Příklad:

Na zadané konstrukci určit
- stupeň statické neurčitosti
- počet neznámých při řešení ODM
  - pruty VV
  - pruty VV + VK
  - obecné zatížení
  - symetrické zat.

ZDM — II —

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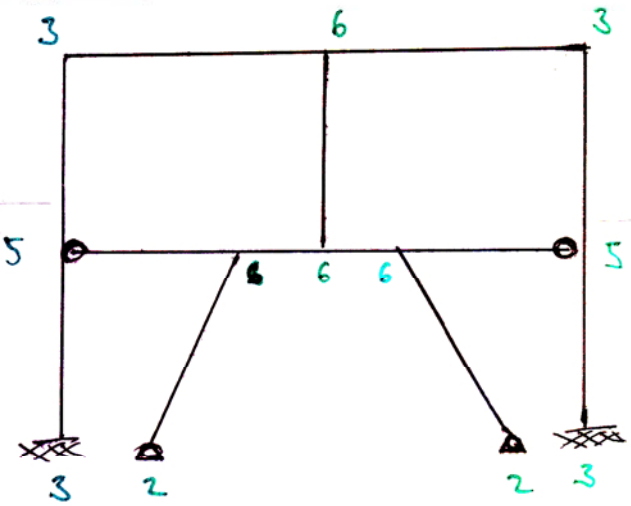
1) STUPEŇ STATICKÉ NEURČITOSTI



