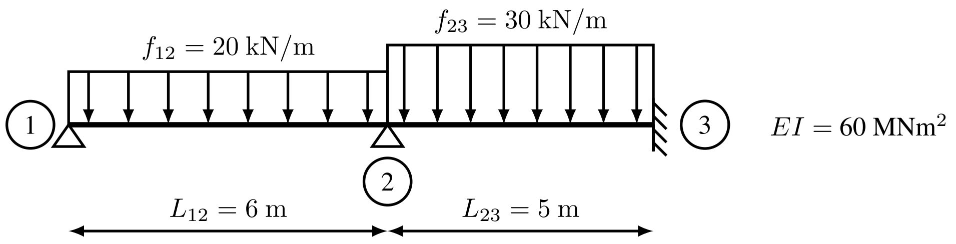
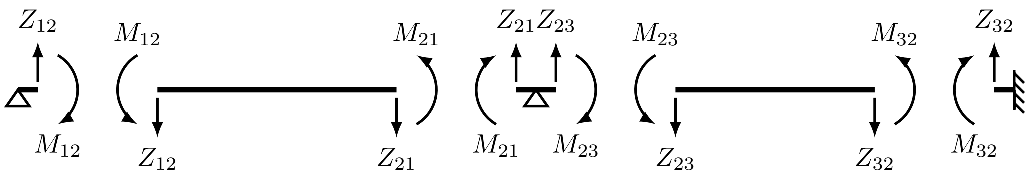
Příklad 1





rozdíl konvence vnitřních sil a koncových sil

neznámé (φ_1 a φ_2 nebo pouze φ_2) $\rightarrow$ podmínky rovnováhy

$$k_{12} = \frac{2 \cdot EI}{L_{12}} = \frac{2 \cdot 60}{6} = 20 \text{ MNm}$$
$$k_{23} = \frac{2 \cdot EI}{L_{23}} = \frac{2 \cdot 60}{5} = 24 \text{ MNm}$$

$$\overline{M}_{12} = 0 \text{ kNm}$$
$$\overline{M}_{21} = \frac{\left(-f_{12} \cdot {L_{12}}^2\right)}{8} = \frac{\left(-20 \cdot 6^2\right)}{8} = (-90) \text{ kNm}$$
$$\overline{M}_{23} = \frac{f_{23} \cdot {L_{23}}^2}{12} = \frac{30 \cdot 5^2}{12} = 62.5 \text{ kNm}$$
$$\overline{M}_{32} = \frac{\left(-f_{23} \cdot {L_{23}}^2\right)}{12} = \frac{\left(-30 \cdot 5^2\right)}{12} = (-62.5) \text{ kNm}$$

$$\overline{Z}_{12} = \frac{(-3 \cdot f_{12} \cdot L_{12})}{8} = \frac{(-3 \cdot 20 \cdot 6)}{8} = (-45) \text{ kN}$$
$$\overline{Z}_{21} = \frac{(-5 \cdot f_{12} \cdot L_{12})}{8} = \frac{(-5 \cdot 20 \cdot 6)}{8} = (-75) \text{ kN}$$
$$\overline{Z}_{23} = \frac{(-f_{23} \cdot L_{23})}{2} = \frac{(-30 \cdot 5)}{2} = (-75) \text{ kN}$$
$$\overline{Z}_{32} = \frac{(-f_{23} \cdot L_{23})}{2} = \frac{(-30 \cdot 5)}{2} = (-75) \text{ kN}$$

$$M_{12}^{\varphi} = 0 \text{ kNm}$$
$$M_{21}^{\varphi} = \frac{3 \cdot k_{12}}{2} \cdot \varphi_2$$
$$M_{23}^{\varphi} = k_{23} \cdot 2 \cdot \varphi_2$$
$$M_{32}^{\varphi} = k_{23} \cdot \varphi_2$$

$$Z_{12}^{\varphi} = \left(-\frac{3 \cdot k_{12}}{2 \cdot L_{12}}\right) \cdot \varphi_2$$
$$Z_{21}^{\varphi} = \frac{3 \cdot k_{12}}{2 \cdot L_{12}} \cdot \varphi_2$$
$$Z_{23}^{\varphi} = \left(-\frac{3 \cdot k_{23}}{L_{23}}\right) \cdot \varphi_2$$
$$Z_{32}^{\varphi} = \frac{3 \cdot k_{23}}{L_{23}} \cdot \varphi_2$$

$$M_{21} = \overline{M}_{21} + M_{21}^{\varphi}$$
$$M_{23} = \overline{M}_{23} + M_{23}^{\varphi}$$

