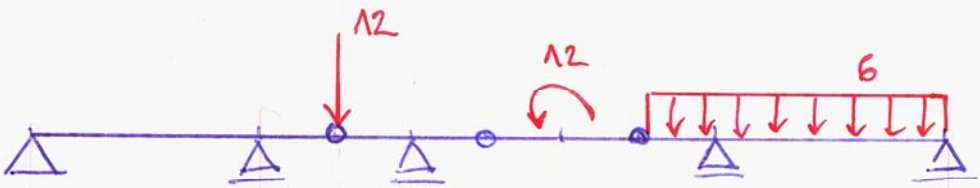
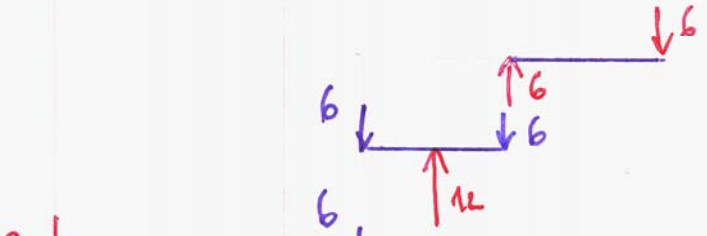
Předpříklad

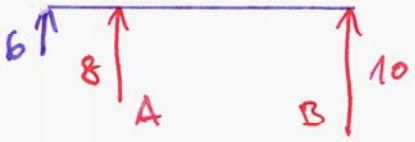
Vše [kN]; [kNm]

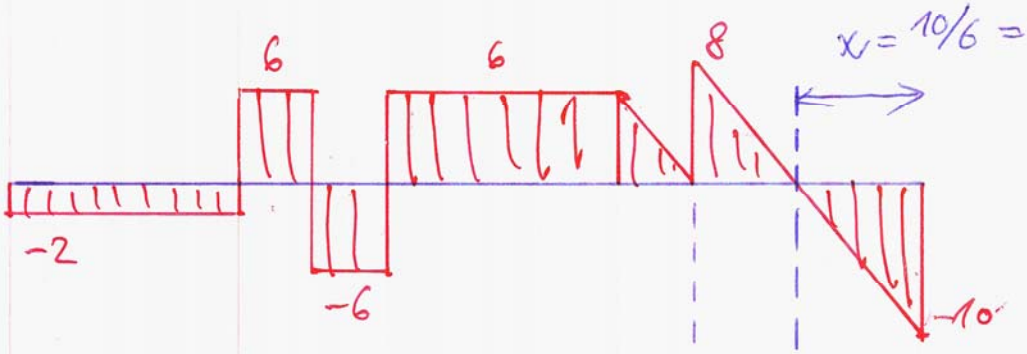




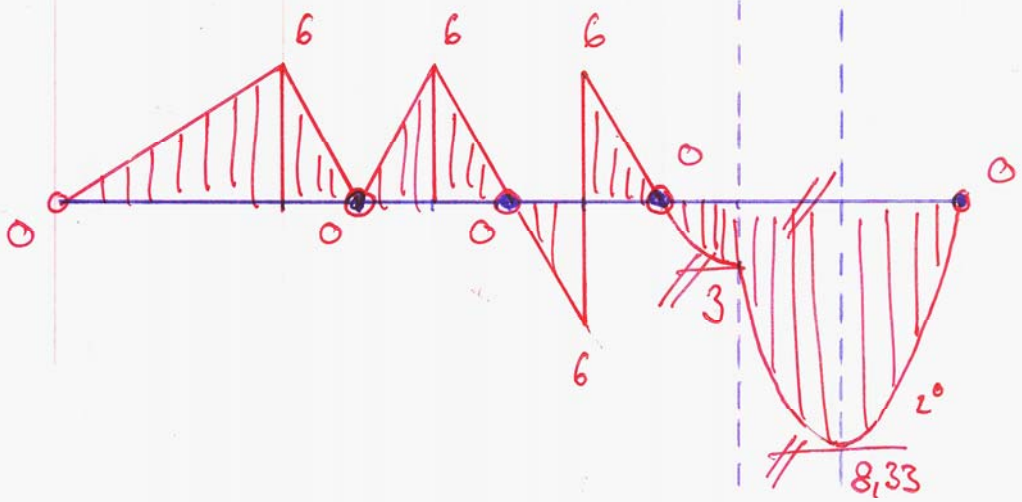
$$A = \frac{-6 \cdot 4 + 6 \cdot 4 \cdot 2}{3} = 8$$