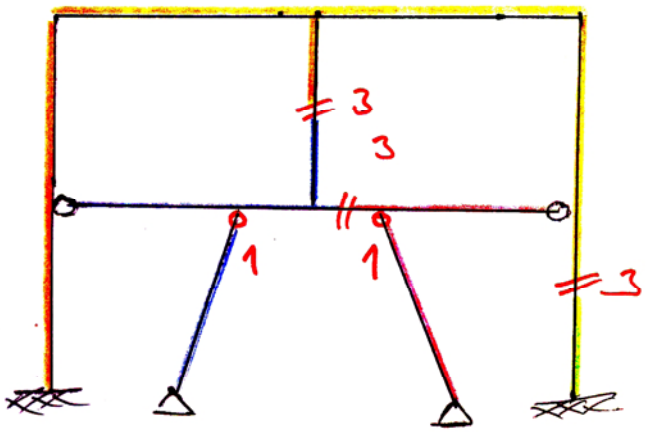
Σ 12
Σ 28
Σ 10
Σ 50

13 desek × 3 SV = 39 SV

konstrukce je $(50 - 39) \times$ staticky neurčitá

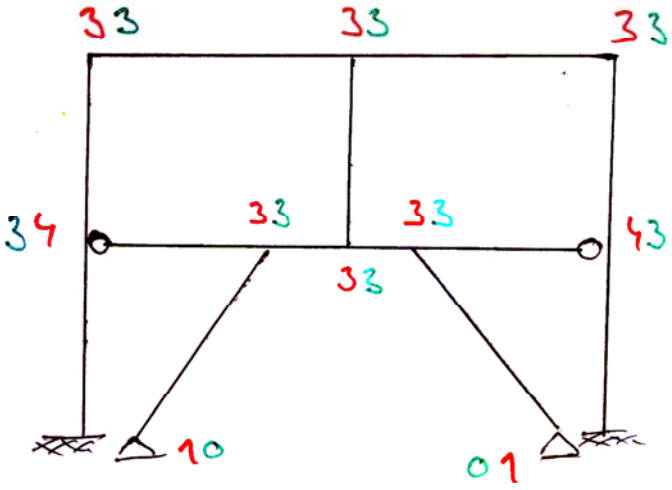
$n = 11$

ODLIŠNÝ POSTUP: VYTVOŘÍME STATICKY URČITOU KONSTRUKCI



$3 \times 3 + 2 \times 1 = 11$

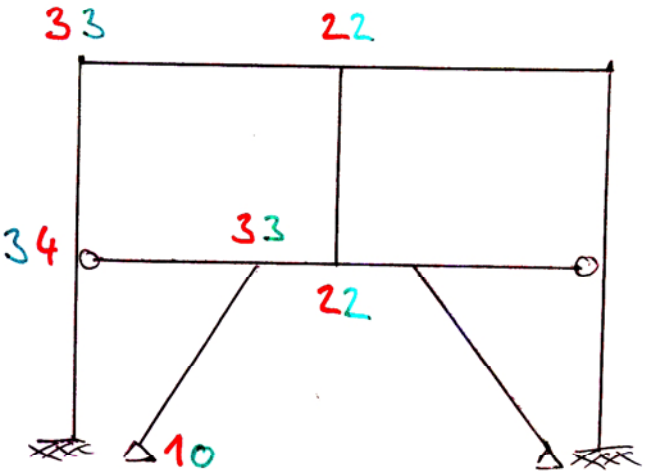
- ODM



prvky VV
Σ = 28 neznámých

prvky VK
Σ = 24 neznámých

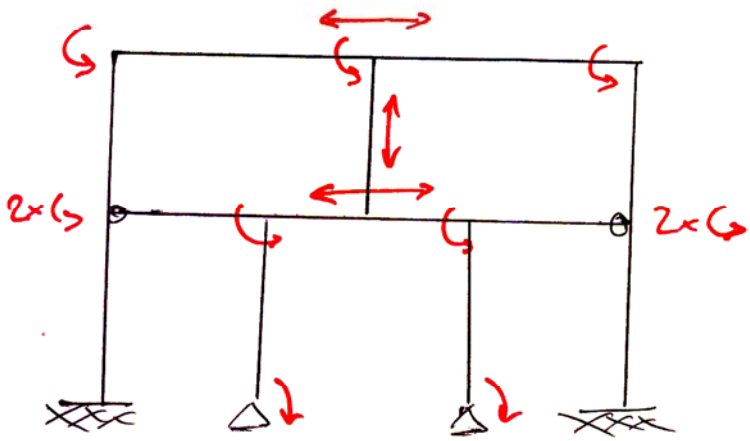
- ODM + symetrické zatížení



prvky VV
15 neznámých

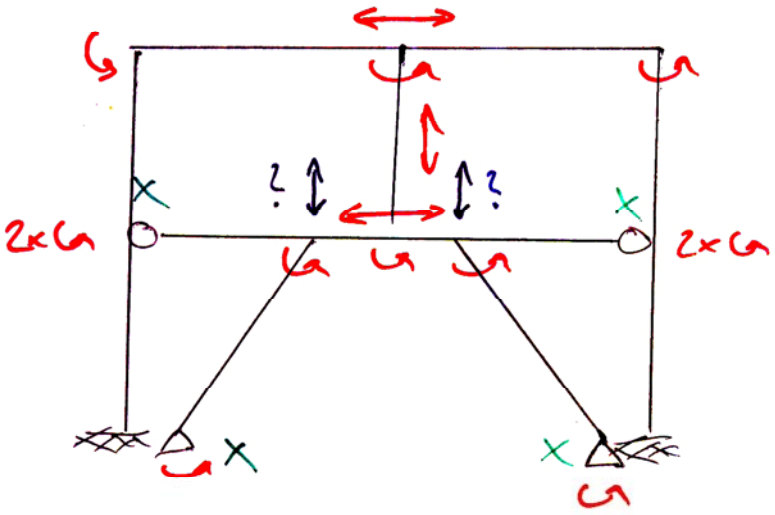
prvky VK
13 neznámých

- ZDM – příprava cvičení



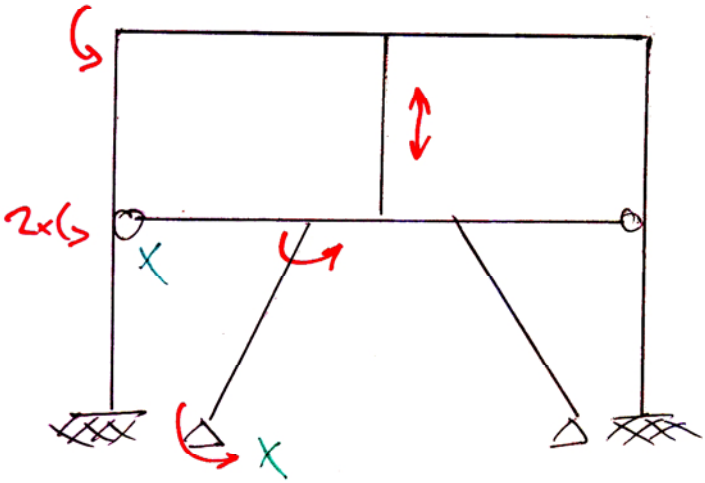
14 neznámých pro VV a obecné zatížení

ZDM obecné zatížení



m prvky VV
15 neznámých
m prvky VK
11 neznámých

ZDM symetrické zatížení



m prvky VV
6 neznámých
m prvky VK
4 neznámé

Cviceni 8 **ZDM**

Domain type: 2dbeam, default ndofs per node is 3, per side is 0

DofManager output:

Node 1 (1): dof 1 d 8.02464539e-03 dof 2 d 2.56397657e-14 dof 3 d -1.30700857e-03

Node 2 (2): dof 1 d 8.02464539e-03 dof 2 d 3.03612170e-14 dof 3 d 8.06898598e-04

Node 3 (3): dof 1 d 6.28590961e-03 dof 2 d 2.11866489e-14 dof 3 d -8.74150459e-04

Node 4 (4): dof 1 d 6.28590961e-03 dof 2 d 2.54807875e-14 dof 3 d -2.86141531e-04

Node 5 (5): dof 1 d 2.92103990e-03 dof 2 d 1.25624644e-14 dof 3 d -1.44446120e-03

Node 6 (6): dof 1 d 2.92103990e-03 dof 2 d 1.54378933e-14 dof 3 d -2.60425544e-04

Node 7 (7): dof 1 d 0.00000000e+00 dof 2 d 0.00000000e+00 dof 3 d 0.00000000e+00

Node 8 (8): dof 1 d 0.00000000e+00 dof 2 d 0.00000000e+00 dof 3 d 0.00000000e+00

ODM

DofManager output:

Node 1 (1): dof 1 d 8.10981322e-03 dof 2 d 5.12871705e-04 dof 3 d -1.32215806e-03

Node 2 (2): dof 1 d 8.08511934e-03 dof 2 d 6.07128295e-04 dof 3 d 7.99590585e-04

Node 3 (3): dof 1 d 6.32122421e-03 dof 2 d 4.23802043e-04 dof 3 d -8.85346942e-04

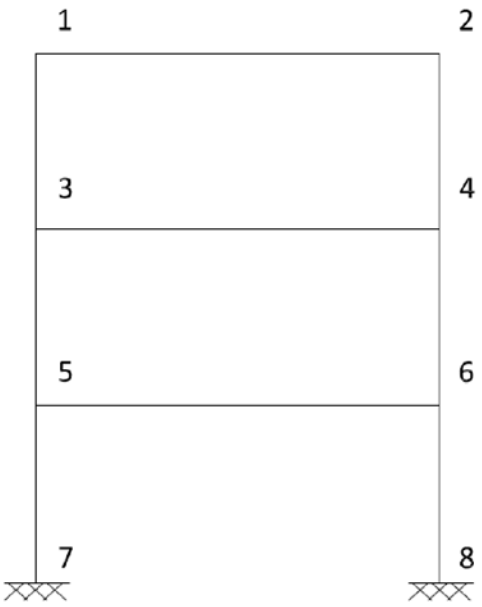
Node 4 (4): dof 1 d 6.32341903e-03 dof 2 d 5.09531290e-04 dof 3 d -2.94260173e-04

