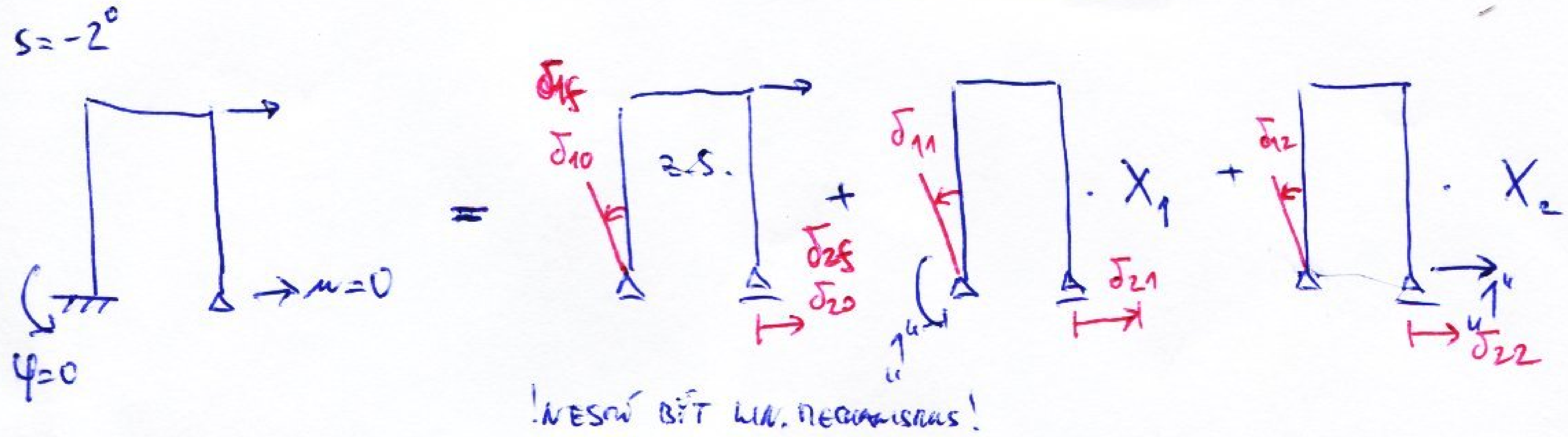
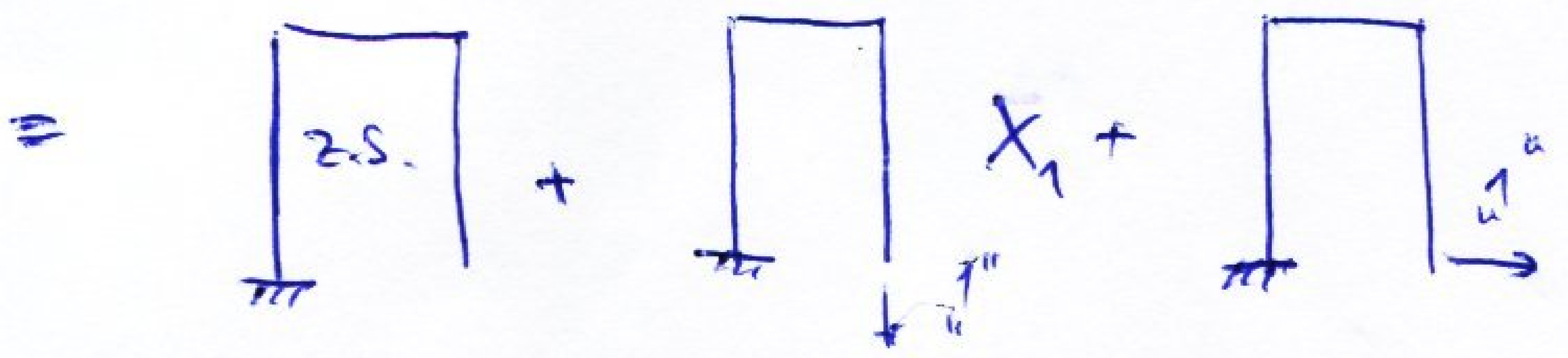
Silová metoda (superpozice, proporcionalita)



!NESMÍ BÝT LIN. MECHANISMUS!



PŘETVÁRNÉ ROVNICE

$\varphi_0 + \varphi_1 X_1 + \varphi_2 X_2 = 0$

$u_0 + u_1 X_1 + u_2 X_2 = 0$

$\delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0$

$\delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0$

V MÍSTĚ — OD ZATÍŽENÍ

δ_{ij} – PRVKY MATICE PODDAJNOSTI
$i \neq 0$ ČTVERCOVÁ, SYMETRICKÁ,
$j \neq 0$ POZ. DEFINITNÍ

$$\delta_{ij} = \int \frac{m_i m_j}{EI} dx + \ldots$$

OBECNÁ ROVNICE – VIZ STR. 5

PVP

$$W_{ext} + W_{int} = 0$$

$$W_{ext} = \sum \bar{F}_i \delta_i + \sum \bar{M}_i \varphi_i$$

$$W_{int} = -\left[\int \bar{M} d\varphi + \int \bar{V} dw + \int \bar{N} du \right]$$

$$d\varphi = \frac{M}{EI} dx + \alpha \frac{\Delta t}{h} dx$$

$$dw = \frac{V\beta}{\cancel{k}GA} dx$$

$$du = \frac{N}{EA} dx + \alpha t_s dx$$

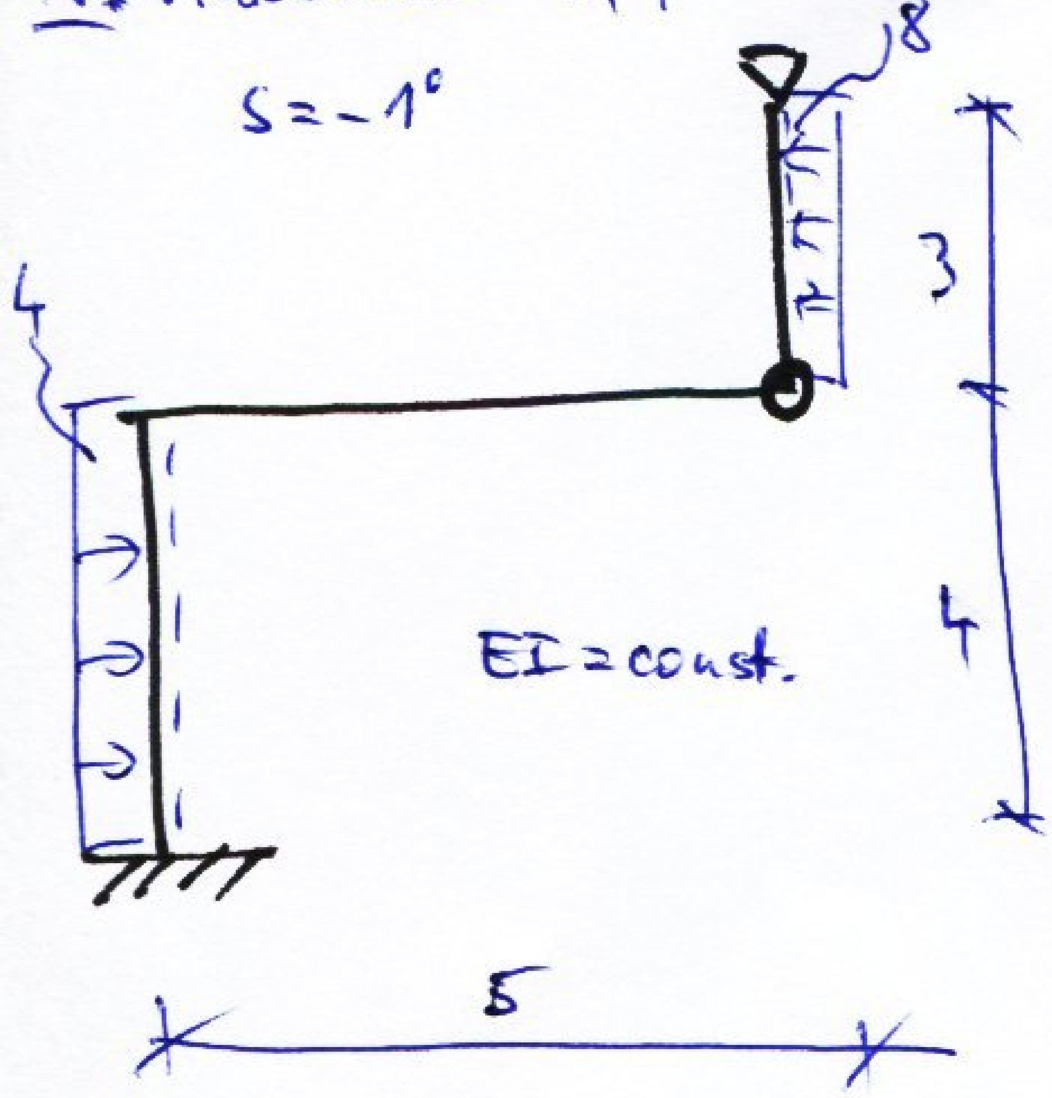
$$\bar{1} \cdot \delta = \int \frac{M\bar{M}}{EI} dx + \beta \int \frac{V\bar{V}}{GA} dx + \int \frac{N\bar{N}}{EA} dx + \int \bar{M} \alpha \frac{t_d - t_h}{h} dx + \int \bar{N} \alpha \frac{t_d + t_h}{2} dx - \sum \bar{R}_i r_i$$

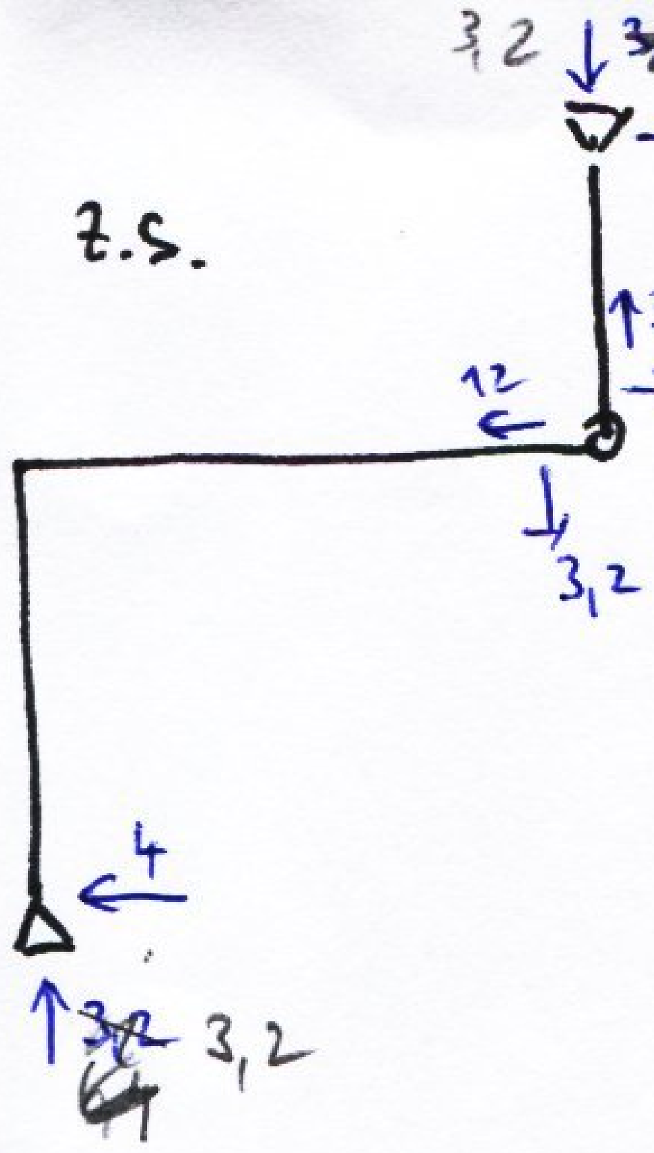
ZADANÉ

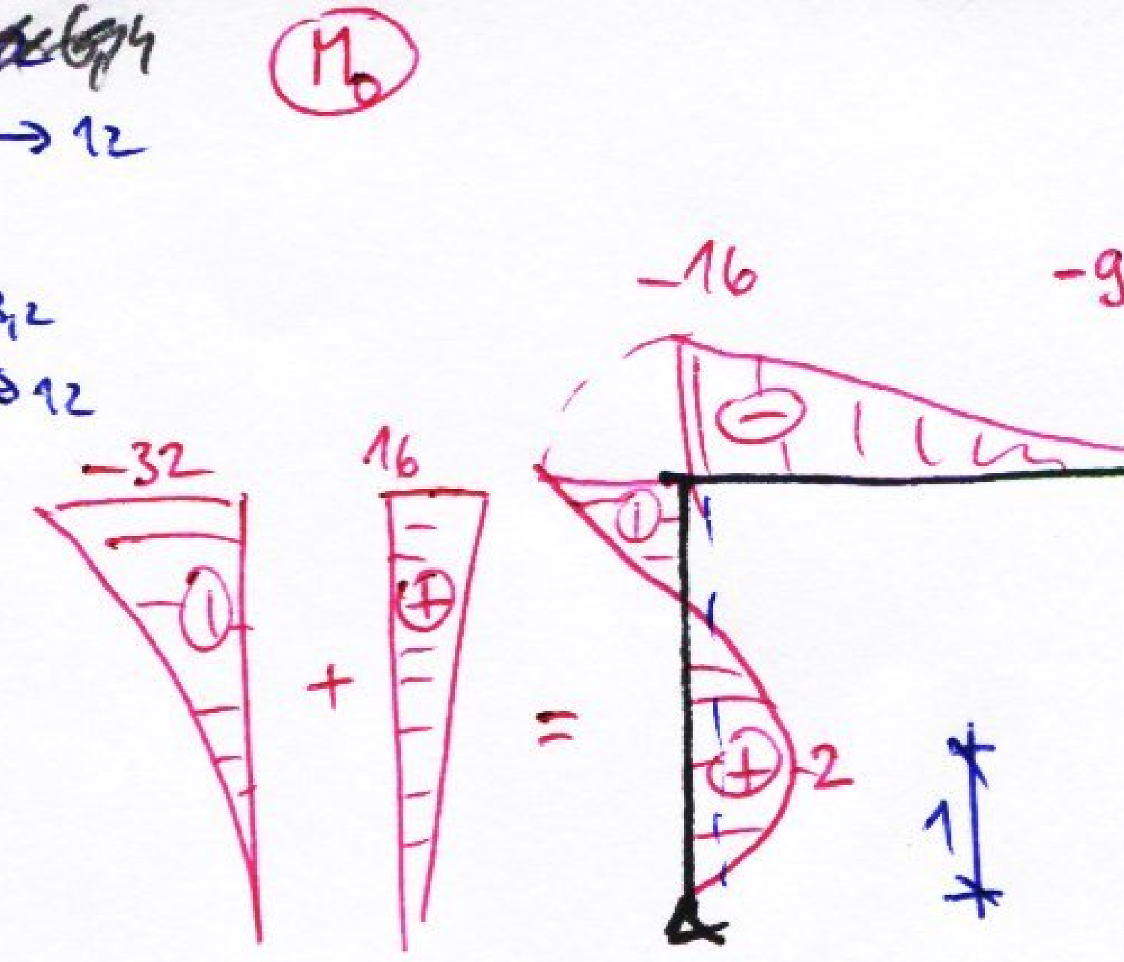
NEJEN VIRTUÁLNÍ REAKCE, ALE I VIRTUÁLNÍ ZATÍŽENÍ $(\bar{1})$

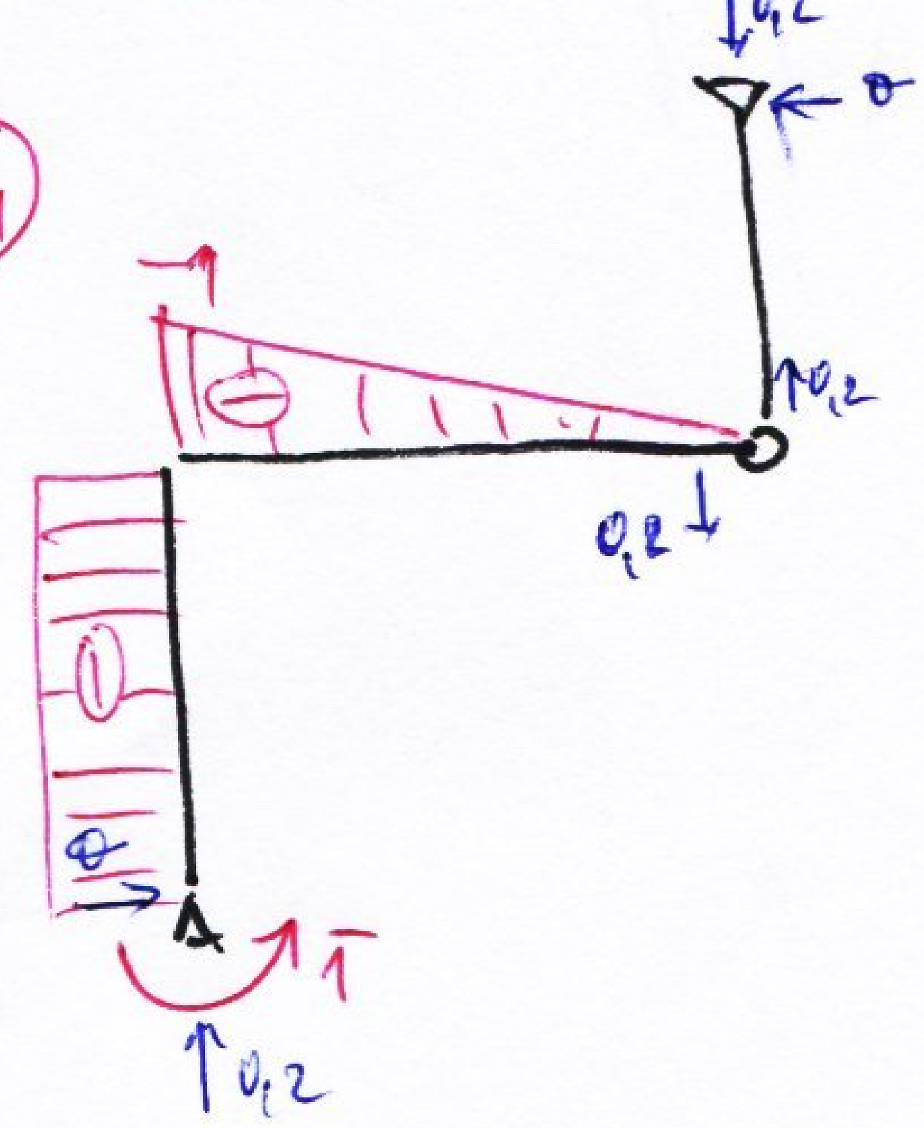
Př. VYKRESLETE N, V, M

$S = -1°$





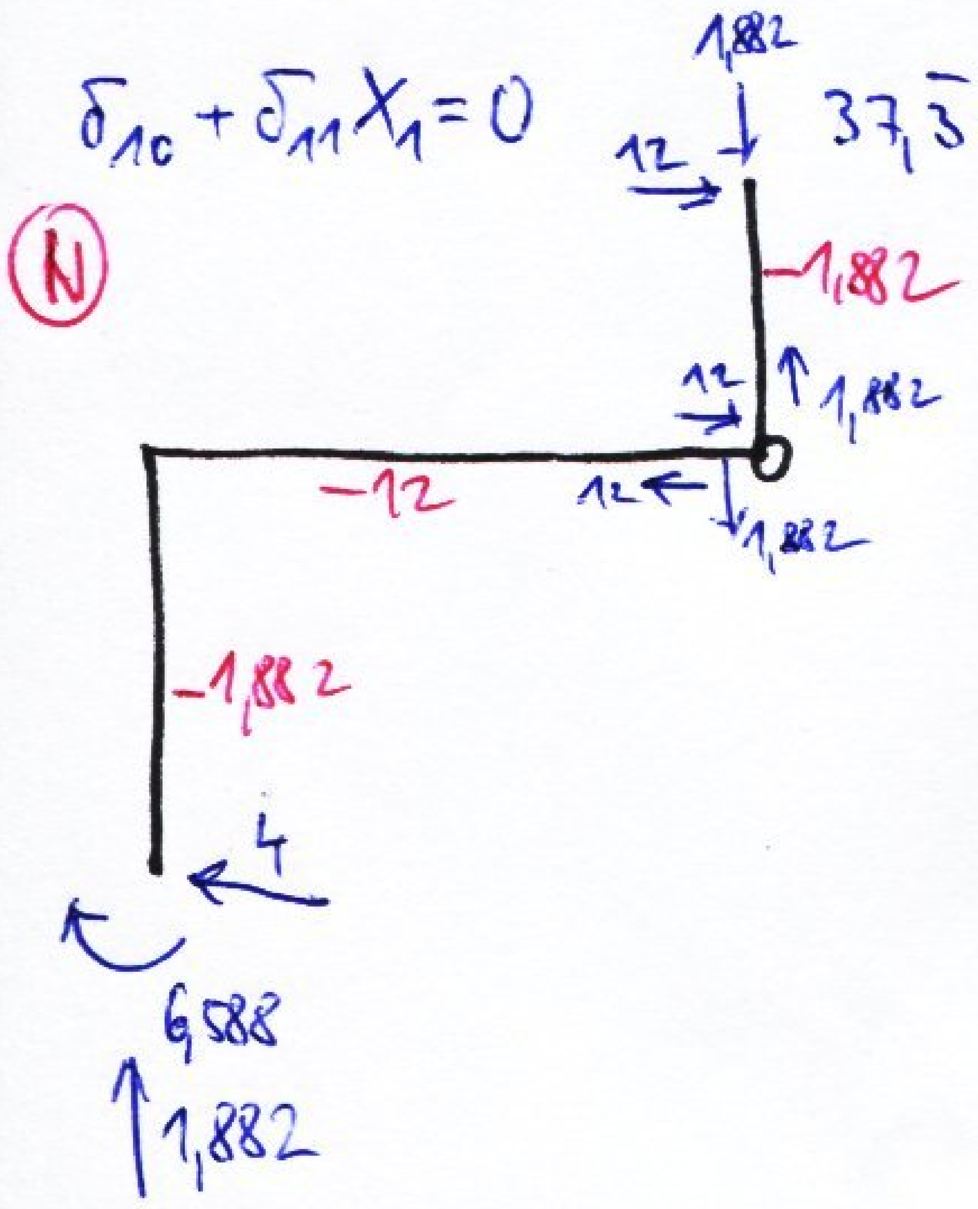


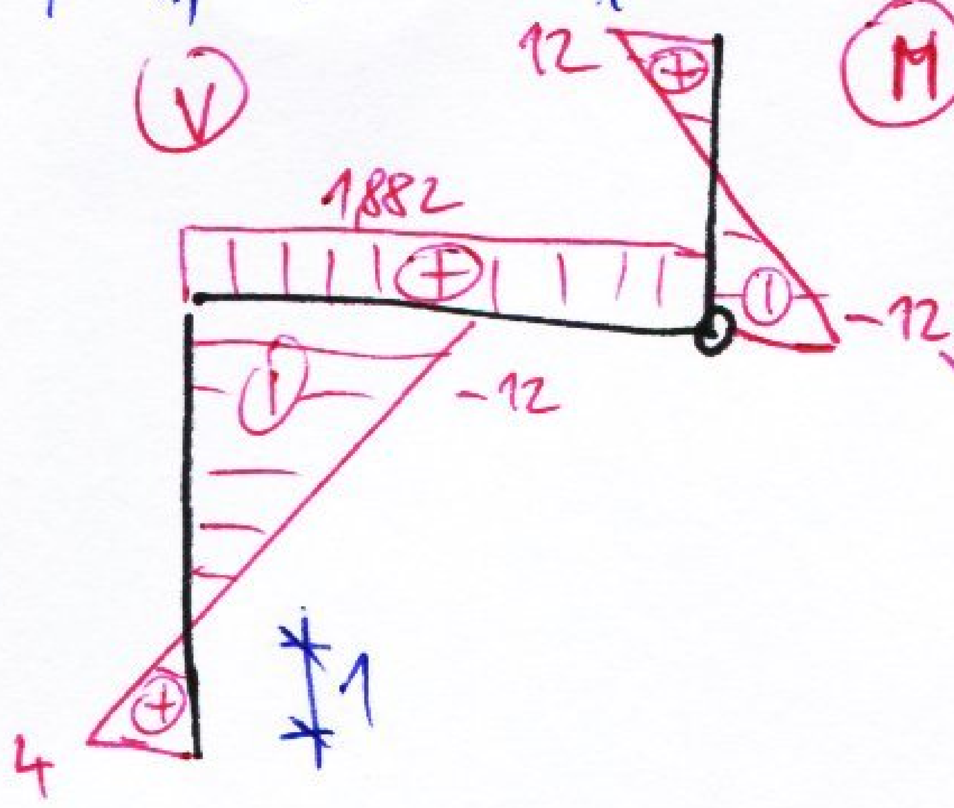


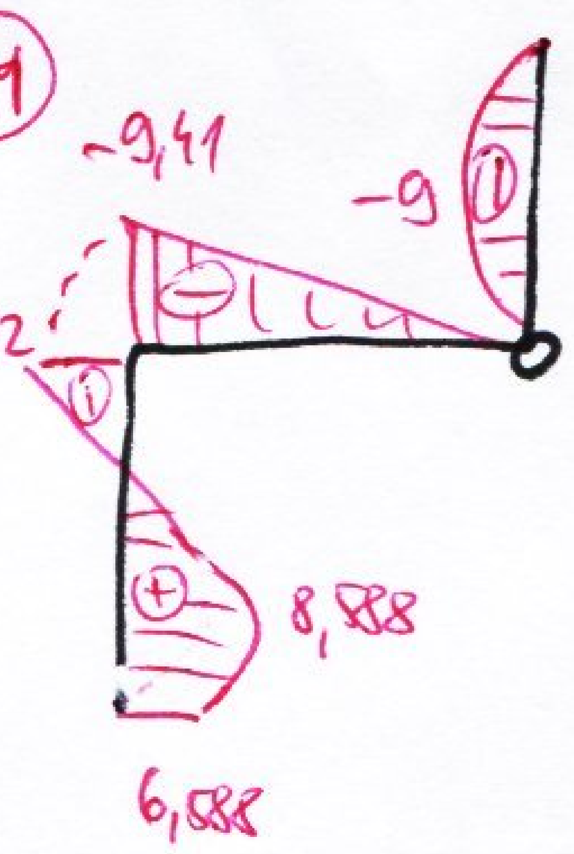
$$\delta_{10} = \frac{1}{EI}\left[\frac{1}{3}\cdot 32\cdot 1\cdot 4 + \frac{1}{2}\cdot(-1)\cdot 16\cdot 4 + \frac{1}{3}\cdot 16\cdot 1\cdot 5\right] = \frac{1}{EI}\left[42{,}\bar{6} - 32 + 26{,}\bar{6}\right] = \frac{37{,}\bar{3}}{EI}$$

$$\delta_{11} = \frac{1}{EI}\left[1^2\cdot 4 + \frac{1}{3}\cdot 1^2\cdot 5\right] = \frac{5{,}\bar{6}}{EI}$$

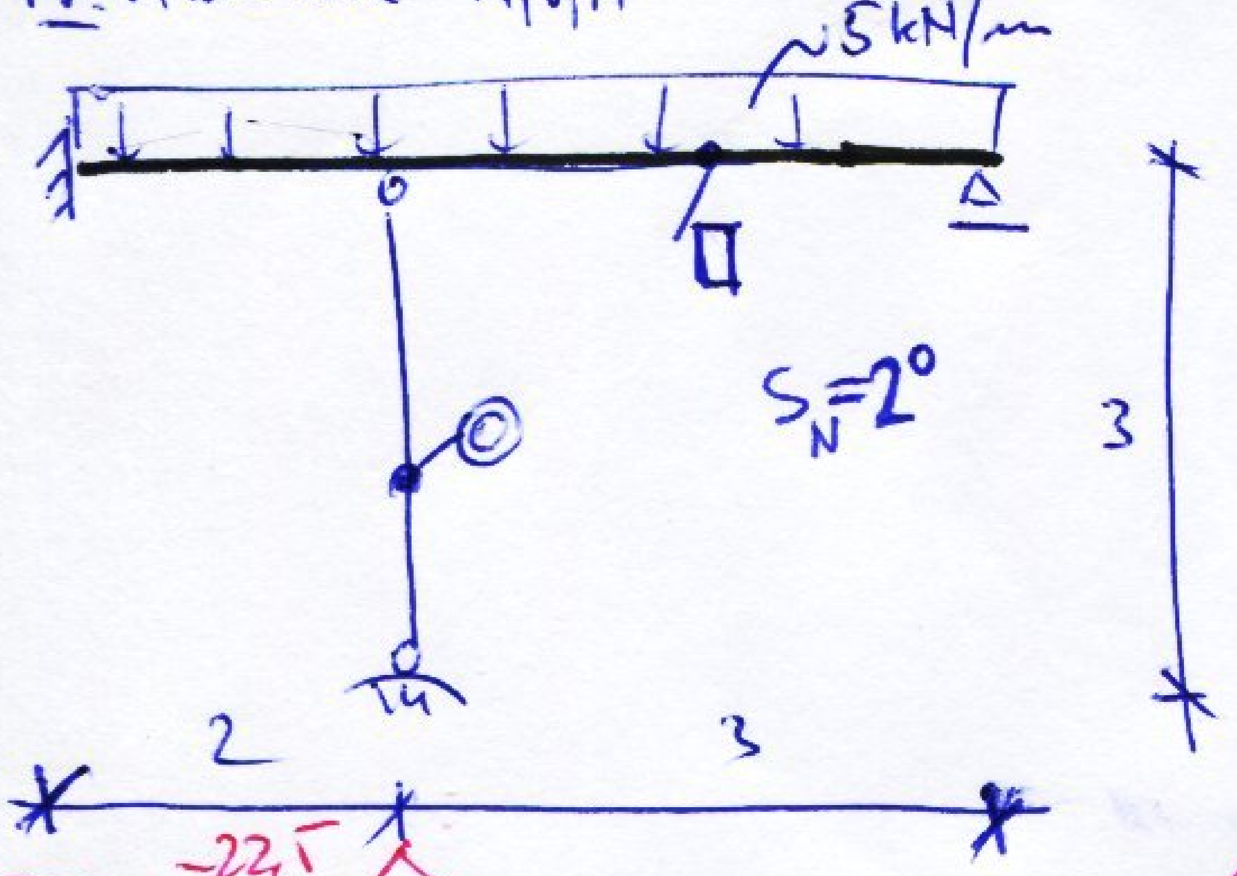
$\delta_{10} + \delta_{11}X_1 = 0$ $\quad 37{,}\bar{3} + 5{,}\bar{6}\,X_1 = 0$ $\quad X_1 = 6{,}588\ \mathrm{kNm}$

