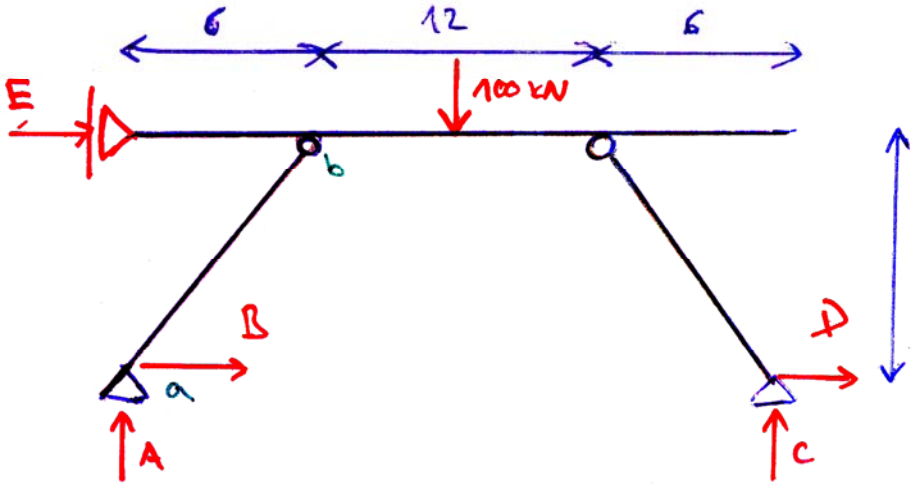
Předpříklad



počet s.v.? (vazby ↔ podpory)

Řešení: 1) Výpočet reakcí

$\uparrow: A + C - 100\,kN = 0$; Symetrie $\rightarrow$ $A = 50\,kN$, $C = 50\,kN$

$\rightarrow: B + D + E = 0$, z mom. podmínky $\circlearrowleft_a$ $24 \cdot C - 12 \cdot 100\,kN - 6E = 0$

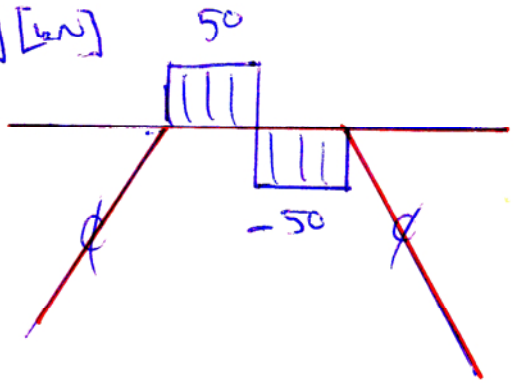
$\Rightarrow E = 0\,kN$

$\circlearrowleft_b$ $6B - 6A = 0$

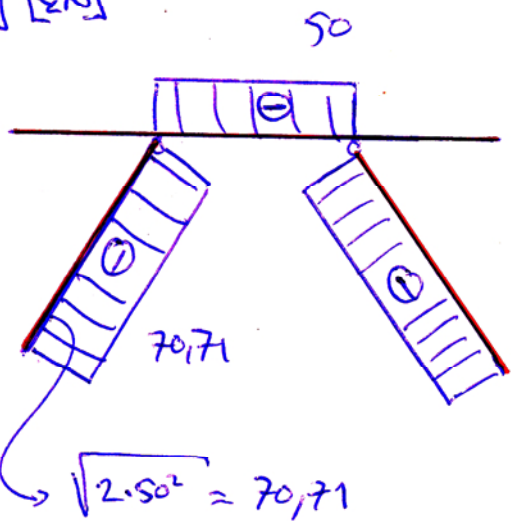
$\Rightarrow B = A = 50\,kN$

$\Rightarrow D = -50\,kN$

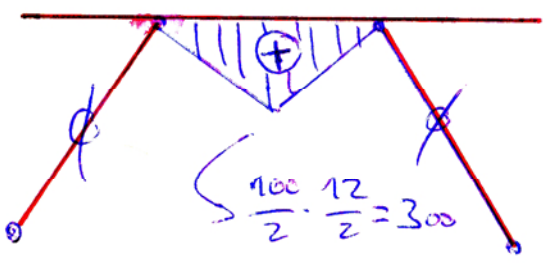
V [kN]