Node 5 (5): dof 1 d 2.93107047e-03 dof 2 d 2.51294928e-04 dof 3 d -1.44982396e-03

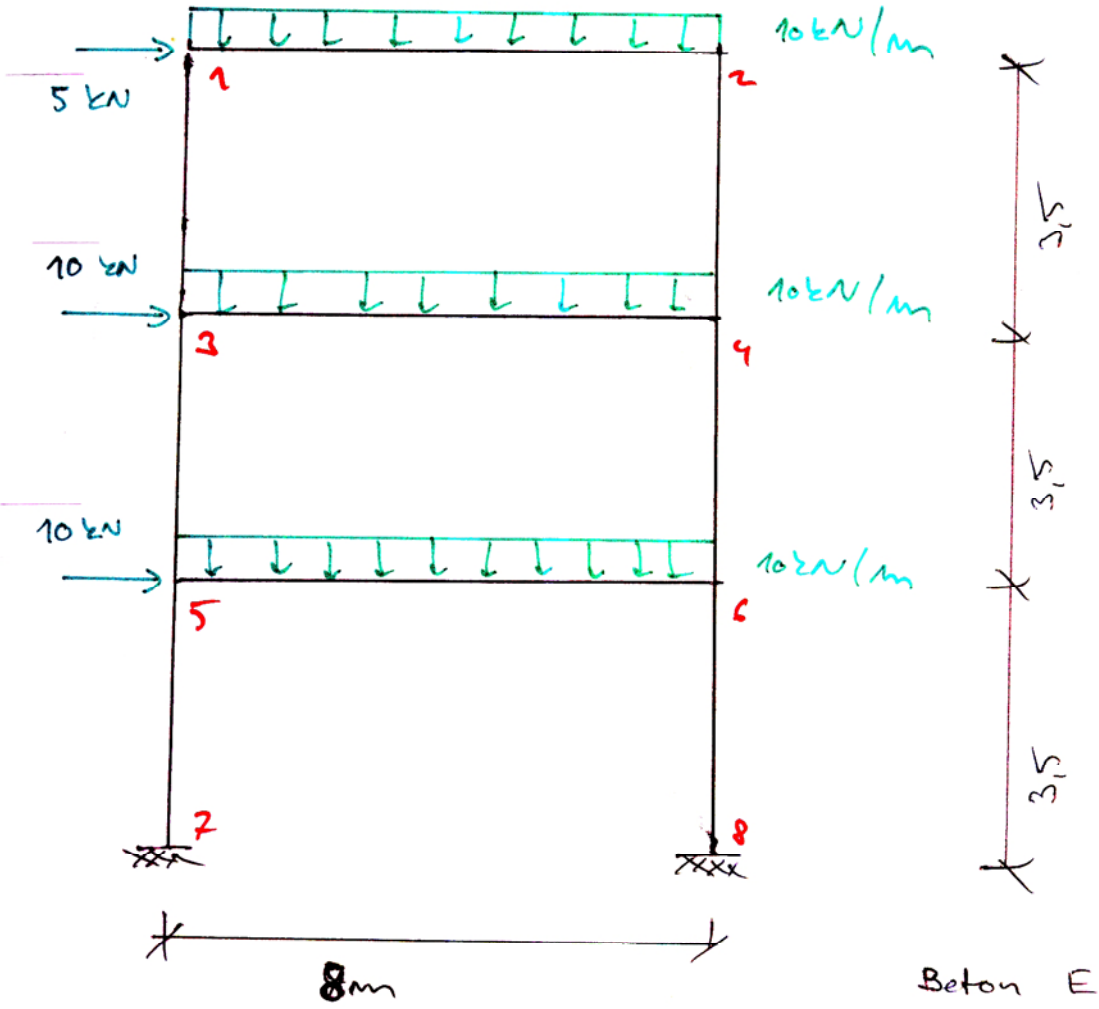
Node 6 (6): dof 1 d 2.93057871e-03 dof 2 d 3.08705072e-04 dof 3 d -2.66879023e-04