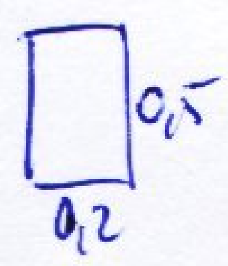






Př. Vykreslete N, V, M

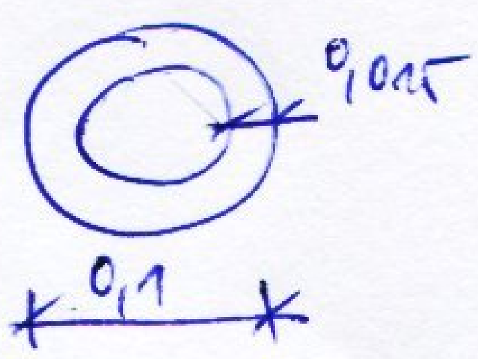




$E = 25\ GPa$

$I = \frac{1}{12} \cdot 0{,}2 \cdot 0{,}5^3 = 2{,}083 \cdot 10^{-3}\ m^4$

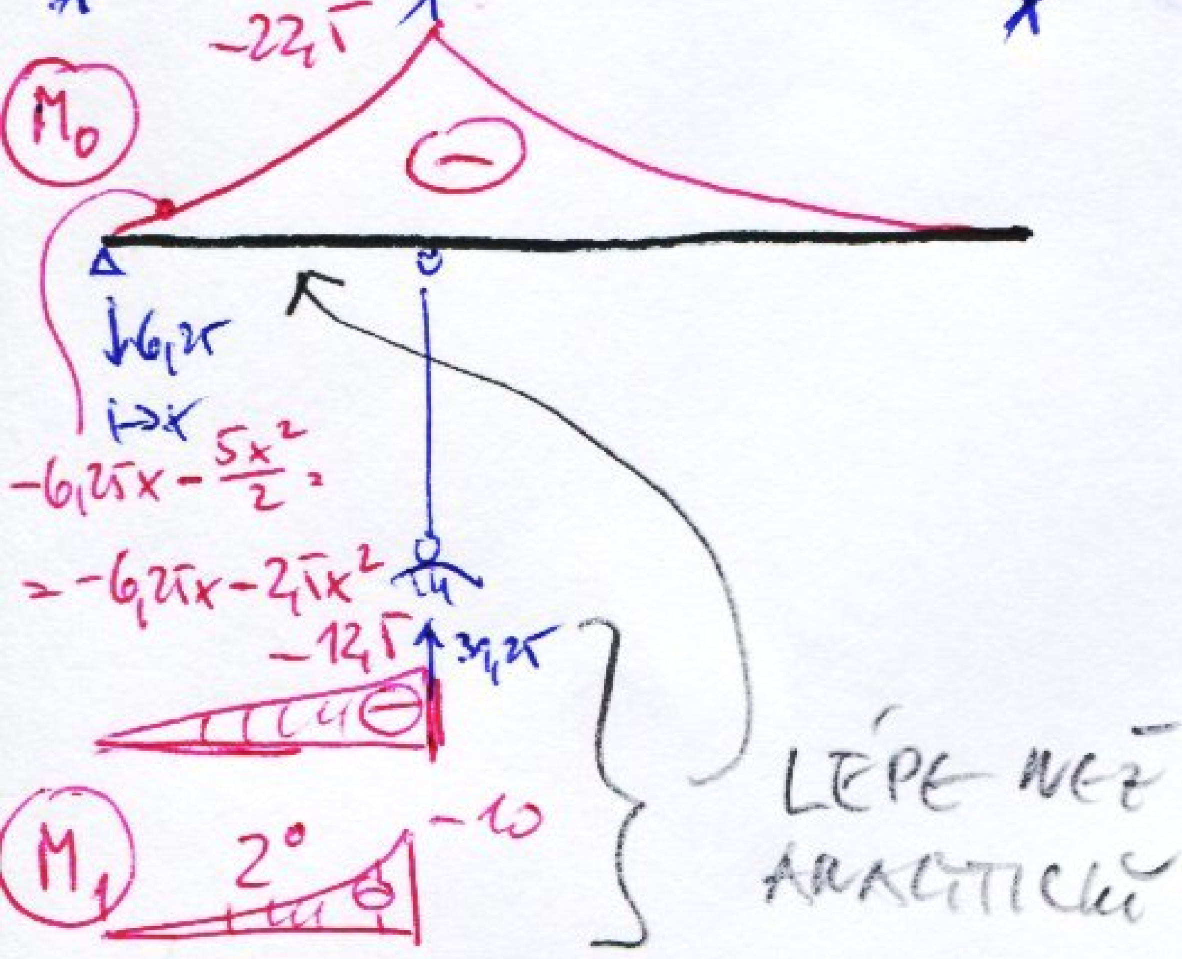
$EI = 52083\ kNm^2$

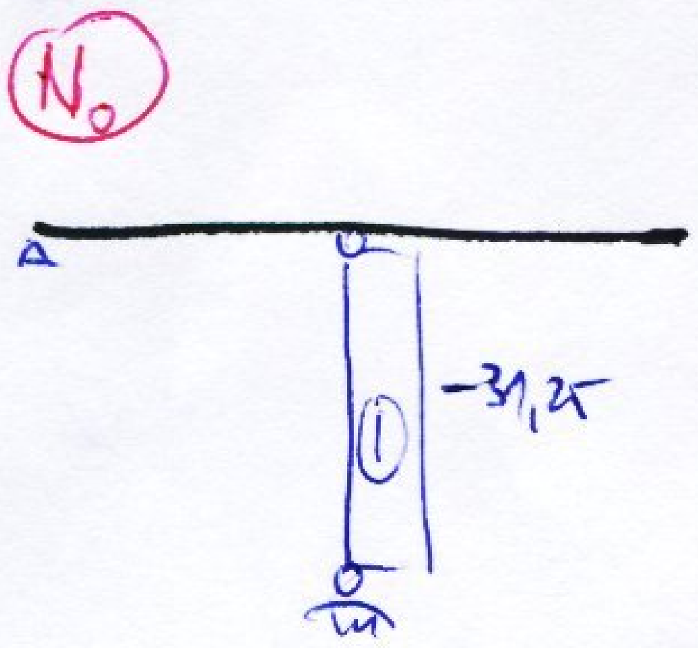


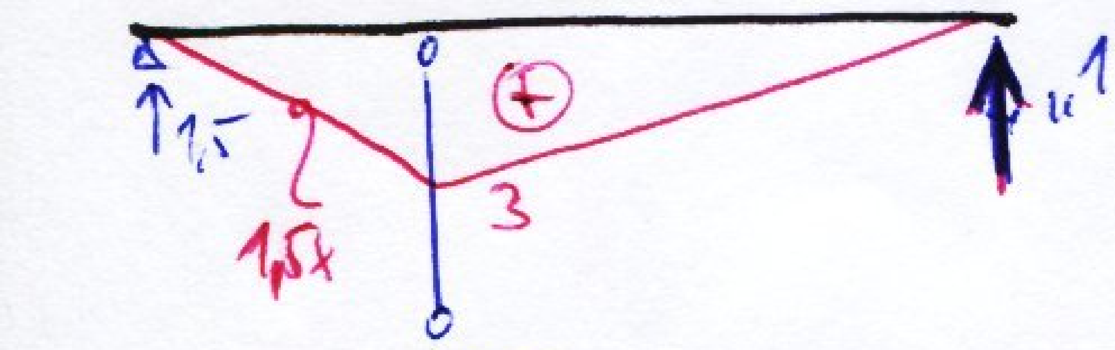
$E = 210\ GPa$

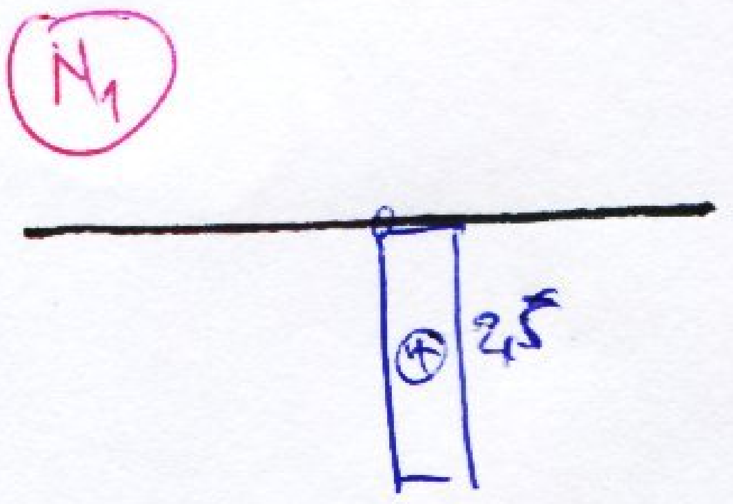
$A = \frac{\pi}{4}(d_1^2 - d_2^2) = \frac{\pi}{4}(0{,}1^2 - 0{,}07^2) = 4 \cdot 10^{-3}\ m^2$

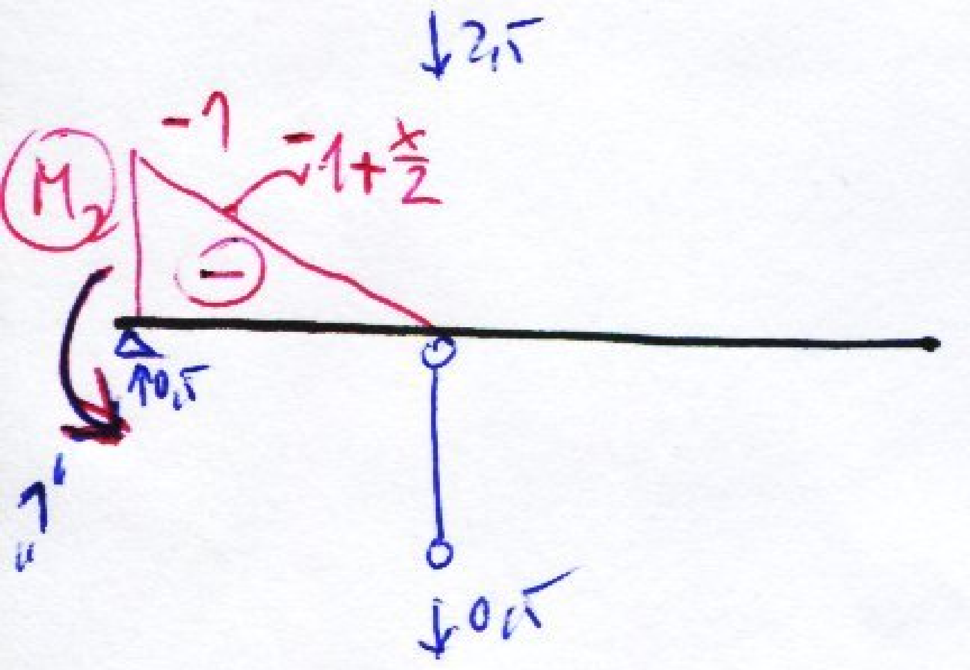
$EA = 841\,155\ kN$

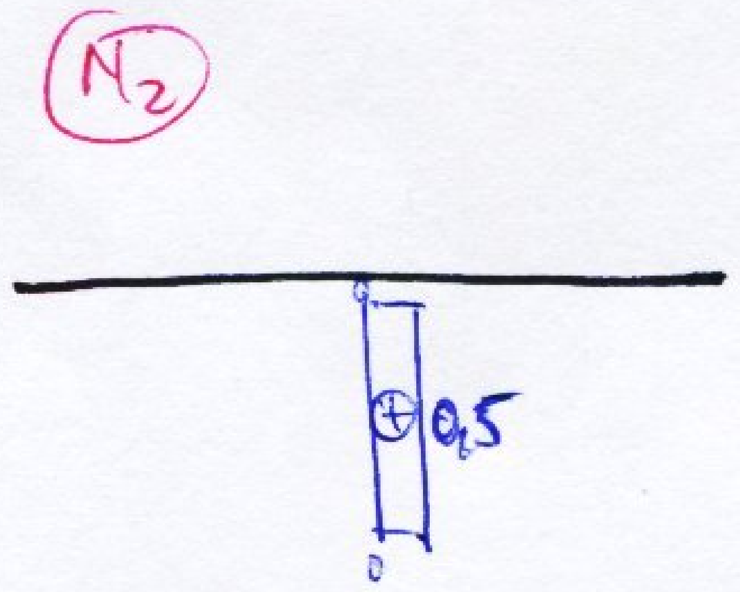












$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{3} \cdot 3^2 \cdot 2 + \frac{1}{3} \cdot 3^3\right] + \frac{1}{EA}\left[2{,}5^2 \cdot 3\right] = \frac{15}{EI} + \frac{18{,}75}{EA} = 310 \cdot 10^{-6}$$

$$\delta_{12} = \frac{1}{EI}\left[\frac{1}{6} \cdot (-1) \cdot 3 \cdot 2\right] + \frac{1}{EA}\left[2{,}5 \cdot 0{,}5 \cdot 3\right] = -\frac{1}{EI} + \frac{3{,}75}{EA} = -14{,}74 \cdot 10^{-6}$$

$$\delta_{22} = \frac{1}{EI}\left[\frac{1}{3} \cdot 2\right] + \frac{1}{EA}\left[0{,}5^2 \cdot 3\right] = \frac{0{,}\overline{6}}{EI} + \frac{0{,}75}{EA} = 13{,}69 \cdot 10^{-6}\ rad$$

$$\delta_{10} = \frac{1}{EI}\left[\underbrace{\int_0^2 1{,}5x(-6{,}25x - 2{,}5x^2)\,dx}_{-9{,}375x^2 - 3{,}75x^3} + \frac{1}{4} \cdot (-22{,}5) \cdot 3 \cdot 3\right] + \frac{1}{EA}\left[-31{,}25 \cdot 2{,}5 \cdot 3\right] =$$

$$= \frac{1}{EI}\left[\left[-3{,}125x^3 - 0{,}9375x^4\right]_0^2 - 50{,}625\right] + \frac{1}{EA}\left[-234{,}375\right] =$$

$$= -\frac{90{,}625}{EI} - \frac{234{,}375}{EA} = -2{,}0186 \cdot 10^{-3}$$

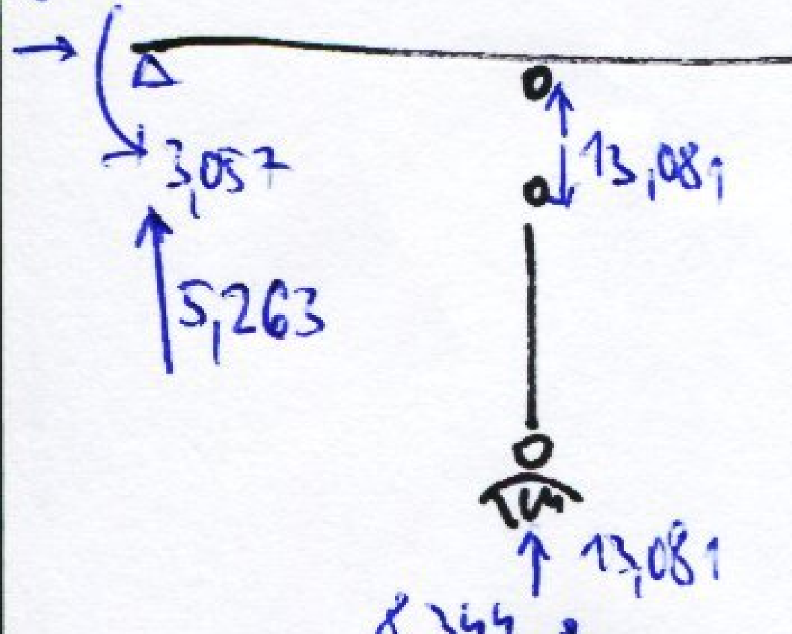
$$\delta_{20} = \frac{1}{EI}\left[\int_0^2 \underbrace{\left(-1 + \frac{x}{2}\right)(-6{,}25x - 2{,}5x^2)}_{+6{,}25x + 2{,}5x^2 - 3{,}125x^2 - 1{,}25x^3 = +6{,}25x - 0{,}625x^2 - 1{,}25x^3}\,dx\right] + \frac{1}{EA}\left[-31{,}25 \cdot 0{,}5 \cdot 3\right] = \frac{1}{EI}\left[3{,}125x^2 - 0{,}208\overline{3}x^3 - 0{,}3125x^4\right]_0^2 - \frac{46{,}875}{EA} = \frac{5{,}8\overline{3}}{EI} - \frac{46{,}875}{EA} =$$

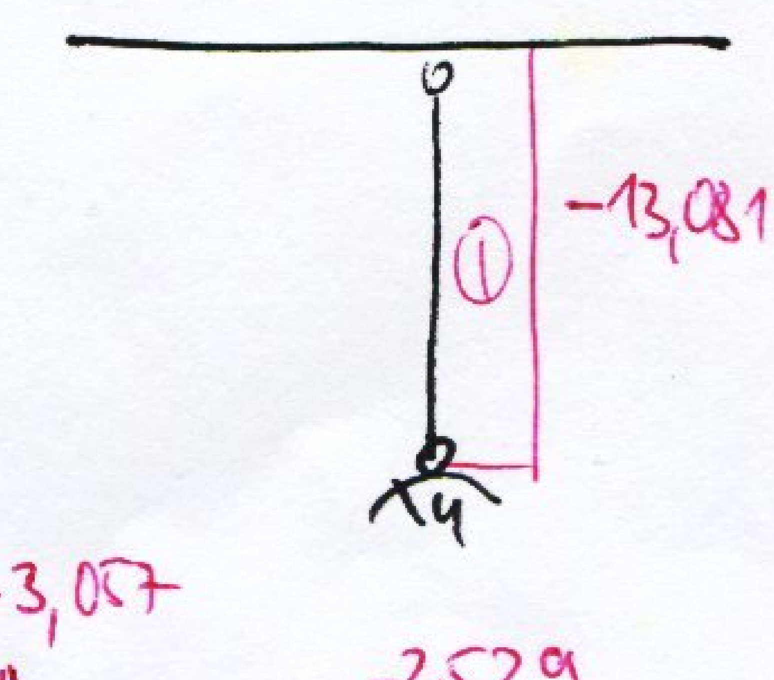
$$= 56{,}273 \cdot 10^{-6}\ kNm$$

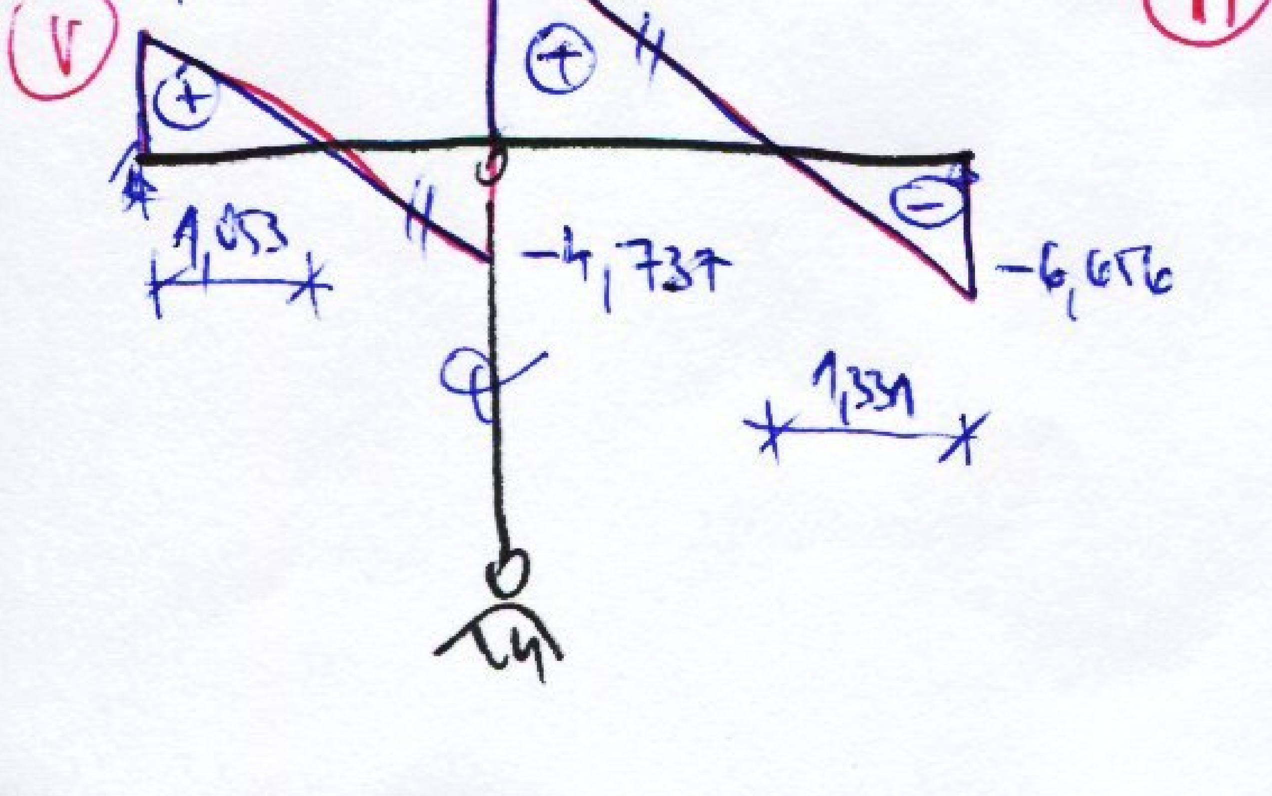
$$\begin{pmatrix} 310 & -14{,}74 \\ -14{,}74 & 13{,}69 \end{pmatrix} \cdot 10^{-6} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} -2{,}0186 \cdot 10^{-3} \\ 56{,}273 \cdot 10^{-6} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

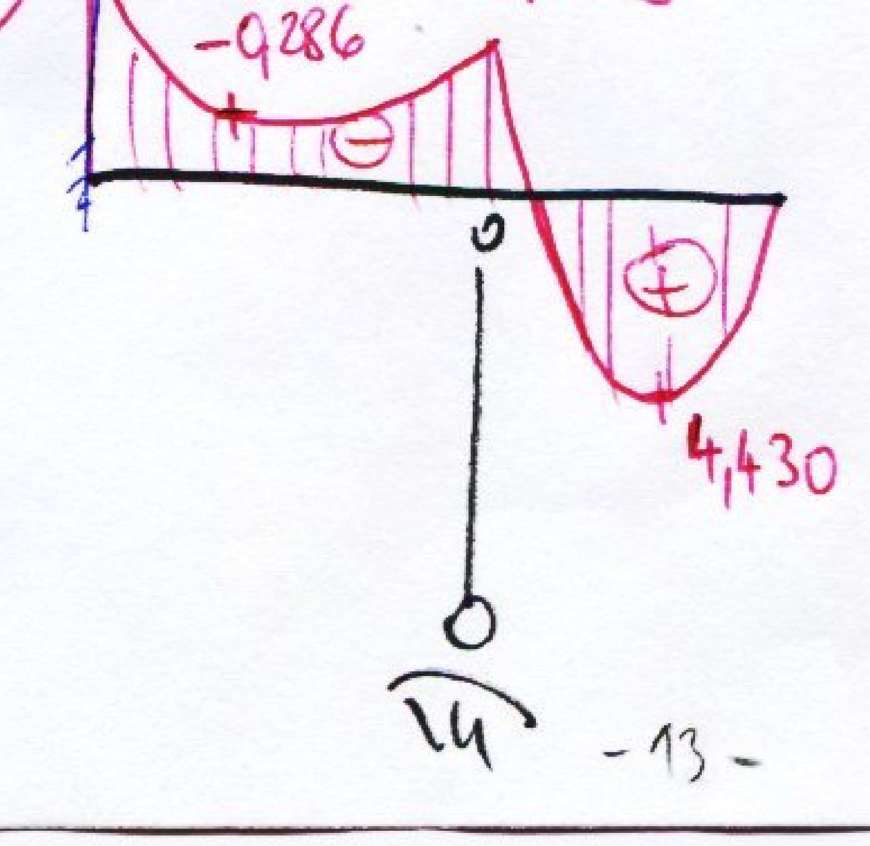
$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{10^{12} \cdot 10^{-6} \cdot 10^{-6}}{4027} \begin{pmatrix} 13{,}69 & 14{,}74 \\ 14{,}74 & 310 \end{pmatrix} \begin{pmatrix} 201{,}86 \\ -56{,}273 \end{pmatrix} = \begin{pmatrix} 6{,}656\ kN \\ 3{,}057\ kNm \end{pmatrix}$$

↑ KONEC 4. CV.