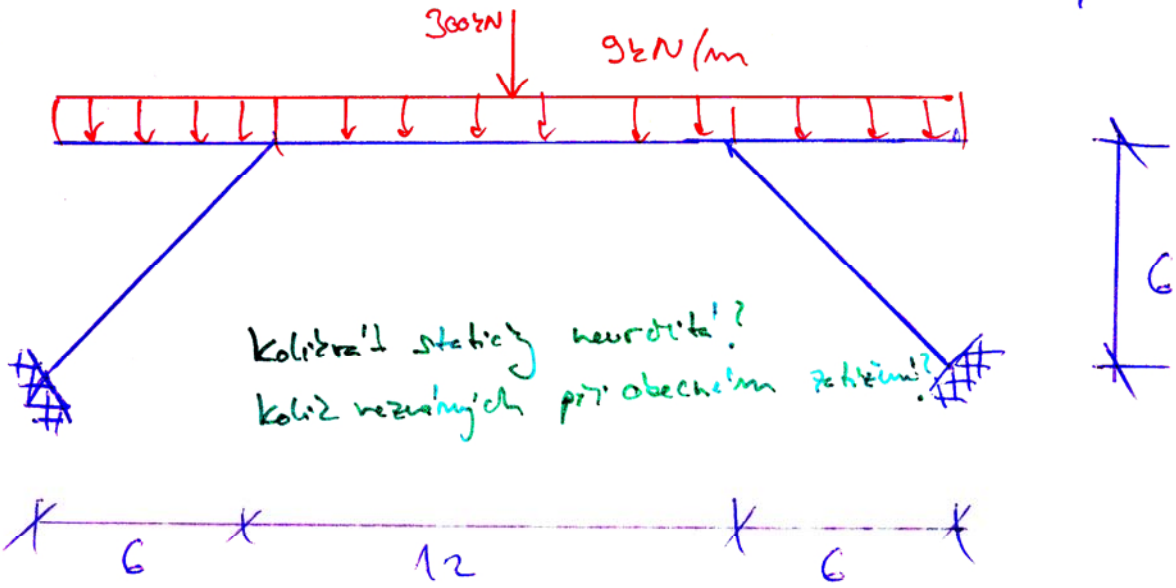
N [kN]



M [kNm]

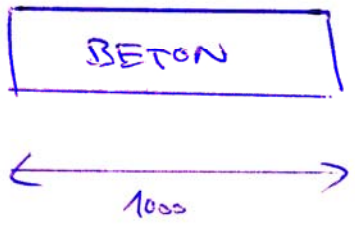


Př. 1



→ Řešíme na 1 bm konstrukce

Průřez:



$$I = \frac{1}{12} \cdot 1 \cdot 0{,}8^3 = 0{,}04267\,m^4$$

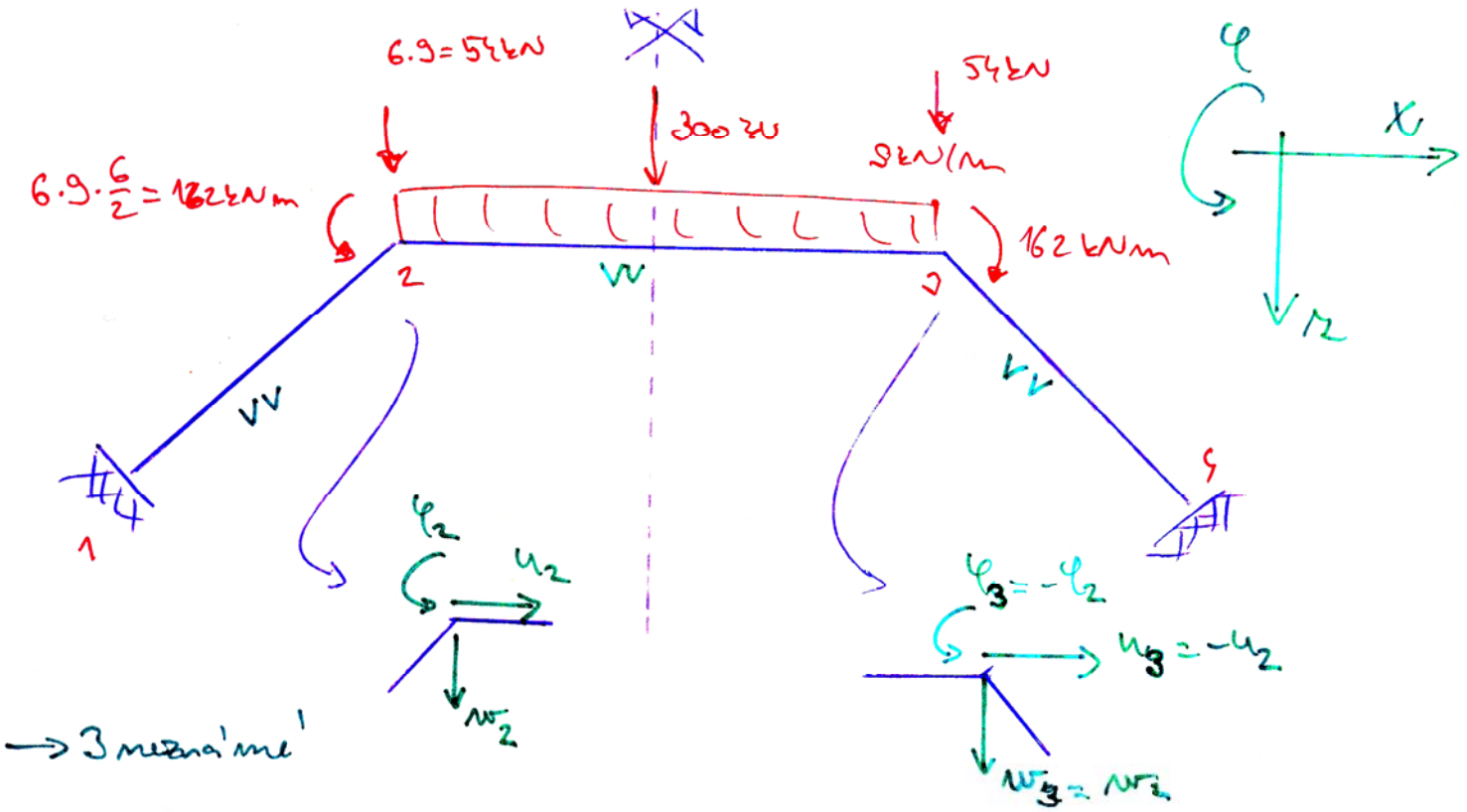
$$A = 0{,}8\,m^2$$

$$E = 30\,GPa$$

$$EI = 1{,}28\,GNm^2$$

$$EA = 30 \cdot 0{,}8\,GN = 24\,GN$$

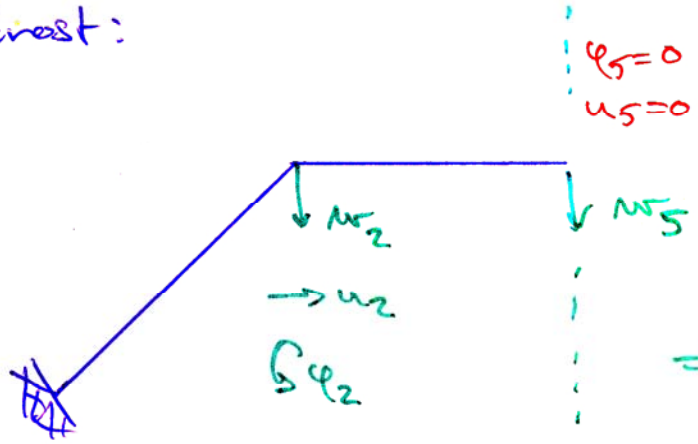
Pro uvažované zatížení (symetrické) se konstrukce zjednoduší:



→ 3 neznámé

SM3, cv. 6, str. 3

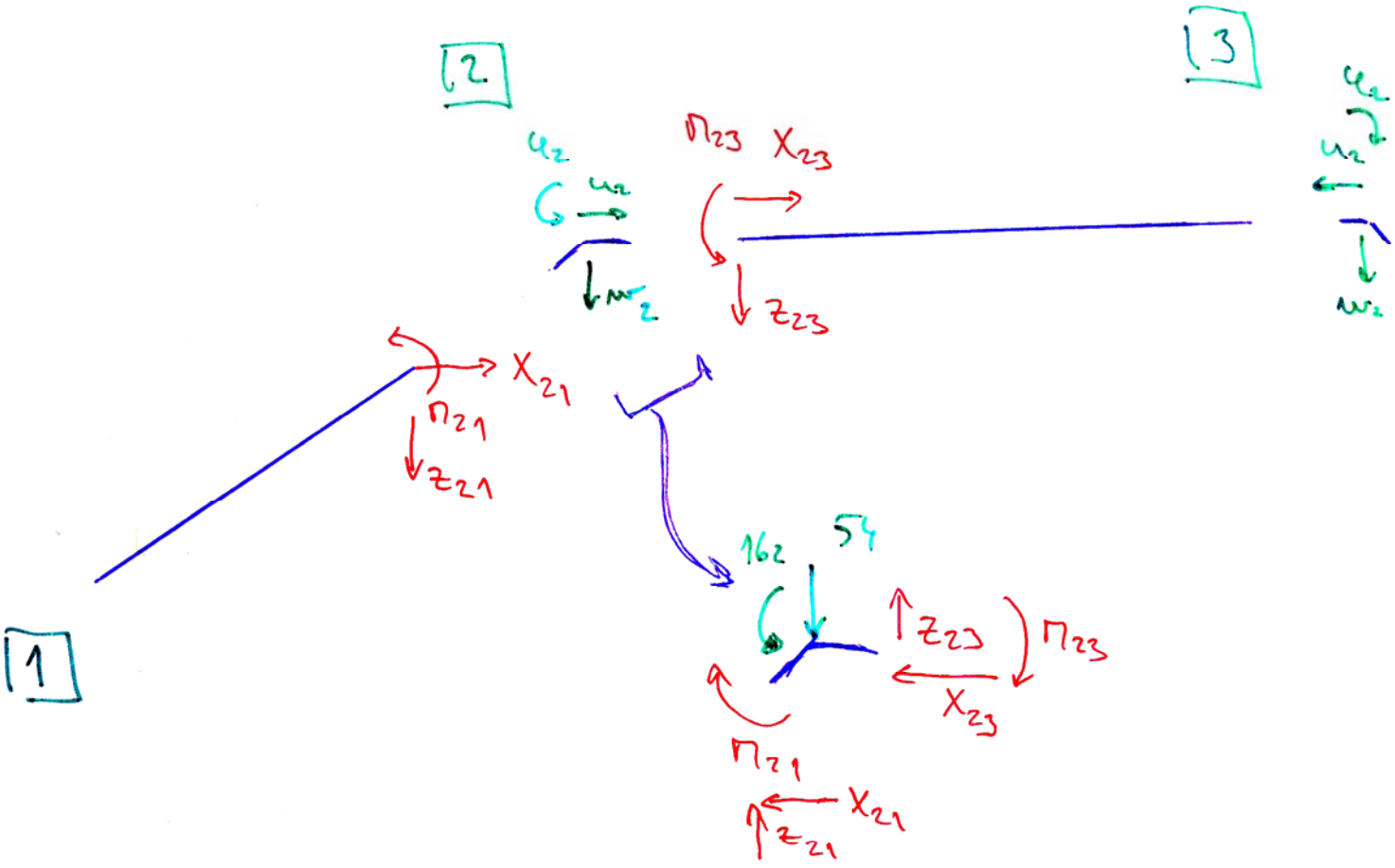
2. možnost:



=> 4 neznámé

Kolik nezadaných při ZDD?

Budeme řešit 1. variantu s neznámými u_2, w_2, φ_2



Podmínky rovnováhy styčníku:

↑: $Z_{23} + Z_{21} = 54$

→: $-X_{21} - X_{23} = 0 \rightarrow X_{21} = -X_{23}$

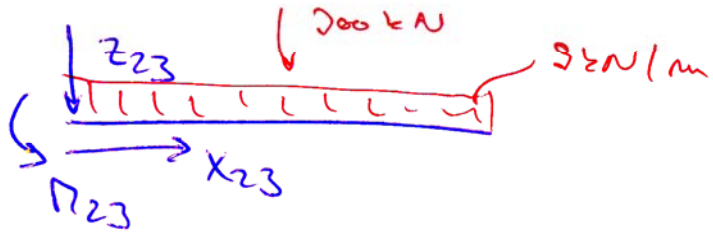
↺: $M_{23} + M_{21} = 162$

Nejprve se vyjádří globelním koncové síly pomocí lokálních.

Lokální koncové síly definujeme součtem příspěvků od

1) SILOVÉHO MIMOSTYČNÍKOVÉHO ZATÍŽENÍ
2) POSUNŮ A NATOČENÍ STYČNÍKŮ

1) ~~Prut~~ 2-3 : $\alpha = 0$, transformovat se nemusí



$$\rightarrow M_{23} = \overline{\Pi}_{23} + k \cdot \left(2\cdot\varphi_2 + \varphi_3 + \frac{3(w_3 - w_2)}{L_{23}} \right)$$

$$\overline{\Pi}_{23} = \frac{1}{12} \cdot g \cdot L_{23}^2 + \frac{300 \cdot L_{23}}{8} = \frac{9 \cdot 12^2}{12} + \frac{300 \cdot 12}{8} = 558 \text{ kNm}$$

$$k = \frac{2EI}{L_{23}} = \frac{2 \cdot 1280}{12} = 213{,}33 \text{ MNm} \quad \text{(natočení v rad)}$$

$$\varphi_3 = -\varphi_2$$

$$w_3 = w_2$$

$$\Pi_{23} = 558 + 213{,}33 \left(\varphi_2 + \frac{3(0)}{12} \right) = 558 + 213{,}33\,\varphi_2$$

$$\rightarrow z_{23} = \overline{z}_{23} - \frac{3k}{L_{23}} \left(\underbrace{\varphi_2 + \varphi_3}_{0} + \underbrace{\frac{2(w_3 - w_2)}{L}}_{0} \right)$$