Node 7 (7): dof 1 d 0.00000000e+00 dof 2 d 0.00000000e+00 dof 3 d 0.00000000e+00

Node 8 (8): dof 1 d 0.00000000e+00 dof 2 d 0.00000000e+00 dof 3 d 0.00000000e+00



Příklad – rám s posuvnými styčníky



Beton $E = 30$ GPa

patra: 250 / 1000

$I = 0{,}0013 \text{ m}^4 \Rightarrow EI_p = 39{,}06 \text{ MNm}^2$

$A = 0{,}25 \text{ m}^2 \Rightarrow EA_p = 7500 \text{ MN}$

sloupy: 500 / 100

$I = 0{,}00104 \text{ m}^4 \Rightarrow EI_s = 31{,}26 \text{ MNm}^2$

$A = 0{,}05 \text{ m}^2 \Rightarrow EA_s = 1500 \text{ MN}$

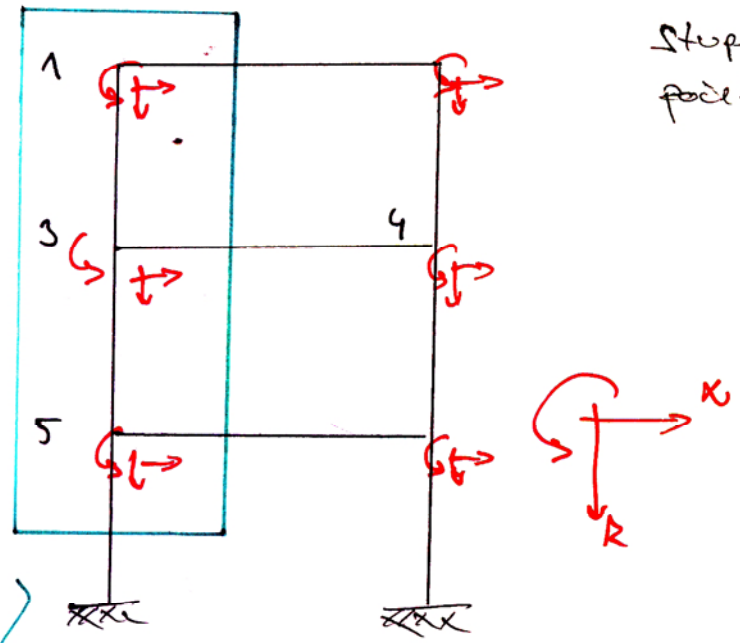
→ VLIV VÝPOČETNÍ METODY NA NASAZENÉ POSUNY/POTOČE A ROZLOŽENÍ VNITŘNÍCH SIL

1) ODM

2) ZDM

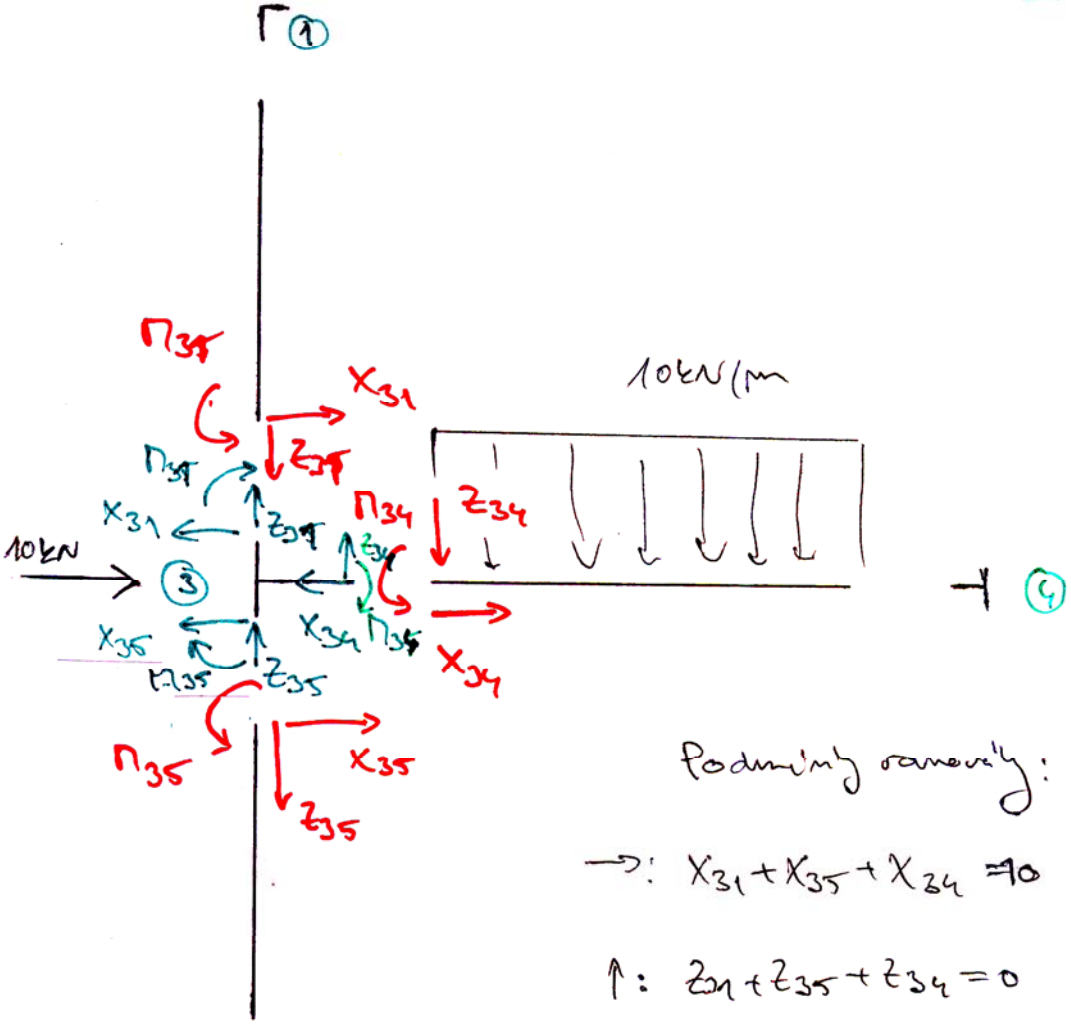
3) ZDM + $EI_p = \infty$

1) ŘEŠENÍ KONSTRUKCE ODT



Stupeň statické neurčitosti? 9
počet neznámých? $6 \times 3 = 18$

→ budeme sestavovat podmínky rovnováhy pro 3. styčník



Podmínky rovnováhy:

$\rightarrow: X_{31} + X_{35} + X_{34} = 10$

$\uparrow: Z_{31} + Z_{35} + Z_{34} = 0$

$\circlearrowleft: \Pi_{31} + \Pi_{34} + \Pi_{35} = 0$

part 3-1

$$z_{31}^{l} = X_{31} = \bar{z}_{31}^{l} - \frac{3k_{13}}{L_{13}}\left(\varphi_3 + \varphi_1 + \frac{2(w_1^l - w_3^l)}{L_{13}}\right)$$

