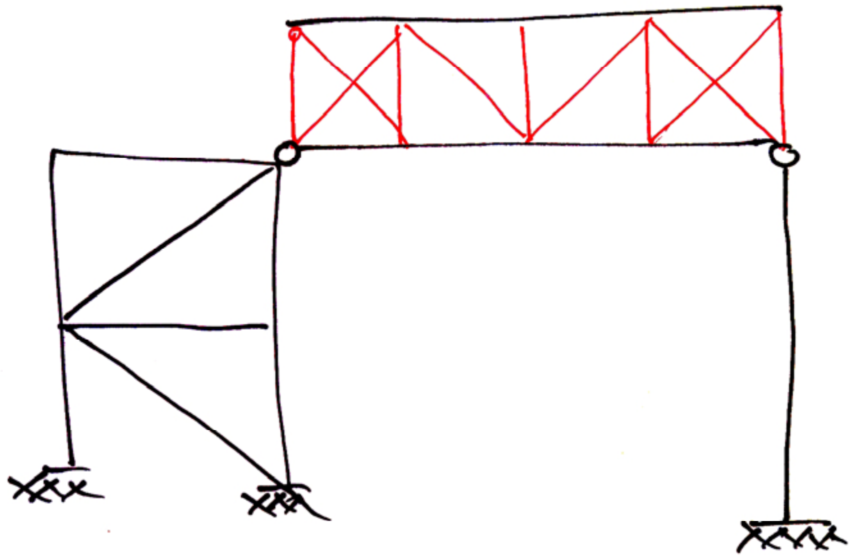
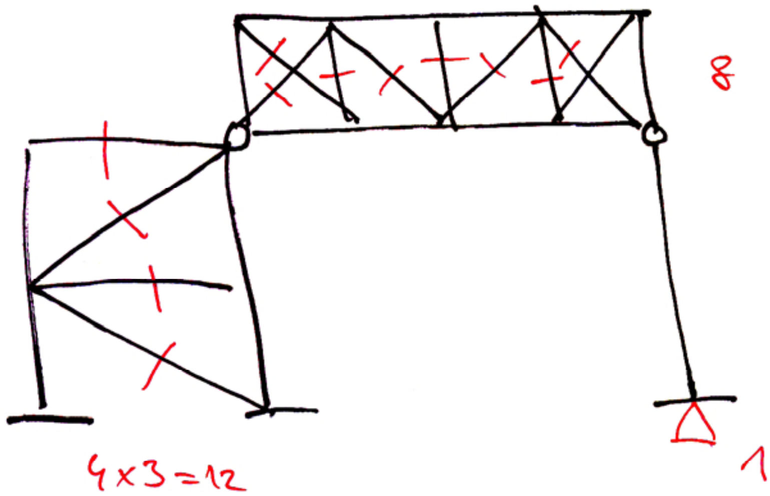
Příklad na rozehřátí

červené pruty jsou tuhé

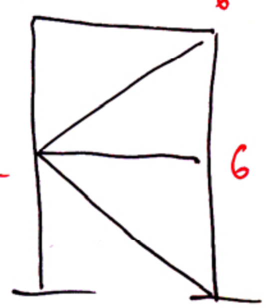


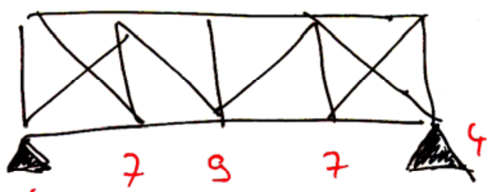
→ Stupeň statické neurčitosti



Σ = 21 SN

Nebo po deskách





57 SV − 66 SV = 9 × SN

$\boxed{9 + 12 = 21 \times SN}$

SU.

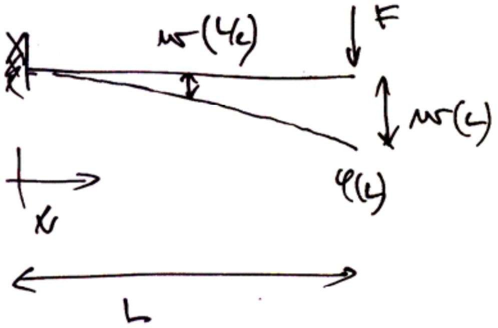
24 SV − 36 SV = 12 × SN

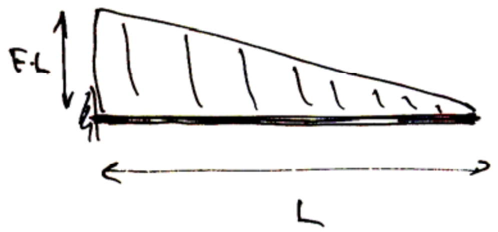
Princip virtuálních sil

$$\int_0^L \delta M(x)\,\varkappa(x)\,dx = \int_0^L \delta f(x)\,w(x)\,dx + \sum_i \delta F_i\, w_i + \sum_j \delta M_j\, \varphi_j$$