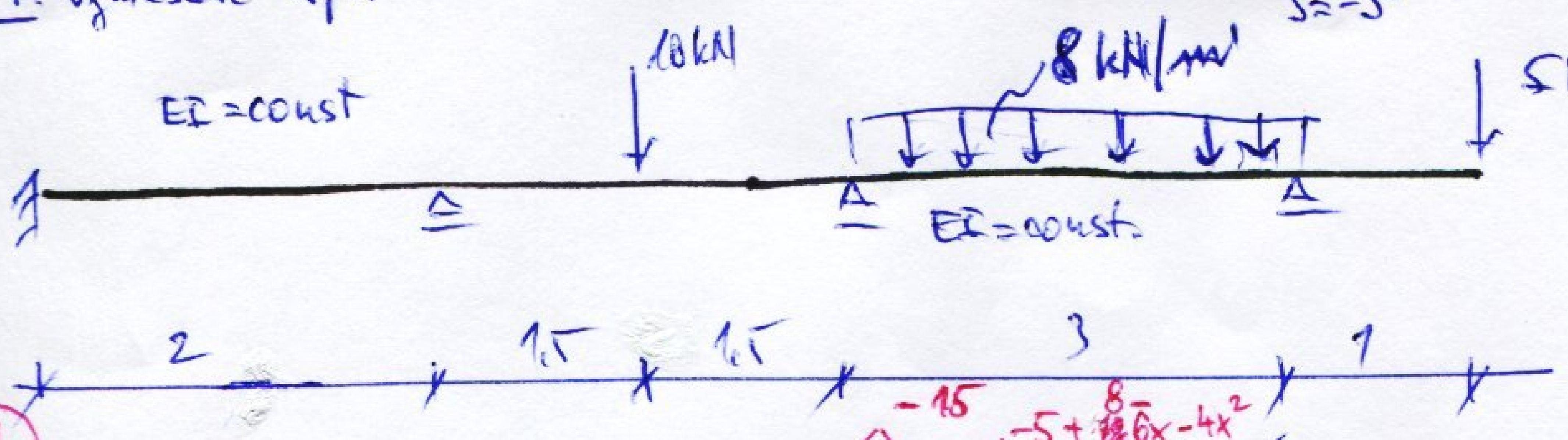


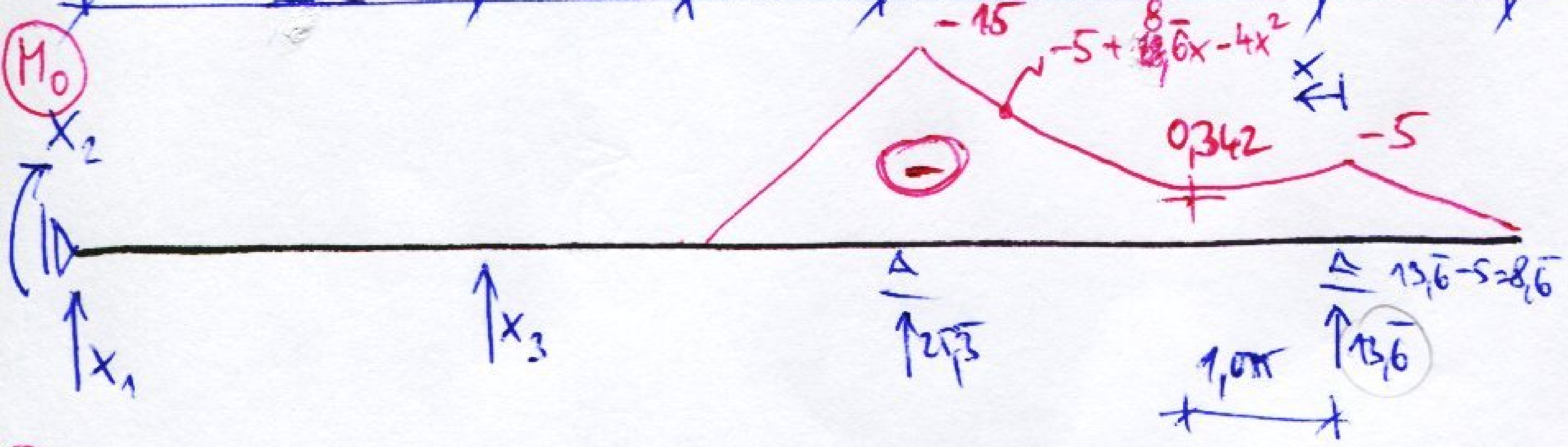


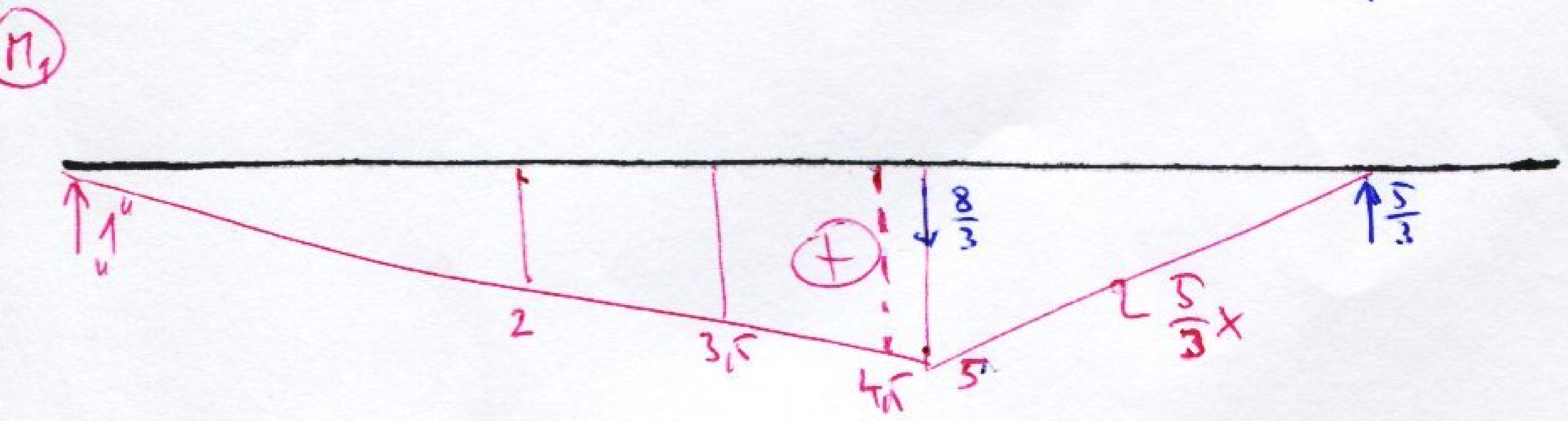


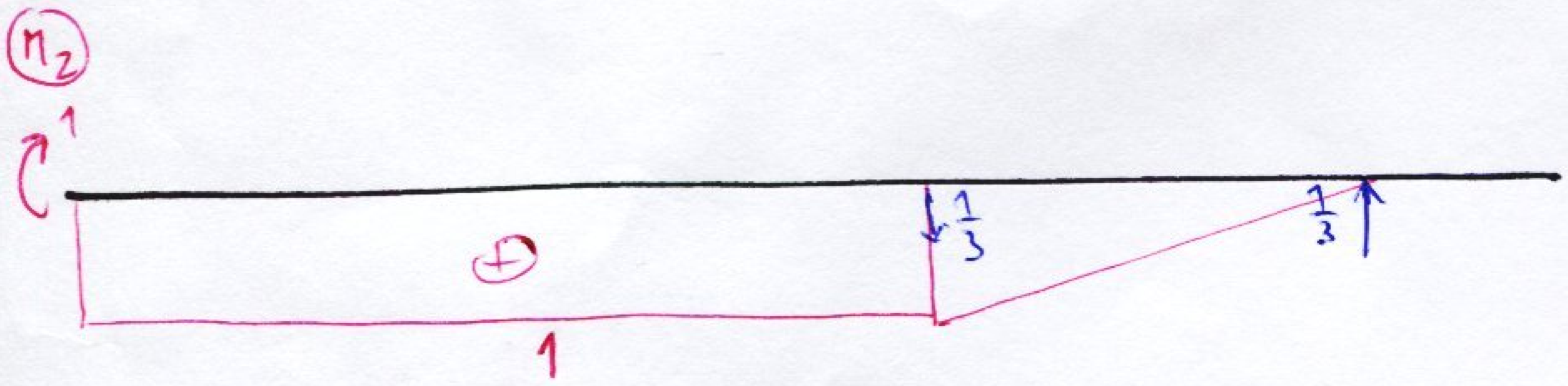


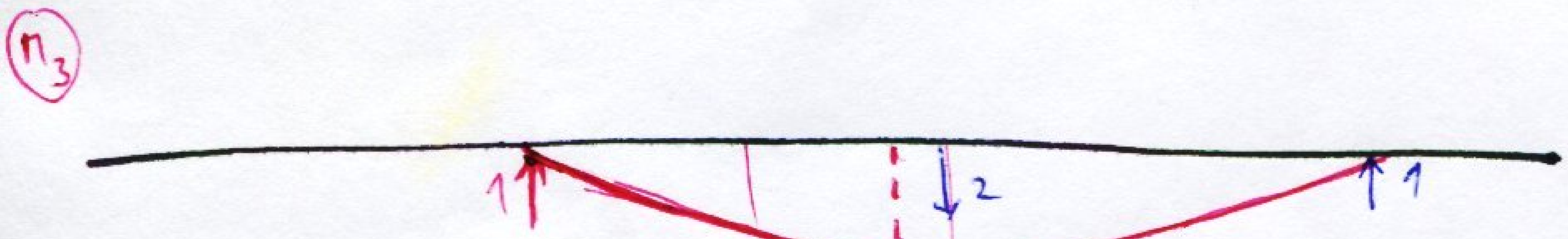
Př. Vykreslete N, M





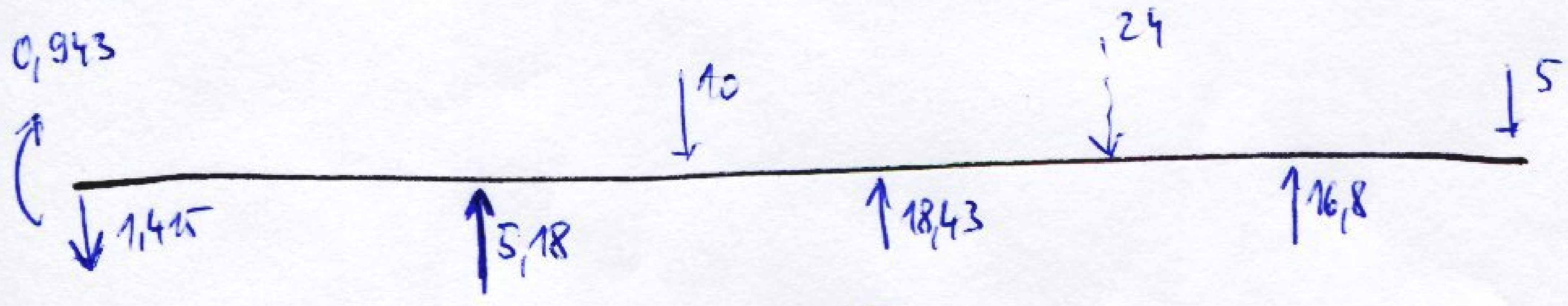


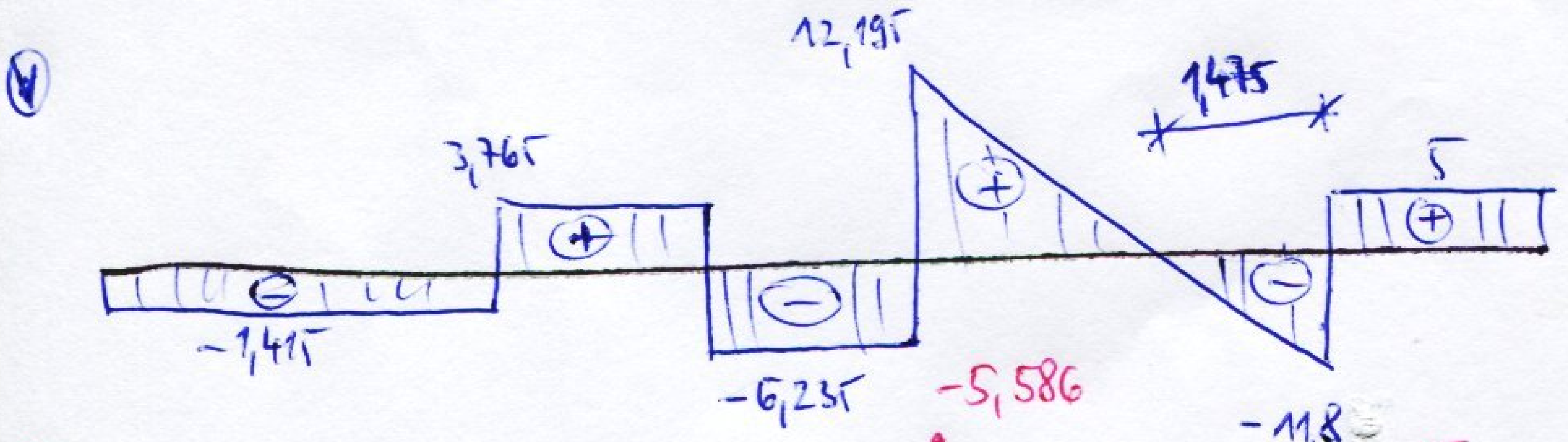


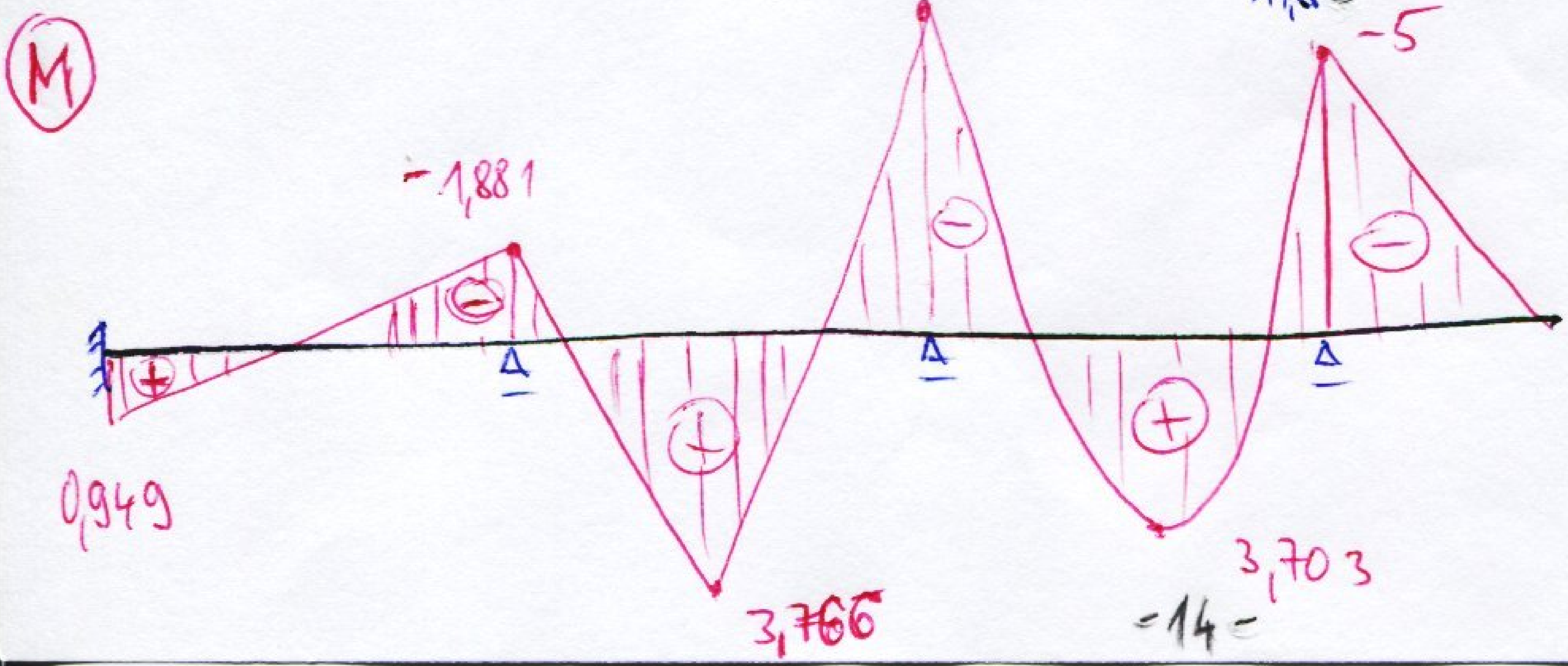


$$\begin{pmatrix} 66,\bar{6} & 17,5 & 33 \\ & 6 & 7,5 \\ \text{SYM} & & 18 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 93,12 \\ 19,75 \\ 53,645 \end{pmatrix}$$

$x_1 = -1,415$ kN
$x_2 = 0,943$ kN
$x_3 = 5,18$ kN







$$\delta_{10} = \frac{1}{EI}\left[-15\cdot 1,5\cdot \frac{1}{2}\cdot 4,5 + \int_0^3 \underbrace{\frac{5}{3}x\left(-5+8,6x-4x^2\right)}_{-\frac{25x}{3}+14,4x^2-\frac{20}{3}x^3}dx\right] =$$

$$= \frac{1}{EI}\left[-50,625 + \left[-\frac{25x^2}{6}+\frac{14,4x^3}{3}-\frac{20}{12}x^4\right]_0^3\right] =$$

$$= \frac{1}{EI}\left[-50,625 - 42,5\right] = -\frac{93,12}{EI}$$

$$\delta_{20} = \frac{1}{EI}\left[\frac{1}{2}\cdot 1\cdot(-15)\cdot 1,5 - 42,5\cdot\frac{1}{5}\right] = -\frac{19,75}{EI}$$

$$\delta_{30} = \frac{1}{EI}\left[\frac{1}{2}\cdot(-15)\cdot 1,5\cdot 2,5 - 42,5\cdot\frac{3}{5}\right] =$$

$$= \frac{1}{EI}\left[-28,125 - 25,5\right] = -\frac{53,625}{EI}$$

$$\delta_{12} = \frac{1}{EI}\left[\frac{1}{2}\cdot 5\cdot 1\cdot 5 + \frac{1}{3}\cdot 5\cdot 1\cdot 3\right] = \frac{17,5}{EI}$$

$$\delta_{13} = \frac{1}{EI}\left[\frac{1}{2}\cdot 3\cdot 3\cdot 4 + \frac{1}{3}\cdot 5\cdot 3\cdot 3\right] = \frac{33}{EI}$$

$$\delta_{23} = \frac{1}{EI}\left[\frac{1}{2}\cdot 1\cdot 3\cdot 3 + \frac{1}{3}\cdot 1\cdot 3\cdot 3\right] = \frac{7,5}{EI}$$

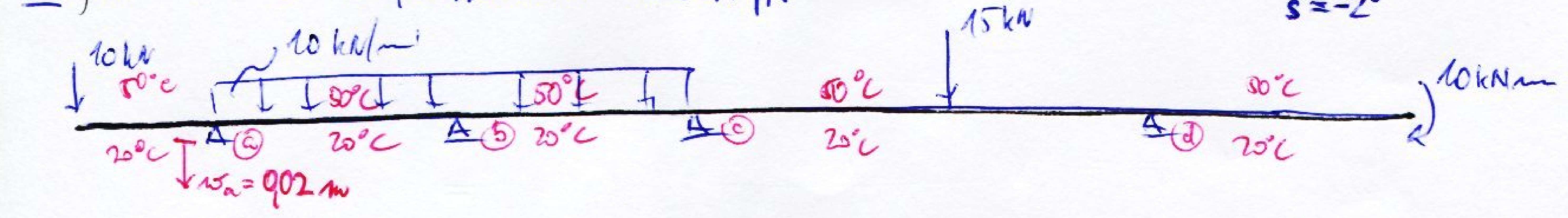
$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{3}\cdot 5^3 + \frac{1}{3}\cdot 5^2\cdot 3\right] = \frac{66,\bar{6}}{EI}$$

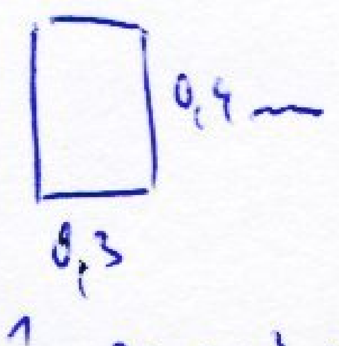
$$\delta_{22} = \frac{1}{EI}\left[1^2\cdot 5 + \frac{1}{3}\cdot 1^2\cdot 3\right] = \frac{6}{EI}$$

$$\delta_{33} = \frac{1}{EI}\left[\frac{1}{3}3^3 + \frac{1}{3}\cdot 3^3\right] = \frac{18}{EI}$$

Př. a) URČETE REAKCE, VYKRESLETE PRŮBĚH V, M

$s = -2°$



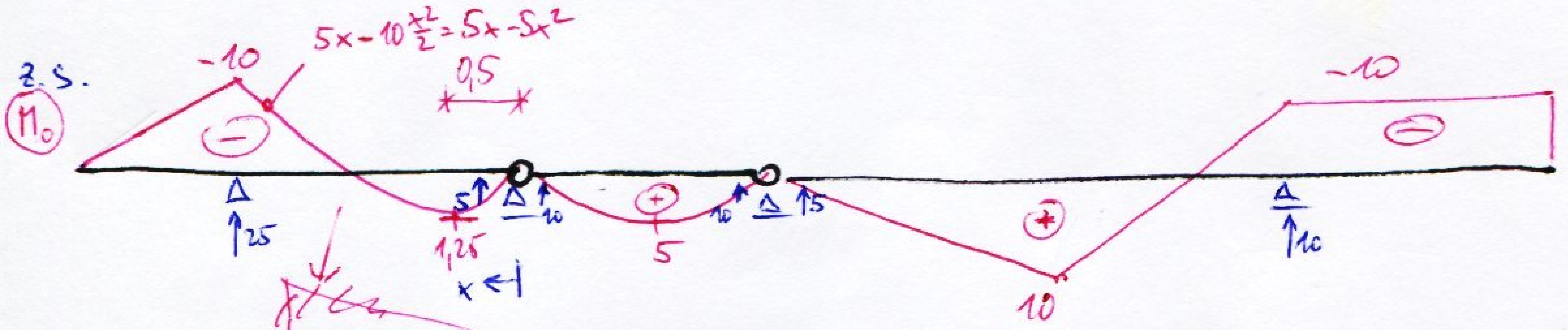


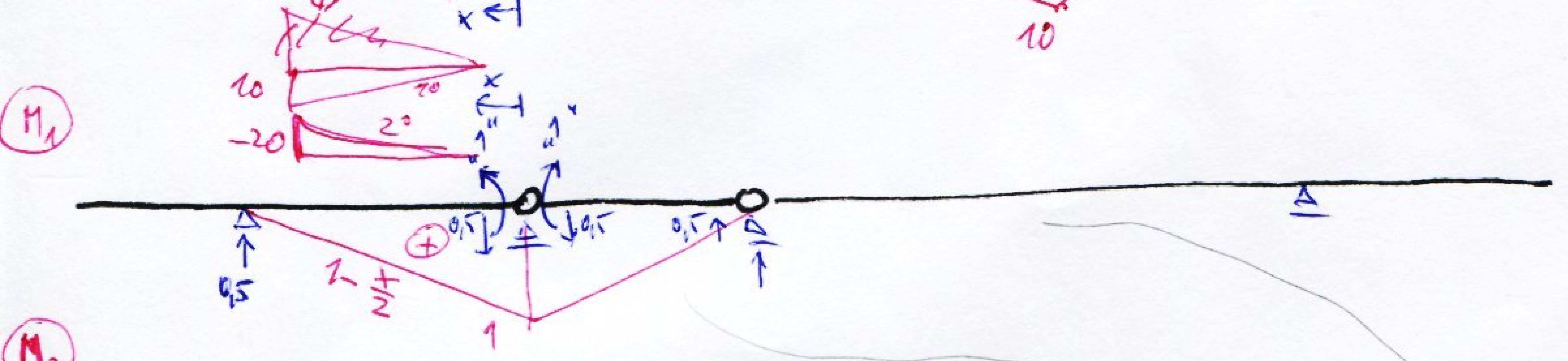
$E = 30$ GPa

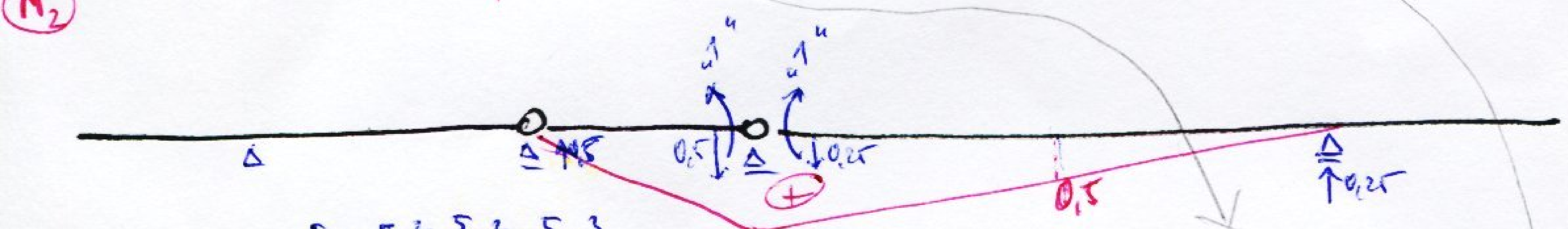
$I = \frac{1}{12} \cdot 0{,}3 \cdot 0{,}4^3 = 1{,}6 \cdot 10^{-3}\ m^4$

$EI = 48\,000\ kNm^2$

$\alpha = 12 \cdot 10^{-6}\ \frac{1}{K}$







$$\delta_{10} = \frac{1}{EI}\left[\int_0^2 \overbrace{\left(1-\frac{x}{2}\right)(5x-5x^2)}^{5x-5x^2-\frac{5}{2}x^2+\frac{5}{2}x^3} dx + \frac{1}{3}\cdot 1\cdot 5\cdot 2\right] + \frac{20-50}{0{,}4}\cdot\alpha\cdot\left[\frac{1}{2}\cdot 1\cdot 2 + \frac{1}{2}\cdot 1\cdot 2\right] - \left(0{,}5\cdot(-0{,}02)\right) =$$

$$= \frac{1}{48\,000}\left[\left[\frac{5x^2}{2} - \frac{15x^3}{6} + \frac{5x^4}{8}\right]_0^2 + \frac{10}{3}\right] - 1{,}8\cdot 10^{-3} + 0{,}01 = \frac{1}{48000}\left[0 + \frac{10}{3}\right] - 1{,}8\cdot 10^{-3} + 0{,}01 = 8{,}269\cdot 10^{-3}\ kNm$$

$$\delta_{20} = \frac{1}{EI}\left[\underbrace{\frac{1}{3}\cdot 5\cdot 1\cdot 2}_{10/3} + \underbrace{\frac{1}{6}\, 10\,(1+2\cdot 0{,}5)\cdot 2}_{20/3} + \underbrace{\frac{1}{6}\cdot 0{,}5\cdot(2\cdot 10 - 10)\cdot 2}_{5/3}\right] + \frac{20-50}{0{,}4}\cdot\alpha\left[\frac{1}{2}\cdot 1\cdot 2 + \frac{1}{2}\cdot 4\cdot 1\right] = \frac{11{,}\overline{6}}{EI} - 2{,}7\cdot 10^{-3} = -2{,}458\cdot 10^{-3}\ kN$$

$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{3}\cdot 1\cdot 1\cdot 2 \times 2\right] = \frac{4}{3EI} = 27{,}\overline{7}\cdot 10^{-6}$$

$$\delta_{22} = \frac{1}{EI}\left[\frac{1}{3}\cdot 1\cdot 1\cdot 2 + \frac{1}{3}\cdot 1\cdot 1\cdot 4\right] = \frac{2}{EI} = 41{,}\overline{6}\cdot 10^{-6}$$

$$\delta_{12} = \frac{1}{EI}\left[\frac{1}{6}\cdot 1\cdot 1\cdot 2\right] = \frac{1}{3EI} = 6{,}9\overline{4}\cdot 10^{-6}$$

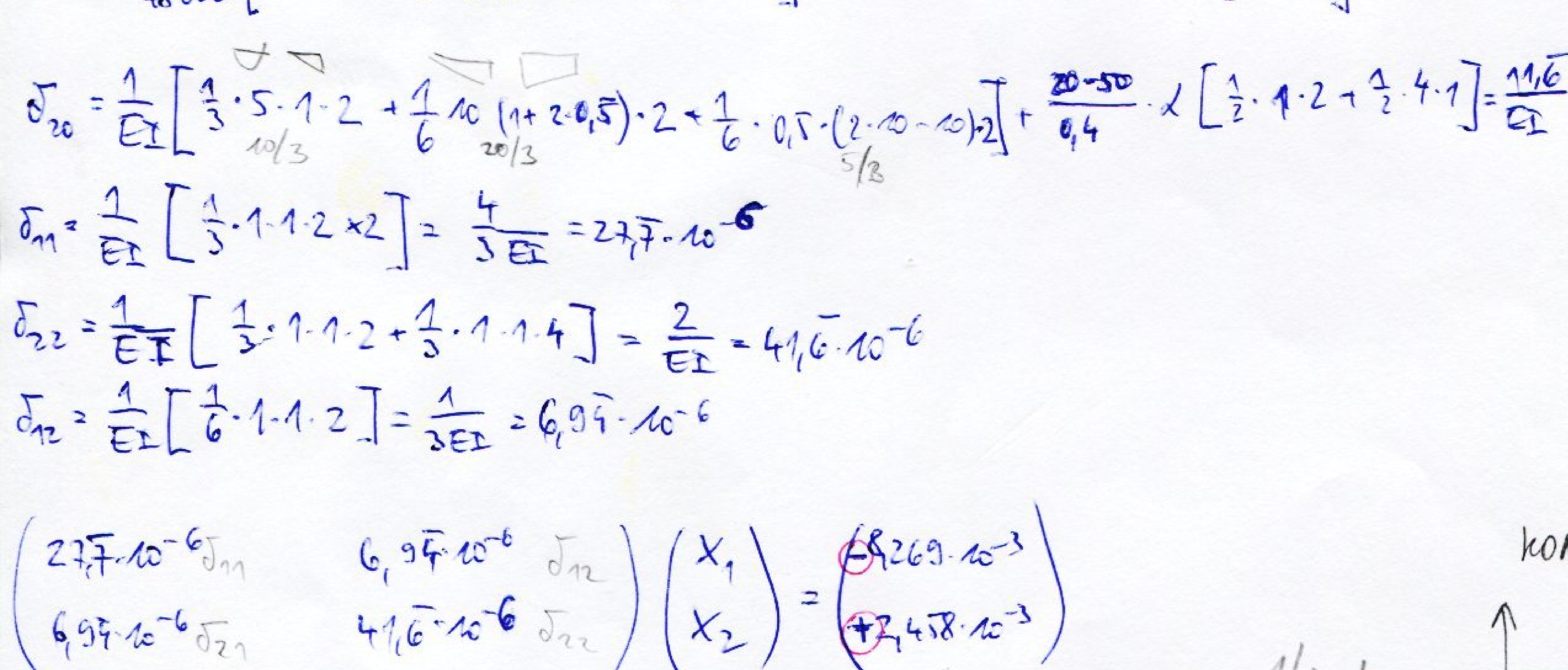
$$\underbrace{\begin{pmatrix} 27{,}\overline{7}\cdot 10^{-6}\ \delta_{11} & 6{,}9\overline{4}\cdot 10^{-6}\ \delta_{12} \\ 6{,}9\overline{4}\cdot 10^{-6}\ \delta_{21} & 41{,}\overline{6}\cdot 10^{-6}\ \delta_{22} \end{pmatrix}}_{A} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} -8{,}269\cdot 10^{-3} \\ +2{,}458\cdot 10^{-3} \end{pmatrix}$$

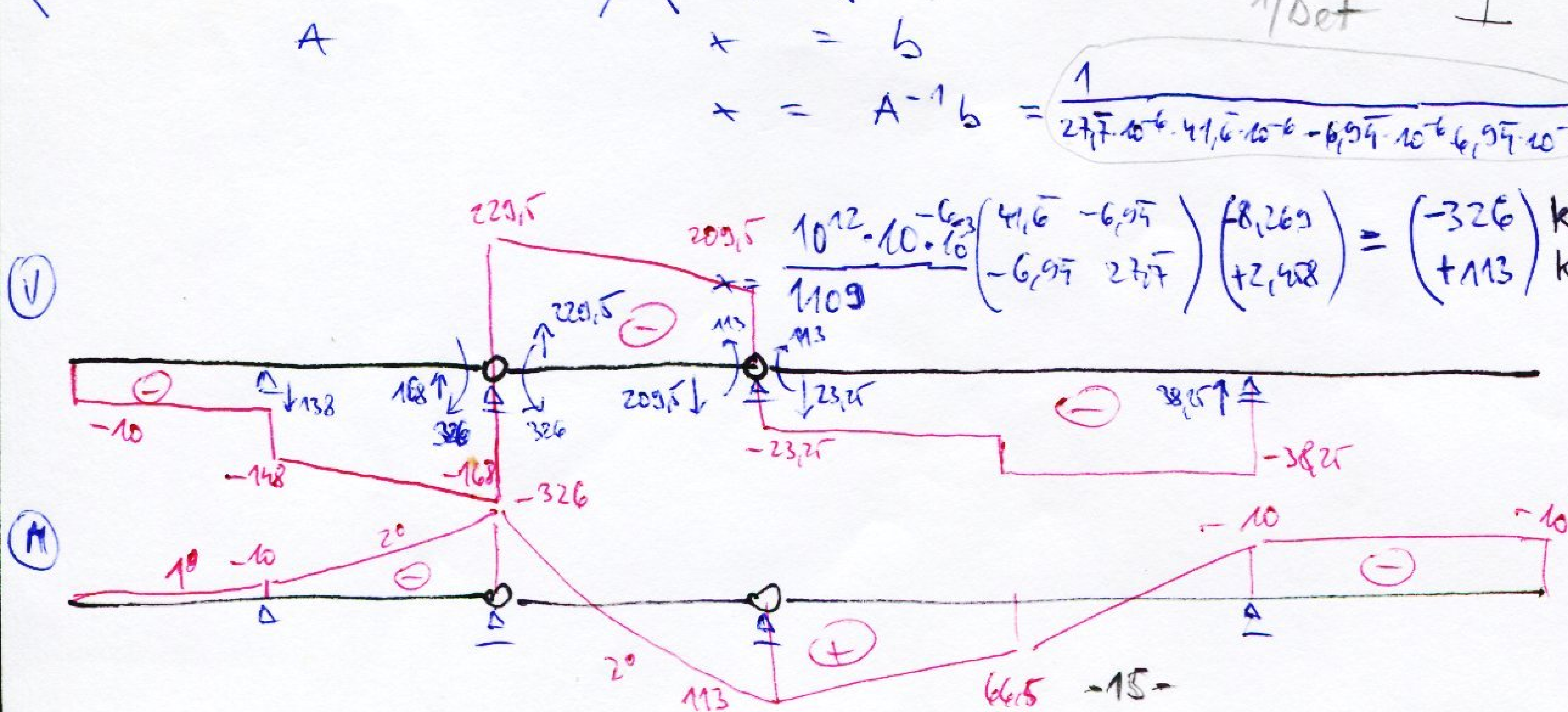
KONEC 5. CV.

$$x = b$$

$$x = A^{-1} b = \underbrace{\frac{1}{27{,}\overline{7}\cdot 10^{-6}\cdot 41{,}\overline{6}\cdot 10^{-6} - 6{,}9\overline{4}\cdot 10^{-6}\cdot 6{,}9\overline{4}\cdot 10^{-6}}}_{1/Det} \overbrace{\begin{pmatrix} 41{,}\overline{6}\cdot 10^{-6} & -6{,}9\overline{4}\cdot 10^{-6} \\ -6{,}9\overline{4}\cdot 10^{-6} & 27{,}\overline{7}\cdot 10^{-6} \end{pmatrix}}^{\text{ADJUNGOVANÁ MATICE}} \cdot b$$

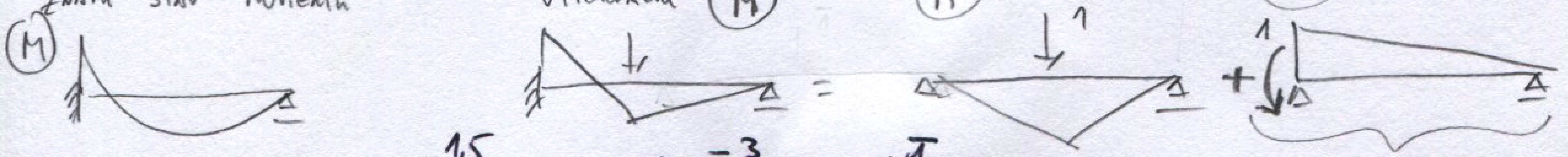
$$= \frac{10^{12}\cdot 10^{-6}\cdot 10^{-3}}{1109}\begin{pmatrix} 41{,}\overline{6} & -6{,}9\overline{4} \\ -6{,}9\overline{4} & 27{,}\overline{7} \end{pmatrix}\begin{pmatrix} -8{,}269 \\ +2{,}458 \end{pmatrix} = \begin{pmatrix} -326 \\ +113 \end{pmatrix} \begin{matrix} kN \\ kN \end{matrix}$$





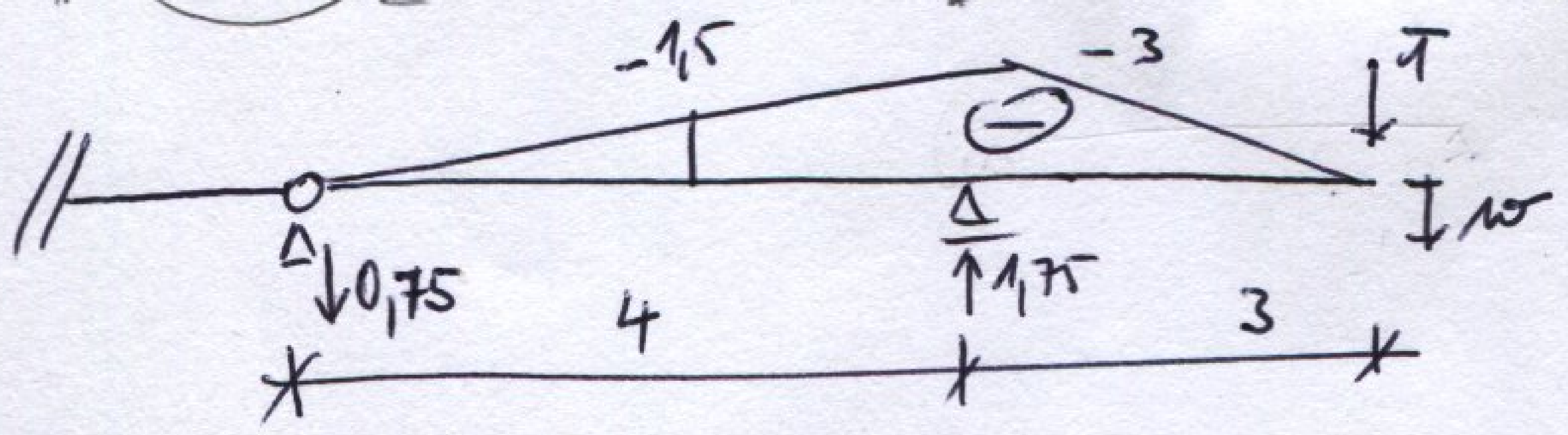
b) VYUŽITÍM REDUKČNÍ VĚTY URČETE PRŮHYB ~~KON~~ PRAVÉHO KONCE NOSNÍKU (NEDĚLAT – TEPLOTA)