$$B = 6 \cdot 4 - 6 - 8 = 10$$





$x = 10/6 = 1{,}667\,m$

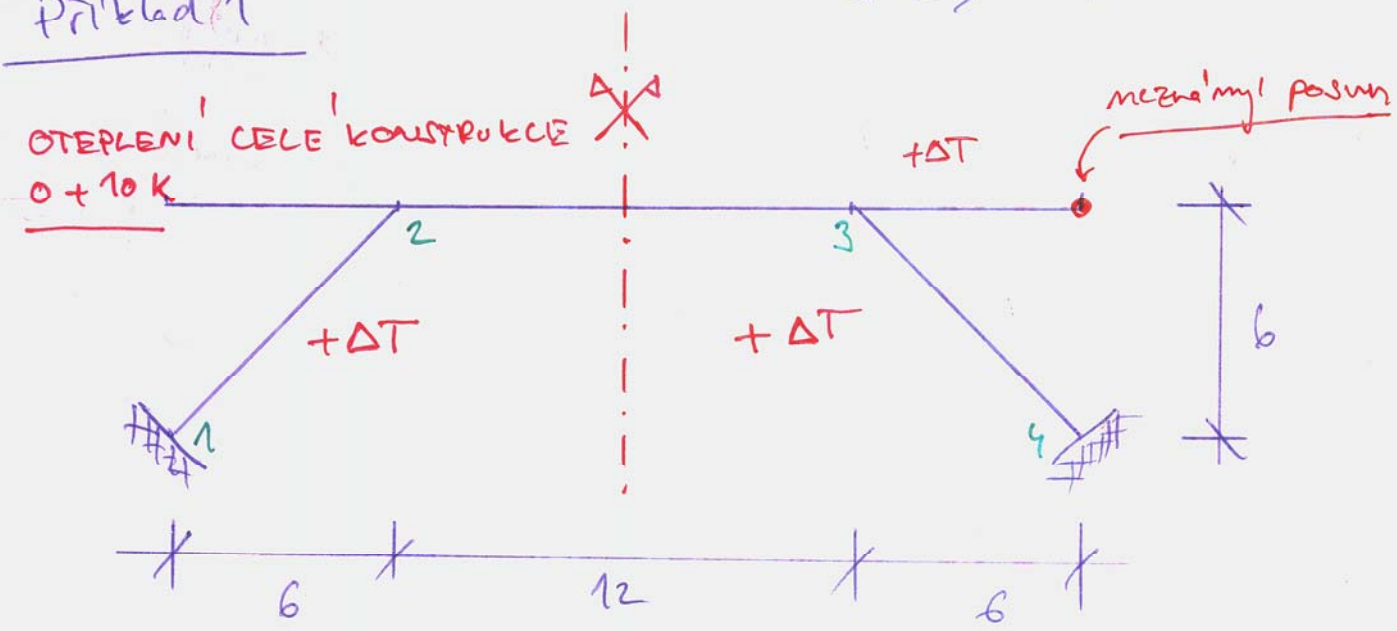


$$M_{max} = 10 \cdot 1{,}667 - \frac{6 \cdot 1{,}667^2}{2} =$$

$$= 8{,}333\ kNm$$

SOS, cv 7, str. 2

Příklad 1



Průřez: 1000 x 600 mm $I = 0{,}04267\ m^4$

$E = 30\ GPa$ $EI = 1{,}28\ GN\,m^2$

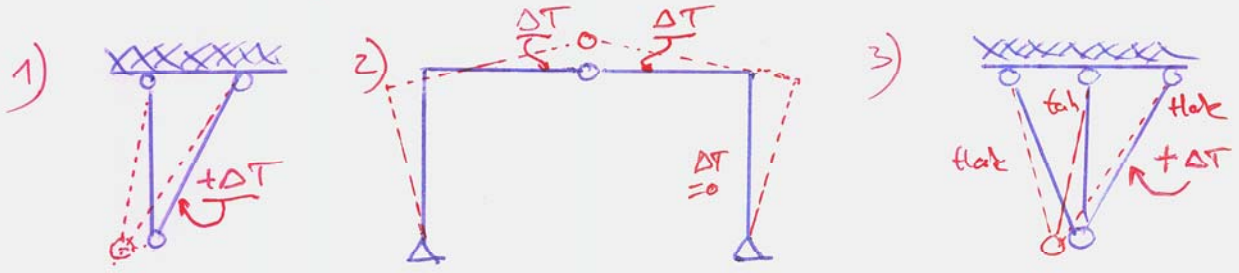
$\alpha_T = 12 \cdot 10^{-6}\ K^{-1}$ $EA = 24\ GN$

ODBOČKA

1) Staticky určitě příhradové konstrukce
při oteplení středníku o ΔT_S → vznikají posuny
→ ∅ osové síly způsobené oteplením

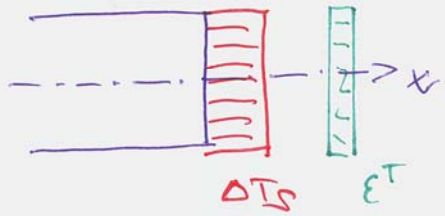
2) Staticky určitě konstrukce — " —

Staticky neurčité konstrukce
- příhradové → kombinace posunů i osových sil (jen NORMÁLOVÉ SÍLY)
  3) – záleží na tuhosti EA
- rámové → kombinace posunů, natočení, osových sil, posouvajících sil, momentů, vliv EA, EI
  4)



Vliv teploty

Opakování: rovnoměrné oteplení střednice o ΔT_S



Staticky určité:

$\varepsilon = \varepsilon_T = \alpha \Delta T_S$

$\sigma_x = 0;\ N = 0$

Staticky neurčité konstrukce

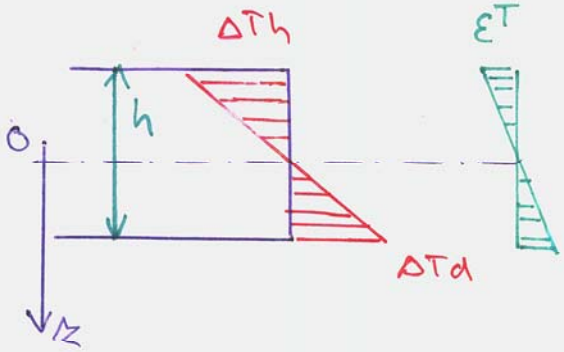
$\varepsilon = 0 = \varepsilon^f + \varepsilon^T$

$\downarrow \alpha\Delta T$

$\Rightarrow \varepsilon^f = -\alpha\Delta T = \frac{\sigma}{E}$

$N = \sigma \cdot A = -EA\alpha\Delta T$

Nerovnoměrné oteplení konstrukce

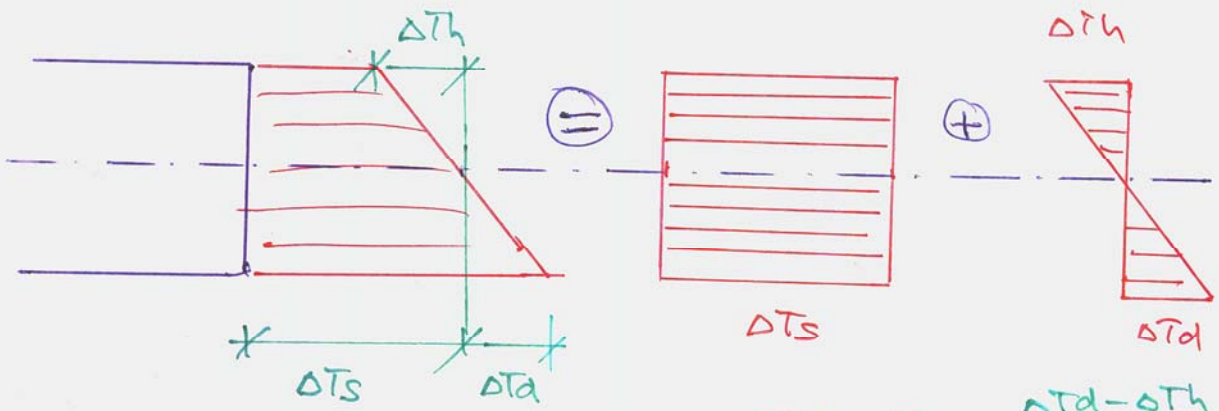


popis pole teploty: $\Delta T(z) = \frac{\Delta T_d - \Delta T_h}{h} \cdot z$

deformace:

$$\varepsilon^T(z) = \alpha_T \Delta T(z) = \alpha_T \cdot z \cdot \frac{\Delta T_d - \Delta T_h}{h}$$

Obecné oteplení konstrukce



$$\Delta T(z) = \Delta T_S + \frac{\Delta T_d - \Delta T_h}{h} \cdot z$$

Celkové deformace

$$\varepsilon = \varepsilon^T + \varepsilon^k = \underbrace{\alpha \Delta T_S}_{\varepsilon^T} + \underbrace{\frac{\Delta T_d - \Delta T_h}{h} \cdot z}_{\varepsilon^k} + \frac{N}{EA} + \frac{M \cdot z}{EI}$$

$\varkappa^T$

- normálové, osové účinky
- ohybové, momentové účinky

$$N_x(x) = \int_A \sigma_x(x,z)\,dy\,dz = EA\left[\varepsilon_S(x) - \varepsilon_S^T\right] \qquad \varepsilon_S = \varepsilon^k + \varepsilon^T$$

$$M_y(x) = \int_A z\,\sigma_x(x,z)\,dy\,dz = EI\left[\varkappa(x) - \varkappa^T\right] \qquad \varkappa = \varkappa^k + \varkappa^T$$

Příklad

$$\varkappa_T = \frac{\Delta T_d - \Delta T_h}{h} \cdot z = \text{konst}$$

$$\varkappa = \varkappa^k + \varkappa^T = 0 + \varkappa^T$$

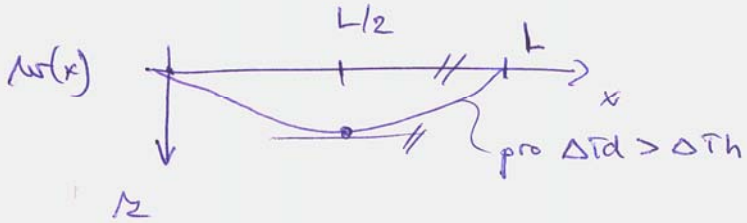
$$w'' = -\varkappa$$

$$w' = -\varkappa^T x + c_1$$

$$w = -\frac{\varkappa^T x^2}{2} + c_1 x + c_2 \qquad w(0) = 0 \Rightarrow c_2 = 0$$