$\bar{z}_{31}^{l} = 0$

$L_{13} = 3{,}5\,m \qquad k_{13} = \frac{2EI_S}{L_{13}} \qquad w_1^l = u_1 \qquad w_3^l = u_3$

$$X_{31}^{l} = -z_{31} = \bar{X}_{31}^{l} - n_{13}(u_1^l - u_3^l)$$

$\bar{X}_{31}^{l} = 0$

$n_{13} = \frac{EA_S}{L_{13}} \qquad u_1^l = -w_1 \qquad u_3^l = -w_3$

$$\Pi_{31} = \bar{\Pi}_{31} + k\left(2\varphi_3 + \varphi_4 + \frac{3(w_4^l - w_3^l)}{L_{13}}\right)$$

$\bar{\Pi}_{31} = 0$

part 3-4

$$X_{34}^{l} = \bar{X}_{34}^{l} - n_{34}(u_4^l - u_3^l) = X_{34}$$

$\bar{X}_{34}^{l} = 0$

$n_{34} = \frac{EA_P}{L_{34}} \qquad u_4^l = u_4 \qquad u_3^l = u_3$

$$z_{34}^{l} = z_{34} = \bar{z}_{34}^{l} - \frac{3k_{34}}{L_{34}}\left(\varphi_3 + \varphi_4 + \frac{2(w_4^l - w_3^l)}{L_{34}}\right)$$

$L_{34} = 8\,m \qquad w_4^l = w_4 \qquad w_3^l = w_3$

$$\bar{z}_{34}^{l} = -\frac{10 \cdot L_{34}}{2}$$

$$M_{34}^{l} = \Pi_{34} = \bar{\Pi}_{34}^{l} + k_{34}\left(2\varphi_3 + \varphi_4 + \frac{3(w_4^l - w_3^l)}{L_{34}}\right)$$

$$\bar{\Pi}_{34}^{l} = \frac{10 \cdot L_{34}^2}{12}$$

Part 3-5

... me mouille

Prut 3-5

$$M_{35} = \overline{M}_{35} + k_{35}\left(\varphi_5 + 2\varphi_3 + \frac{3(w_3^l - w_5^l)}{L_{35}}\right)$$

$\overline{M}_{35} = 0$; $w_3^l = u_3$; $w_5^l = u_5$; $L_{35} = 3{,}5\,m$

$$k_{35} = \frac{2EI_5}{L_{13}} = \frac{2 \cdot 31{,}26}{3{,}5} = 17{,}8629\ MNm \quad (= k_{31})$$

$$z_{35}^l = X_{35} = \overline{z}_{35}^l + \frac{3k_{35}}{L_{35}}\left(\varphi_3 + \varphi_5 + \frac{2(w_3^l - w_5^l)}{L_{35}}\right)$$

$\overline{z}_{35}^l \downarrow 0$

$$X_{35}^l = -z_{35} = \overline{X}_{35}^l + M_{35}(u_3^l - u_5^l)$$

$\overline{X}_{35}^l \downarrow 0$

$w_3^l = -w_3$

$w_5^l = -w_5$

$$M_{35} = \frac{EA_5}{L_{35}} = \frac{1000}{3{,}5} = 428{,}57\ \frac{MN}{m}$$

Kontrola výsledků: pro stykník č. 3

1) Vodorovný směr

Prut (13)

$$X_{31} = -\frac{3 \cdot 17{,}8629}{3{,}5}\left(-8{,}8535 \cdot 10^{-7} + -1{,}32216 + \frac{2 \cdot (8{,}1098 - 6{,}3212)}{3{,}5}\right) =$$

(φ_3, φ_1, u_1, u_3)

$$= 18{,}1505\ kN$$

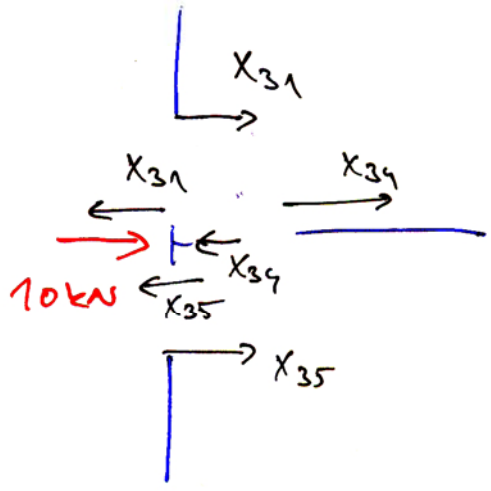
(34) $$X_{34} = -937{,}5 \cdot (6{,}3234 - 6{,}3212) = -2{,}0625\ kN$$

(u_4, u_3; $\frac{EA_p}{L_{34}}$)

(35) $$X_{35} = \frac{3 \cdot 17{,}8629}{3{,}5}\left(-0{,}88534 + -1{,}4498 + \frac{2(6{,}321 - 2{,}931)}{3{,}5}\right) = -6{,}093\ kN$$

(φ_3, φ_5, w_3, w_5)

rovnováha:



$X_{31} + X_{34} + X_{35} = 10$

$18{,}15 - 2{,}063 - 6{,}093 = 10$

$9{,}994 = 10$

OK

- Kontrola výsledků

Podmínka rovnováhy pro svislý směr:

Zmínit přesnost pro normálové síly !?