$$M_{21} + M_{23} = 0$$
$$(-90) + \frac{3 \cdot 20}{2} \cdot \varphi_2 + 62.5 + 24 \cdot 2 \cdot \varphi_2 = 0$$
$$(-27.5) + 78 \cdot \varphi_2 = 0$$

$$\varphi_2 = 0.352564 \text{ mrad}$$

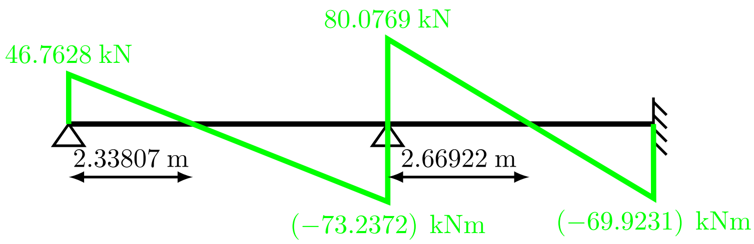
$$M_{12}^{\varphi} = 0 \text{ kNm}$$
$$M_{21}^{\varphi} = \frac{3 \cdot k_{12}}{2} \cdot \varphi_2 = \frac{3 \cdot 20}{2} \cdot 0.352564 = 10.5769 \text{ kN}$$
$$M_{23}^{\varphi} = k_{23} \cdot 2 \cdot \varphi_2 = 24 \cdot 2 \cdot 0.352564 = 16.9231 \text{ kN}$$
$$M_{32}^{\varphi} = k_{23} \cdot \varphi_2 = 24 \cdot 0.352564 = 8.46154 \text{ kN}$$

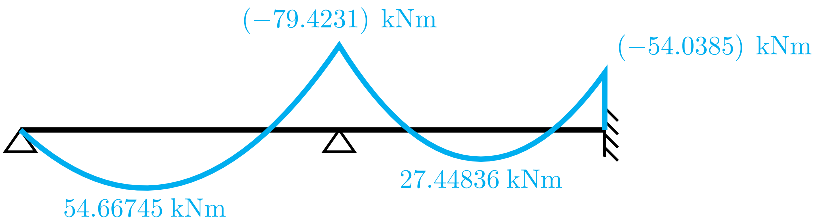
$$Z_{12}^{\varphi} = \left(-\frac{3 \cdot k_{12}}{2 \cdot L_{12}}\right) \cdot \varphi_2 = \left(-\frac{3 \cdot 20}{2 \cdot 6}\right) \cdot 0.352564 = (-1.76282) \text{ kN}$$
$$Z_{21}^{\varphi} = \frac{3 \cdot k_{12}}{2 \cdot L_{12}} \cdot \varphi_2 = \frac{3 \cdot 20}{2 \cdot 6} \cdot 0.352564 = 1.76282 \text{ kN}$$
$$Z_{23}^{\varphi} = \left(-\frac{3 \cdot k_{23}}{L_{23}}\right) \cdot \varphi_2 = \left(-\frac{3 \cdot 24}{5}\right) \cdot 0.352564 = (-5.07692) \text{ kN}$$
$$Z_{32}^{\varphi} = \frac{3 \cdot k_{23}}{L_{23}} \cdot \varphi_2 = \frac{3 \cdot 24}{5} \cdot 0.352564 = 5.07692 \text{ kN}$$

$$M_{12} = \overline{M}_{12} + M_{12}^{\varphi} = 0 + 0 = 0 \text{ kNm}$$
$$M_{21} = \overline{M}_{21} + M_{21}^{\varphi} = (-90) + 10.5769 = (-79.4231) \text{ kNm}$$
$$M_{23} = \overline{M}_{23} + M_{23}^{\varphi} = 62.5 + 16.9231 = 79.4231 \text{ kNm}$$
$$M_{32} = \overline{M}_{32} + M_{32}^{\varphi} = (-62.5) + 8.46154 = (-54.0385) \text{ kNm}$$

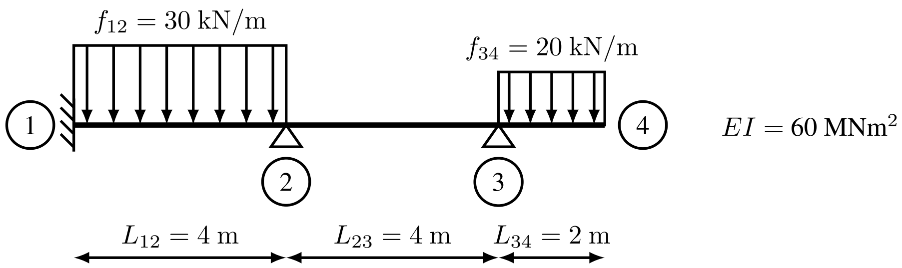
$$Z_{12} = \overline{Z}_{12} + Z_{12}^{\varphi} = (-45) + (-1.76282) = (-46.7628) \text{ kNm}$$
$$Z_{21} = \overline{Z}_{21} + Z_{21}^{\varphi} = (-75) + 1.76282 = (-73.2372) \text{ kNm}$$
$$Z_{23} = \overline{Z}_{23} + Z_{23}^{\varphi} = (-75) + (-5.07692) = (-80.0769) \text{ kNm}$$
$$Z_{32} = \overline{Z}_{32} + Z_{32}^{\varphi} = (-75) + 5.07692 = (-69.9231) \text{ kNm}$$