$$w(L) = 0 \Rightarrow c_1 = \frac{\varkappa^T L}{2}$$

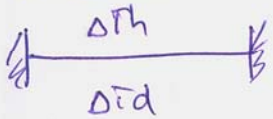
$$w = -\frac{\varkappa^T x^2}{2} + \frac{\varkappa^T L}{2} x$$



$$\Rightarrow \frac{\Delta T_d - \Delta T_h}{h} \cdot \alpha_T = -\frac{\Pi}{EI}$$

$$\Rightarrow \Pi = -\frac{EI\alpha_T(\Delta T_d - \Delta T_h)}{h}$$

Příklad



$w(0) = 0 = w(L)$

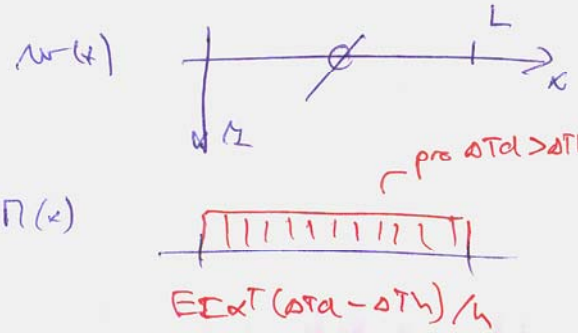
$w'(0) = 0 = w'(L)$

$$w'' = -\varkappa = -(\varkappa^k + \varkappa^T)$$

$$w' = -x(\varkappa^k + \varkappa^T) + c_1 \to 0$$

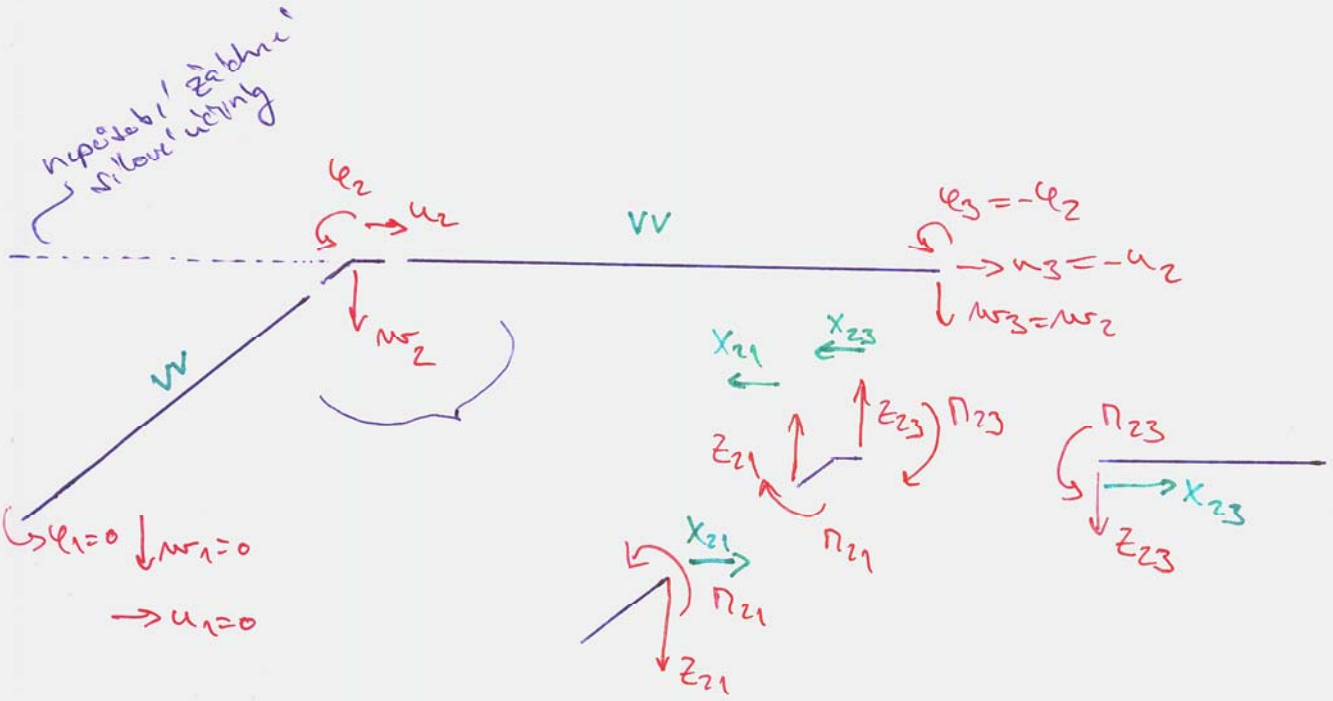
$$w = -\frac{x^2}{2}(\varkappa^T + \varkappa^k) + c_2 \to 0$$

$$w(L) = 0 \Rightarrow -\varkappa^T = \varkappa^k(L)$$



Pokračování př. 1

Symetrie $\Rightarrow \varphi_2 = -\varphi_3;\ w_2 = w_3;\ u_2 = -u_3$



Sestavíme podmínky rovnováhy pro těleso 2:

$\uparrow: Z_{21} + Z_{23} = 0$

$\rightarrow: X_{21} + X_{23} = 0$

$\circlearrowleft\ M_{21} + M_{23} = 0$

1) Prut 2-3: koncové momenty a síly

$$M_{23} = \bar{M}_{23} + k\left(2\cdot\varphi_2 + \varphi_3 + \frac{3(w_3 - w_2)}{L_{23}}\right) = 213{,}33\cdot(\varphi_2) \qquad (= 2\varphi_2 - \varphi_2)$$

$\bar{M}_{23} = \bar{M}^{I}_{23} + \bar{M}^{II}_{23} = 0$ (0 + 0)

$$k = \frac{2EI}{L_{23}} = \frac{2\cdot 1280}{12} = 213{,}33\ MNm$$

$$Z_{23} = \bar{Z}_{23} - \frac{3k}{L_{23}}\left(\underbrace{\varphi_2 + \varphi_3}_{0} + \frac{2(w_3 - w_2)}{L}\right) = 0$$

($\bar{Z}_{23} = 0$)

$$X_{23} = \bar{X}_{23} - n_{23}(u_3 - u_2) = 2880 - 2000(-2u_2) = 2880 + 4000\,u_2$$

$$n_{23} = \frac{EA}{L_{23}} = \frac{24\,000}{12} = 2000\ MN/m$$

$$\bar{X}_{23} = EA\,\alpha_T\,\Delta T_S = 24000\cdot 12\cdot 10^{-6}\cdot 10 = 2{,}88\ MN$$

2) Prut 1-2 – koncové síly a momenty – 1) LOKÁLNÍ

$$\boxed{\Pi_{21}} = \overline{\Pi}_{21} + k_{12}\left(\varphi_1 + 2\varphi_2 + \frac{3(w_2 - w_1)}{L_{12}}\right) = \boxed{301{,}7 \cdot \left(2\varphi_2 + \frac{3w_2^l}{\sqrt{2}\cdot 6}\right)}$$

$$k_{12} = \frac{2EI}{L_{12}} = \frac{2\cdot 1280}{\sqrt{2}\cdot 6} = 301{,}7\ MN\,m$$

$$\boxed{Z_{21}^l} = \overline{Z}_{21}^l + \frac{3k_{12}}{L_{12}}\left(\varphi_1 + \varphi_2 + 2\frac{w_2^l - w_1^l}{L_{12}}\right) = \frac{3\cdot 301{,}7}{\sqrt{2}\cdot 6}\left(0 + \varphi_2 + \frac{2w_2^l}{\sqrt{2}\cdot 6}\right) =$$

$$= \boxed{106{,}67\varphi_2 + 25{,}1417 w_2^l}$$

$$X_{21}^l = \overline{X}_{21}^l + n_{12}(u_2^l - u_1^l) \qquad n_{12} = \frac{EA}{L_{12}} = \frac{24000}{8{,}485} = 2828{,}43\ MN/m$$

$$\overline{X}_{21}^l = -EA\alpha_T \Delta T_S = -2{,}88\ MN$$

$$\boxed{X_{21}^l = -2880 + 2828{,}43\, u_2^l}$$

2) TRANSFORMACE

- SÍLY

$$X_{21} = X_{21}^l \cos\alpha - Z_{21}^l \sin\alpha \quad \Big\| \quad \alpha = -45^\circ$$

$$Z_{21} = X_{21}^l \sin\alpha + Z_{21}^l \cos\alpha$$

$$\Pi_{21} = \Pi_{21}^l$$

$$\cos(-45^\circ) = \frac{\sqrt{2}}{2}$$

$$\sin(-45^\circ) = -\frac{\sqrt{2}}{2}$$

- POSUNY

$$u_2^l = u_2 \cos\alpha + w_2 \sin\alpha$$

$$w_2^l = -u_2 \sin\alpha + w_2 \cos\alpha$$

- $X_{21} = X_{21}^{l}\cos\alpha - Z_{21}^{l}\sin\alpha$

$X_{21}^{l} = -2880 + 2828{,}43\,u_2^{l}$

$u_2\cos\alpha + w_2\sin\alpha$

$\frac{\sqrt{2}}{2}u_2 - \frac{\sqrt{2}}{2}w_2$

$Z_{21}^{l} = 106{,}674\varphi_2 + 25{,}1417\,w_2^{l}$

$-u_2\sin\alpha + w_2\cos\alpha$

$\frac{\sqrt{2}}{2}u_2 + \frac{\sqrt{2}}{2}w_2$

$$X_{21} = \frac{\sqrt{2}}{2}\left[-2880 + 2828{,}43\left(\frac{\sqrt{2}}{2}u_2 - \frac{\sqrt{2}}{2}w_2\right) + 106{,}674\varphi_2 + 25{,}1417\cdot\frac{\sqrt{2}}{2}\left(u_2 + w_2\right)\right]$$

$$X_{21} = -2036{,}47 + 1414{,}22\,(u_2 - w_2) + 75{,}4274\varphi_2 + 12{,}57\,(u_2 + w_2) =$$

$$X_{21} = \boxed{-2036{,}47} + 1426{,}79\,u_2 = 1401{,}64\,w_2 + 75{,}4274\varphi_2$$