→ užití
- výpočet zvoleného posunu / natočení
- kontrola kompatibility

Př. 1 – průhyb a natočení konce konzole



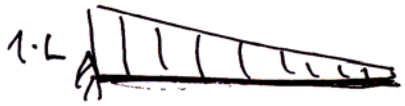


"Skutečný" stav

$$M(x) = -F\cdot L + F\cdot x$$

$$\varkappa(x) = (-FL + Fx)/(EI)$$

1. virtuální stav – průhyb na konci

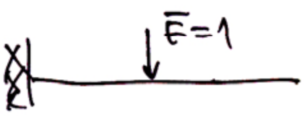


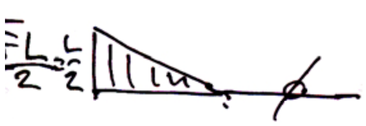
$$\bar{M}(x) = -L + x$$

$$\int_0^L \bar{M}(x)\,\varkappa(x)\,dx = 1\cdot w(L)$$

$$w(L) = \frac{F}{EI}\int_0^L (x-L)^2\,dx = \frac{F}{EI}\left[\frac{x^3}{3} - x^2 L + L^2 x\right]_0^L = \frac{FL^3}{3EI}$$

2. virtuální stav – průhyb v polovině





$$\bar{M}(x) = -\frac{\bar{F}\cdot L}{2} + \bar{F}\cdot x = x - \frac{L}{2} \quad \text{pro } x \in \langle 0; L/2 \rangle$$

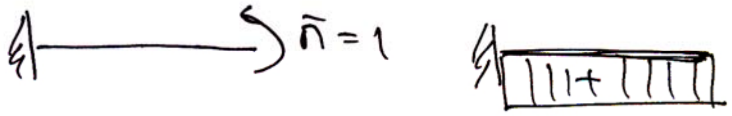
$$\bar{M}(x) = 0 \quad \text{pro } x \in \langle L/2; L \rangle$$

$$w\left(\frac{L}{2}\right) = \int_0^{L/2} \bar{M}(x)\,\varkappa(x)\,dx + \cancel{\int_{L/2}^{L} \bar{M}(x)\,\varkappa(x)\,dx}$$

$$w\left(\frac{L}{2}\right) = \int_0^{L/2} \left(x-\frac{L}{2}\right)\left(\frac{Fx-FL}{EI}\right)dx = \frac{F}{EI}\int_0^{L/2} x^2 - xL - \frac{xL}{2} + \frac{L^2}{2}\,dx = \frac{F}{EI}\left[\frac{x^3}{3} - \frac{x^2L}{2} - \frac{x^2L}{4} + \frac{L^2x}{2}\right]_0^{L/2}$$

$$= \frac{F}{EI}\left[\frac{L^3}{24} - \frac{L^3}{8} - \frac{L^3}{16} + \frac{L^3}{4}\right] = \underline{\underline{\frac{5}{48}\frac{F}{EI}}}$$

3. virtuální stav – natočení konce

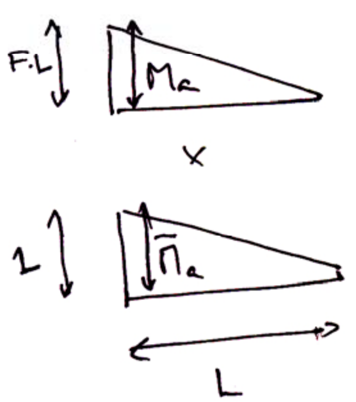


$$\int_0^L 1\cdot\frac{F}{EI}\left(x-L\right)dx = \varphi(L)$$

$$\varphi(L) = \frac{F}{EI}\left[\frac{x^2}{2} - xL\right]_0^L = \frac{F}{EI}\left[\frac{L^2}{2} - L^2\right] = \underline{\underline{-\frac{FL^2}{2EI}}}$$

Diagramy pro výpočet integrálů:

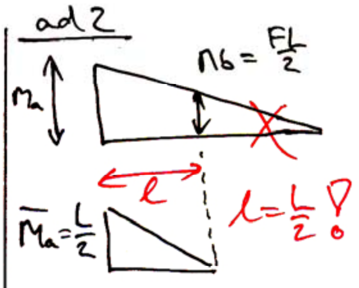
ad 1



$$\int = \frac{1}{3}M_a\bar{M}_a\cdot L = \frac{1}{3}FL\cdot L\cdot L = \frac{FL^3}{3}$$

$$w = \frac{\int}{EI} = \frac{FL^3}{3EI}$$

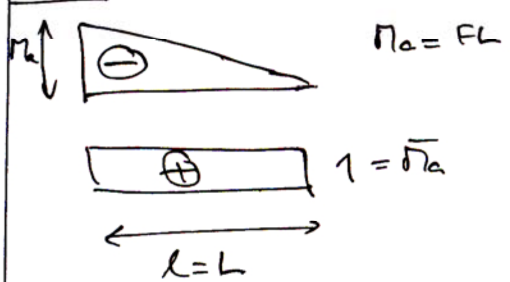
ad 2



$$\int = \frac{1}{6}\left(2FL + \frac{FL}{2}\right)\frac{L}{2}\cdot\frac{L}{2} = \frac{5}{48}FL^3$$

$$w = \frac{\int}{EI} = \frac{5FL^3}{48EI}$$

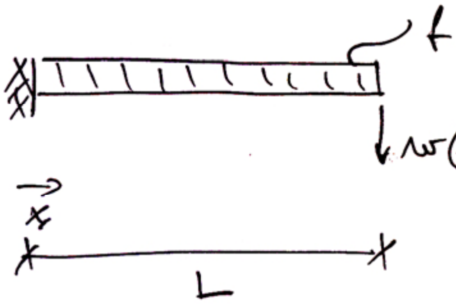
ad 3

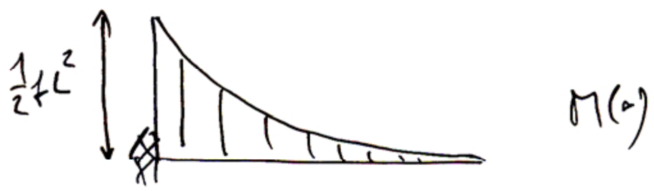


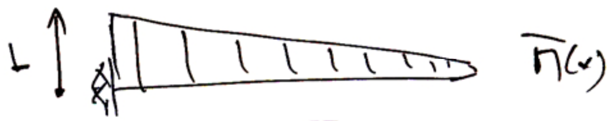
$$\int = -\frac{1}{2}\cdot 1\cdot FL^2 = -\frac{FL^2}{2}$$

$$\varphi = -\frac{FL^2}{2EI}$$

Př. 2 Výpočet průhybu na konzole zatížené spojitým zat.

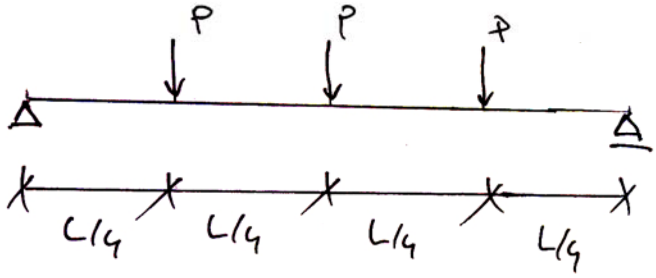






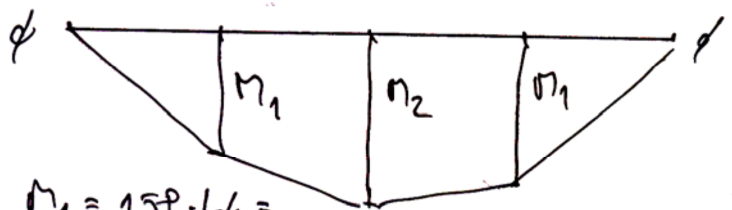
$$w(L) = \int \frac{\bar{M}(x)\, M(x)}{EI}\, dx = \frac{1}{EI}\left[\frac{1}{4}\cdot\frac{1}{2} fL^2\cdot L\cdot L\right] = \frac{fL^4}{EI}\cdot\frac{1}{8}$$