síly Z též možno určit z podmínek rovnováhy na prutech

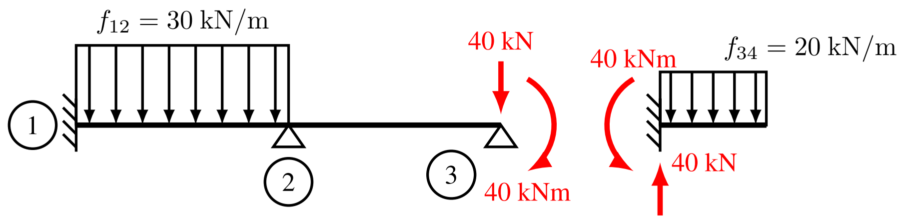

46.7628 kN
80.0769 kN
2.33807 m
2.66922 m
(−73.2372) kNm
(−69.9231) kNm


(−79.4231) kNm
(−54.0385) kNm
27.44836 kNm
54.66745 kNm

Příklad 2



Vyřešení staticky určité části



$$k_{12} = \frac{2 \cdot EI}{L_{12}} = \frac{2 \cdot 60}{4} = 30 \text{ MNm}$$
$$k_{23} = \frac{2 \cdot EI}{L_{23}} = \frac{2 \cdot 60}{4} = 30 \text{ MNm}$$

$$\overline{M}_{12} = \frac{f_{12} \cdot {L_{12}}^2}{12} = \frac{30 \cdot 4^2}{12} = 40 \text{ kNm}$$
$$\overline{M}_{21} = \frac{(-f_{12} \cdot {L_{12}}^2)}{12} = \frac{(-30 \cdot 4^2)}{12} = (-40) \text{ kNm}$$
$$\overline{M}_{23} = 0 \text{ kNm}$$
$$\overline{M}_{32} = 0 \text{ kNm}$$

$$\overline{Z}_{12} = \frac{(-f_{12} \cdot L_{12})}{2} = \frac{(-30 \cdot 4)}{2} = (-60) \text{ kN}$$
$$\overline{Z}_{21} = \frac{(-f_{12} \cdot L_{12})}{2} = \frac{(-30 \cdot 4)}{2} = (-60) \text{ kN}$$
$$\overline{Z}_{23} = 0 \text{ kN}$$
$$\overline{Z}_{32} = 0 \text{ kN}$$

$$M_{12}^{\varphi} = k_{12} \cdot \varphi_2$$
$$M_{21}^{\varphi} = 2 \cdot k_{12} \cdot \varphi_2$$
$$M_{23}^{\varphi} = k_{23} \cdot (2 \cdot \varphi_2 + \varphi_3)$$
$$M_{32}^{\varphi} = k_{23} \cdot (\varphi_2 + 2 \cdot \varphi_3)$$

$$Z_{12}^{\varphi} = \left(-\frac{3 \cdot k_{12}}{L_{12}}\right) \cdot \varphi_2$$
$$Z_{21}^{\varphi} = \frac{3 \cdot k_{12}}{L_{12}} \cdot \varphi_2$$
$$Z_{23}^{\varphi} = \left(-\frac{3 \cdot k_{23}}{L_{23}}\right) \cdot (\varphi_2 + \varphi_3)$$
$$Z_{32}^{\varphi} = \frac{3 \cdot k_{23}}{L_{23}} \cdot (\varphi_2 + \varphi_3)$$

$$M_{21} = \overline{M}_{21} + M_{21}^{\varphi}$$
$$M_{23} = \overline{M}_{23} + M_{23}^{\varphi}$$
$$M_{32} = \overline{M}_{32} + M_{32}^{\varphi}$$

$$M_{21} + M_{23} = 0$$
$$M_{32} = -40$$

$$(-40) + 2 \cdot 30 \cdot \varphi_2 + 0 + 30 \cdot (2 \cdot \varphi_2 + \varphi_3) = 0$$
$$0 + 30 \cdot (\varphi_2 + 2 \cdot \varphi_3) = -40$$

$$(-40) + 120 \cdot \varphi_2 + 30 \cdot \varphi_3 = 0$$
$$60 \cdot \varphi_3 + 30 \cdot \varphi_2 = -40$$