Prut 13

$$z_{31} = n_{13}(-w_1 + w_3) = \frac{1500}{3,5}(-0,51287 + 0,4238) = -38,17\ kN$$

Prut 34

$$z_{34} = \bar{z}_{34} - \frac{3k_{34}}{L_{34}}\left(\varphi_3 + \varphi_4 + \frac{2(w_4 - w_3)}{L_{34}}\right) =$$

$$= -40 - \frac{3 \cdot \frac{2EI_y}{L_{34}}}{L_{34}}\left(-0,8853 - 0,2943 + \frac{2(0,5095 - 0,424)}{L_{34}}\right) =$$

$$= -40 - 3,662(-0,8853 - 0,2943 + 0,02138) = -35,76\ kN$$

Prut 35

$$z_{35} = -n_{35}(-w_3 + w_5) = 428,57(0,4238 - 0,2573) = 73,93\ kN$$

rovnováha:

$$z_{31} + z_{34} + z_{35} = 0$$

$$-38,17 - 35,76 + 73,93 = 0 \quad \text{ok}$$

Momentové podmínka rovnováhy

$$M_{31} = k_{13}\left(2\varphi_3 + \varphi_1 + \frac{3(u_1 - u_3)}{L_{13}}\right) =$$

$$= 17{,}8629\left(2(-0{,}8853) - 1{,}3222 + \frac{3(8{,}1098 - 6{,}3212)}{3{,}5}\right) = -27{,}86 \text{ kNm}$$

$$M_{34} = \overline{M}^{\ell}_{34} + k_{34}\left(2\varphi_3 + \varphi_4 + \frac{3(w_4 - w_3)}{L_{34}}\right) =$$

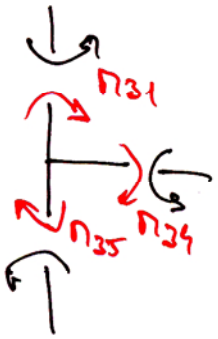
$$= \frac{10 \cdot 8^2}{12} + 9{,}765\left(2 \cdot (-0{,}8853) - 0{,}2942 + \frac{3(0{,}5095 - 0{,}4238)}{8}\right) =$$

$$= 33{,}48 \text{ kNm}$$

$$M_{35} = k_{35}\left(\varphi_5 + 2\varphi_3 + \frac{3(u_3 - u_5)}{3{,}5}\right) =$$

$$= 17{,}8629\left(-1{,}4498 - 2 \cdot 0{,}8853 + \frac{3(6{,}3212 - 2{,}931)}{3{,}5}\right) = -5{,}62 \text{ kNm}$$

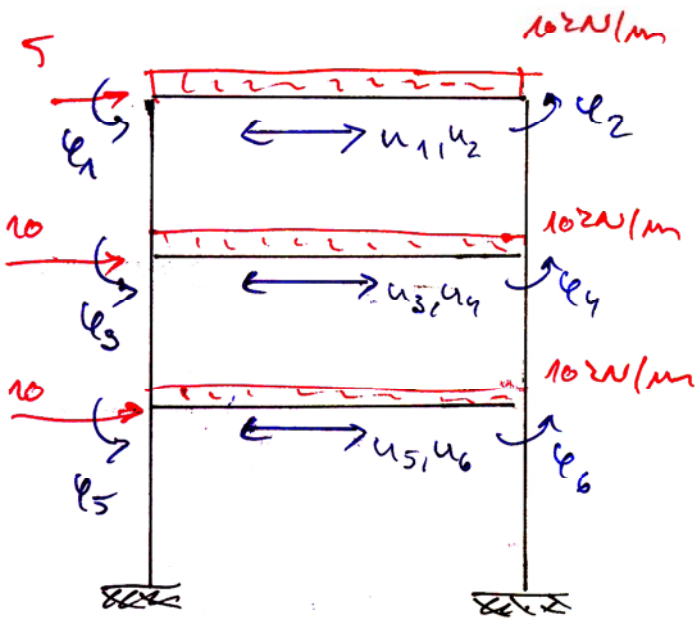
Rovnováha:



$$M_{31} + M_{34} + M_{35} = 0$$

$$-27{,}86 + 33{,}48 - 5{,}62 = 0 \quad \text{OK}$$

2) ŘEŠENÍ KONSTRUKCE POMOCÍ ZDM



Důsledky ZDM

$$\begin{cases} u_1 = u_2 \\ u_3 = u_4 \\ u_5 = u_6 \end{cases}$$

$$\left[w_{1\dots6} = 0 \right.$$

Nesymetrické zatížení

$\rightarrow \varphi_1 \neq \varphi_2$

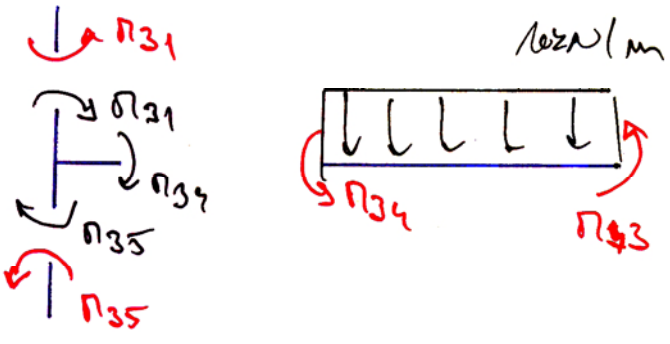
$\varphi_3 \neq \varphi_4$

$\varphi_5 \neq \varphi_6$

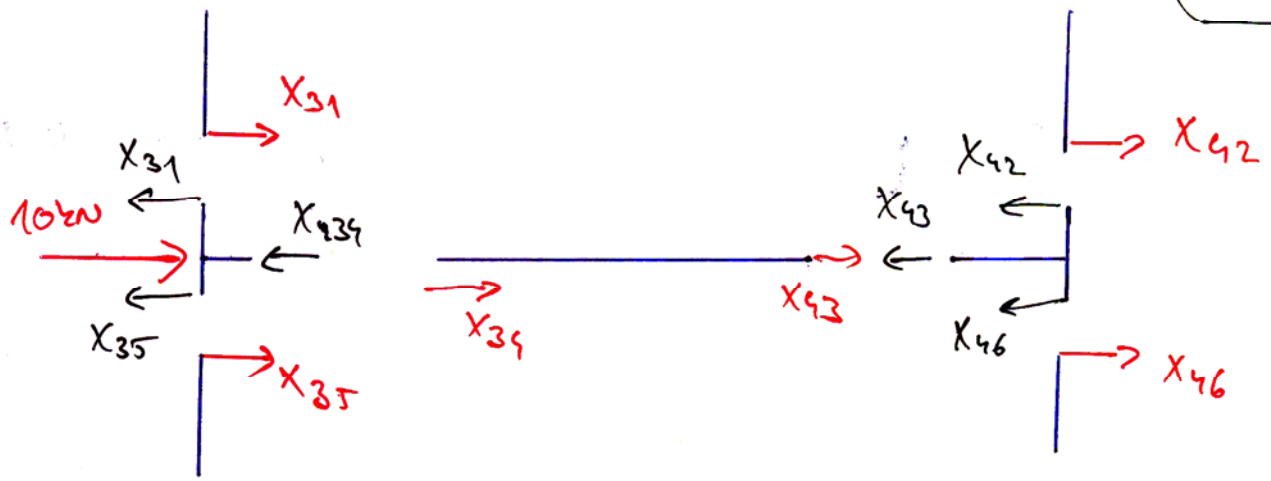
$\rightarrow$ 3 neznámé PATROVÉ POSUNY + 6 neznámých natočení

Kromě momentových podmínek se zřícených se setkáví PATROVÉ ROVNICE

Budeme řešit prostřední patro:



Momentové podmínky



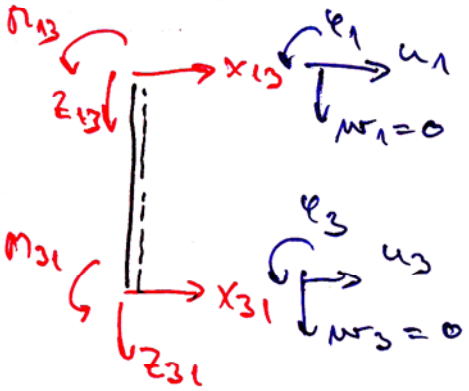
Vodorovné podmínky

(1) $X_{31} + X_{34} + X_{35} = 10$ | STYČNÍKOVÉ PODMÍNKY ROVNOVÁHY

(2) $X_{42} + X_{43} + X_{46} = 0$ |

(3) $X_{34} + X_{23} = 0$ | PODMÍNKA ROVNOVÁHY NA PRUTU

→ $X_{31} + X_{35} + X_{42} + X_{46} = 10$ Patrová rovnice



$$z^{l}_{ab} = \bar{z}^{l}_{ab} - \frac{3k}{L}\left(\varphi_a + \varphi_b + \frac{2(M^{l}_{rb} - M^{l}_{ra})}{L}\right)$$

$$z^{l}_{ba} = \quad + \quad$$

→ $X_{31} \equiv z^{l}_{31} = \bar{z}^{l}_{31} - \frac{3k_{13}}{L_{13}}\left(\varphi_3 + \varphi_1 + \frac{2(u_1 - u_3)}{L_{13}}\right)$

$\bar{z}^{l}_{31} = 0$

$L_{13} = 3{,}5$ m

$k_{13} = \frac{2EI_S}{L_{13}} = \frac{2 \cdot 31{,}26}{3{,}5} = 17{,}863$ MNm

$-\frac{3 \cdot 17{,}863}{3{,}5} = -15{,}311$ MN

$$X_{31} = -15{,}311\left(\varphi_3 + \varphi_1 + \frac{u_1 - u_3}{1{,}75}\right)$$

→ $X_{35} \equiv z^{l}_{35} = \bar{z}^{l}_{35} + \frac{3k_{35}}{L_{35}}\left(\varphi_3 + \varphi_5 + \frac{2(u_3 - u_5)}{L_{35}}\right)$

$$X_{35} = 15{,}311\left(\varphi_3 + \varphi_5 + \frac{u_3 - u_5}{1{,}75}\right)$$

→ $X_{42} \equiv z^{l}_{42} = -15{,}311\left(\varphi_4 + \varphi_2 + \frac{u_1 - u_3}{1{,}75}\right)$

→ $X_{46} \equiv z^{l}_{46} = 15{,}311\left(\varphi_4 + \varphi_6 + \frac{u_3 - u_5}{1{,}75}\right)$

$$X_{31} = -15{,}311\left(-0{,}8742 - 1{,}3070 + \frac{8{,}0246 - 6{,}286}{1{,}75}\right) = 18{,}18\ \text{kN}$$

$$X_{42} = -15{,}311\left(-0{,}2861 + 0{,}8069 + \frac{8{,}0246 - 6{,}286}{1{,}75}\right) = -23{,}185\ \text{kN}$$

$$X_{35} = 15{,}311\left(-0{,}87415 - 1{,}4444 + \frac{6{,}2859 - 2{,}921}{1{,}75}\right) = -6{,}0532\ \text{kN}$$

$$X_{46} = 15{,}311\left(-0{,}2861 - 0{,}2604 + \frac{6{,}2859 - 2{,}921}{1{,}75}\right) = 21{,}0725\ \text{kN}$$