Př. 3



úkol – vypočítat průhyb pod prostřední silou

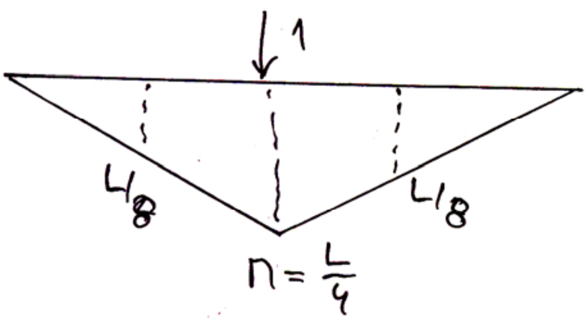
- Průběh skutečných momentů



$M_1 = 1{,}5P\cdot L/4 =$
$= 0{,}375\, PL = \frac{3}{8}$

$M_2 = 1{,}5P\cdot L/2 - P\cdot\frac{L}{4} = 0{,}5\, PL$

- Průběh virtuálních momentů



Výpočet průhybu:

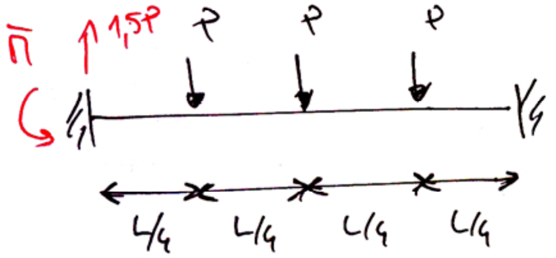
$$w\left(\frac{L}{2}\right) = \int_0^L \bar{M}(x)\, M(x)\, dx =$$

$$= 2\cdot\int_0^{L/4} \bar{M}(x)\, M(x)\, dx + 2\int_{L/4}^{L/2} \bar{M}(x)\, M(x)\, dx =$$

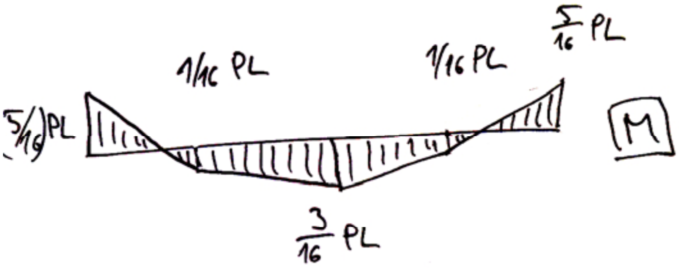
$$= 2\cdot\frac{1}{3}\cdot 0{,}375\cdot P\cdot L\cdot\frac{L}{8}\cdot\frac{L}{4} + 2\cdot\frac{1}{6}\left[0{,}375PL\cdot\left(2\cdot\frac{L}{8}+\frac{L}{4}\right) + 0{,}5PL\left(2\cdot\frac{L}{4}+\frac{L}{8}\right)\right]\cdot\frac{L}{4} \Big/ EI$$

$$= \frac{\frac{1}{128}PL^3 + \frac{1}{24}PL^3}{EI} = \frac{\frac{19}{384}PL^3}{EI}$$

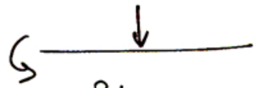
Pr. 4 průhyb uprostřed rozpětí



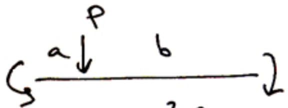
$$\bar{M} = PL\left(\frac{1}{8} + \frac{3}{64} + \frac{9}{64}\right) = \underline{\underline{\frac{5}{16} PL}}$$



