N}{EA} + L \alpha_T \Delta T$$

→ ALE podmínky rovnováhy se sestavují pro koncové síly



$$X_{ab}^l = \bar{X}_{ab}^l - n_{ab} (u_b^l - u_a^l)$$

$$L \rightarrow EA \alpha_T \Delta Ts$$

$$X_{ba}^l = \bar{X}_{ba}^l + n_{ab} (u_b^l - u_a^l)$$

$$L \rightarrow -EA \alpha_T \Delta Ts$$



Podmínky rovnováhy

STEJNE!  $\uparrow: N_{14} \cdot \underbrace{\cos \alpha'}_{0,894} + N_{34} \cos \alpha' + N_{24} - 100 \text{ kN} = 0$

$\rightarrow: -N_{14} \sin \alpha' + N_{34} \sin \alpha'$   
0,447

OROVNĚ SILY / KONCOVĚ SILY

$N_{24} = X_{42}^l$

$X_{42}^l = m_{24} (w_4 - w_2) = \frac{2}{3\sqrt{5}} \frac{6N}{m} \cdot w_4$   
0

$N_{14} = X_{41}^l$

$X_{41}^l = m_{14} (u_4 - u_1) = \frac{1}{3\sqrt{5}} \frac{6N}{m} \left[ u_4 \underbrace{\cos(90-\alpha')}_{0,447} + w_4 \underbrace{\sin(90-\alpha')}_{0,894} \right]$

$N_{34} = X_{43}^l$

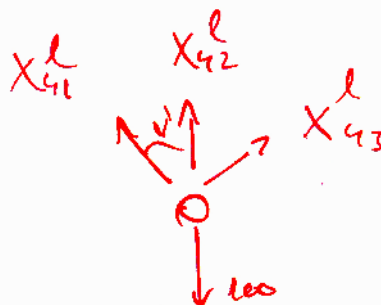
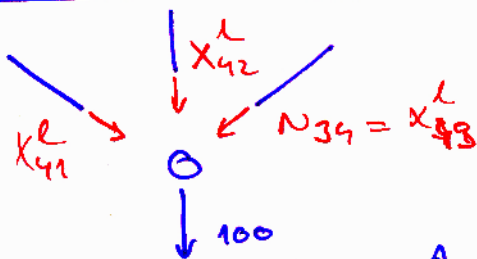
$X_{43}^l = \bar{X}_{43}^l + m_{34} (u_4 - u_3) = -3EA \cdot \alpha_T \cdot 0,75 + m_{34} (u_4 \cos(90+\alpha') + w_4 \sin(90+\alpha'))$   
 $= \underbrace{-36N \cdot 12 \cdot 10^{-6} \text{ k}^{-1} \cdot 90k}_{6 \cdot 10^{-4}} + \frac{1}{\sqrt{5}} \frac{6N}{m} (u_4 \cos(90+\alpha') + w_4 \sin(90+\alpha'))$   
 -1,8 MN

SOUBRAVA:

$\uparrow: N_{14} \cos \alpha' + N_{24} + N_{34} \cos \alpha' - 100 \text{ kN} = 0$

$\rightarrow: -N_{14} \sin \alpha' + N_{34} \sin \alpha' = 0 \quad \rightarrow N_{34} - N_{14} = 0$

POŘADÍ KONCOVÝCH SIL



$\uparrow: -5 \cdot X_{41}^l + X_{42}^l + 6X_{43}^l = 100 \rightarrow -5X_{41}^l + 6X_{43}^l = 100$

$$\uparrow: \frac{1}{305} \frac{6N}{m} \left[ 0,447 u_4 + w_4 \cdot 0,894 \right] \cdot 0,894 + \frac{2}{305} \frac{6N}{m} \cdot w_4 +$$

$$+ 0,894 \left[ -1,8 \text{ kN} + \frac{1}{15} \frac{6N}{m} \left[ u_4 \cdot (-0,447) + w_4 (0,894) \right] \right] - 100 \text{ kN} = 0$$

$$\rightarrow: \frac{1}{305} \frac{6N}{m} \left[ 0,447 u_4 + 0,894 w_4 \right] + (-1,8 \text{ kN}) + \frac{1}{15} \frac{6N}{m} \left[ -0,447 u_4 + 0,894 w_4 \right] = 0$$

$$\left[ \begin{array}{l} u_4 = -5,373 \text{ mm} \\ w_4 = 1,38 \text{ mm} \end{array} \right]$$

VÝHODNOUJEME! OSOŤŤICH SIL (KONCOVŤICH)

$$X_{42}^l = \frac{2}{305} \frac{6N}{m} \cdot 1,38 \text{ mm} = 0,4114 \text{ MN} = \boxed{411,4 \text{ kN}}$$

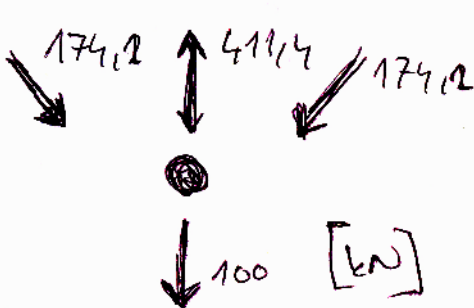
$$X_{41}^l = \frac{1}{305} \frac{6N}{m} \left[ -5,373 \text{ mm} \cdot 0,447 + 1,38 \text{ mm} \cdot 0,894 \right] = -0,1741 \text{ MN} =$$

$$= \boxed{-174,1 \text{ kN}}$$

$$X_{43}^l = \frac{1}{15} \frac{6N}{m} \left[ -5,373 \text{ mm} \cdot (-0,447) + 1,38 \text{ mm} \cdot 0,894 \right] - 1,8 \text{ kN} = -0,1741 \text{ MN}$$

$$= \boxed{174,1 \text{ kN}}$$

KONTROLA - SILOVE! PODMIŤKY ROVNOUŤIJE VE STYČNÍKŤ 4



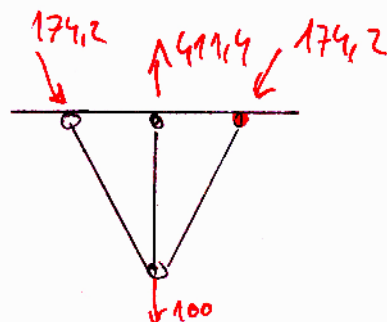
rozdrcovna!  OK

smrc!:

$$411,4 - 100 - 2 \cdot 174,1 \cdot 0,894 =$$

$$= -0,0966 \text{ kN} \quad \boxed{\text{OK}}$$

MOMENTOVŤ!



$$3(100 - 411,4) + 174,1 \cdot 6 \cdot 0,894 = 0,209 \text{ kNm}$$

OK