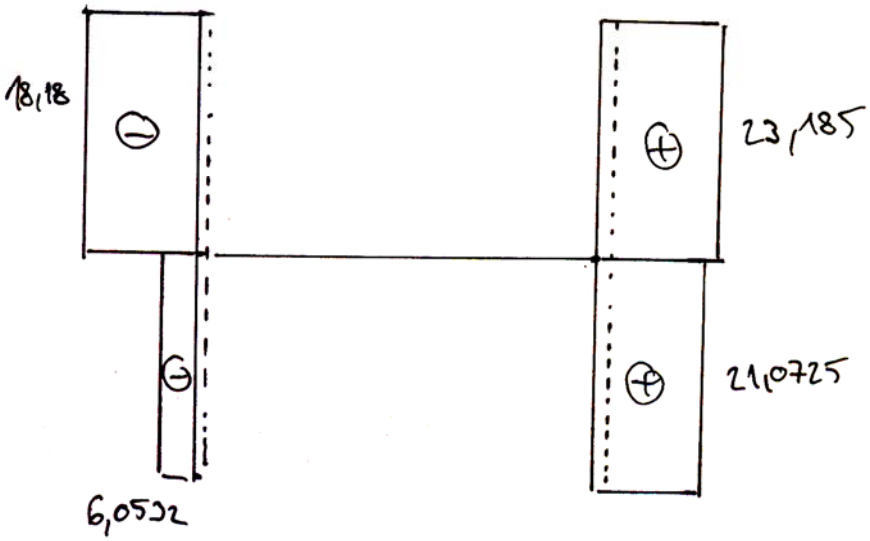
Ověření podmínky rovnováhy:

$$X_{31} + X_{42} + X_{35} + X_{46} = 10$$

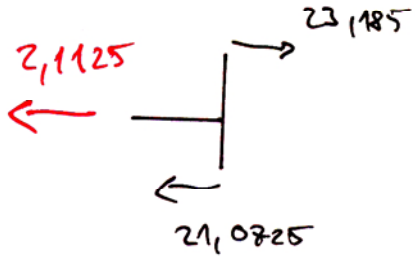
$$18{,}18 - 23{,}185 - 6{,}0532 + 21{,}0725 = 10{,}01 \ldots \text{OK}$$

INTERPRETACE VÝSLEDKŮ

1) Vyznačení posouvajících sil



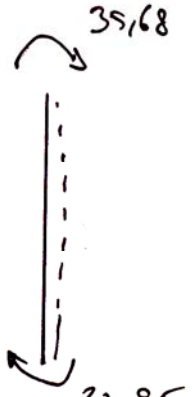
2) Normálová síla v prutu 3-4?



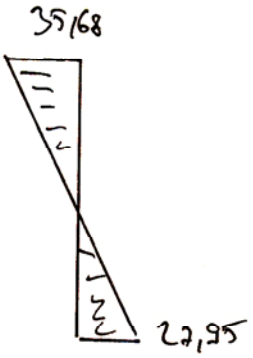
3) Průběh momentů na prutu 3-1

$$M_{31} = \bar{M}_{31} + k_{13}\left(2\varphi_3 + \varphi_1 + \frac{3(w_1^l - w_3^l)}{L_{13}}\right) \qquad w_1^l = u_1 \quad w_3^l = u_3$$

$$M_{31} = 17{,}863\left(2(-0{,}8742) - 1{,}307 + \frac{3(8{,}0246 - 6{,}2859)}{3{,}5}\right) = -27{,}95 \text{ kNm}$$



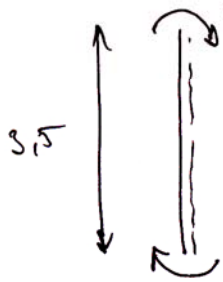
$$27{,}95 - 18{,}18 \times 3{,}5 = -35{,}168$$



4) Průběh momentů na prutu 5-3

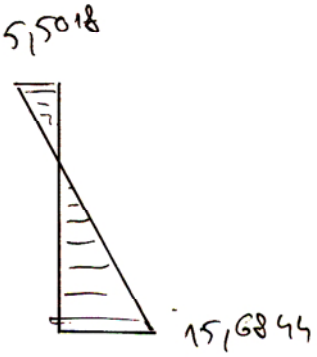
$$M_{35} = k_{35}\left(\varphi_5 + 2\varphi_3 + \frac{3(u_3 - u_5)}{3{,}5}\right) =$$

$$= 17{,}863\left(-1{,}444 - 2 \times 0{,}8741 + \frac{3(6{,}2859 - 2{,}921)}{3{,}5}\right) = -5{,}5018 \text{ kNm}$$

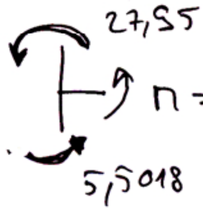


parametr = −6,0532

$$M = -5{,}5018 + 3{,}5 \cdot 6{,}0532 = 15{,}6844 \text{ kNm}$$



5) Průběh momentů na prutu 3-4



$M = -27,55 - 5,5018 = -33,46$ kNm

→ Kontrola → $M_{34} = M^{l}_{34} + k_{34}\left(2\varphi_3 + \varphi_4 + \frac{3(\not{w}_4 - \not{w}_3)}{L_{34}}\right)$ (with $w_4 \to 0$, $w_3 \to 0$)

$M^{l}_{34} = \frac{1}{12}\cdot 10 \cdot 8^2 = \frac{160}{3}$

$k_{34} = \frac{2EI_p}{L_{34}} = \frac{39,06}{4}$ MNm

$M_{34} = \frac{160}{3} + \frac{39,06}{4}\left(2\cdot(-0,8741) - 0,2861\right) = 33,46$ kNm ⇒ OK