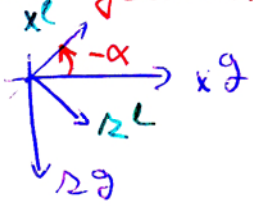
$$\overline{z}_{23} = -\frac{9 \cdot 12}{2} - \frac{300}{2} = -204 \text{ kN} \quad \rightarrow \text{Nezávisí na deformacích (= posunech)}$$

$$\rightarrow X_{23} = \overline{X}_{23} - n_{23}(u_3 - u_2) = -2000\left(-2u_2\right) = 4000\,u_2$$

$$\downarrow 0 \quad n_{23} = \frac{EA}{L_{23}} = \frac{24000}{12} = 2000 \text{ MN/m} \quad \text{(posun v mm)}$$

2) Prut 1-3

Tento prut je nutné transformovat. oproti globálnímu systému souřadnic je natočen o úhel $-45°$



1) Transformace koncových sil Glob(lokální)

$$X_{21} = X_{21}^{l} \cos\alpha - Z_{21}^{l} \sin\alpha$$

$$Z_{21} = X_{21}^{l} \cdot \sin\alpha + Z_{21}^{l} \cdot \cos\alpha$$

$$M_{21} = M_{21}^{l}$$

Mimostředová zatížení není, proto

$\Rightarrow \bar{X}_{21} = \bar{Z}_{21} = \bar{M}_{21} = 0$

2) koncové lokální síly

- $X_{21}^{l} = \bar{X}_{21}^{l} + n_{21}(u_2^{l} - u_1^{l})$; $\quad n_{21} = \frac{EA}{L_{12}} = \frac{24\,000}{8{,}485} = 2828{,}43$ MN/m

$$X_{21}^{l} = 2828{,}43\, u_2^{l}$$

- $Z_{21}^{l} = \bar{Z}_{21}^{l} + \frac{3k_{21}}{L_{12}}\left(\varphi_1 + \varphi_2 + 2\frac{w_2^{l} - w_1^{l}}{L_{12}}\right)$; $\quad k_{12} = \frac{2EI}{L_{12}} = \frac{2 \cdot 1280}{\sqrt{2}\cdot 6} = 301{,}7$ MNm

$$Z_{21}^{l} = \phi + \underbrace{\frac{3 \cdot 301{,}7}{\sqrt{2}\cdot 6}}_{106{,}67}\left(\phi + \varphi_2 + \frac{2 w_2^{l}}{\sqrt{2}\cdot 6}\right) =$$

$$= 106{,}67\,\varphi_2 + 25{,}1417\, w_2^{l}$$

- $M_{21} = \bar{M}_{21} + k_{12}\cdot\left(\varphi_1 + 2\varphi_2 + \frac{3(w_2^{l} - w_1^{l})}{L_{12}}\right) =$

$$= \phi + 301{,}7\left(\phi + 2\varphi_2 + \frac{3 w_2^{l}}{\sqrt{2}\cdot 6}\right) = 603{,}4\cdot\varphi_2 + 106{,}67\cdot w_2^{l}$$

3) transformujeme lokální posuny do globálních souřadnic

$$u_2^{l} = u_2 \cos\alpha + w_2 \sin\alpha$$

$$w_2^{l} = -u_2 \sin\alpha + w_2 \cos\alpha$$

- $X_{21} = X_{21}^{l} \cos\alpha - Z_{21}^{l} \sin\alpha$

$X_{21}^{l} = 2828{,}43\, u_2^{l}$

$u_2 \cos\alpha + w_2 \sin\alpha$

$\frac{\sqrt{2}}{2} u_2 + \left(-\frac{\sqrt{2}}{2}\right) w_2$

$106{,}67\varphi_2 + 25{,}1417\, w_2^{l}$

$-u_2 \sin\alpha + w_2 \cos\alpha$

$\frac{\sqrt{2}}{2} u_2 + \frac{\sqrt{2}}{2} w_2$

$$X_{21} = \frac{\sqrt{2}}{2} \cdot 2828{,}43 \left(\frac{\sqrt{2}}{2} u_2 - \frac{\sqrt{2}}{2} w_2\right) + \frac{\sqrt{2}}{2} \left(106{,}67\varphi_2 + \frac{25{,}1417\sqrt{2}}{2}\left(u_2 + w_2\right)\right)$$