ZNÁMÝ STAV MOMENTŮ M — VIRTUÁLNÍ $\bar{M}$ — $\bar{n}^1$ — $\bar{n}^2$

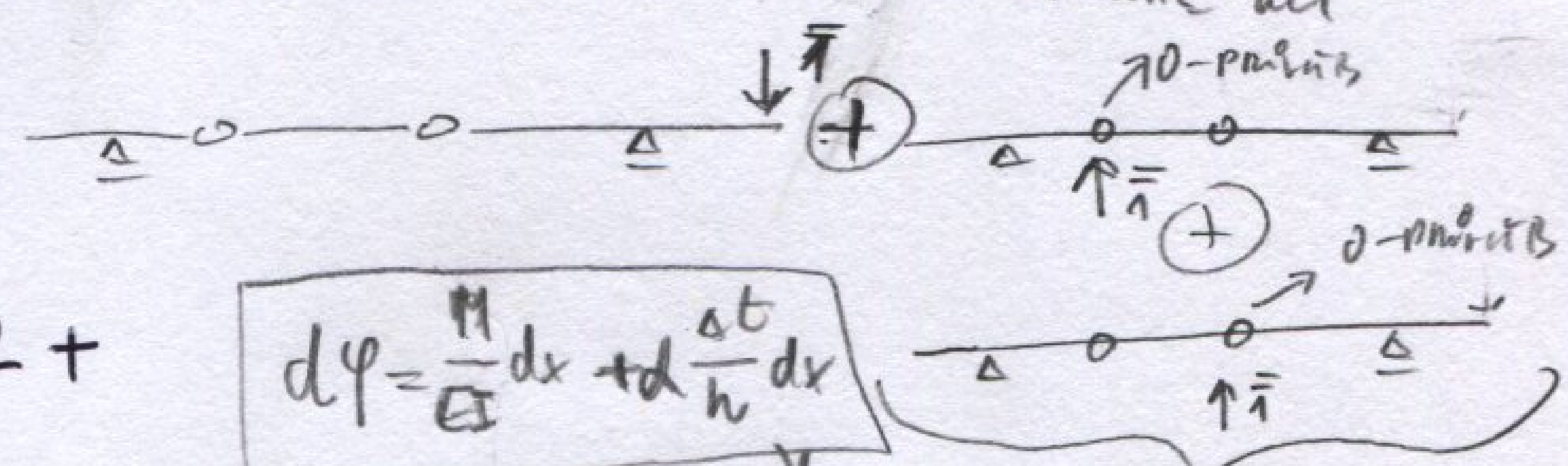


$$\int \frac{1}{EI}\left(M\bar{n}^2\right) = 0$$

NATOČENÍ V LEVÉ PODPOŘE = 0



JAK BY VYPADAL $\bar{1}$ STAV NA STAT. NEURČITÉ ULCI

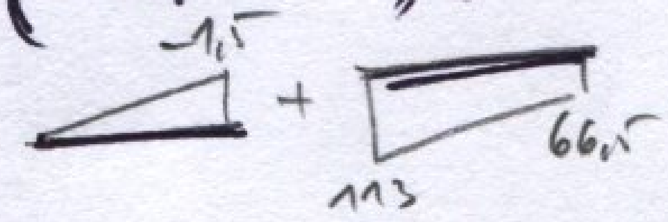


NEUPLATNÍ SE

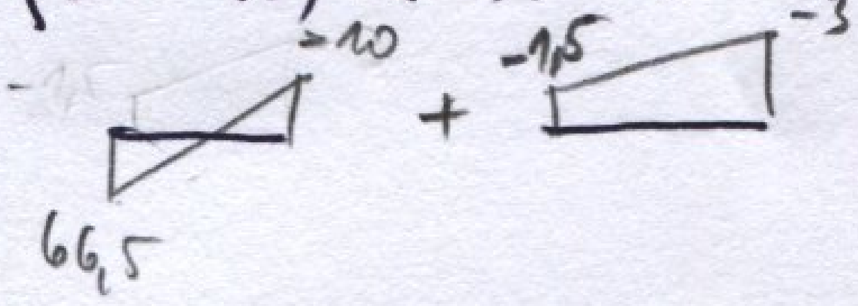
$$\boxed{d\varphi = \frac{M}{EI}dx + \alpha\frac{\Delta t}{h}dx}$$

$$W_{int} = \int \bar{M}\, d\varphi$$

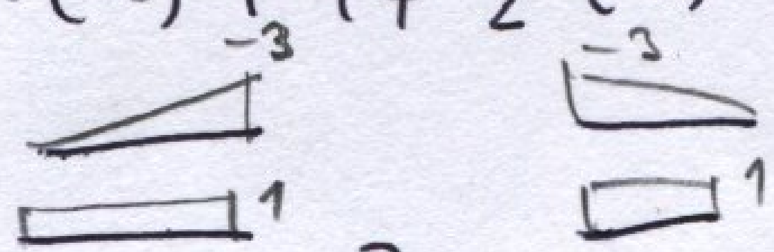
$$\bar{1}\cdot w = \frac{1}{EI}\Bigg\{ \cancel{\frac{1}{6}\left(2\cdot(-10)+66\right)}\ \frac{1}{6}\left(2\cdot 66{,}5 + 113\right)(-1{,}5)\cdot 2 +$$



$$+ \frac{1}{6}\Big[66{,}5\left(2\cdot(-1{,}5)+(-3)\right) + (-10)\cdot\left(2\cdot(-3)+(-1{,}5)\right)\Big]\cdot 2 + \frac{1}{2}\cdot(-10)\cdot(-3)\cdot 3\Bigg\}$$



$$+ \alpha\left(\frac{20-50}{0{,}4}\right)\left\{\frac{1}{2}\cdot(-3)\cdot 1\cdot 4 + \frac{1}{2}\cdot(-3)\cdot 1\cdot 3\right\}$$ VLIV TEPLOTY

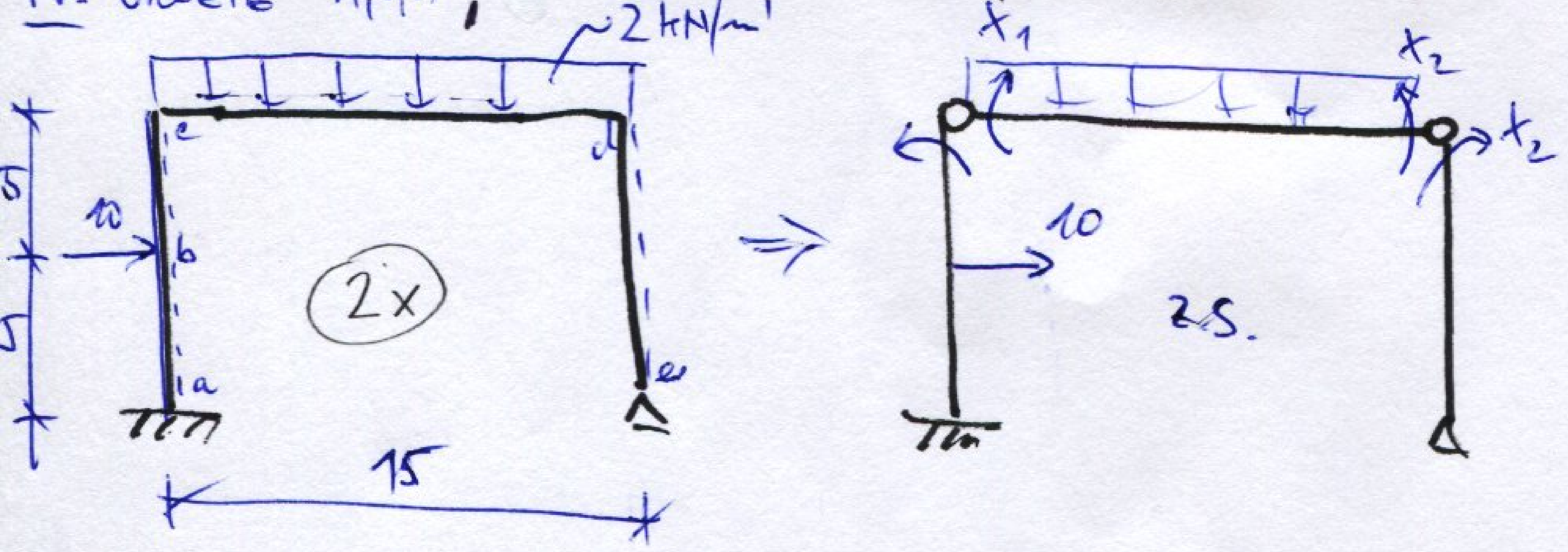


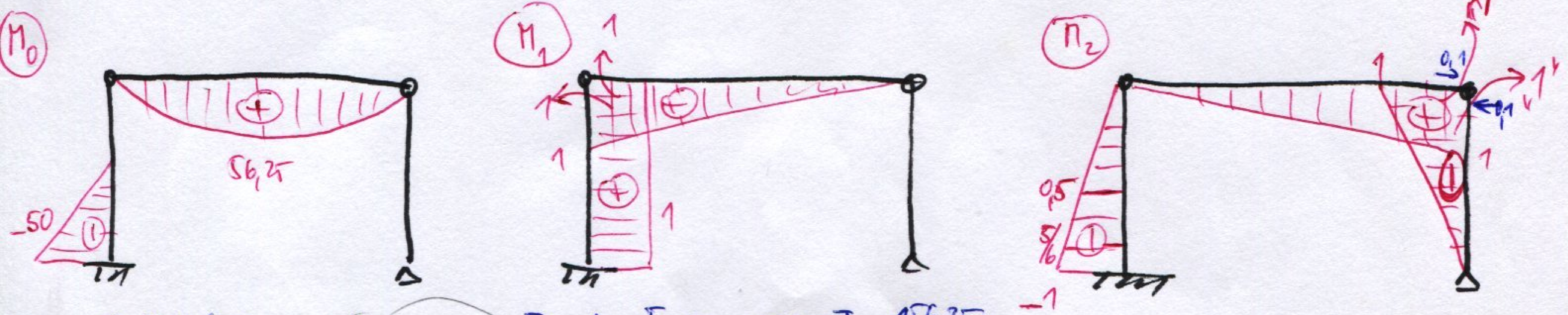
$$= \underbrace{\frac{1}{EI}\left\{-123 - 108 + 45\right\}}_{\text{VLIV M}} + \underbrace{\left(-9\cdot 10^{-4}\right)\left\{-6 - 4{,}5\right\}}_{\text{VLIV TEPLOTY (NORMÁLNĚ SE NEDĚLÁ PŘI REDUKČNÍ VĚTĚ)}} = -0{,}003875 + 0{,}00945\ \text{m} =$$

$$= 5{,}575\ \text{mm}$$

POZN. VLIV TEPLOTY A POKLESU PODPOR SE MUSÍ ZAHRNOUT DO VÝPOČTU REDUKČNÍ VĚTOU

Př. Určete N, V, M ; Určete posun w_d, ověřte nulové natočení φ_a pomocí redukční věty





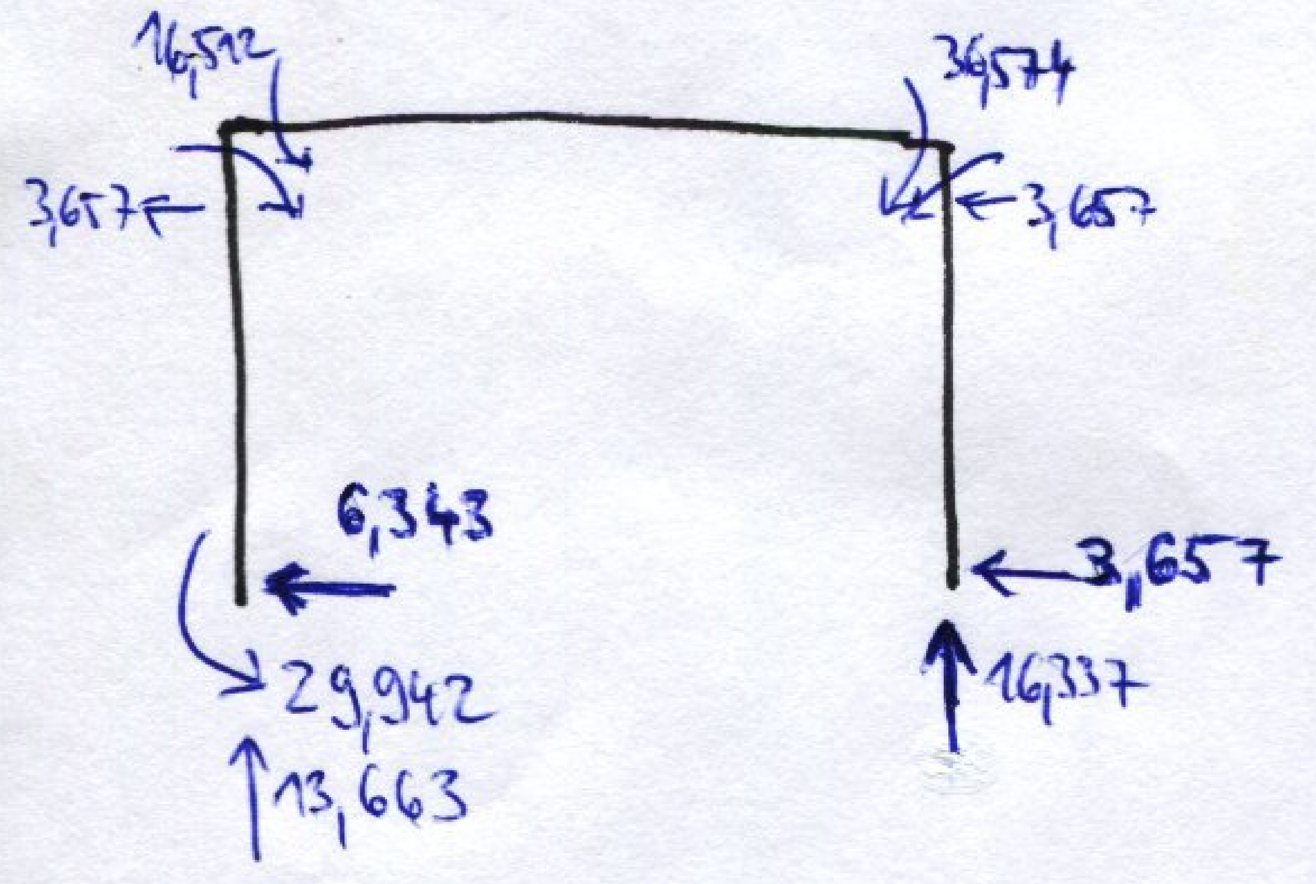
$$\delta_{10} = \frac{1}{EI}\left[\frac{1}{2}\cdot(-50)\cdot 1\cdot 5 + \frac{1}{3}\cdot 1\cdot 56{,}25\cdot 15\right] = \frac{1}{EI}\left[-125 + 281{,}25\right] = \frac{156{,}25}{EI}$$

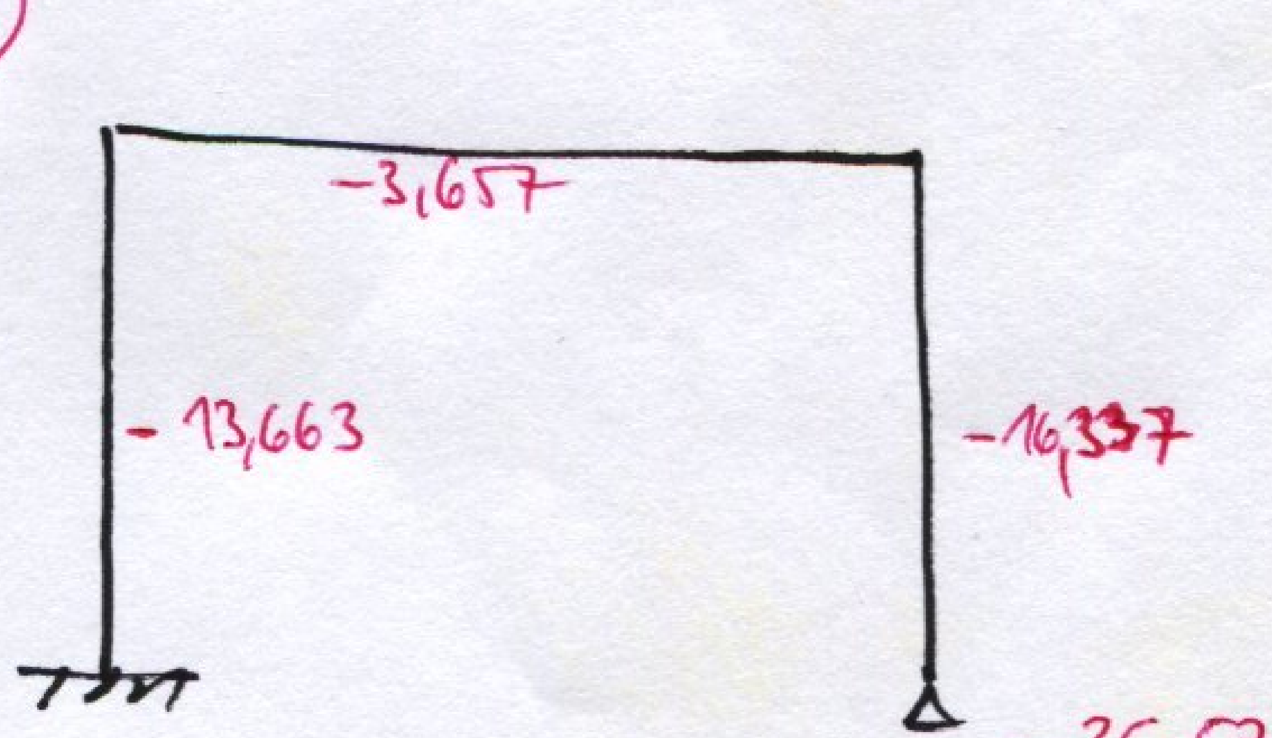
$$\delta_{20} = \frac{1}{EI}\left[\frac{1}{2}\cdot(-50)\cdot\left(-\frac{5}{6}\right)\cdot 5 + \frac{1}{3}\cdot 1\cdot 56{,}25\cdot 15\right] = \frac{1}{EI}\left[104{,}1\bar{6} + 281{,}25\right] = \frac{385{,}41\bar{6}}{EI}$$

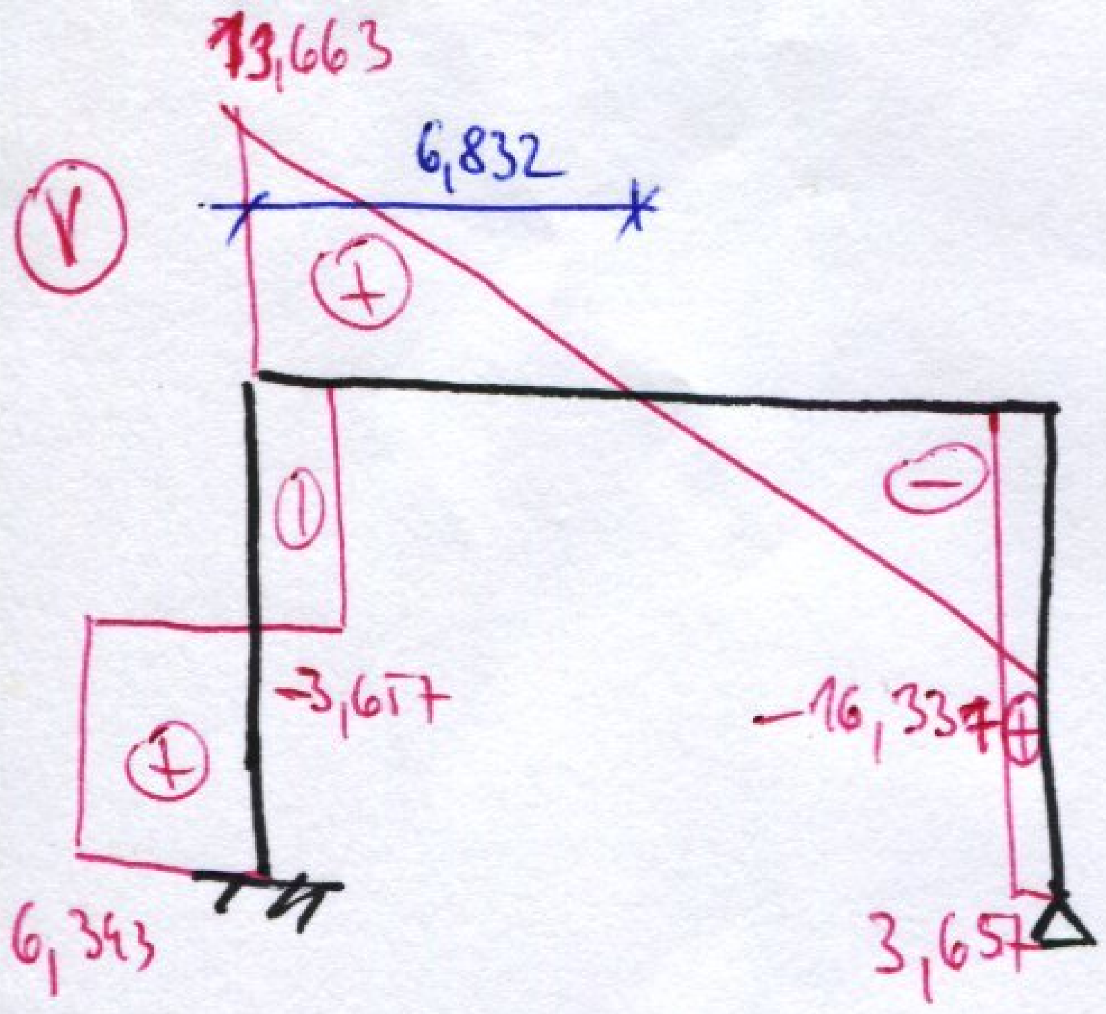
$$\delta_{11} = \frac{1}{EI}\left[1^2\cdot 10 + \frac{1}{3}\cdot 1^2\cdot 15\right] = \frac{15}{EI} \qquad \delta_{22} = \frac{1}{EI}\left[\frac{1}{3}\cdot 1^2\cdot 10 + \frac{1}{3}\cdot 1^2\cdot 15 + \frac{1}{3}\cdot 1^2\cdot 10\right] = \frac{11{,}\bar{6}}{EI}$$

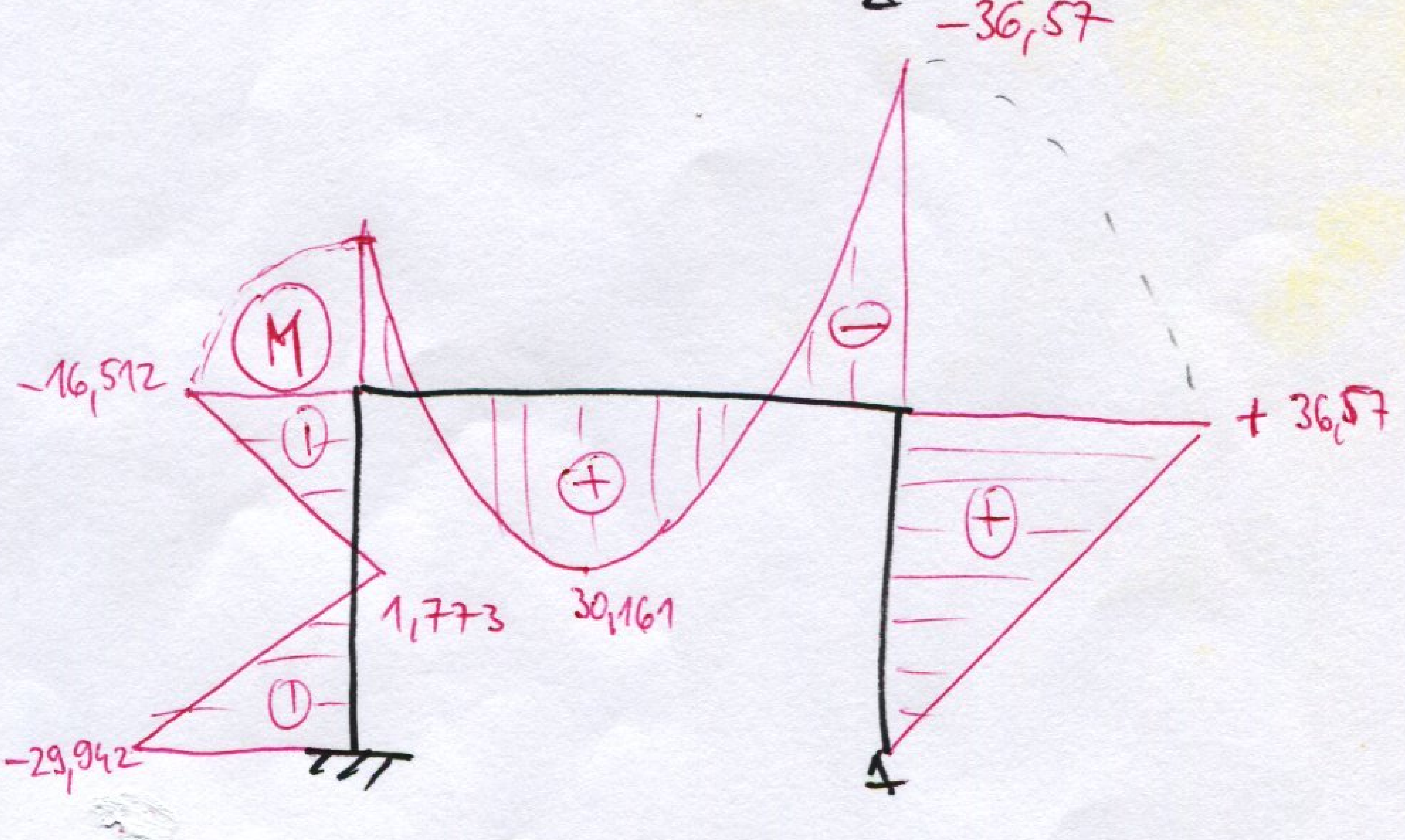
$$\delta_{12} = \frac{1}{EI}\left[\frac{1}{2}\cdot 1\cdot(-1)\cdot 10 + \frac{1}{6}\cdot 1^2\cdot 15\right] = -\frac{2{,}5}{EI}$$

$$\begin{pmatrix} 15 & -2{,}5 \\ -2{,}5 & 11{,}\bar{6} \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} -156{,}25 \\ -385{,}41\bar{6} \end{pmatrix} \qquad \cancel{\begin{pmatrix} 15 & -2{,}5 \\ -2{,}5 & 11{,}\bar{6} \end{pmatrix} /\cdot 6 = \begin{pmatrix} 15 & -2{,}5 \\ -15 & 70 \end{pmatrix} = \begin{pmatrix} 15 & -2{,}5 \\ 0 & 67{,}5 \end{pmatrix}} \quad \left(\begin{array}{cc|c} 15 & -2{,}5 & -156{,}25 \\ 0 & 67{,}5 & -2468{,}7 \end{array}\right)\cdot 6 \rightarrow \begin{array}{l} X_1 = -16{,}512 \\ X_2 = -36{,}574 \end{array}$$

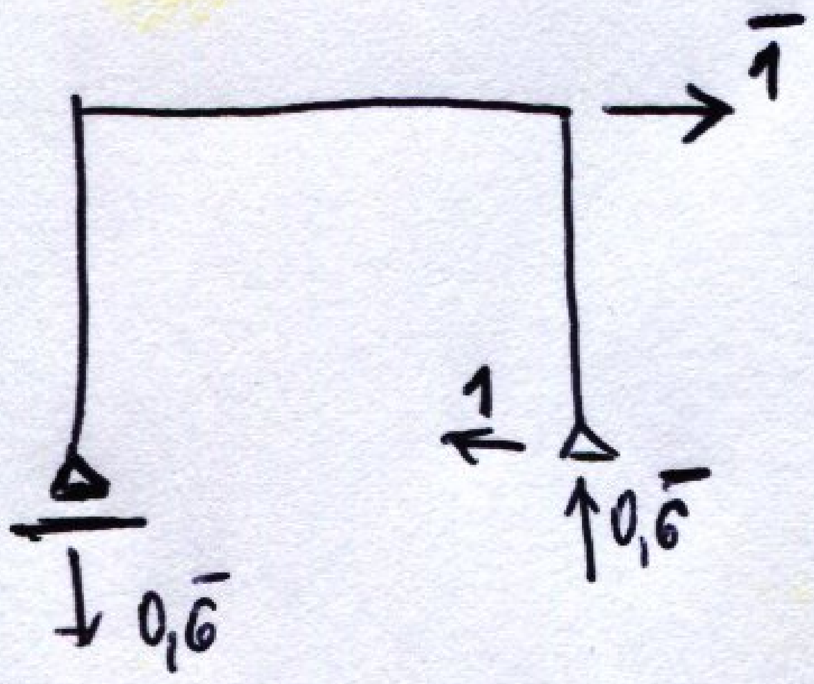


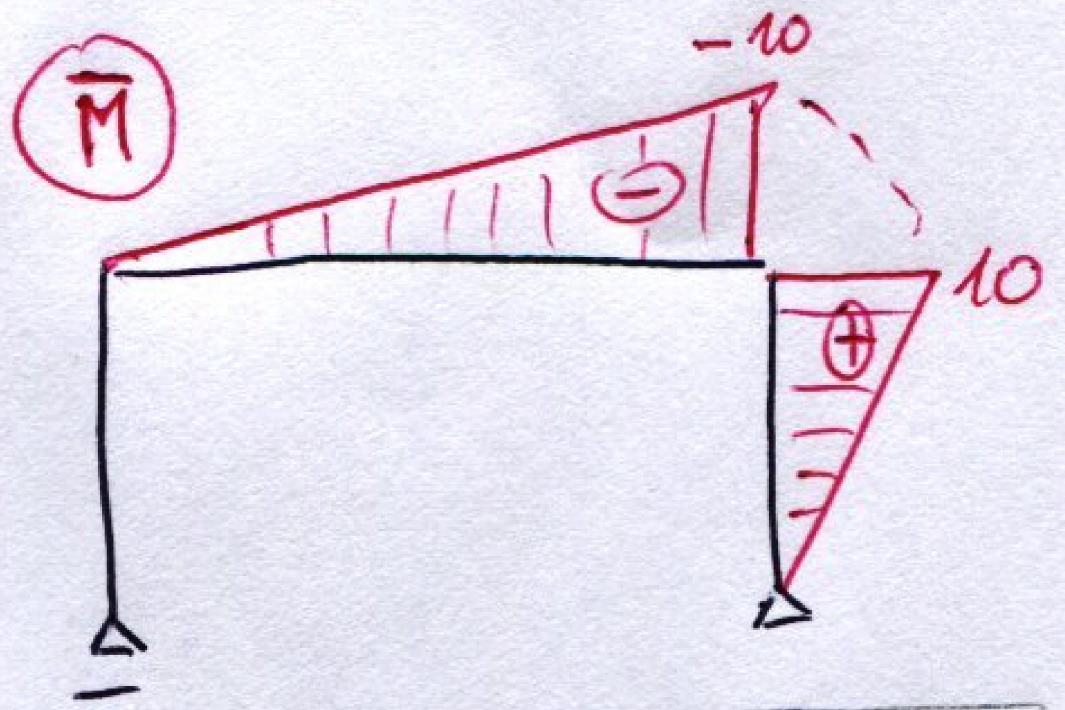




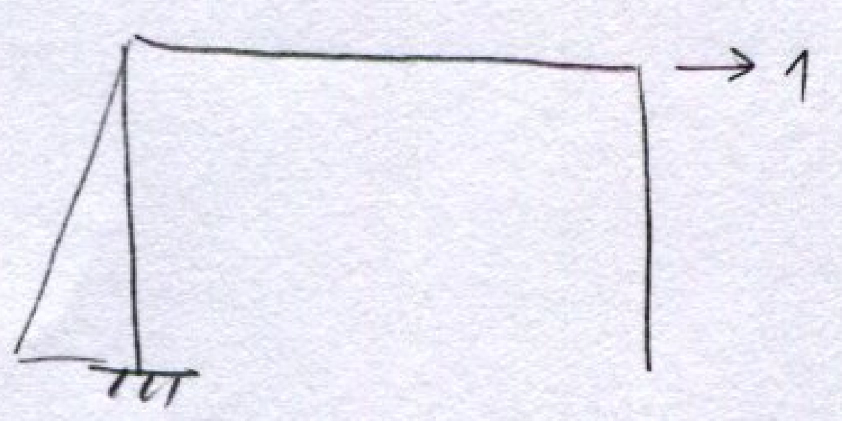


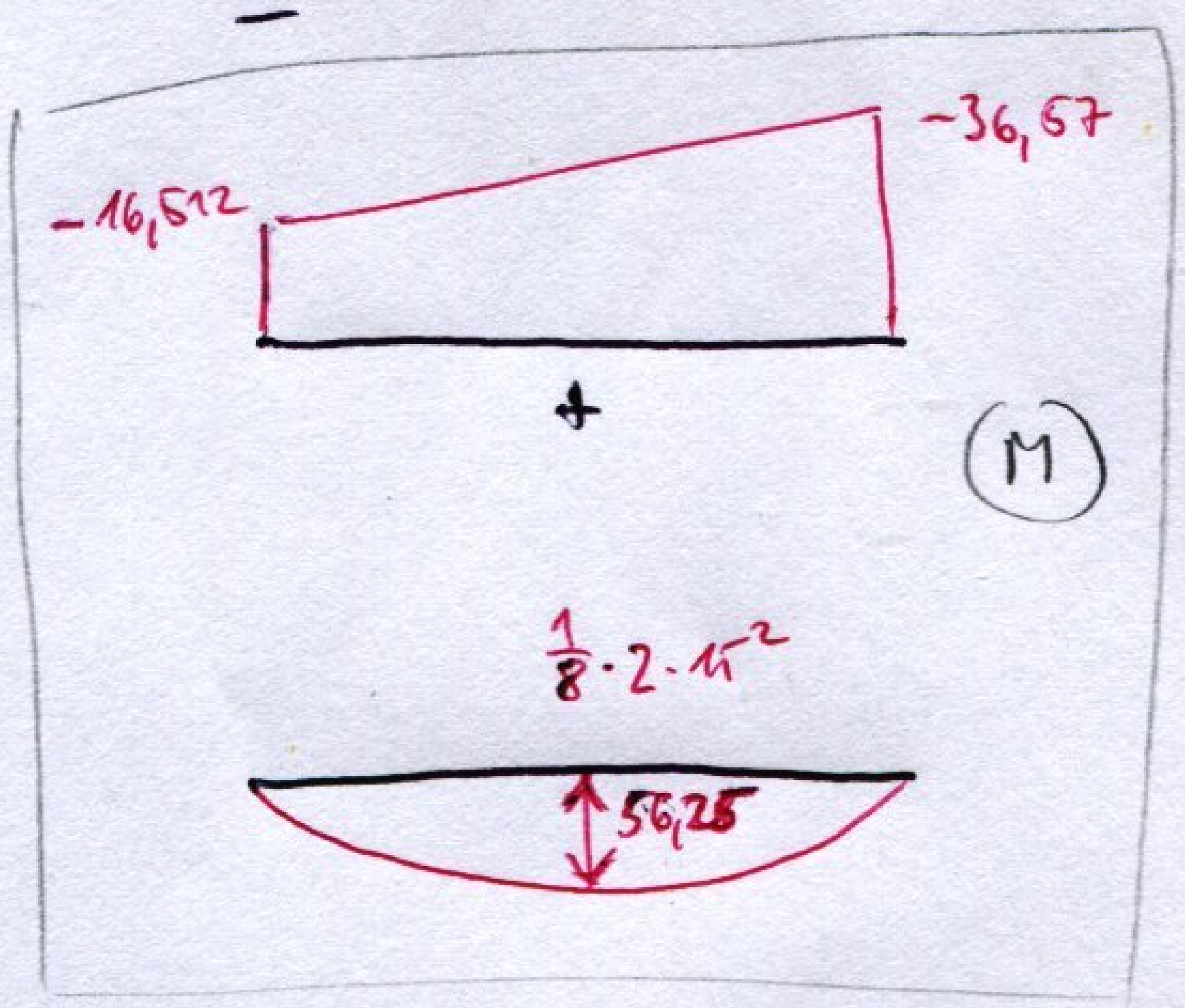
• POSUN w_d – LIBOVOLNÁ Z.S. PŘI REDUKČNÍ VĚTĚ