Skutečný průběh momentů pomocí tabulek pro deformační metodu



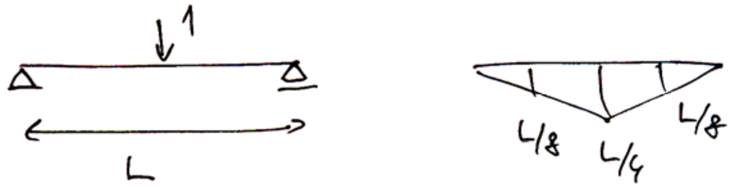
$$M = \frac{P \cdot L}{8}$$



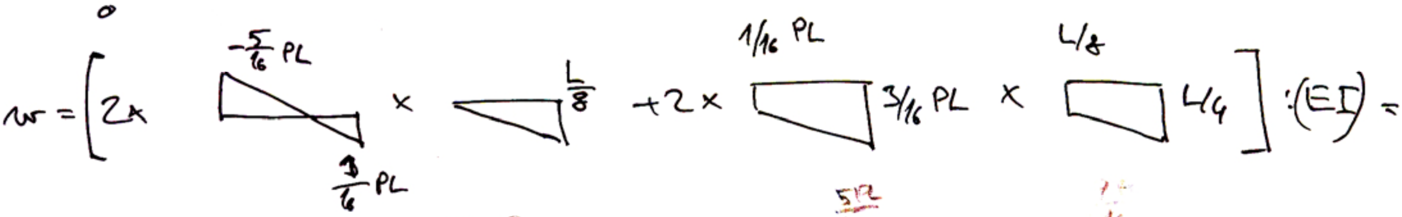
$$\frac{Pab^2}{L^2} = \frac{P \cdot \frac{L}{4} \cdot L^2 \cdot \frac{9}{16}}{L^2} = \qquad \frac{Pa^2b}{L^2} = \frac{P}{L^2} \cdot \frac{L^2}{16} \cdot \frac{3}{4} L = \underline{\frac{3}{64} P \cdot L}$$

$$= \underline{P \cdot L \cdot \frac{9}{64}}$$

Virtuální stav 1



$$1 \cdot w = \int_0^L \bar{M}(x)\, \varkappa(x)\, dx$$

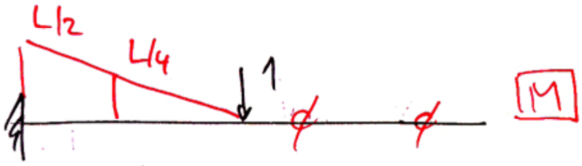


$$= \frac{1}{EI} \cdot 2 \left[\underbrace{\frac{1}{6} \cdot \frac{L}{8} \left(-\frac{5}{16} PL + 2 \cdot \overbrace{\frac{1}{16} PL}^{-3/16\ PL} \right) \cdot \frac{L}{4}}_{-PL^3 \cdot \frac{1}{1024}} + \underbrace{\frac{L}{4} \cdot \frac{1}{6} \cdot \left(\frac{L}{8} \left(\frac{2 PL}{16} + \frac{3 PL}{16} \right) + \frac{L}{4} \left(\frac{2 \cdot 3}{16} PL + \frac{1}{16} PL \right) \right)}_{19\, PL^3 / 3072} \right] =$$

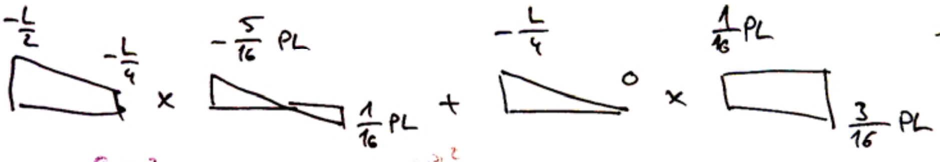
$\frac{5 PL}{16}$ $\frac{1}{16}$ $5\, PL^2/\ldots$ $\ldots L^2/64$

$$= \frac{1}{EI} \cdot 2 \cdot \left[-\frac{1}{1024} \cdot PL^3 + \frac{19}{3072} PL^3 \right] = \underline{\underline{\frac{PL^3}{96 EI}}}$$

Virtuální stav 2

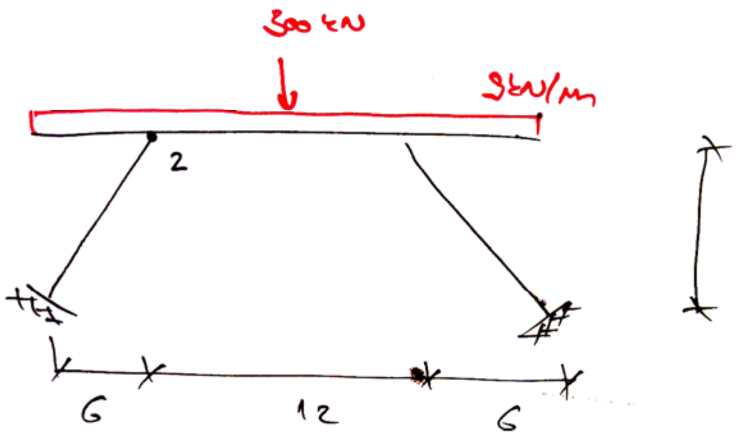


$w = \frac{1}{EI} \cdot$



$$= \frac{1}{EI} \cdot \left[\frac{1}{6} \left[\overbrace{-\frac{L}{2}\left(-\frac{2.5}{16}PL + \frac{1}{16}PL\right)}^{\frac{9}{32}PL^2} \overbrace{-\frac{L}{4}\left(\frac{2.1}{16}PL - \frac{5}{16}PL\right)}^{\frac{PL^2}{64}} \right] \cdot \frac{L}{4} \right. +$$