$$X_{21} = 75{,}4271\varphi_2 + 1426{,}79\, u_2 - 1401{,}64\, w_2$$

- $Z_{21} = X_{21}^{l} \sin\alpha + Z_{21}^{l} \cos\alpha =$

$$= -\frac{\sqrt{2}}{2} 2828{,}43 \left(\frac{\sqrt{2}}{2} u_2 - \frac{\sqrt{2}}{2} w_2\right) + \frac{\sqrt{2}}{2} \left(106{,}67\varphi_2 + \frac{25{,}1417\sqrt{2}}{2}\left(u_2 + w_2\right)\right) =$$

$$= 75{,}4271\varphi_2 - 1401{,}64\, u_2 + 1426{,}79\, w_2$$

- $M_{21} = 603{,}44\varphi_2 + 106{,}67 \cdot w_2^{l}$

$-u_2 \sin\alpha + w_2 \cos\alpha$

$$M_{21} = 603{,}44\varphi_2 + 75{,}4271\, u_2 + 75{,}4271\, w_2$$

S73, cv. 4, str. 7

→ use nadzime do rovnic rovnováhy

○ ↑: $Z_{23} + Z_{21} = 54$

$-204 + 75{,}4271\,\varphi_2 - 1401{,}64\,u_2 + 1426{,}79\,w_2 = 54$

○ → $X_{21} = -X_{23}$; $X_{21} + X_{23} = 0$

$75{,}4271\,\varphi_2 + 1426{,}79\,u_2 - 1401{,}64\,w_2 + 4000\,u_2 = 0$

○ $M_{23} + M_{21} = 162$

$558 + 213{,}334\,\varphi_2 + 603{,}44\,\varphi_2 + 75{,}4271\,u_2 + 75{,}4271\,w_2 = 162$

⇒

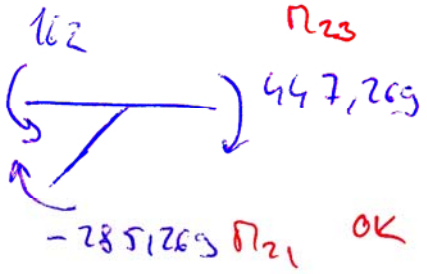
$\varphi_2 = -0{,}51906$ mrad

$u_2 = 0{,}081747$ mm

$w_2 = 0{,}128857$ mm

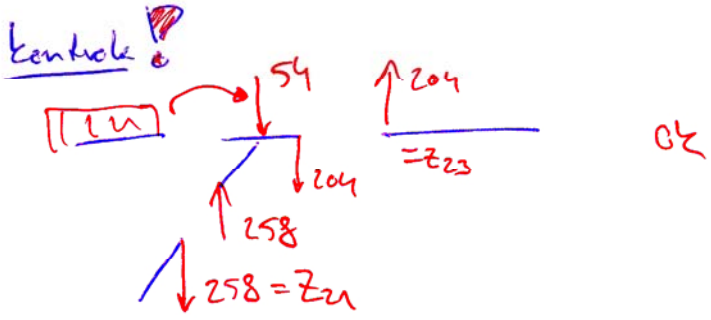
$M_{21} = -285{,}269$ kNm

$M_{23} = 447{,}269$ kNm ⇒ kontrola!