- $Z_{21} = X_{21}^{l}\sin\alpha + Z_{21}^{l}\cos\alpha =$

$$= \frac{\sqrt{2}}{2}\left[+2880 - 2828{,}43\left(\frac{\sqrt{2}}{2}u_2 - \frac{\sqrt{2}}{2}w_2\right) + 106{,}674\varphi_2 + 25{,}1417\left(\frac{\sqrt{2}}{2}u_2 + \frac{\sqrt{2}}{2}w_2\right)\right] =$$

$$Z_{21} = \boxed{+2036{,}47} - 1401{,}64\,u_2 + 1426{,}79\,w_2 + 75{,}427\,\varphi_2$$

- $\Pi_{21} = \bar{\Pi}_{21} + z_{12}\left(\varphi_1 + 2\varphi_2 + \frac{3(w_2 - w_1)}{l_{12}}\right) = 201{,}7\left(2\varphi_2 + \frac{3w_2^{l}}{\sqrt{2}\cdot 6}\right) =$

$$= 603{,}4\varphi_2 + 75{,}4271\,u_2 + 75{,}4271\,w_2$$

Soustava rovnovelhy:

$\curvearrowleft: z_{21} + z_{23} = 0$

$2036{,}47 - 1401{,}64\, u_2 + 1426{,}79\, w_2 + 75{,}427\, \varphi_2 = 0$

$\rightarrow: x_{23} + x_{21} = 0$

$2880 + 4000\, u_2 = 2036{,}47 + 1426{,}79\, u_2 - 1401{,}64\, w_2 + 75{,}427\, \varphi_2$

$\circlearrowright: m_{21} + m_{23} = 0$

$603{,}44\, \varphi_2 + 75{,}427\, u_2 + 75{,}423\, w_2 + 213{,}33\, \varphi_2 = 0$

$\Rightarrow$ řešení: $\varphi_2 = 0{,}26345$ rad $u_2 = -0{,}712$ mm $w_2 = -2{,}141$ mm