POZN. SLOŽITÁ INTEGRACE VLEVO



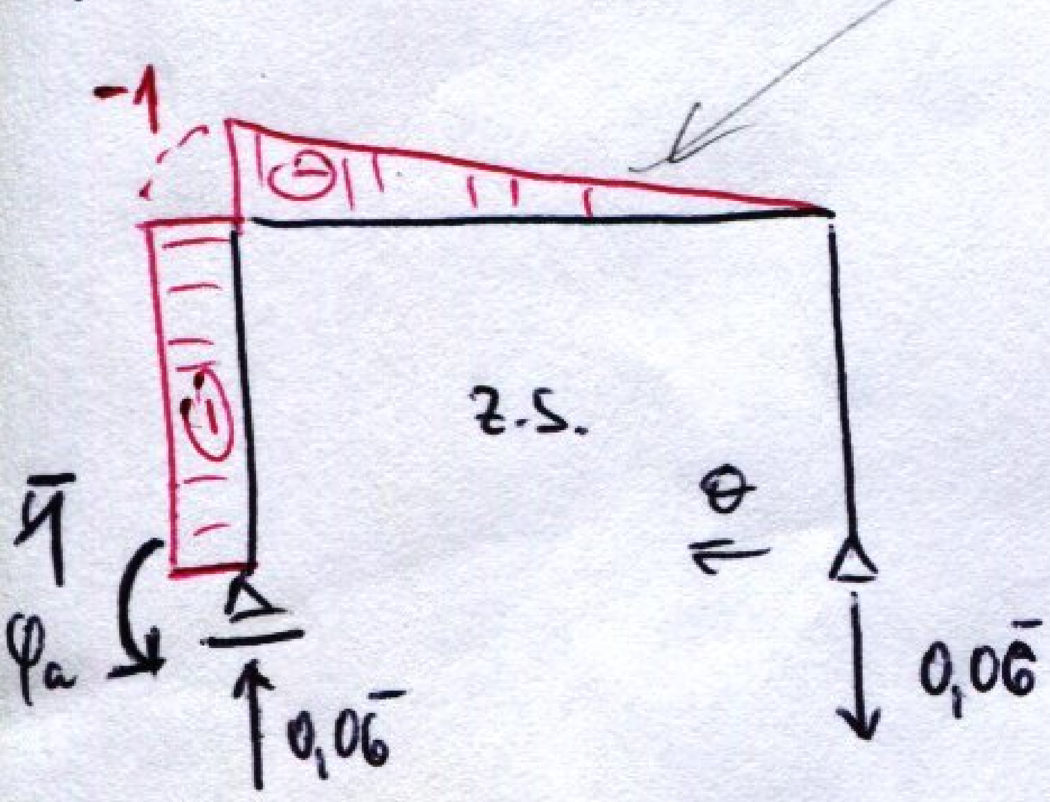


$$1\cdot w_d = \frac{1}{EI}\left[\frac{1}{6}\left(2\cdot(-36{,}57)-16{,}512\right)(-10)\cdot 15 + \frac{1}{3}\cdot 10\cdot 36{,}57\cdot 10\right] = \frac{1}{EI}\left[2241{,}3 + 1219\right] = \frac{\cancel{3460{,}3}\ 648}{EI}$$

$\frac{1}{3}\cdot 56{,}25\cdot(-10)\cdot 15$

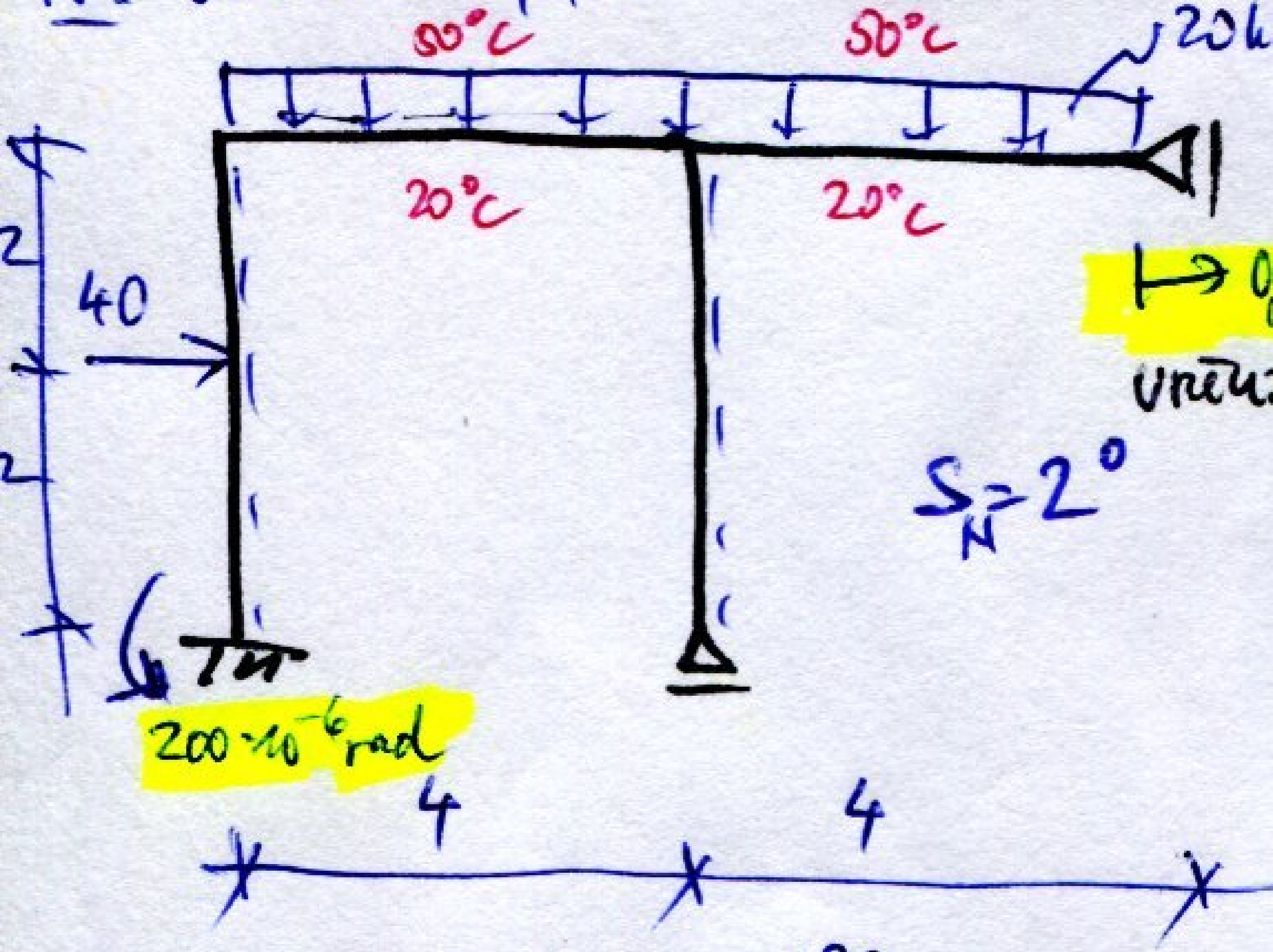
$-2812{,}5$

• $\varphi_a = 0$ – REDUKČNÍ VĚTA



$$1\cdot\varphi_a = \frac{1}{EI}\left[\frac{1}{2}(-29{,}942+1{,}773)\cdot(-1)\cdot 5 + \frac{1}{2}(1{,}773-16{,}512)\cdot(-1)\cdot 5 + \frac{1}{6}\left(2\cdot(-16{,}512)-36{,}57\right)\cdot(-1)\cdot 15 + \frac{1}{3}\cdot(+56{,}25)\cdot(-1)\cdot 15\right] = \frac{1}{EI}\left[70{,}4225 + 36{,}8475 + 173{,}985 - 281{,}25\right] = \frac{0{,}005}{EI} \doteq 0$$

Př. URČETE N, V, M



$E = 25\,GPa$ $\quad EI = 40\,000\,kNm^2$

$I_y = 1{,}6\cdot10^{-3}\,m^4$

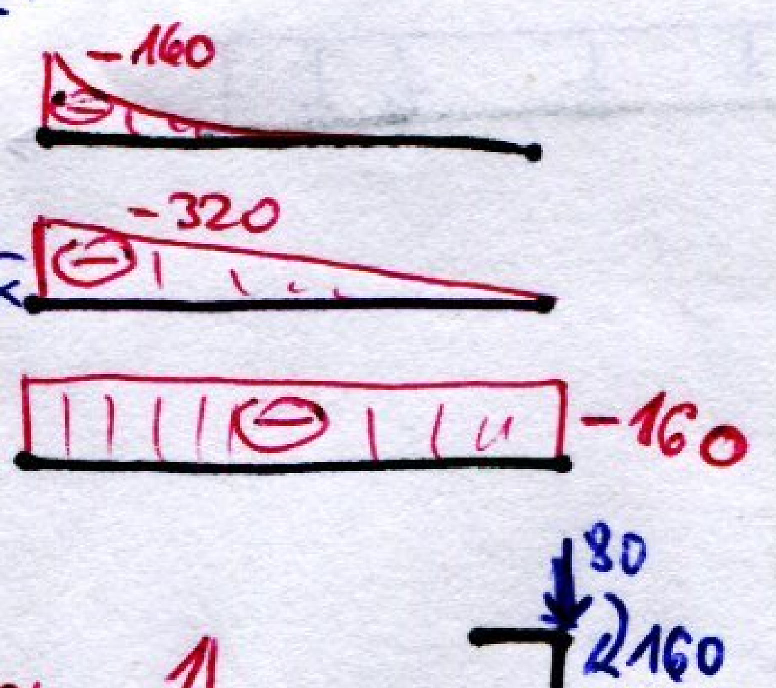
$A = 0{,}12\,m^2$

$h = 0{,}4\,m$

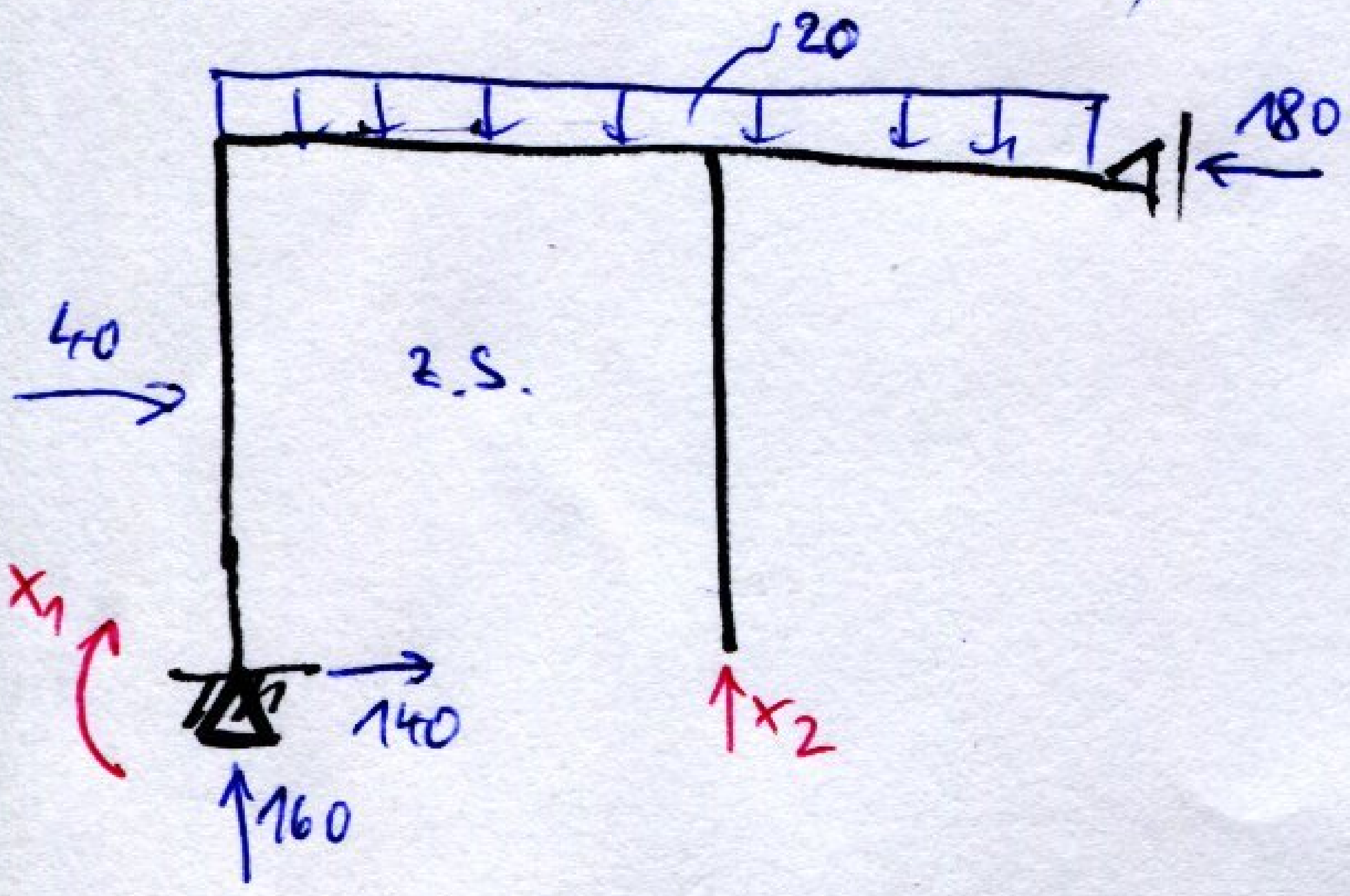
$\alpha = 12\cdot10^{-6}\,\frac{1}{K}$

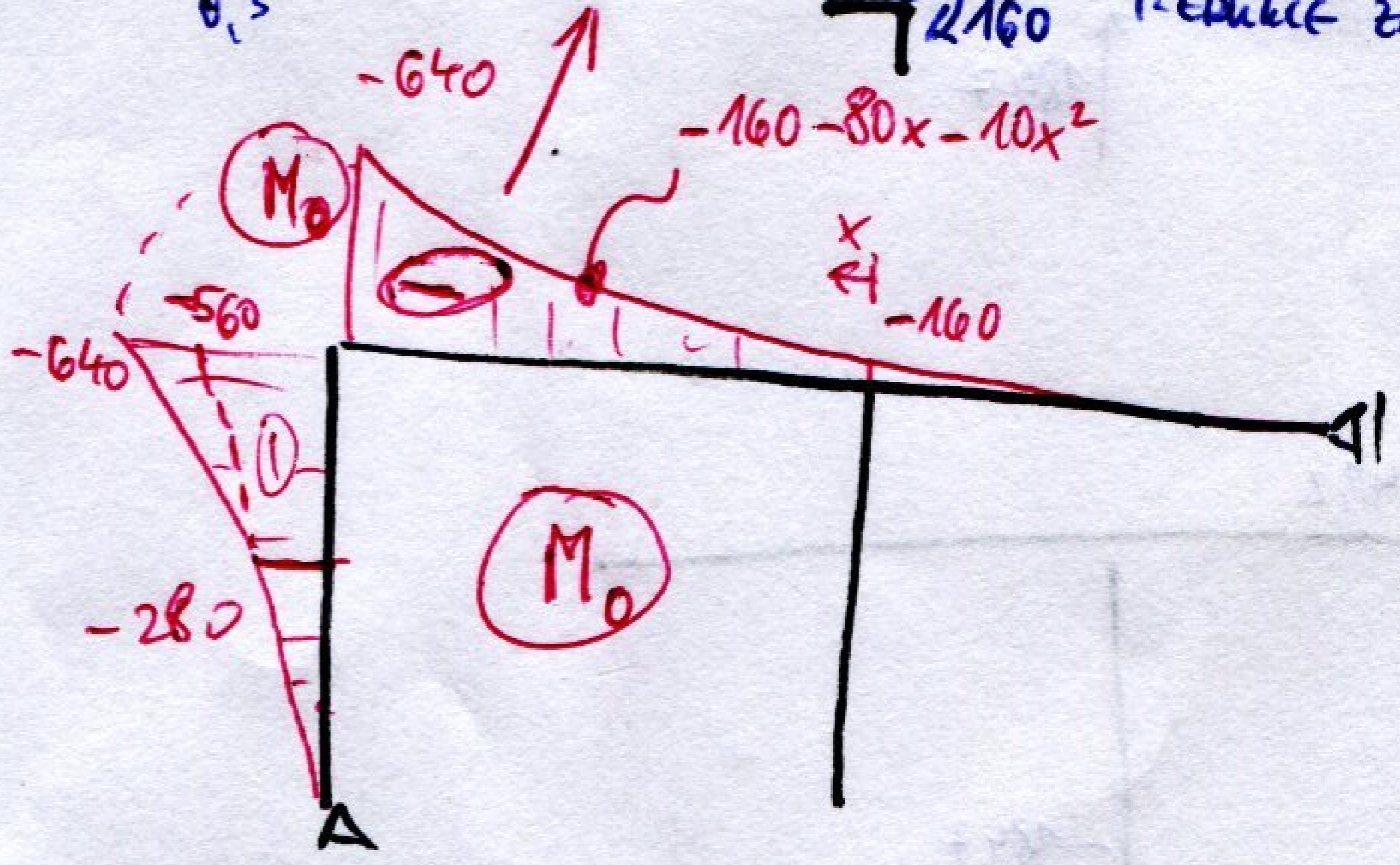
$t_{ref} = 15°C$

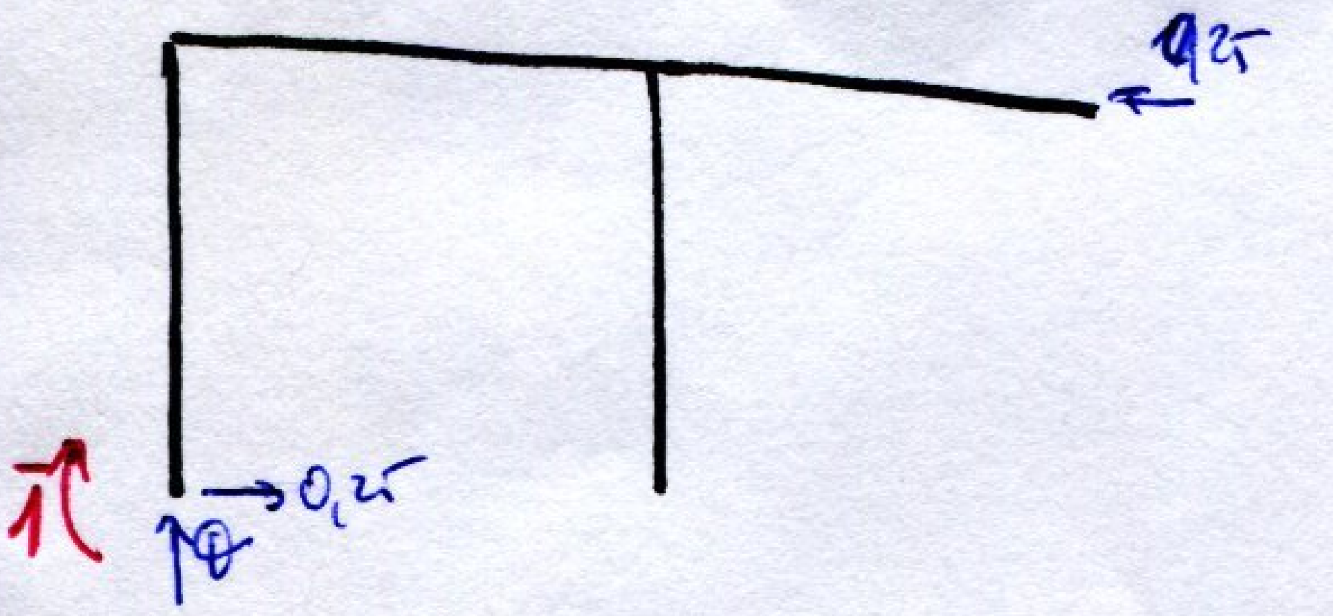
0,4 / 0,3

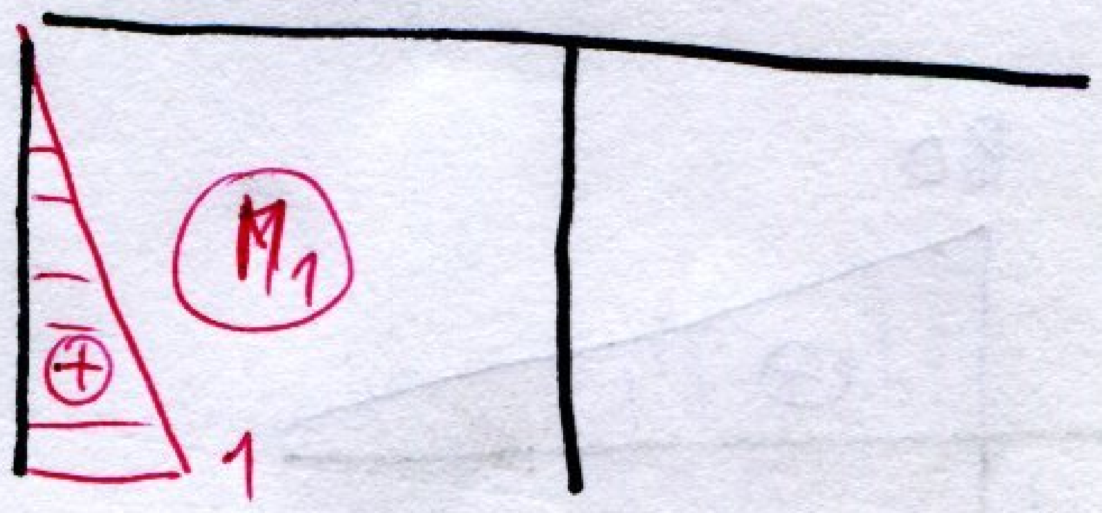


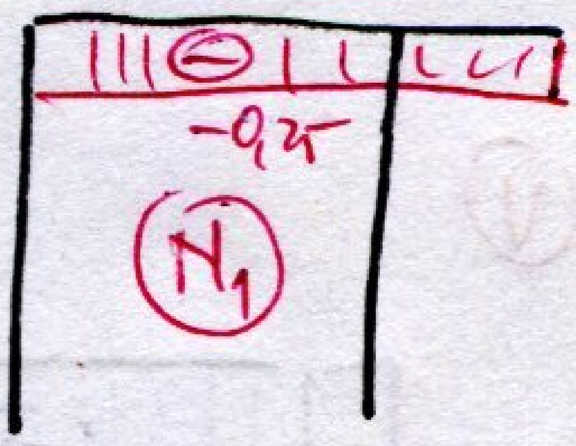
REDUKCE ZATÍŽENÍ K BODU

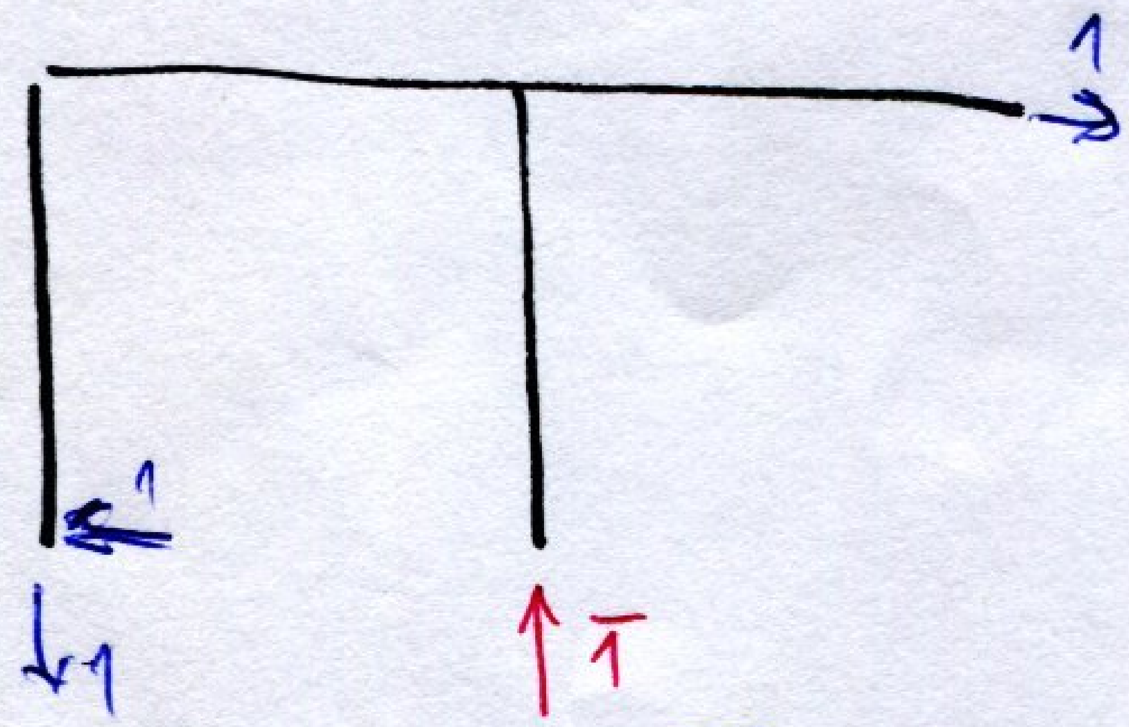


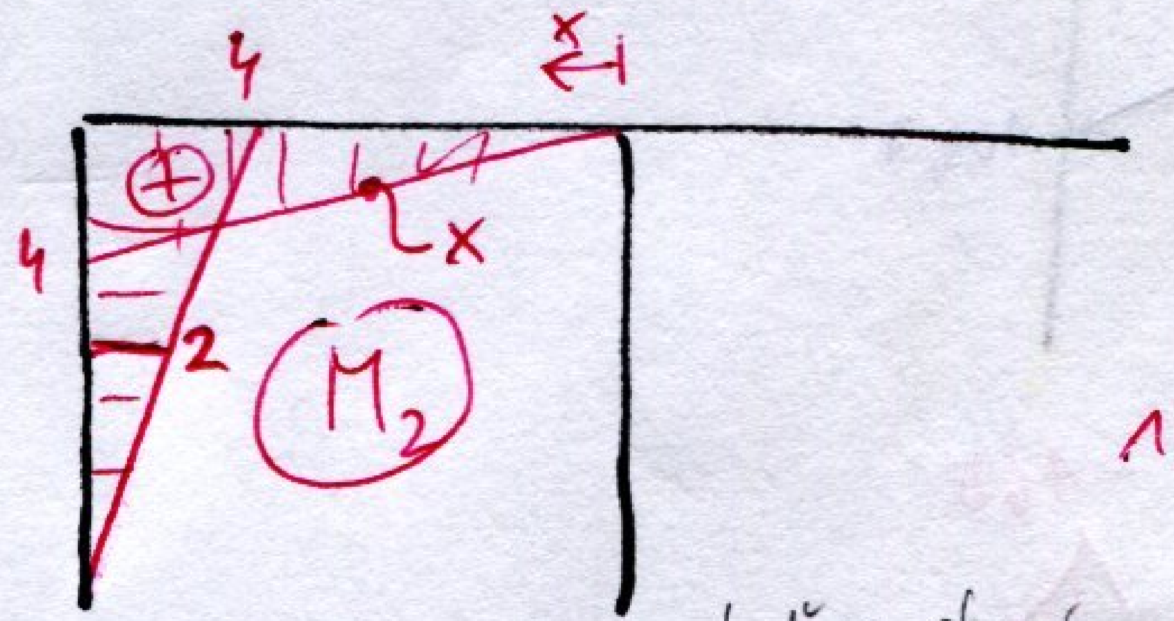


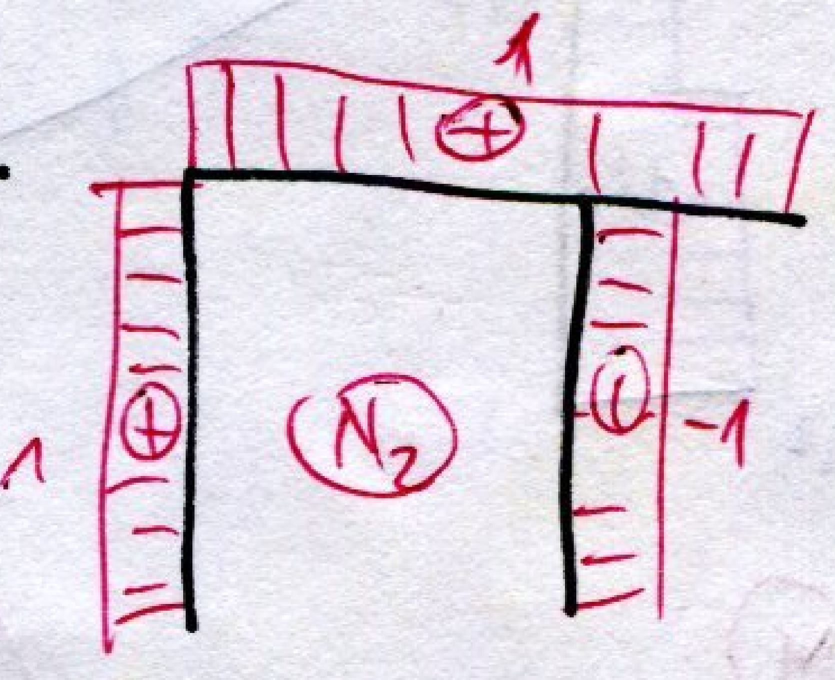












buď se objeví na pravé straně, či zde

$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{3}\cdot1^2\cdot4\right] = \frac{1{,}\bar{3}}{EI} = 33{,}\bar{3}\cdot10^{-6} \quad \delta_{12} = \frac{1}{EI}\left[\frac{1}{6}\cdot4\cdot1\cdot4\right] = \frac{2{,}\bar{6}}{EI} = 66{,}\bar{6}\cdot10^{-6} \quad \delta_{22} = \frac{1}{EI}\left[\frac{1}{3}\cdot4^3\times2\right] = \frac{42{,}\bar{6}}{EI} = 1{,}0\bar{6}\cdot10^{-3}$$

$$\delta_{10} = \frac{1}{EI}\left[\frac{1}{6}\cdot(-560)\cdot1\cdot4 + \frac{1}{6}\cdot(-80)\cdot0{,}5\cdot2\right] + (\overset{35}{t} - \overset{15}{t_{ref}})\cdot\alpha\cdot\left(-\frac{1}{4}\right)\cdot8 - 0{,}01\cdot(-0{,}25) - 1\cdot(-200\cdot10^{-6}) = \frac{1}{EI}\left[-373{,}\bar{3} - 13{,}\bar{3}\right] + 20\cdot12\cdot10^{-6}\cdot(-2) +$$

$$+ 0{,}0025 = -9{,}\bar{6}\cdot10^{-3} - 0{,}48\cdot10^{-3} + 2{,}5\cdot10^{-3} = -7{,}44\bar{6}\cdot10^{-3} \quad -7{,}4466\cdot10^{-3}$$