$$+ \left. \underbrace{\frac{1}{6}\overbrace{\left(\frac{2.1}{16}PL + \frac{3}{16}PL\right)}^{\frac{5PL}{16}} \cdot \left(-\frac{L}{4}\right) \cdot \frac{L}{4}}_{-\frac{5PL^3}{1536}} \right] = \frac{1}{EI} \cdot \left[\frac{L}{24}\left(\frac{9}{32}PL^2 + \frac{3PL^2}{64}\right) - \frac{5PL^3}{1536} \right] = \underline{\underline{\frac{PL^3}{96EI}}}$$



Vypočítali jsme ODM neznámé veličiny v uzlu "2"

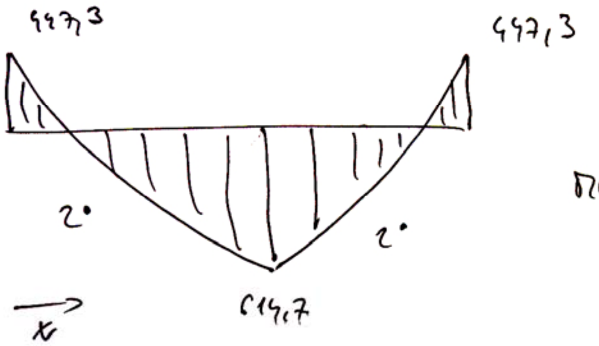
$u_2 = 8{,}175 \cdot 10^{-5}\,m$

$w_2 = 2{,}886 \cdot 10^{-4}\,m$

$\varphi_2 = -5{,}1902 \cdot 10^{-4}\,rad$

Silovou metodou zjistit průhyb uprostřed D!

Skutečný průběh momentů na horní příčli:



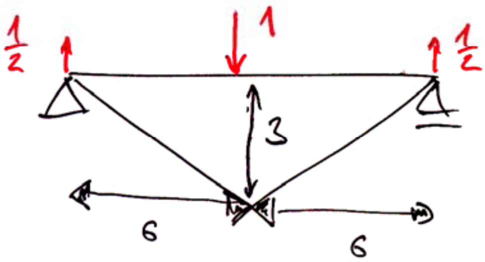
pro $x \in \langle 0;6 \rangle$

$M(x) = -447{,}3 + 204x - \frac{9x^2}{2}$

... kontrola $M(6) = 614{,}7$

Pro násobení obrazců **NELZE** použít tabulku --- parabola ve vrcholu nemá vodorovnou tečnu

Zvolíme virtuální stav:



pro $x \in \langle 0;6 \rangle$

$\bar{M}(x) = \frac{1}{2}x$

PVS:

$$\int_0^L \bar{M}(x)\,\kappa(x)\,dx = \sum_i \delta F_i\, w_i$$

$$= 2\times\int_0^{L/2} \frac{\bar{M}(x)\,M(x)}{EI}\,dx$$

$$\int_0^6 \frac{\bar{M}(x)M(x)}{EI}\,dx = \frac{1}{EI}\int_0^6 \frac{1}{2}x\left(-447{,}3+204x-\frac{9x^2}{2}\right)dx = \frac{1}{EI}\int_0^6 -\frac{447{,}3x}{2}+\frac{204x^2}{2}-\frac{9x^3}{4}\,dx =$$

$$= \frac{1}{EI}\left[-\frac{447{,}3x^2}{4}+\frac{204x^3}{6}-\frac{9x^4}{16}\right]_0^6 = \frac{1}{EI}\left(-4025{,}7+7344-729\right) = \frac{2589{,}3}{EI} =$$

$$= \frac{2589{,}3}{1280} = 2{,}02289$$

$2\times\int = 2\cdot 2{,}02289 = 4{,}04578$ mm ... Proč je to špatně???

$w = 2{,}02289\cdot 2 + 2\cdot\frac{1}{2}\cdot 0{,}2886 = 4{,}334$ mm