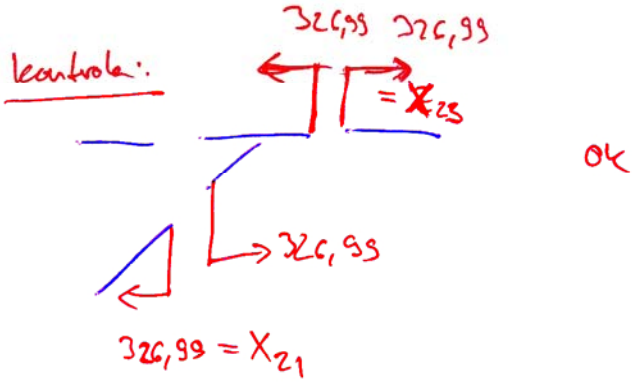
$Z_{21} = 258$ kN

$Z_{23} = -204$

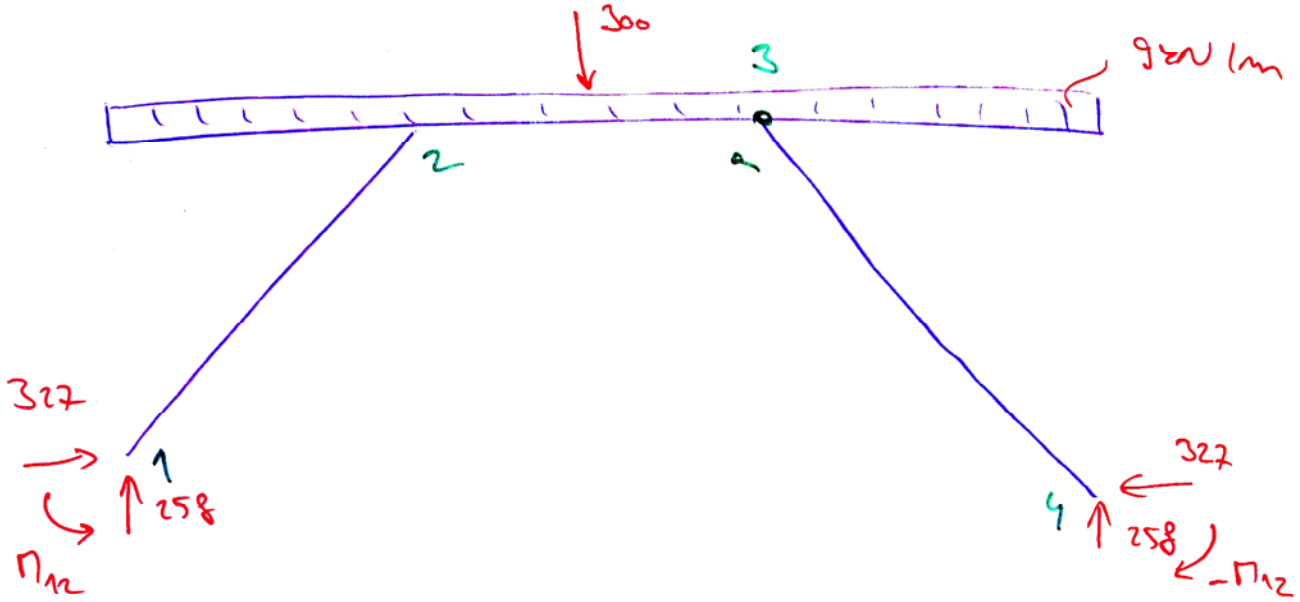


$X_{21} = -326{,}99$

$X_{23} = 326{,}99$



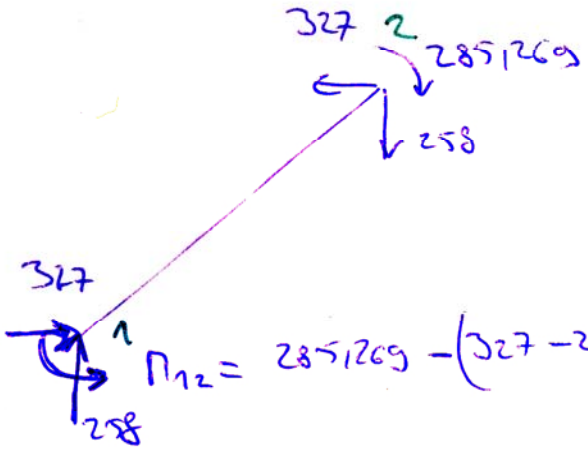
Kontrola: Globální podmínky rovnováhy



$\rightarrow: 327 - 327 = 0$ ok

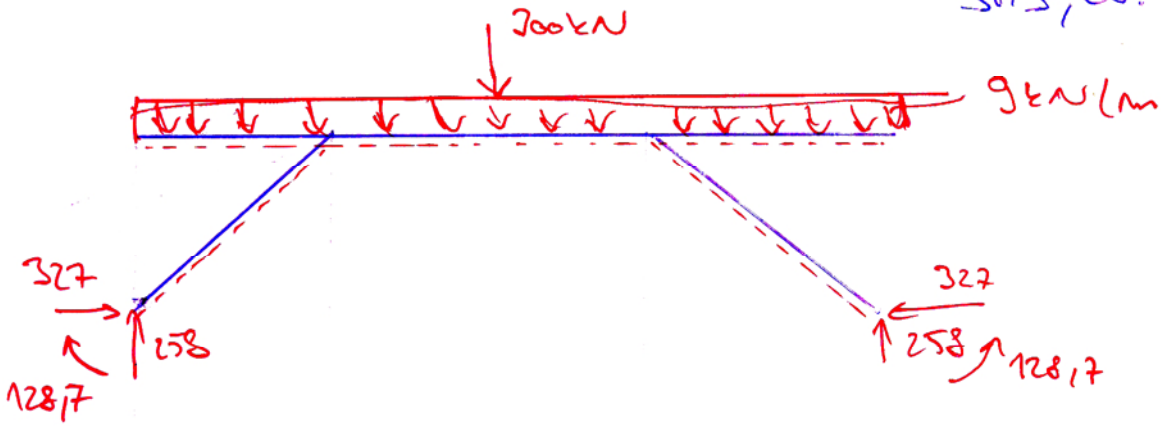
$\uparrow: 2 \cdot 258 - 300 - 24 \cdot 9 = 0$ ok