KONEC 6.CV

$$\delta_{20} = \frac{1}{EI}\left[\frac{1}{3}\cdot(-560)\cdot4\cdot4 + \frac{1}{2}\cdot(-80)\cdot3{,}\bar{3}\cdot4 + \int_0^4 x\left(-160 - 80x - 10x^2\right)dx\right] + 20\cdot\alpha\cdot1\cdot8 + \frac{20-50}{0{,}4}\cdot\alpha\cdot\frac{1}{2}\cdot4\cdot4 - 1\cdot0{,}01 =$$

$$= \frac{1}{EI}\left[-2986{,}\bar{6} - 266{,}\bar{6} + \left[-80x^2 - \frac{80}{3}x^3 - 2{,}5x^4\right]_0^4\right] + 1{,}92\cdot10^{-3} - 7{,}2\cdot10^{-3} - 10\cdot10^{-3} = \frac{1}{EI}\left[-3253{,}\bar{3} - 3626{,}\bar{6}\right] -$$

$$- 15{,}28\cdot10^{-3} = -0{,}1873$$

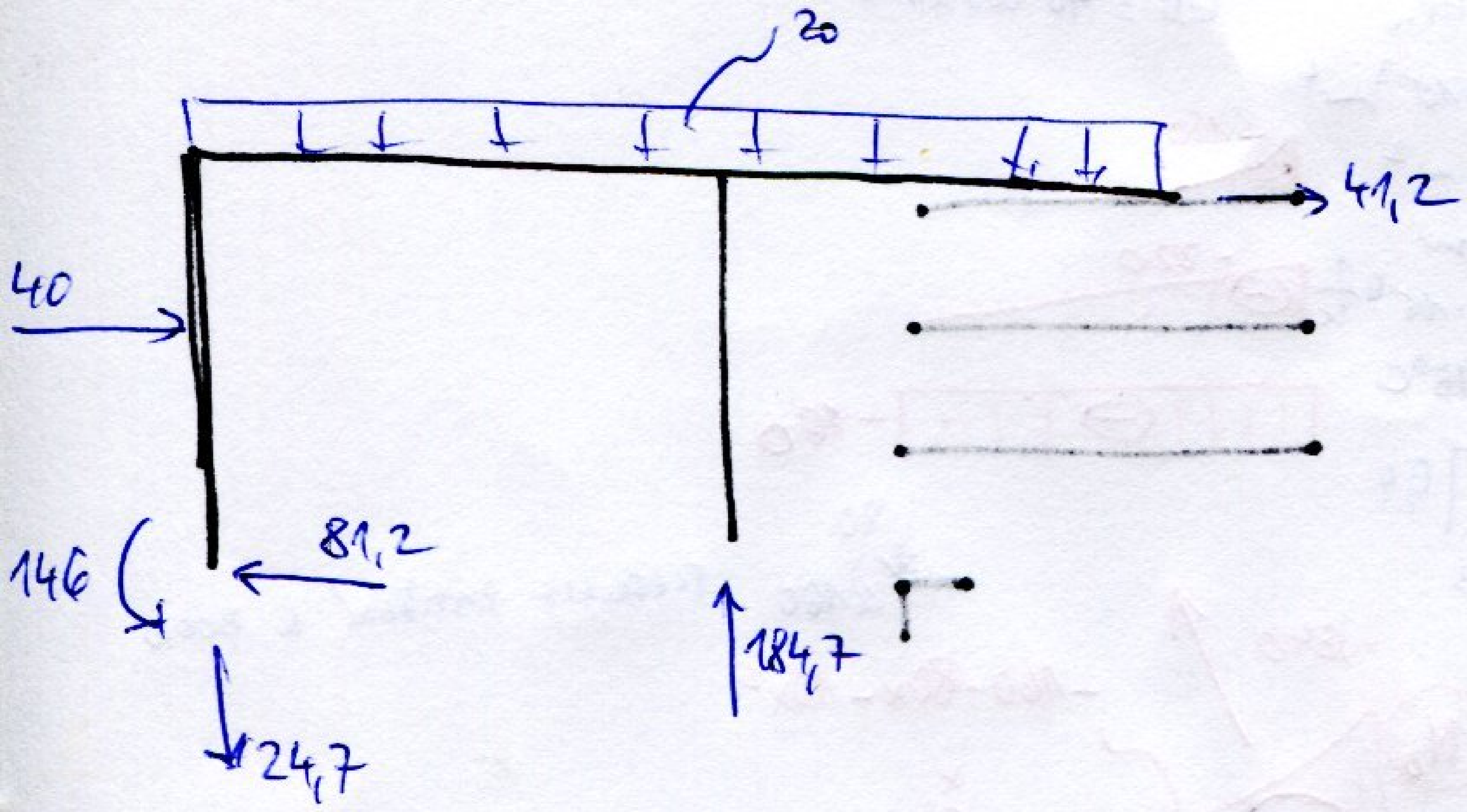
$$33{,}\bar{3}\cdot10^{-6}X_1 + 66{,}\bar{6}\cdot10^{-6}X_2 - 7{,}44\bar{6}\cdot10^{-3} = 0$$

$$66{,}\bar{6}\cdot10^{-6}X_1 + 1{,}0\bar{6}\cdot10^{-3}X_2 - 0{,}1873 = 0$$

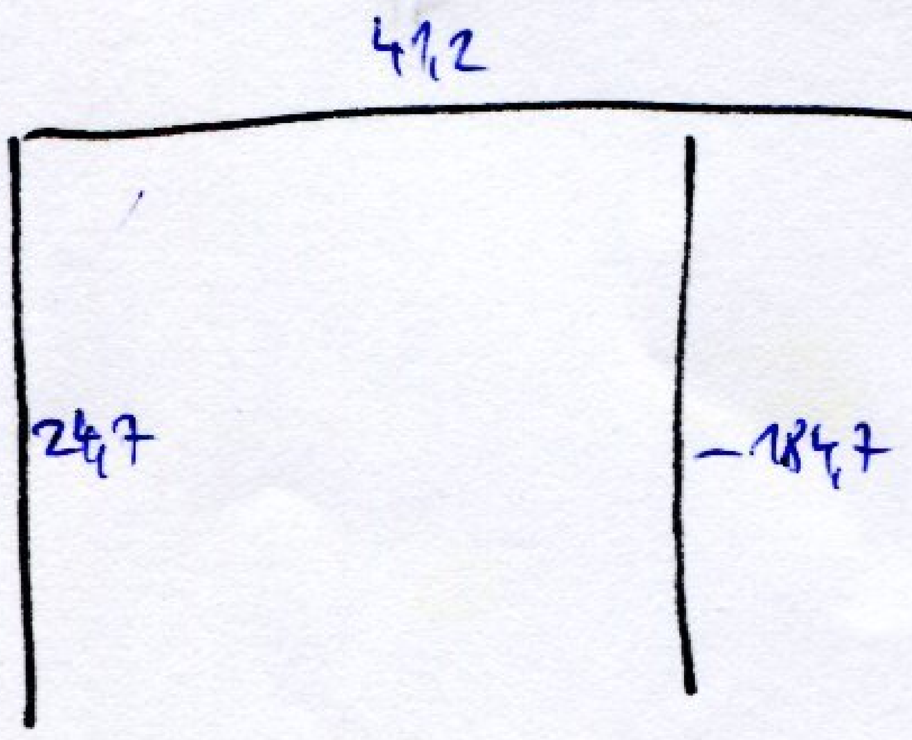
$$10^{-6}\begin{pmatrix}33{,}\bar{3} & 66{,}\bar{6}\\ 66{,}\bar{6} & 1066{,}\bar{6}\end{pmatrix}\begin{pmatrix}X_1\\X_2\end{pmatrix} = 10^{-6}\begin{pmatrix}7446{,}\bar{6}\\187300\end{pmatrix} \quad /\cdot(-2) \quad \left(\begin{array}{cc|c}0 & 933{,}\bar{3} & 172\,407\end{array}\right)$$

$X_1 = -146\ kN$

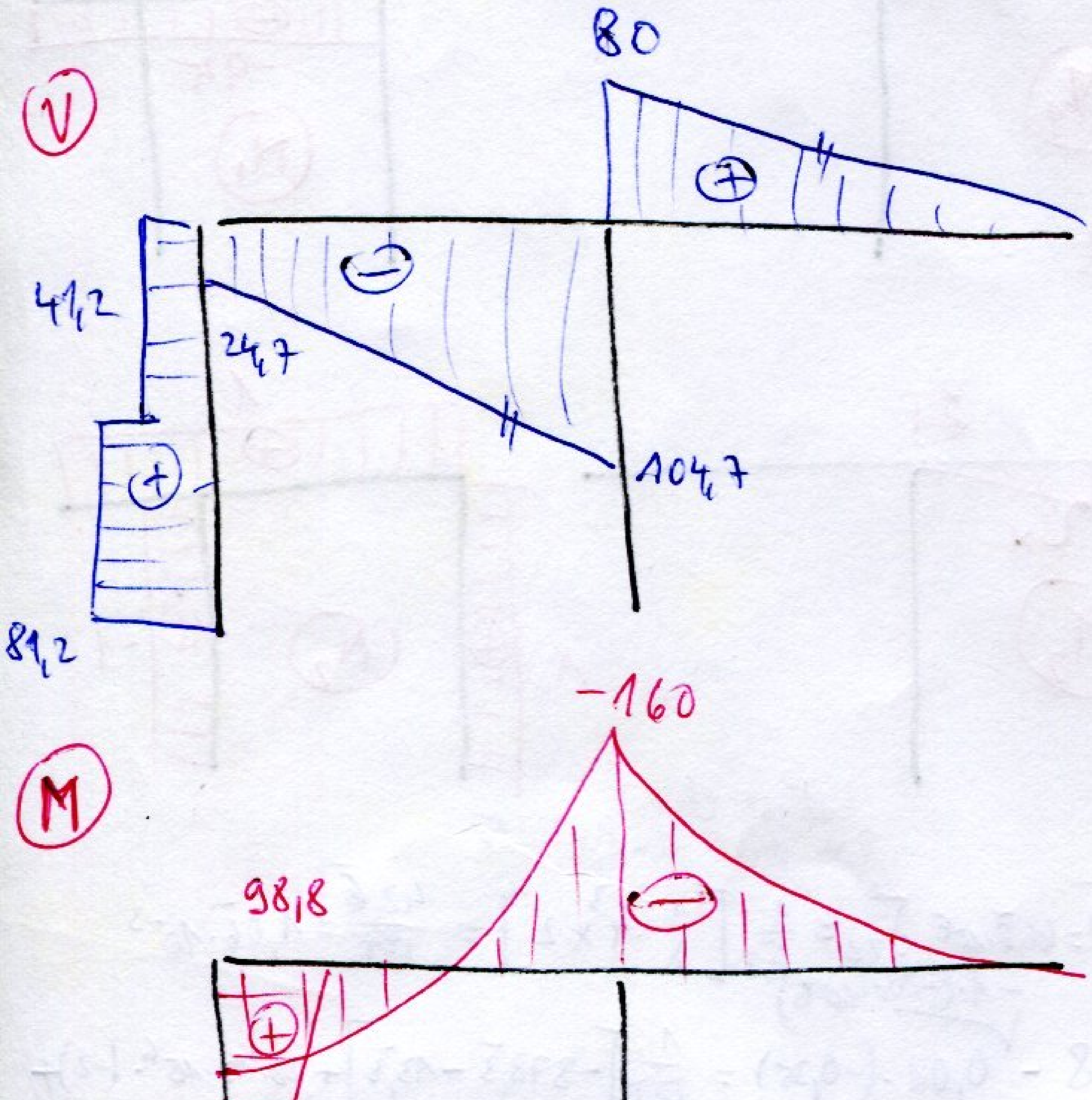
$\rightarrow X_2 = 184{,}7\ kN$


20
41,2
40
146
81,2
184,7
24,7

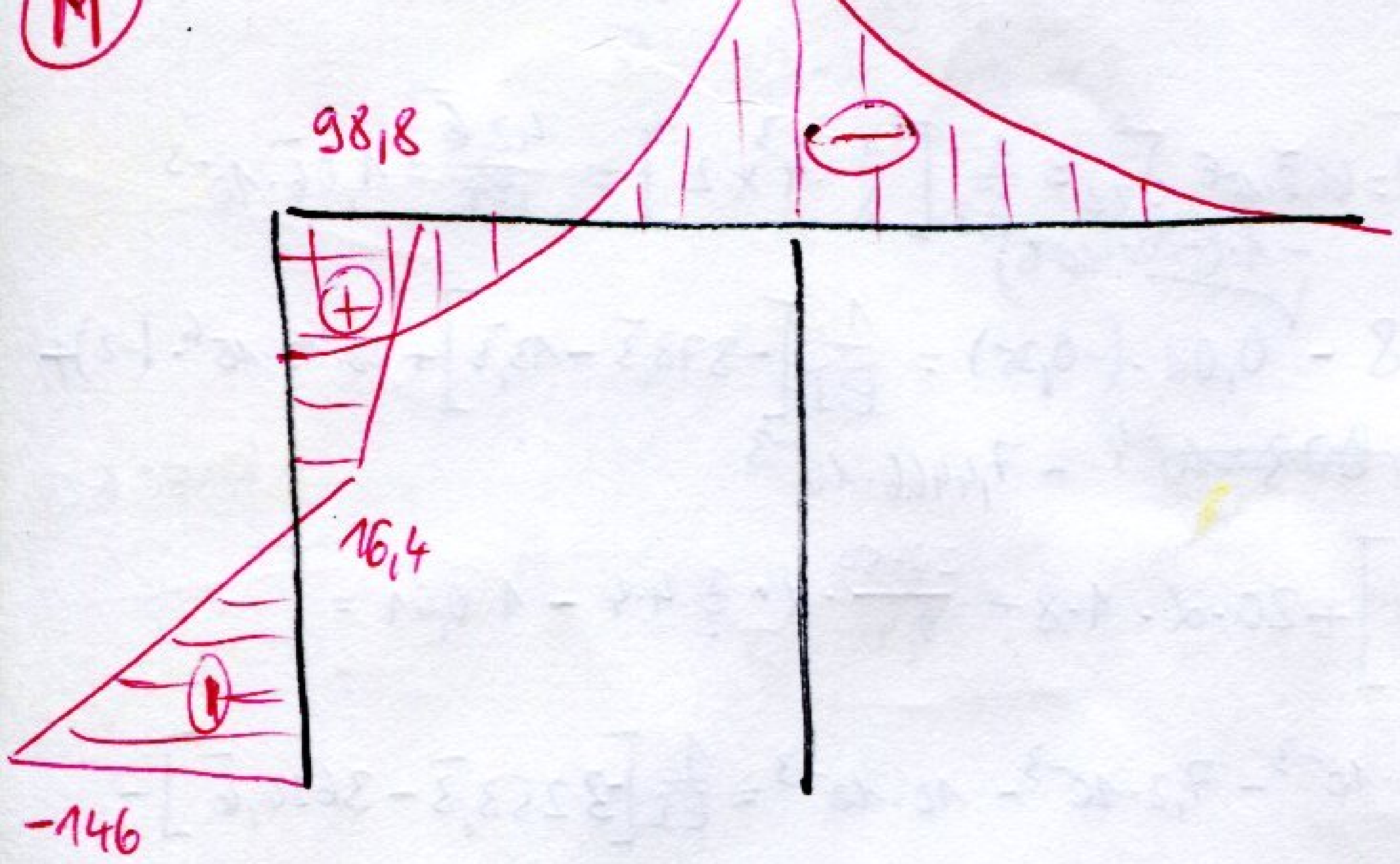
N


41,2
24,7
-184,7

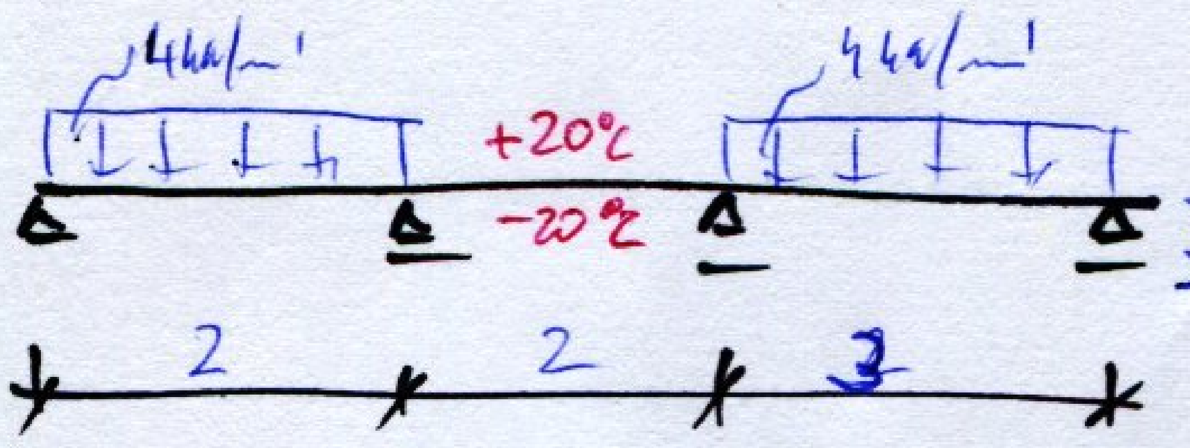
V


80
41,2
24,7
104,7
81,2

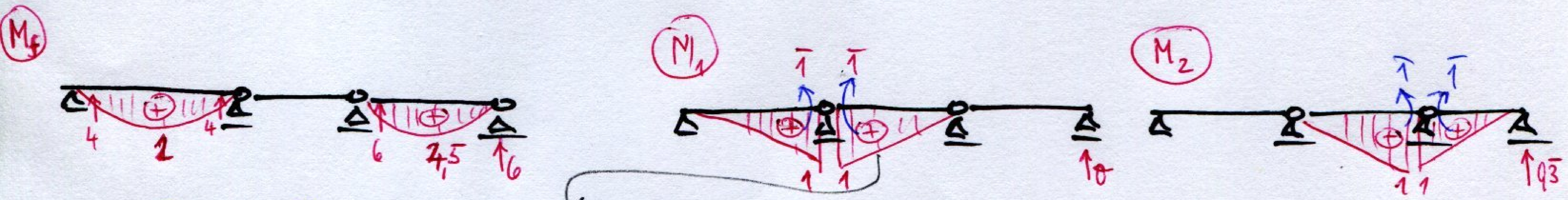
M


-160
98,8
16,4
-146

Př. Řešte silovou metodou spojitý nosník   Příklad do zápočtového testu



EI = 10 000 kNm^2
h = 0,4 m
$\alpha = 12\cdot 10^{-6}$ 1/K



$$\delta_{1f} = \frac{1}{EI}\left[\frac{1}{3}\cdot 2\cdot 1\cdot 2\right] + \frac{-20-20}{0,4}\alpha\left[\frac{1}{2}\cdot 1\cdot 2\right] = \frac{1}{10\,000}\; 0,1\bar{3}\cdot 10^{-3} - 1,2\cdot 10^{-3} = -1,0\bar{6}\cdot 10^{-3}$$

$$\delta_{2f} = \frac{1}{EI}\left[\frac{1}{3}\cdot 4,5\cdot 1\cdot 3\right] + \frac{-20-20}{0,4}\cdot\alpha\left[\frac{1}{2}\cdot 1\cdot 2\right] - (-0,\bar{3}\cdot 0,1) = 0,45\cdot 10^{-3} - 1,2\cdot 10^{-3} + 33,\bar{3}\cdot 10^{-3} = 32,58\bar{3}\cdot 10^{-3}$$

$$\delta_{11} = \frac{1}{EI}\left[2\times\frac{1}{3}\cdot 1^2\cdot 2\right] = 0,1\bar{3}\cdot 10^{-3}$$

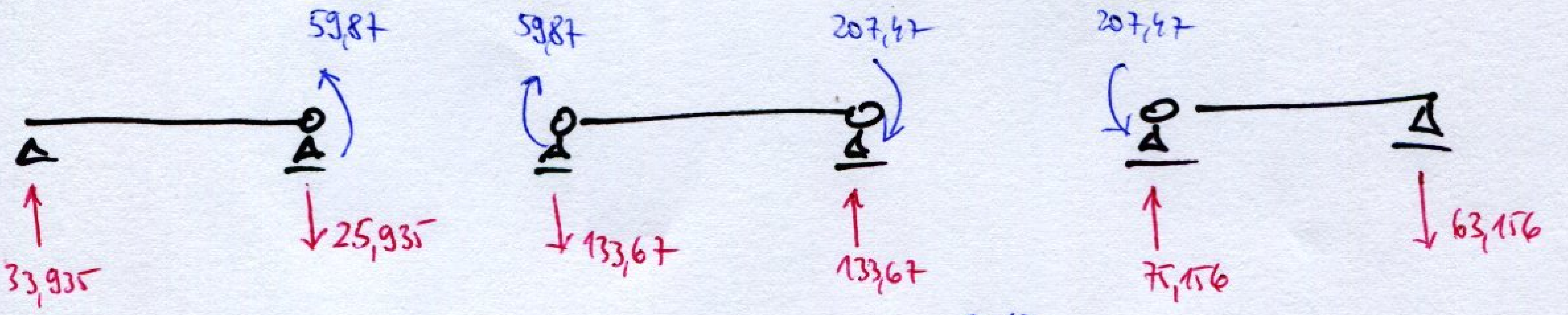
$$\delta_{12} = \frac{1}{EI}\left[\frac{1}{6}\cdot 1^2\cdot 2\right] = 0,0\bar{3}\cdot 10^{-3}$$

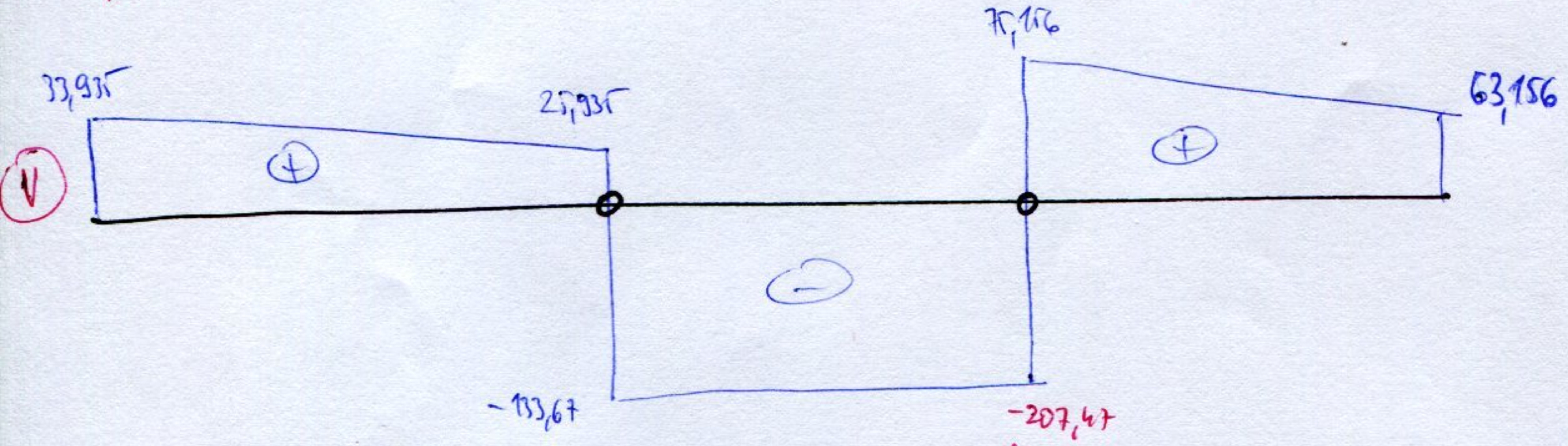
$$\delta_{22} = \frac{1}{EI}\left[\frac{1}{3}\cdot 1^2\cdot 2 + \frac{1}{3}\cdot 1^2\cdot 3\right] = 0,1\bar{6}\cdot 10^{-3}$$

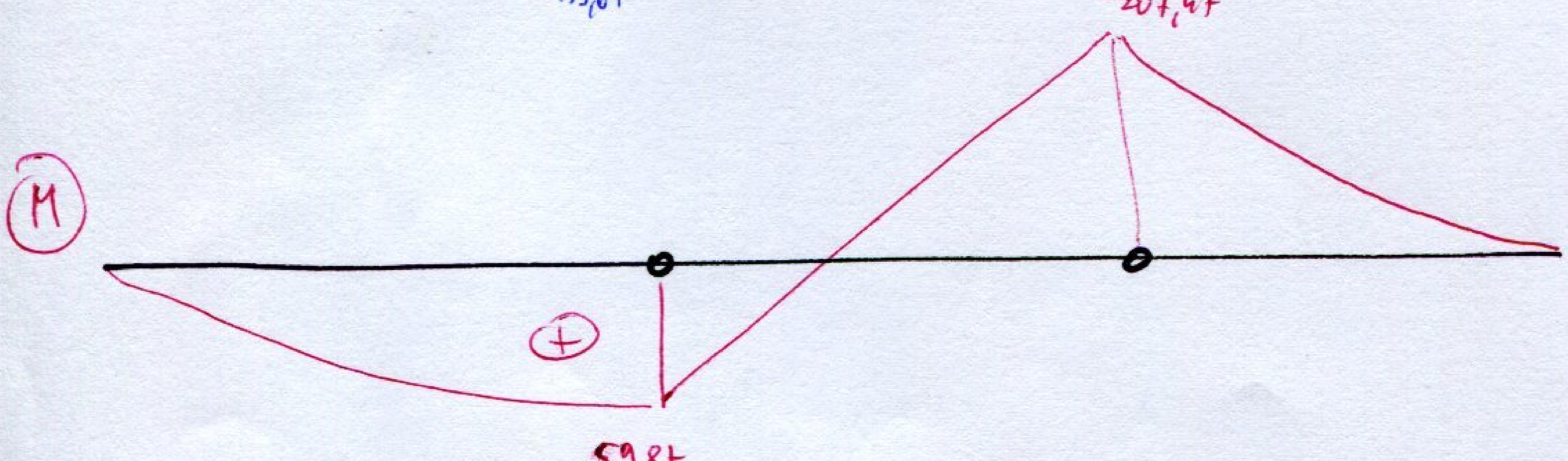
$$\begin{pmatrix} 0,1\bar{3} & 0,0\bar{3} \\ 0,0\bar{3} & 0,1\bar{6} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} +1,0\bar{6} \\ -32,58\bar{3} \end{pmatrix}$$

$X_1 = 59,87$

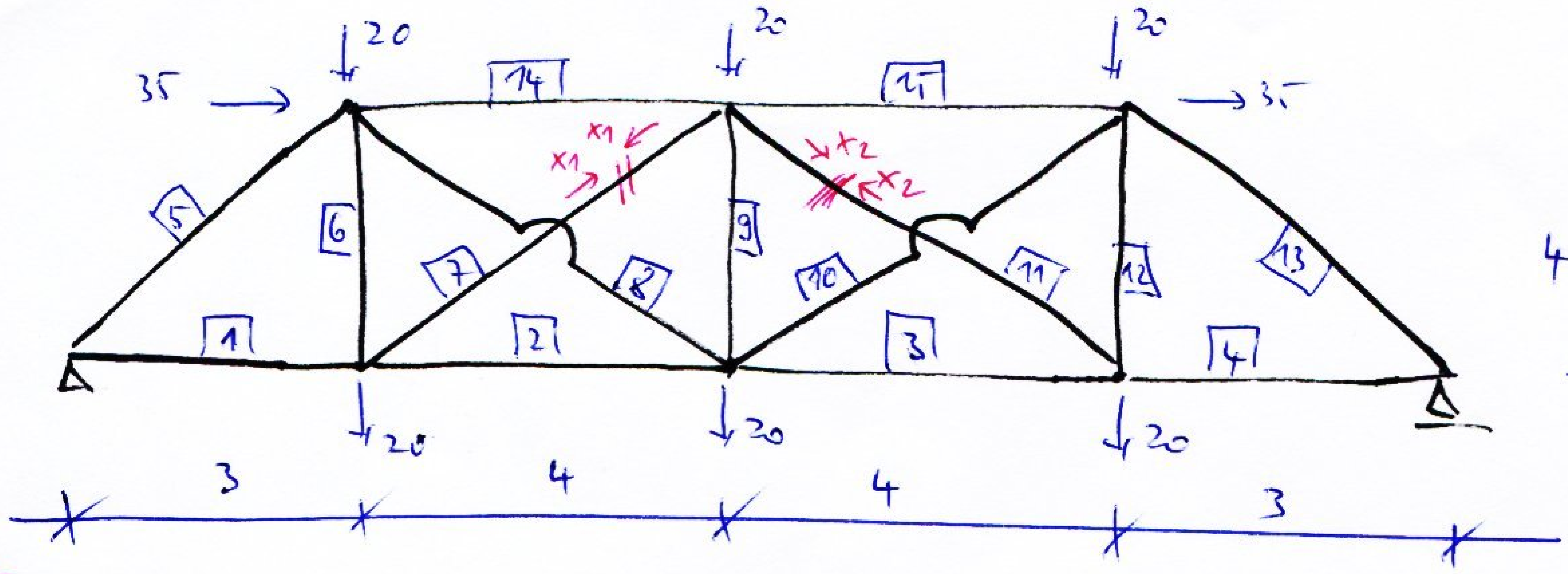
$X_2 = -207,47$







Př. URČETE N



$E = 210\ GPa$

$A_1 = A_{1,2,3,4,14,15} = 0{,}005\ m^2$

$A_5 = A_{ostatní\ 5-13} = 0{,}002\ m^2$

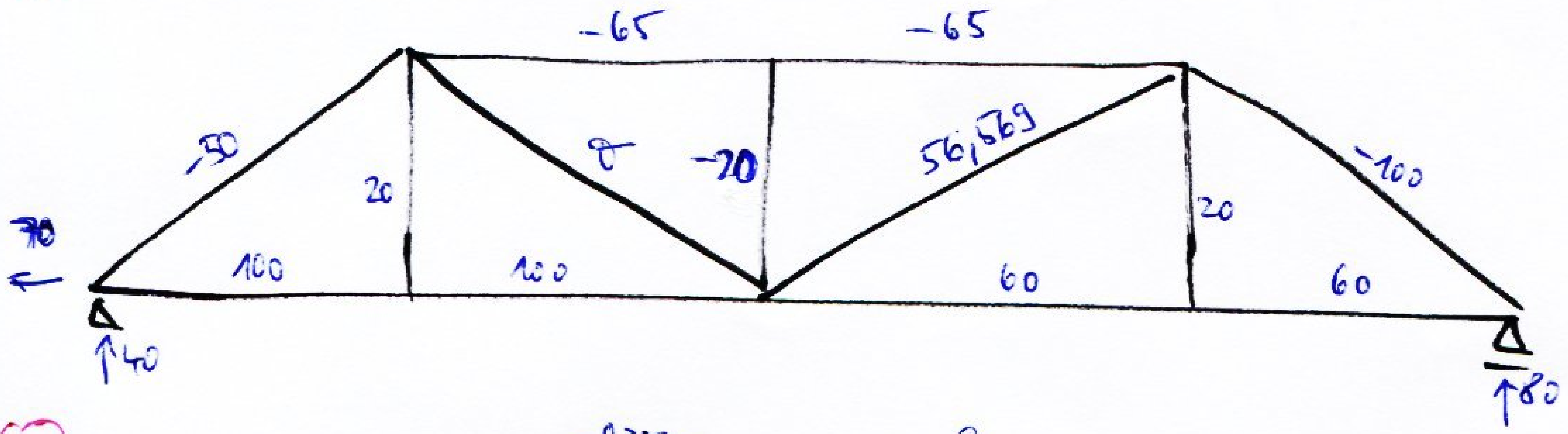
4

$S = 8 \cdot 2 - 3 - 15 = -2^0$

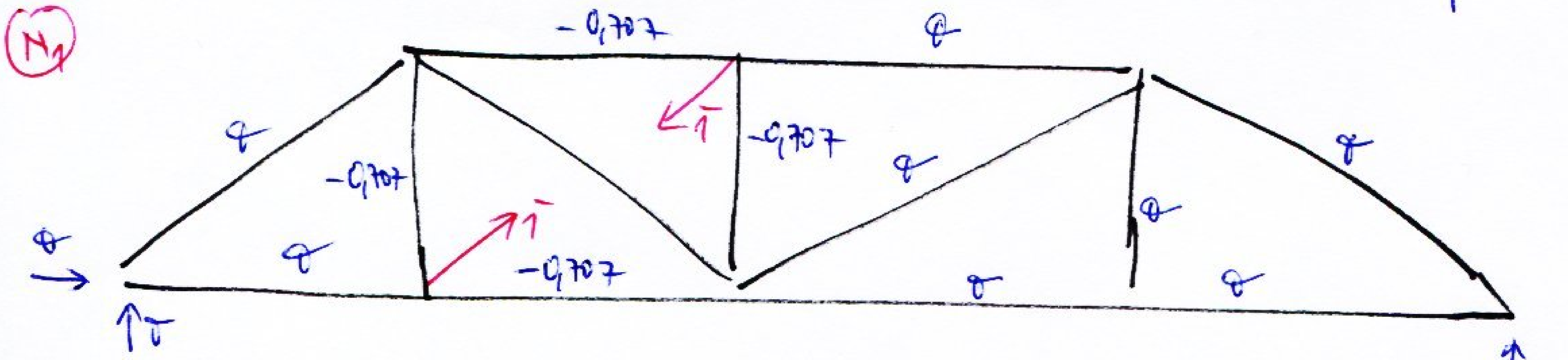
$EA_1 = 1{,}05 \cdot 10^6\ kN$

$EA_5 = 0{,}42 \cdot 10^6\ kN$

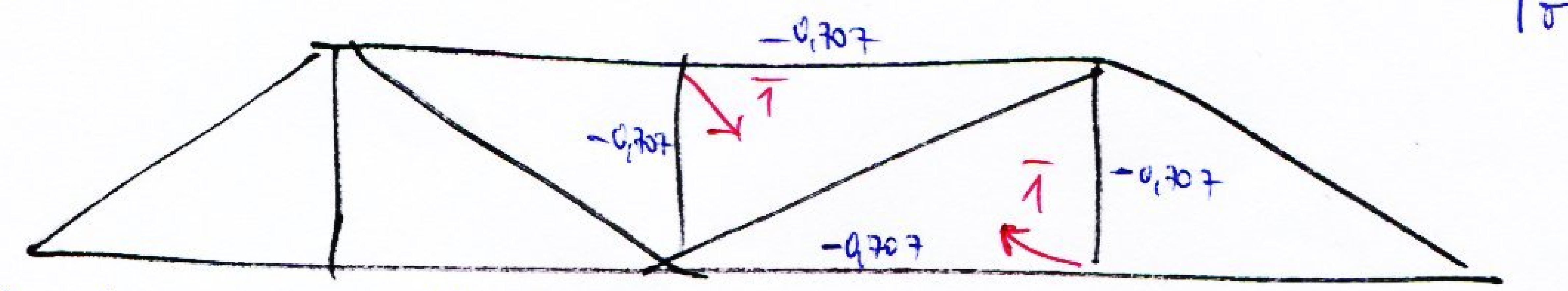
N_0



N_1



N_2



$\delta_{11} = \frac{1}{EA_1}\left(0{,}707^2 \cdot 4 \times 2\right) + \frac{1}{EA_5}\left(0{,}707^2 \cdot 4 \times 2 + 2 \times 1^2 \cdot \sqrt{32}\right) = \mathbf{40{,}27} \cdot 10^{-6}\ kNm$ STEJNÉ $\delta_{22} = 40{,}27 \cdot 10^{-6}\ kNm$

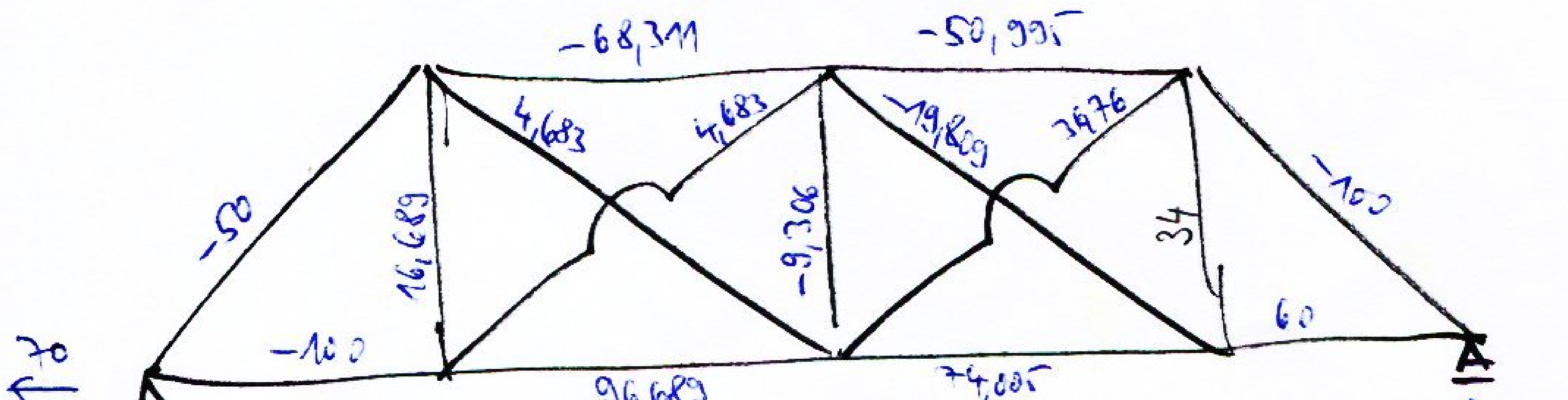
$\delta_{12} = \frac{1}{EA_5}\left(0{,}707^2 \cdot 4\right) = 4{,}761 \cdot 10^{-6}\ kNm$

$\delta_{10} = \frac{1}{EA_1}\left(65 \cdot 0{,}707 \cdot 4 - 0{,}707 \cdot 100 \cdot 4\right) + \frac{1}{EA_5}\left(-20 \cdot 0{,}707 \cdot 4 + 0{,}707 \cdot 20 \cdot 4\right) = -94{,}27 \cdot 10^{-6} + 0\ kNm$

$\delta_{20} = \frac{1}{EA_1}\left(65 \cdot 0{,}707 \cdot 4 - 60 \cdot 0{,}707 \cdot 4\right) + \frac{1}{EA_5}\left(20 \cdot 0{,}707 \cdot 4 + 1 \cdot 56{,}569 \cdot \sqrt{32} - 20 \cdot 0{,}707 \cdot 4\right) = \left[13{,}46 + 761{,}9\right] \cdot 10^{-6} = 775{,}4 \cdot 10^{-6}\ kNm$

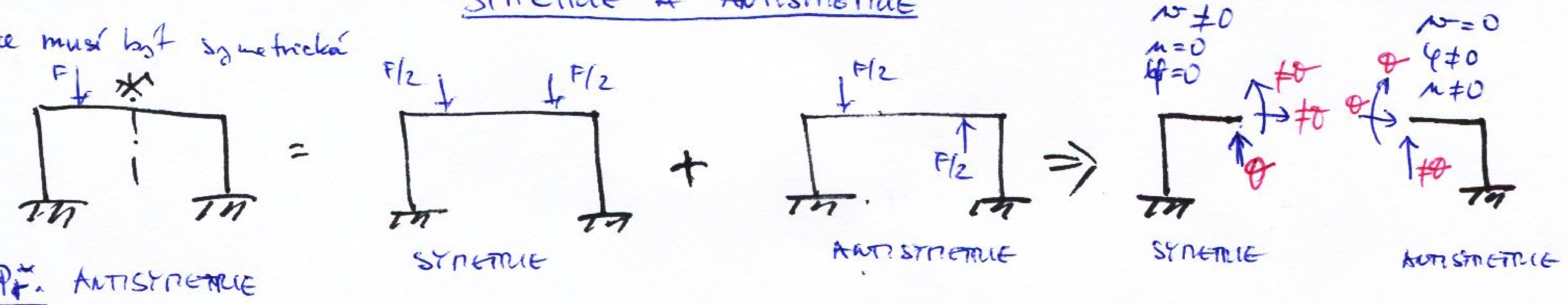
$\begin{pmatrix} 40{,}27 & 4{,}761 \\ 4{,}761 & 40{,}27 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 94{,}27 \\ -775{,}4 \end{pmatrix}$ $\quad \frac{1}{1600} \begin{pmatrix} 40{,}27 & -4{,}761 \\ -4{,}761 & 40{,}27 \end{pmatrix} \begin{pmatrix} 94{,}27 \\ -775{,}4 \end{pmatrix}$ $\quad X_1 = 4{,}683\ kN$

$X_2 = -19{,}809\ kN$

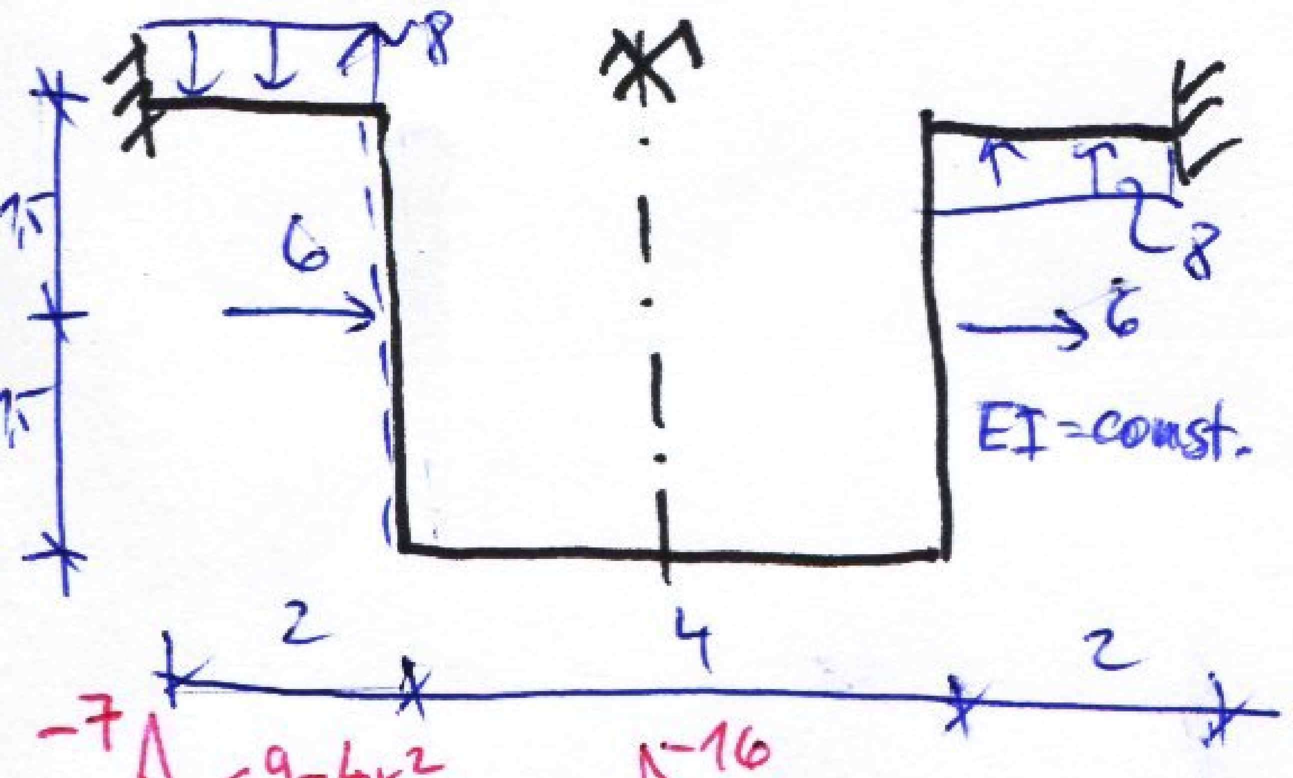


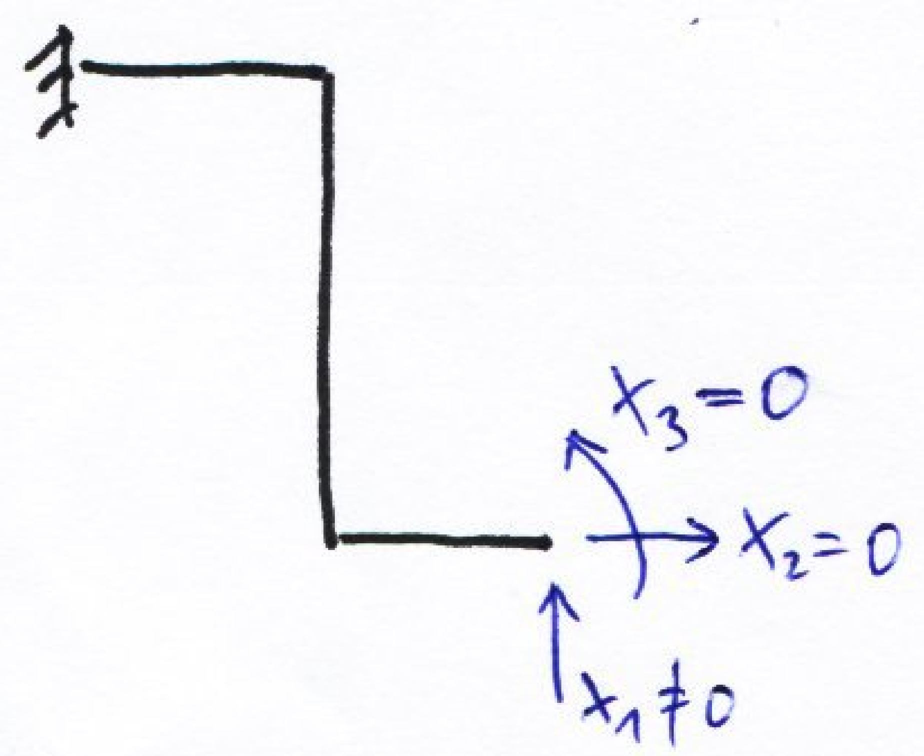
SYMETRIE A ANTISYMETRIE

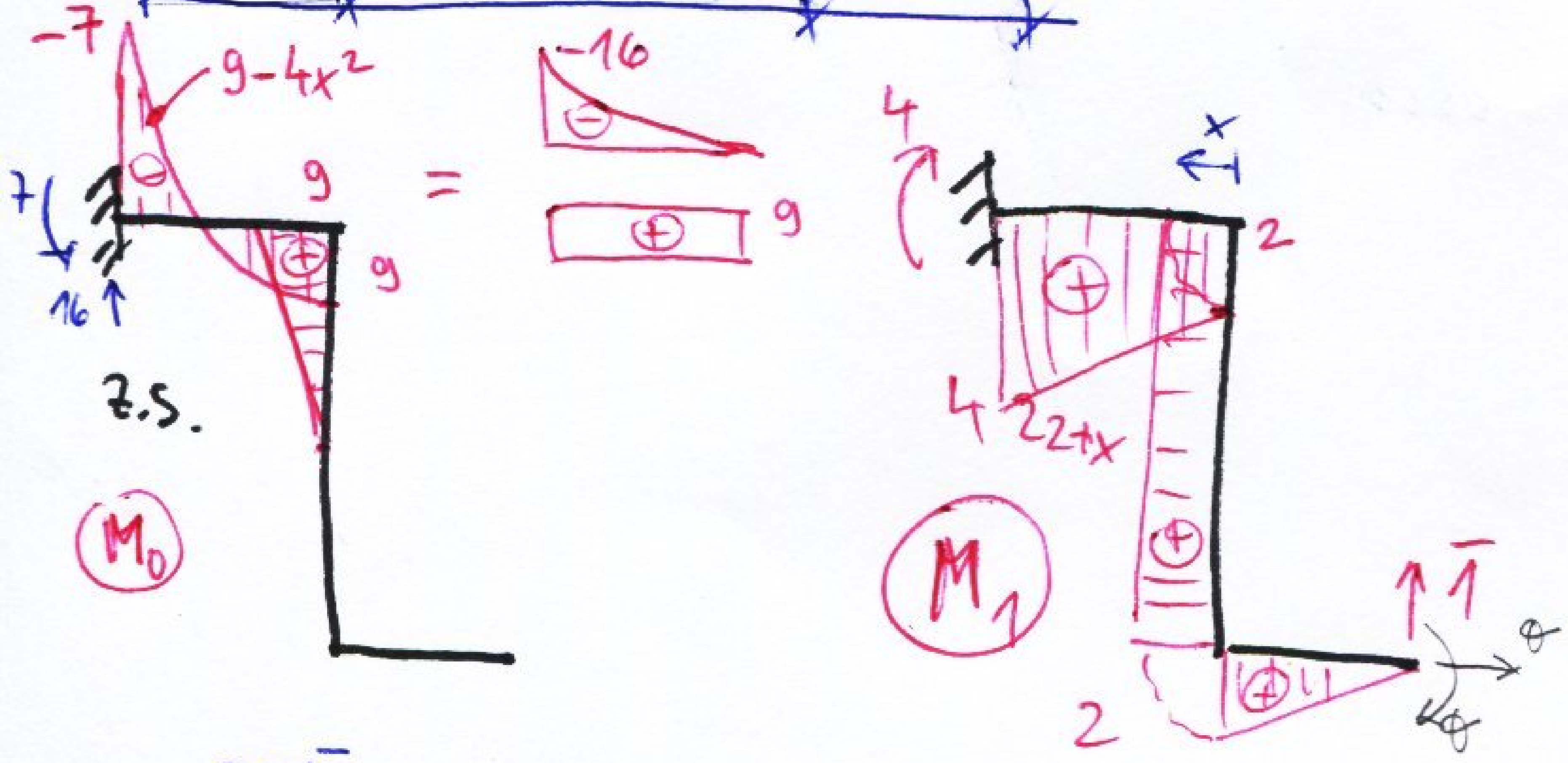
kce musí být symetrická



Př. ANTISYMETRIE





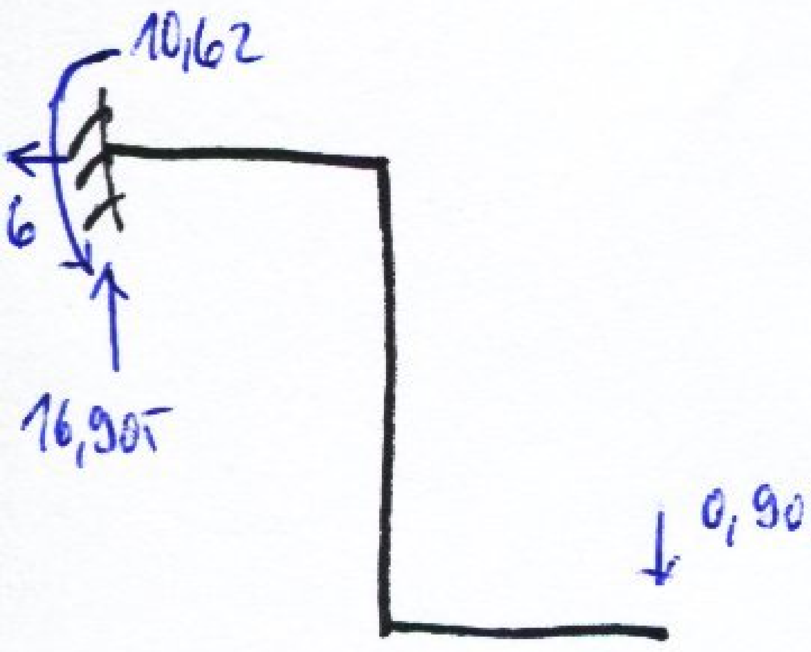


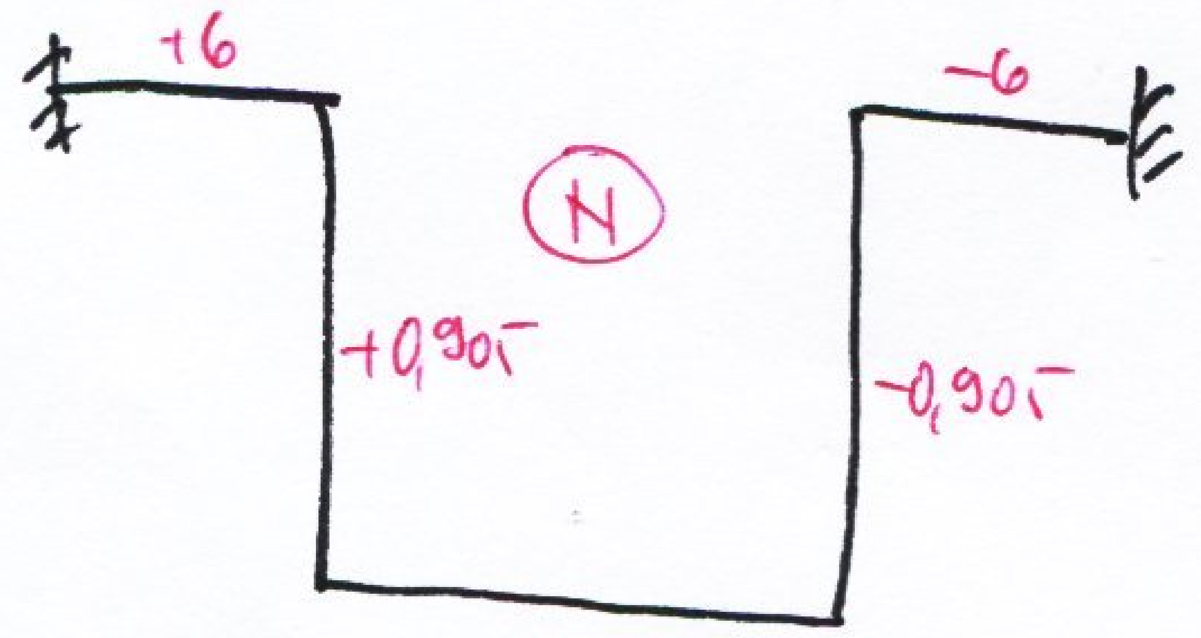
$$\delta_{11} = \frac{1}{EI}\left[\int_0^2 \underbrace{(2+x)^2}_{4+4x+x^2}dx + 2^2\cdot 3 + \frac{1}{3}\cdot 2^2\cdot 2\right] = \frac{1}{EI}\left[\left[4x+2x^2+\frac{x^3}{3}\right]_0^2 + 12 + \frac{8}{3}\right] = \frac{33{,}\bar{3}}{EI}$$

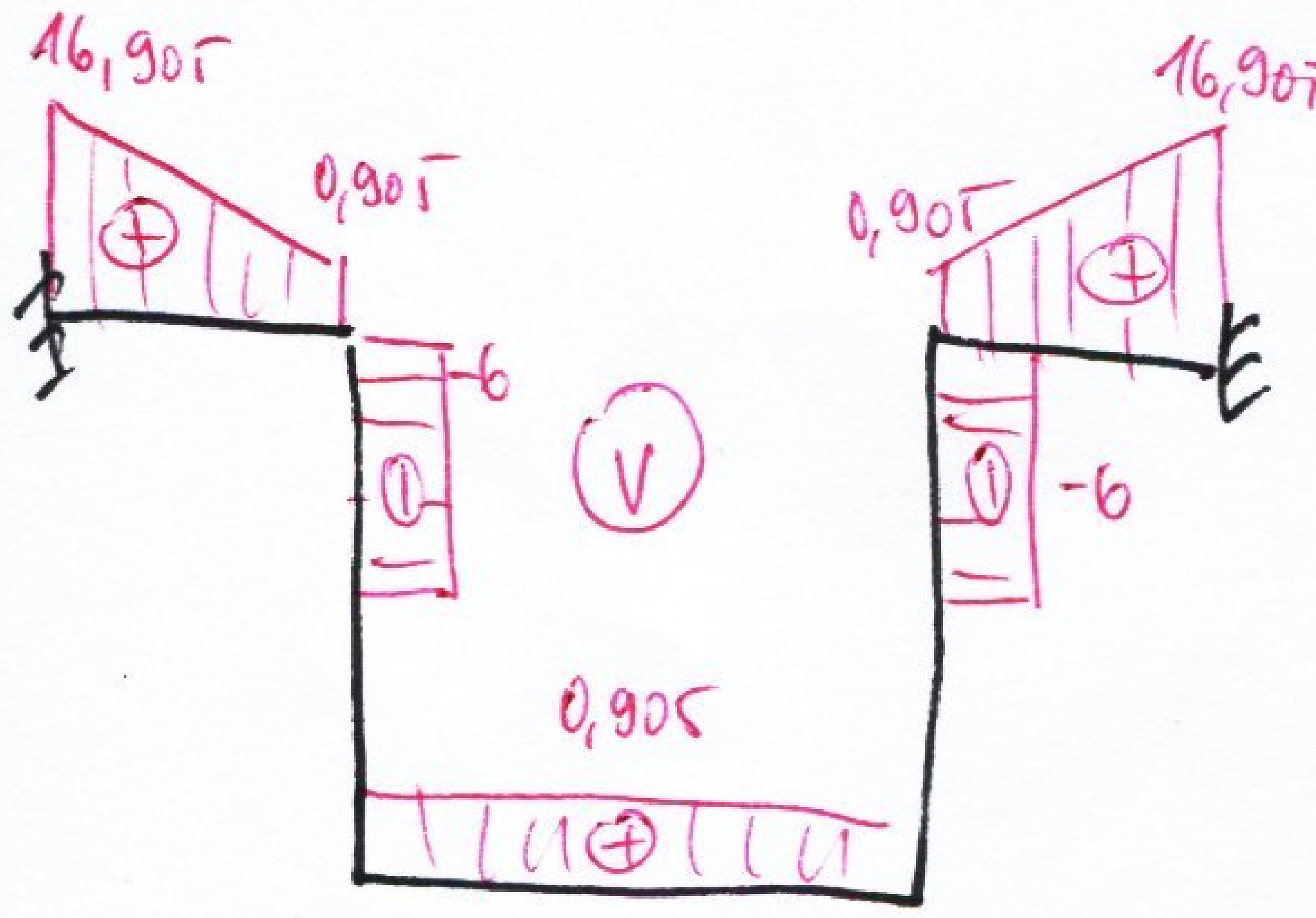
$$\delta_{10} = \frac{1}{EI}\left[\int_0^2 (9-4x^2)(2+x)dx + \frac{1}{2}\cdot 9\cdot 2\cdot 1{,}5\right] = \frac{1}{EI}\left[\int_0^2 (18+9x-8x^2-4x^3)dx + 13{,}5\right] = \frac{1}{EI}\left[\left[18x+4{,}5x^2-\frac{8}{3}x^3-x^4\right]_0^2+13{,}5\right] =$$

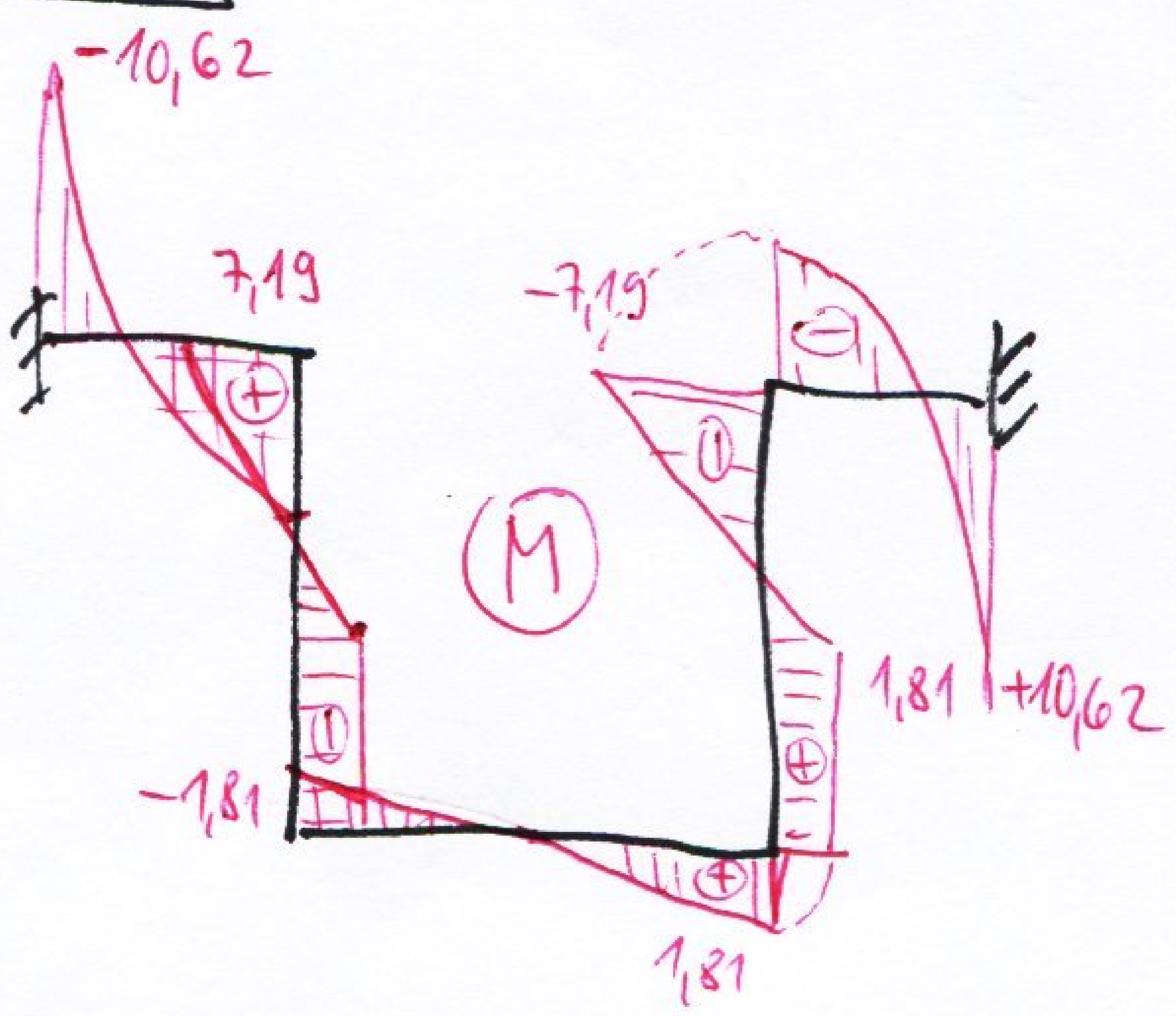
$$= \frac{1}{EI}\left[16{,}\bar{6} + 13{,}5\right] = \frac{30{,}1\bar{6}}{EI}$$