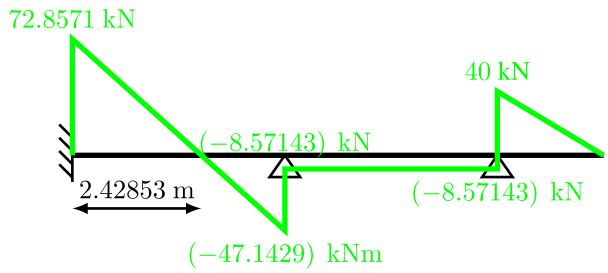
$$\varphi_2 = 0.571429 \text{ mrad}$$
$$\varphi_3 = (-0.952381) \text{ mrad}$$

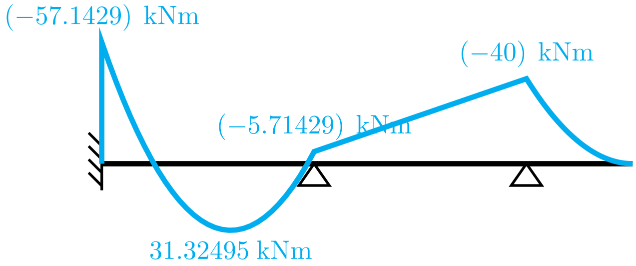
$$M_{12}^{\varphi} = k_{12} \cdot \varphi_2 = 30 \cdot 0.571429 = 17.1429 \text{ kNm}$$
$$M_{21}^{\varphi} = 2 \cdot k_{12} \cdot \varphi_2 = 2 \cdot 30 \cdot 0.571429 = 34.2857 \text{ kNm}$$
$$M_{23}^{\varphi} = k_{23} \cdot (2 \cdot \varphi_2 + \varphi_3) = 30 \cdot (2 \cdot 0.571429 + (-0.952381)) = 5.71429 \text{ kNm}$$
$$M_{32}^{\varphi} = k_{23} \cdot (\varphi_2 + 2 \cdot \varphi_3) = 30 \cdot (0.571429 + 2 \cdot (-0.952381)) = (-40) \text{ kNm}$$

$$Z_{12}^{\varphi} = \left(-\frac{3 \cdot k_{12}}{L_{12}}\right) \cdot \varphi_2 = \left(-\frac{3 \cdot 30}{4}\right) \cdot 0.571429 = (-12.8571) \text{ kN}$$
$$Z_{21}^{\varphi} = \frac{3 \cdot k_{12}}{L_{12}} \cdot \varphi_2 = \frac{3 \cdot 30}{4} \cdot 0.571429 = 12.8571 \text{ kN}$$
$$Z_{23}^{\varphi} = \left(-\frac{3 \cdot k_{23}}{L_{23}}\right) \cdot (\varphi_2 + \varphi_3) = \left(-\frac{3 \cdot 30}{4}\right) \cdot (0.571429 + (-0.952381)) = 8.57143 \text{ kN}$$
$$Z_{32}^{\varphi} = \frac{3 \cdot k_{23}}{L_{23}} \cdot (\varphi_2 + \varphi_3) = \frac{3 \cdot 30}{4} \cdot (0.571429 + (-0.952381)) = (-8.57143) \text{ kN}$$

$$M_{12} = \overline{M}_{12} + M_{12}^{\varphi} = 40 + 17.1429 = 57.1429 \text{ kNm}$$
$$M_{21} = \overline{M}_{21} + M_{21}^{\varphi} = (-40) + 34.2857 = (-5.71429) \text{ kNm}$$
$$M_{23} = \overline{M}_{23} + M_{23}^{\varphi} = 0 + 5.71429 = 5.71429 \text{ kNm}$$
$$M_{32} = 60 \cdot \varphi_3 + 30 \cdot \varphi_2 = 60 \cdot (-0.952381) + 30 \cdot 0.571429 = (-40) \text{ kNm}$$

$$Z_{12} = \overline{Z}_{12} + Z_{12}^{\varphi} = (-60) + (-12.8571) = (-72.8571) \text{ kNm}$$
$$Z_{21} = \overline{Z}_{21} + Z_{21}^{\varphi} = (-60) + 12.8571 = (-47.1429) \text{ kNm}$$
$$Z_{23} = \overline{Z}_{23} + Z_{23}^{\varphi} = 0 + 8.57143 = 8.57143 \text{ kNm}$$
$$Z_{32} = \overline{Z}_{32} + Z_{32}^{\varphi} = 0 + (-8.57143) = (-8.57143) \text{ kNm}$$





V případě nalezení chyb, nejasností či dotazů mi prosím napište na jan.stransky@fsv.cvut.cz

verze 01, 25.2.2015