$m_{21} = -56{,}202$ kNm

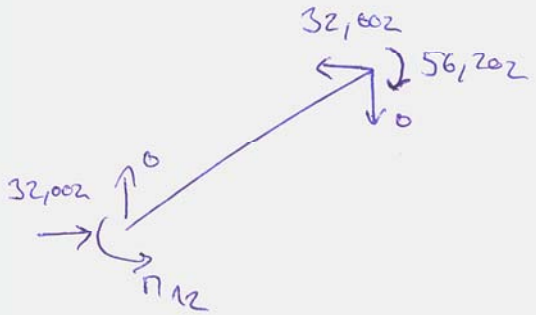
$m_{23} = 56{,}202$ kNm

$z_{21} = \not{0}$ kN

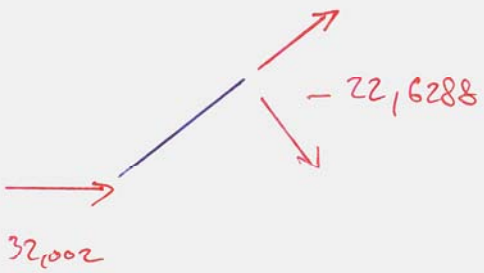
$z_{23} = \not{0}$ kN

$x_{21} = -32{,}002$ kN

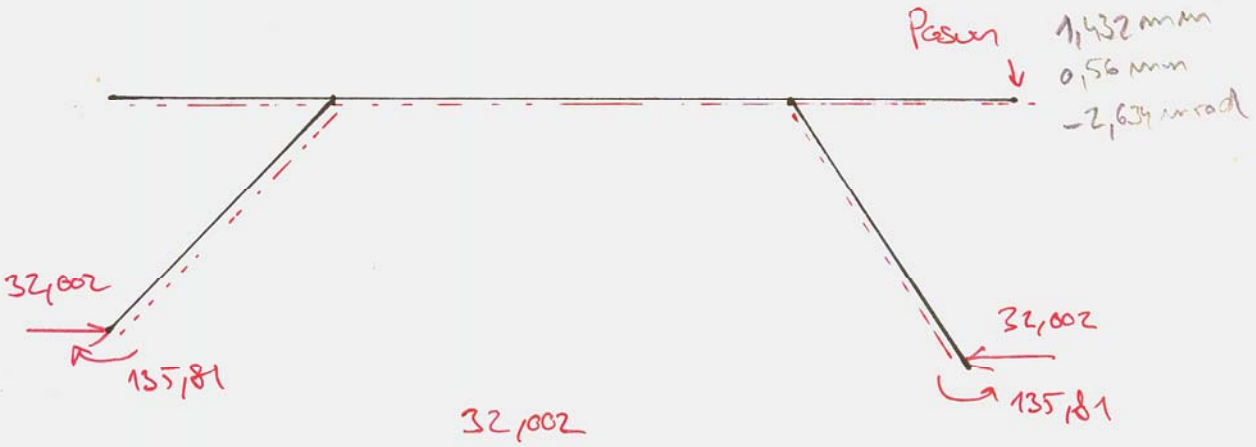
$x_{23} = 32{,}002$ kN



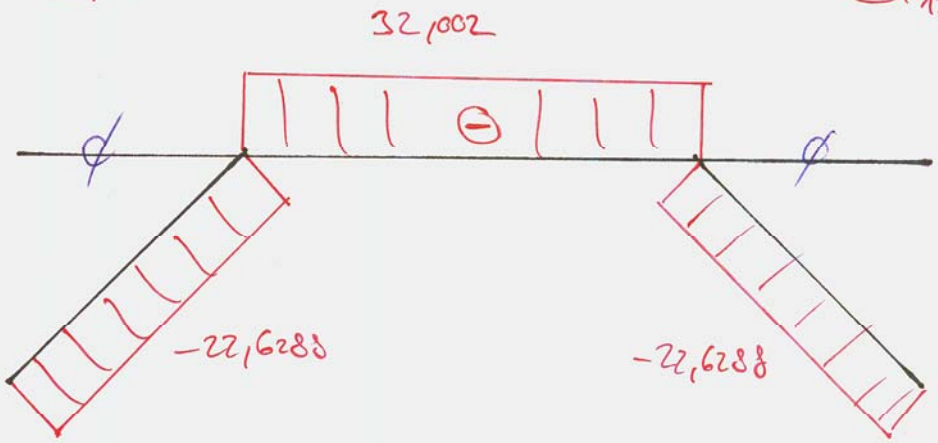
$m_{12} = 56{,}202 - 32{,}002 \cdot 6 = -135{,}81$ kNm



Zatížení, reakce

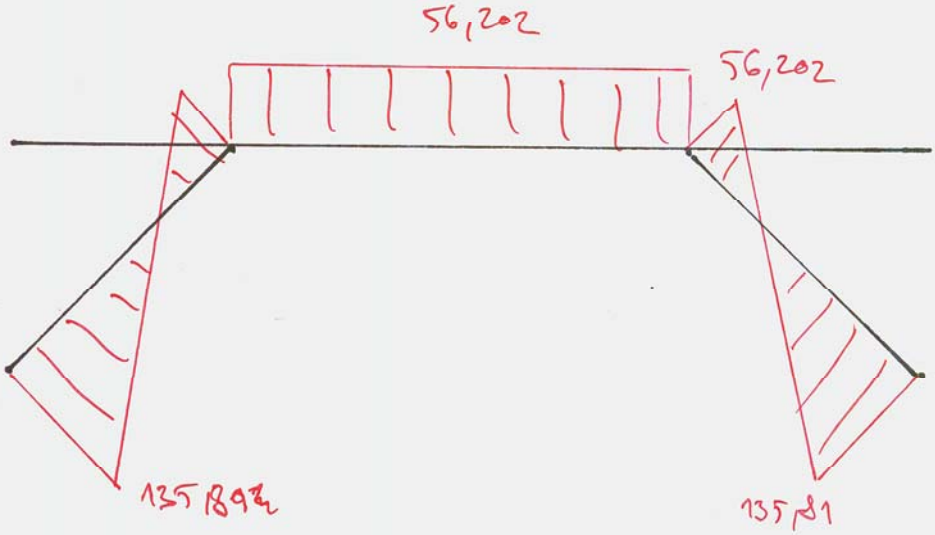


N [kN]

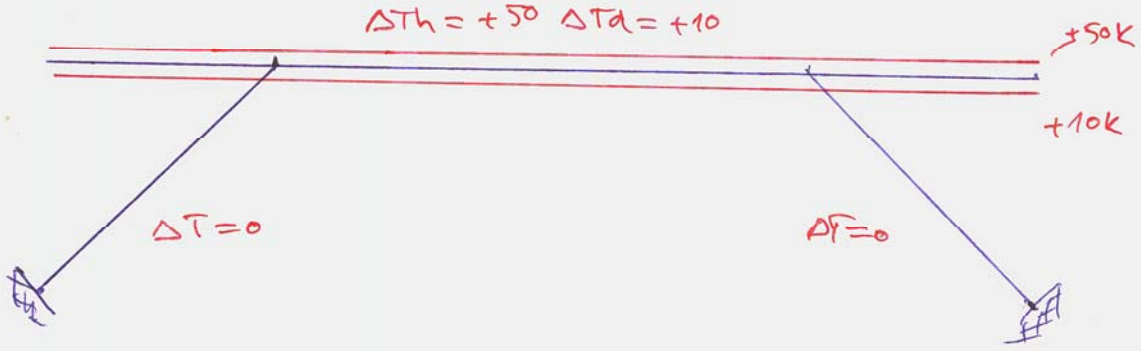


V [kN]

M [kNm]



Příklad 2



$\rightarrow \Delta T_S = +30k$

$\Delta T_h = +20k$

$\Delta T_d = -20k$

Podmínky rovnováhy – viz. př. 1

1) Prut 2-3: koncový moment $\bar{M}_{23}$ — Červená pasivní hodnoty = PŘ. 1

- $M_{23} = \bar{M}_{23} + 213{,}33\,\varphi_2 = -768 + 213{,}33\,\varphi_2$

$$\bar{M}_{23} = \frac{EI}{h} \alpha \Delta t \qquad \Delta t = \Delta T_d - \Delta T_h = -40k$$

$$\bar{M}_{23} = \frac{1280}{0{,}8} \cdot 12 \cdot 10^{-6} \cdot (-40) = -0{,}768\ MNm = -768\ kNm$$

- $Z_{23} = 0$ Proč?