$$X_1 = -\frac{30{,}1\bar{6}}{33{,}\bar{3}} = -0{,}905\,kN$$

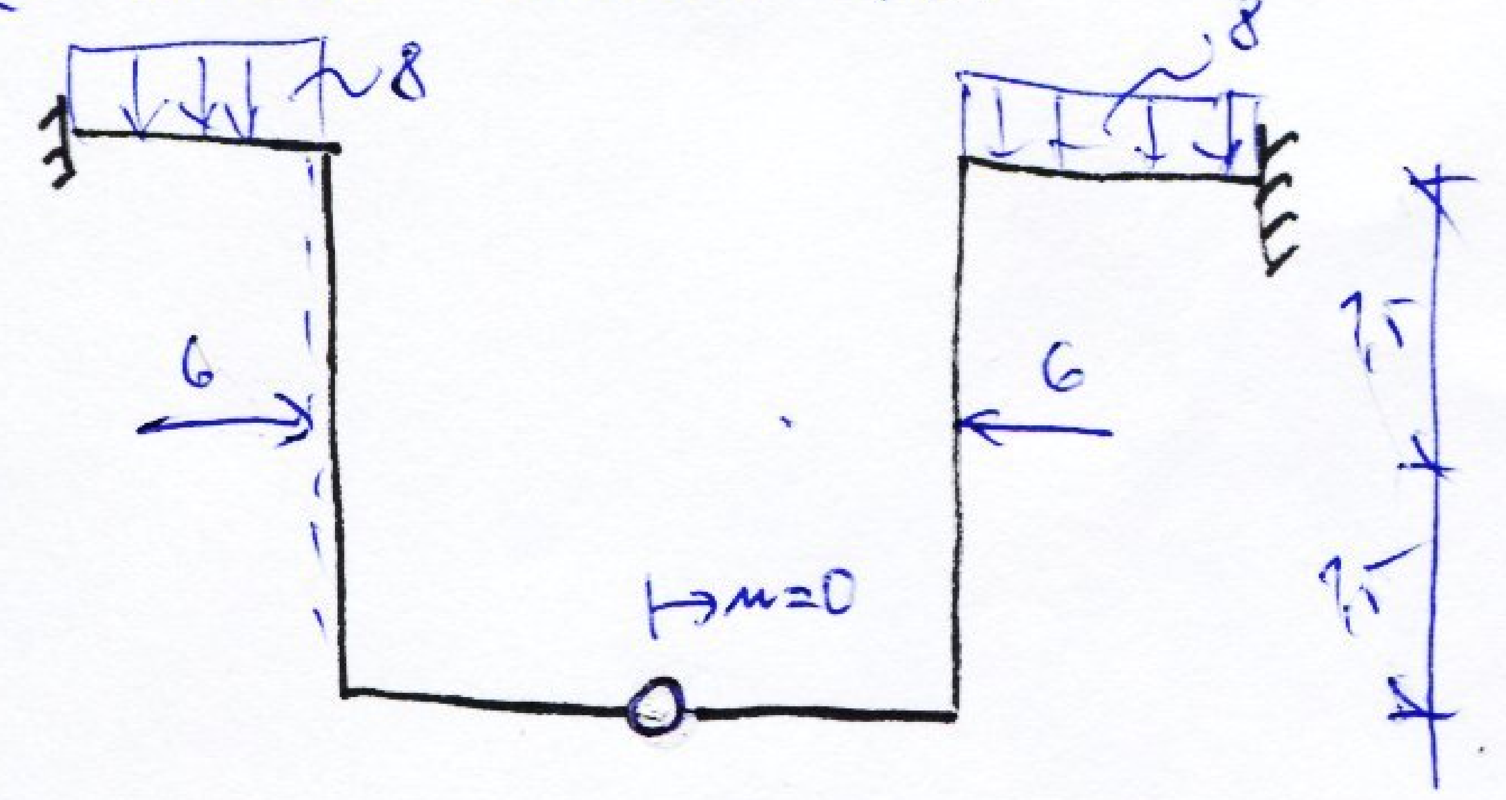


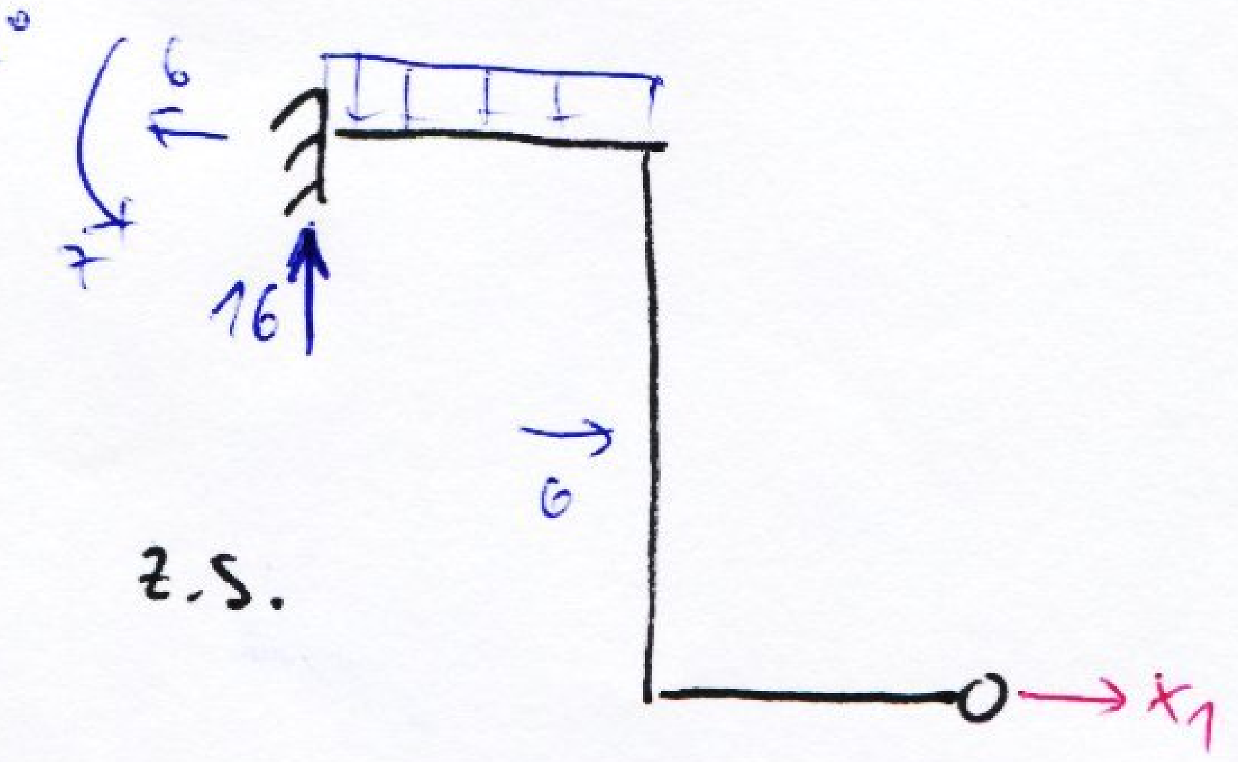




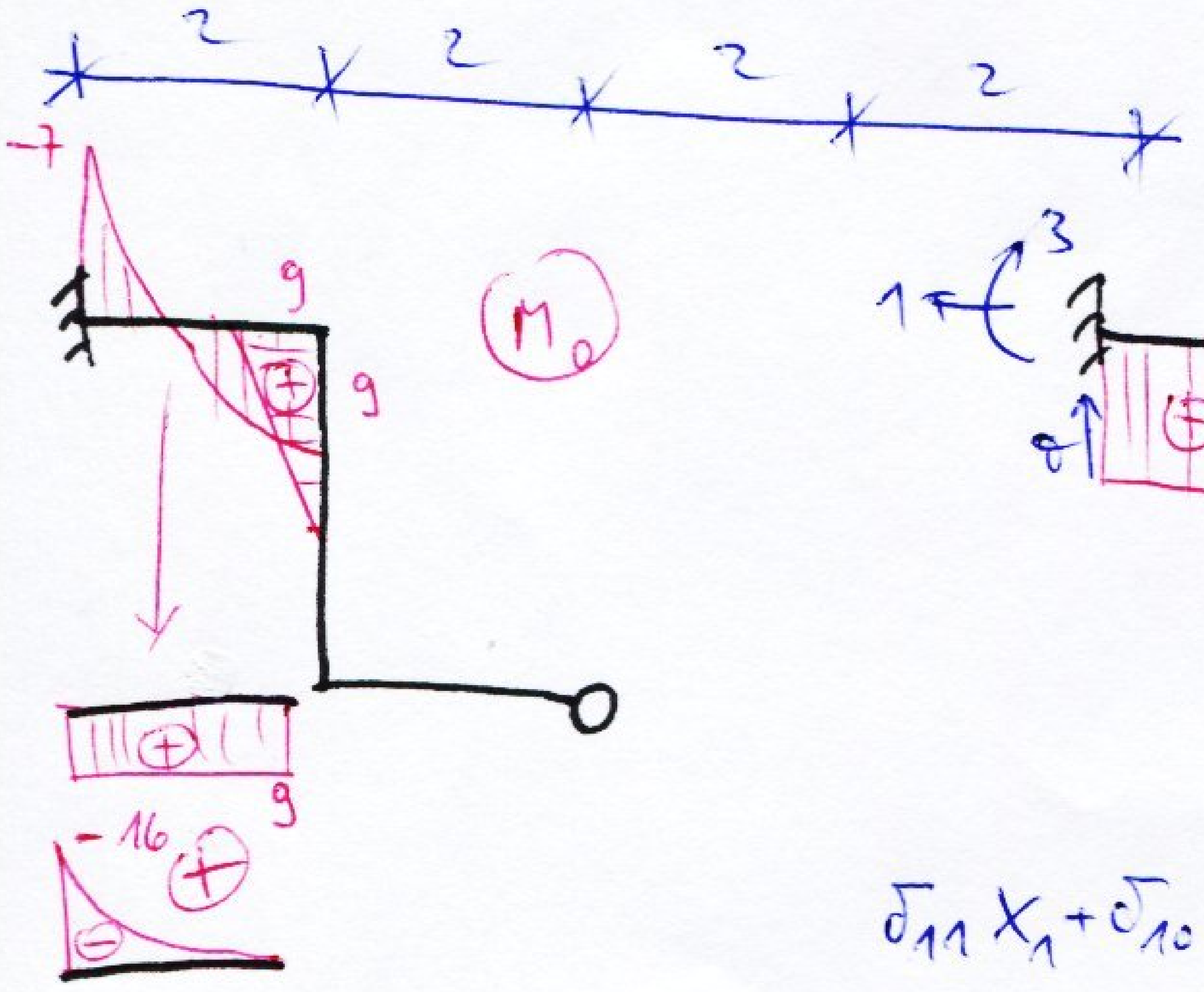


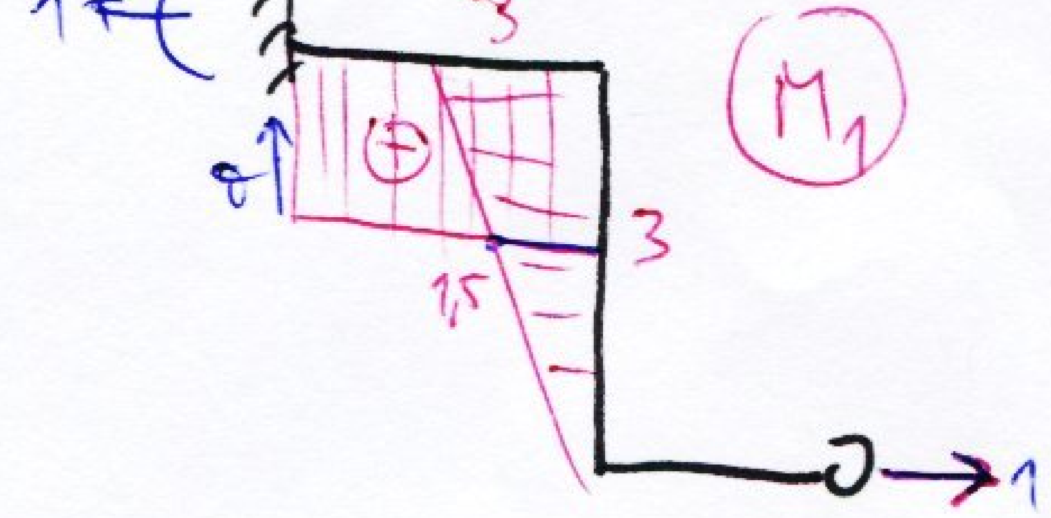
Př. SYMETRIE – VYKRESLETE N, V, M





Z.S.



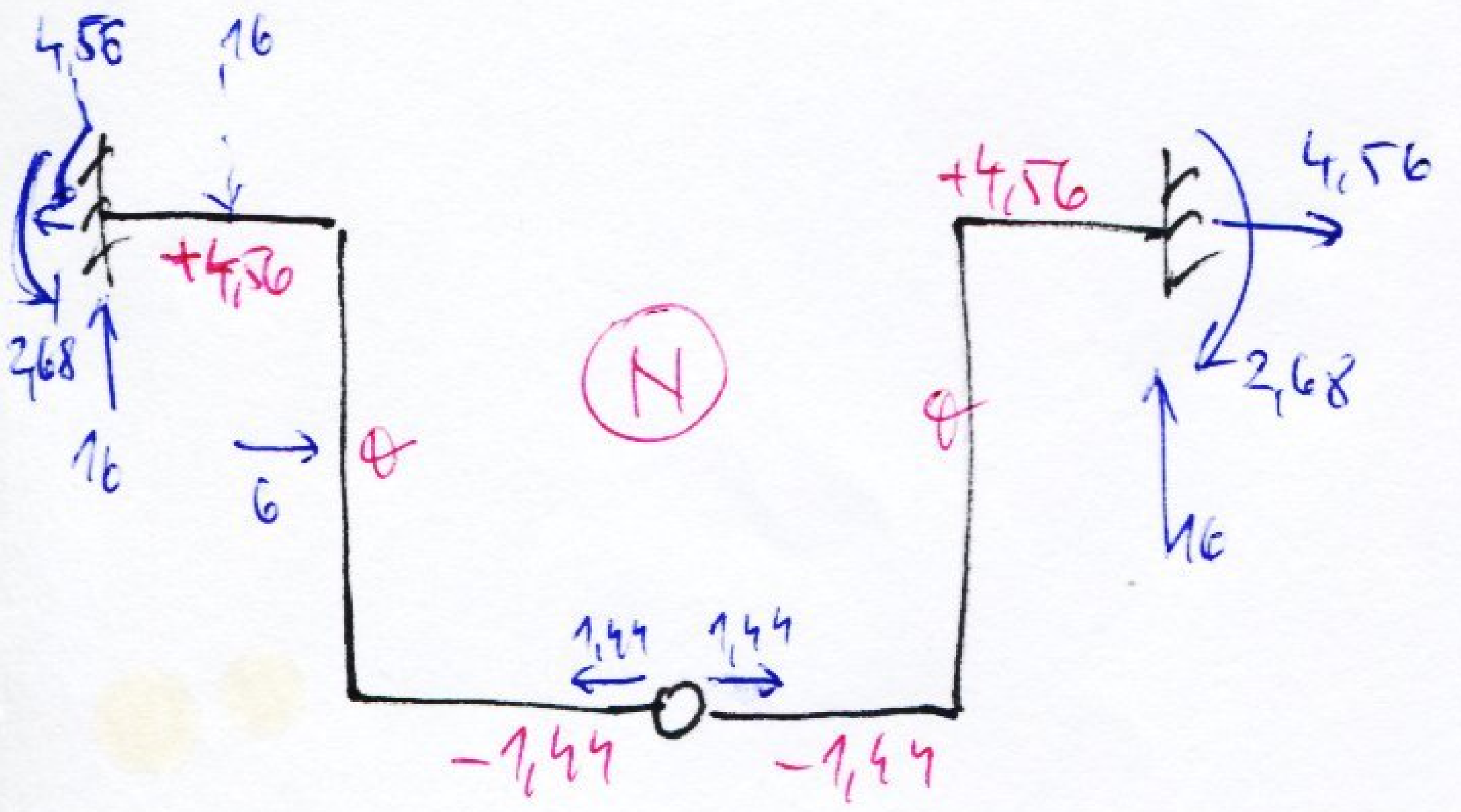


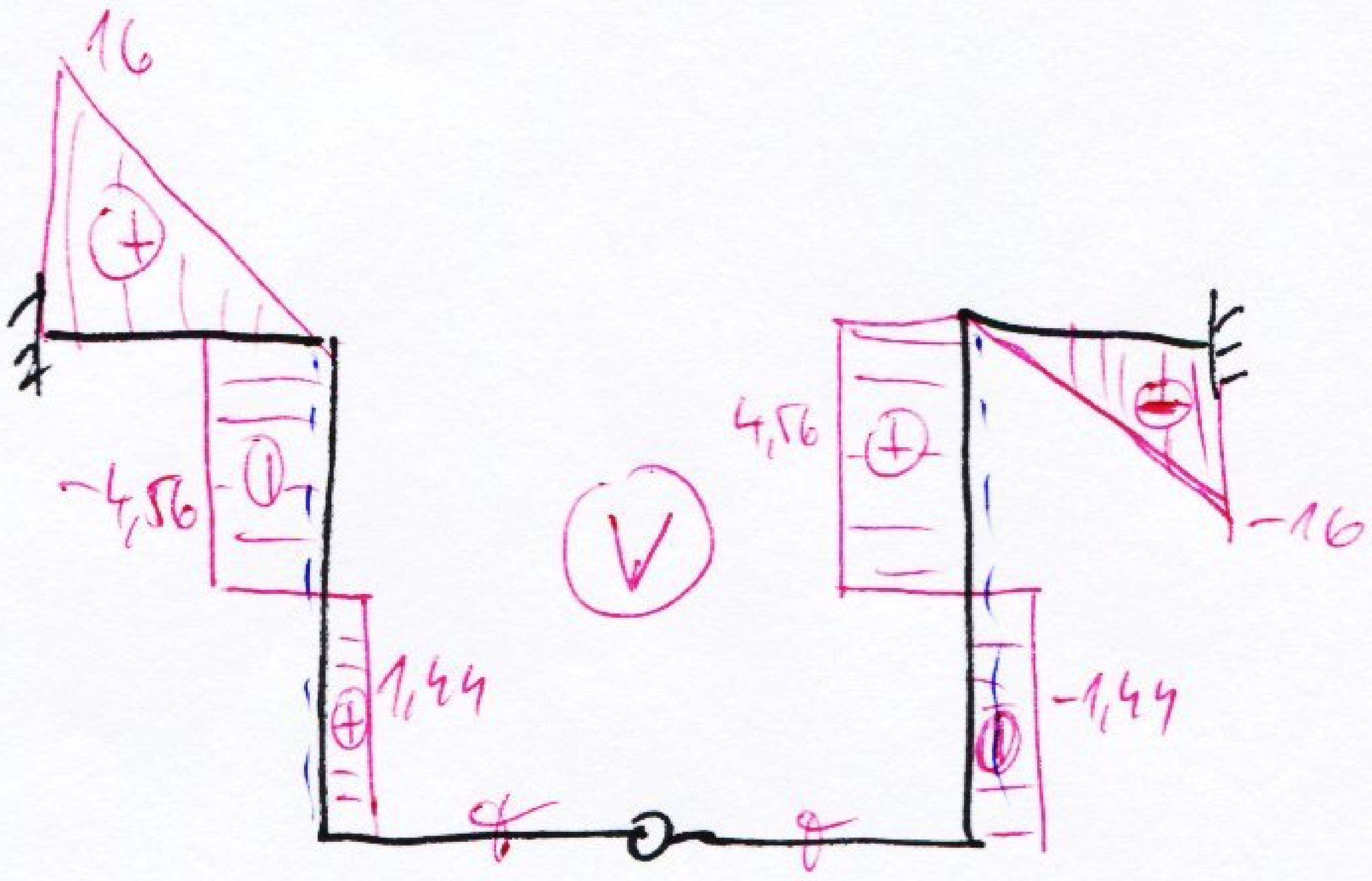
$$\delta_{11} = \frac{1}{EI}\left[3^2 \cdot 2 + \frac{1}{3} \cdot 3^2 \cdot 3\right] = \frac{27}{EI}$$

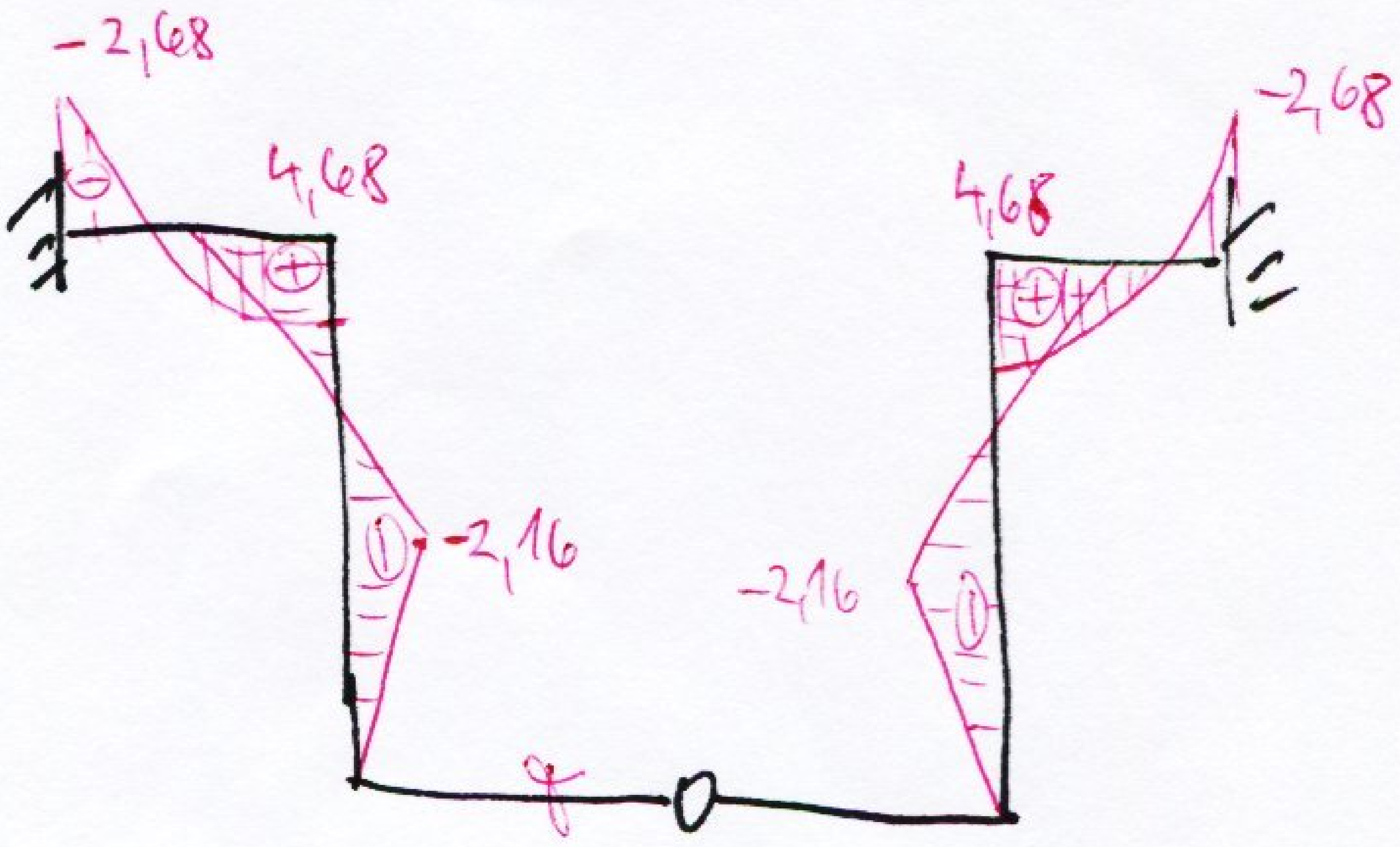
$$\delta_{10} = \frac{1}{EI}\left[3 \cdot 9 \cdot 2 + \frac{1}{3} \cdot 3 \cdot (-16) \cdot 2 + \frac{1}{2} \cdot 9 \cdot 1{,}5 \cdot 2{,}5\right] =$$

$$= \frac{1}{EI}\left[54 - 32 + 16{,}875\right] = \frac{38{,}875}{EI}$$

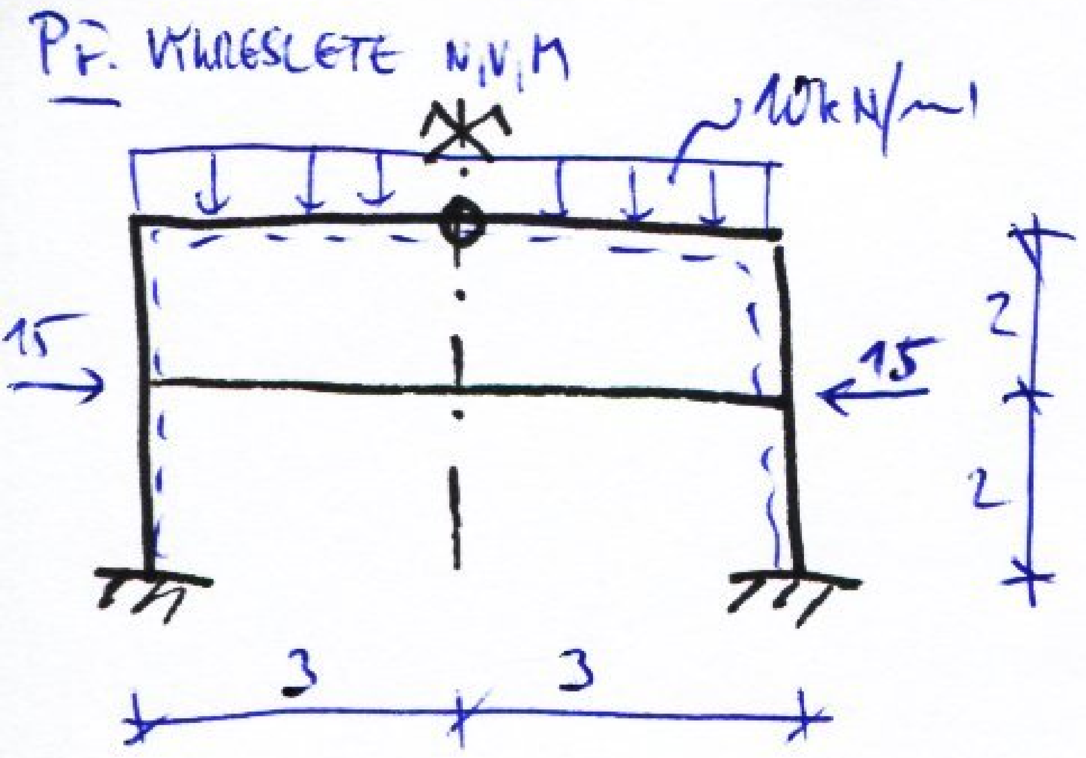
$\delta_{11} X_1 + \delta_{10} = 0$ $\quad X_1 = -1{,}44 \text{ kN}$



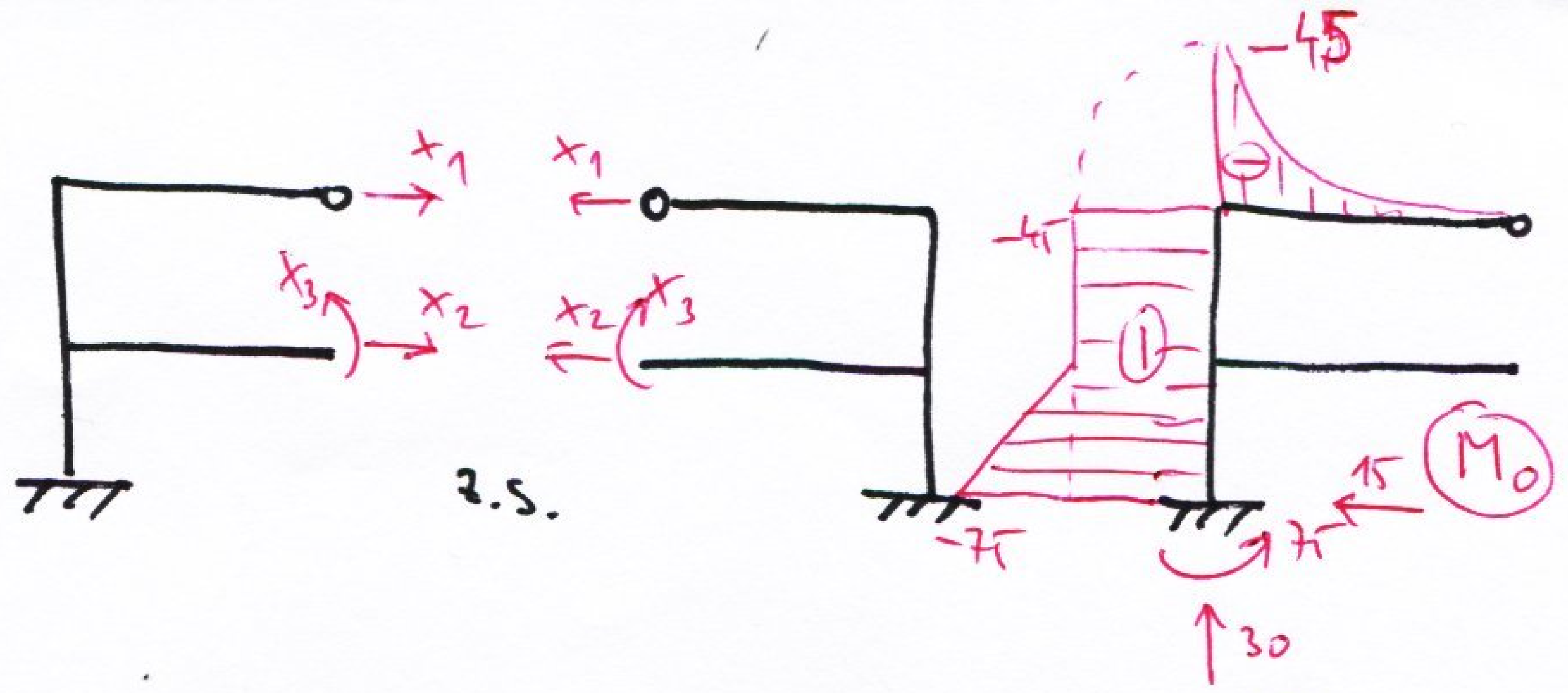


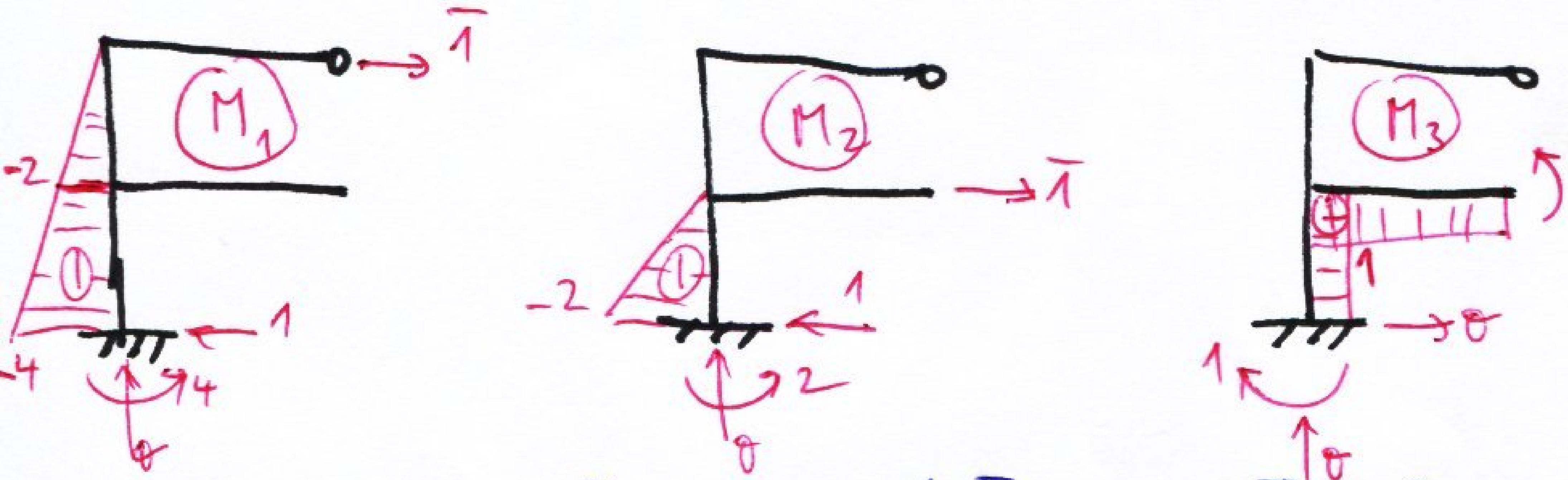


Př. vykreslete N, V, M



$s = -5°$





$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{3}\cdot 4^2 \cdot 4\right] = \frac{21{,}\bar{3}}{EI}$$

$$\delta_{12} = \frac{1}{EI}\left[\frac{1}{2}\cdot 2\cdot 2\cdot 3{,}\bar{3}\right] = \frac{6{,}\bar{6}}{EI}$$

$$\delta_{13} = \frac{1}{EI}\left[1\cdot 2\cdot(-3)\right] = -\frac{6}{EI}$$

$$\delta_{22} = \frac{1}{EI}\left[\frac{1}{3}\cdot 2^2\cdot 2\right] = \frac{2{,}\bar{6}}{EI} \qquad \delta_{23} = \frac{1}{EI}\left[\frac{1}{2}\cdot(-2)\cdot 1\cdot 2\right] = -\frac{2}{EI} \qquad \delta_{33} = \frac{1}{EI}\left[1^2\cdot 2 + 1^2\cdot 3\right] = \frac{5}{EI}$$

$$\delta_{10} = \frac{1}{EI}\left[\frac{1}{2}\cdot 4\cdot 45\cdot 4 + \frac{1}{2}\cdot 30\cdot 3{,}\bar{3}\cdot 2\right] = \frac{460}{EI} \qquad \delta_{20} = \frac{1}{EI}\left[\frac{1}{2}\cdot 2\cdot 2\cdot 65\right] = \frac{130}{EI} \qquad \delta_{30} = \frac{1}{EI}\left[1\cdot 2\cdot(-60)\right] = -\frac{120}{EI}$$

$X_1 = -29{,}353 \qquad X_2 = 23{,}165 \qquad X_3 = -1{,}958$