$\circlearrowleft_a$ $300 \cdot 6 + 9 \cdot 12 \cdot 6 + M_{12} - M_{12} + 258(6-18) + 327(-6) = 0$ ok

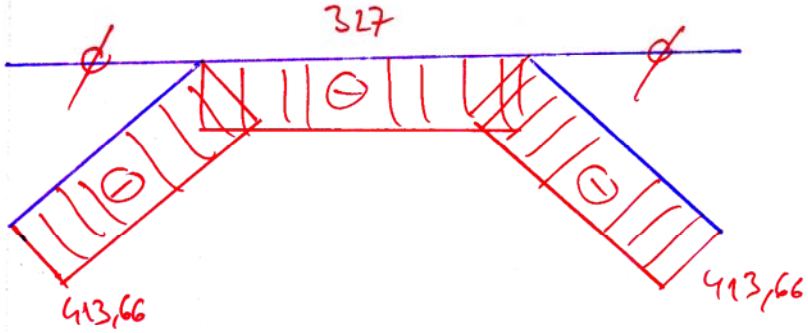


$M_{12} = 285{,}120 - (327 - 258) \cdot 6 = -128{,}731$ kNm

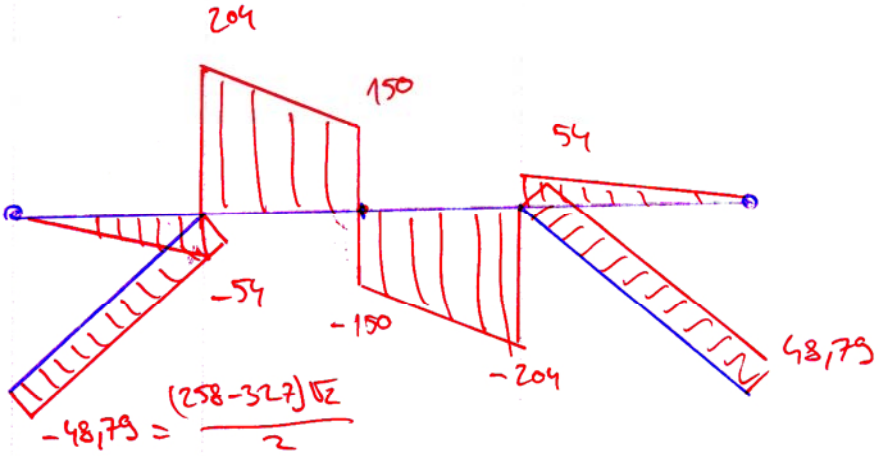
SD3, cv. 7, str. 9



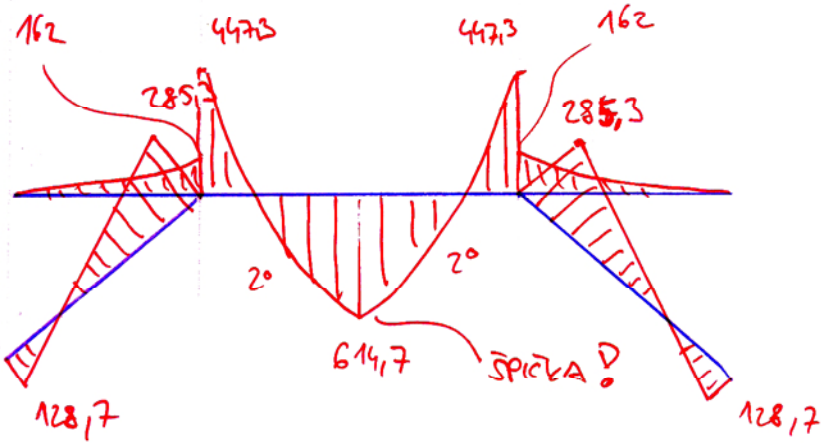
(N) [kN]



(V) [kN]



(M) [kNm]



$M_{max} = 204 \cdot 6 - 9 \cdot 6 \cdot 3 - 447,3 =$