- $X_{23} = 4000\,u_2 + \bar{X}_{23} =$
  $= 4000\,u_2 + 3640$

$\bar{X}_{23} = EA \alpha \Delta T = 24 \cdot 10^6 \cdot 12 \cdot 10^{-6} \cdot 30 = 8640\ kN$

2) Prut 1-2

- $M_{21} = 603{,}4\,\varphi_2 + 75{,}427\,u_2 + 75{,}427\,w_2$ IDENTICKÉ

$$Z_{21}^{l} = 106{,}67\,\varphi_2 + 25{,}1417\,w_2^{l}$$

$$X_{21}^{l} = 2828{,}43\,u_2^{l}$$

- $X_{21} = 1426{,}79\,u_2 - 1401{,}64\,w_2 + 75{,}427\,\varphi_2$
- $Z_{21} = -1401{,}64\,u_2 + 1426{,}79\,w_2 + 75{,}427\,\varphi_2$

} BEZ OSOVÝCH SIL OD OHŘEVU

Výsledky: $\varphi_2 = 1{,}3466$ mrad

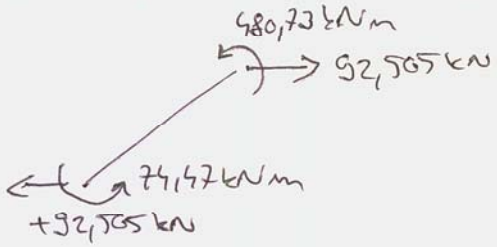
$u_2 = -2{,}1831$ mm

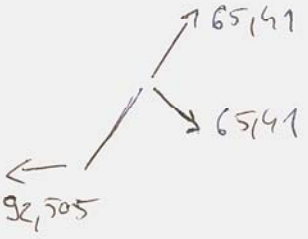
$w_2 = -2{,}2158$ mm

$M_{21} = 480{,}73$ kNm $\quad M_{23} = -480{,}73$ kNm

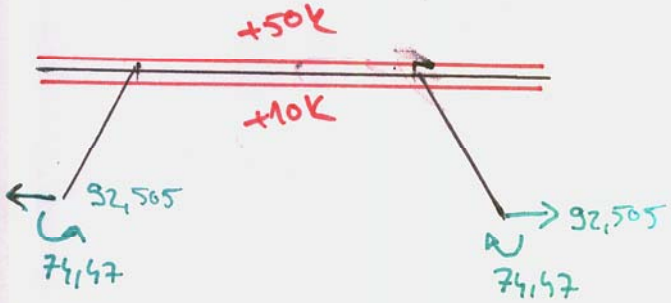
$Z_{21} = 0$ kN $\quad Z_{23} = 0$ kN

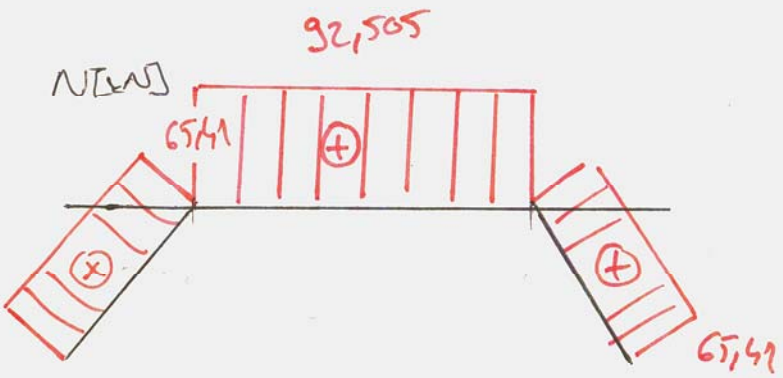
$X_{21} = 92{,}505$ kN $\quad X_{23} = -92{,}505$ kN $\quad M_{12} = 74{,}47$ kNm



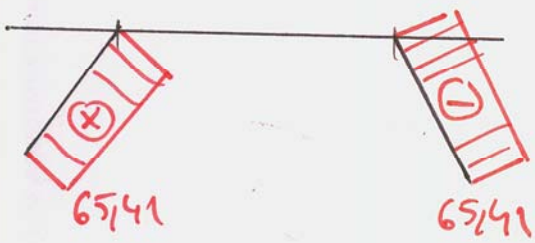


Reakce

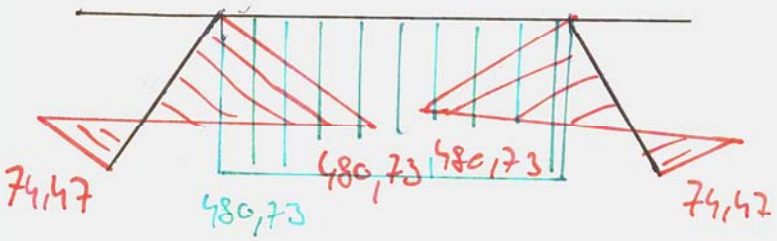




V[kN]



M[kNm]



Tvar zdeformované konstrukce: