Subscale Enrichment of Discontinuity for XFEM Crack Tip Element

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In the present contribution, we propose a new XFEM based enrichment of the displacement field to allow for cracks that end or kink within an element. The basic concept relies on the fact that the crack tip element is treated on a subscale, where, in addition to the macroscopic continuous ($\boldsymbol{\varphi}_c$) and discontinuous ($\boldsymbol{\varphi}_d$) displacement fields, a discontinuous fluctuation field $\boldsymbol{\varphi}_d^f$ with Dirichlet boundary conditions is introduced to allow for proper representation of the discontinuous kinematics. Hence, the finite element approximation of the total deformation map is obtained as:

$$\boldsymbol{\varphi}^h = \underbrace{\sum_{i\in I} N^i \hat{\boldsymbol{\varphi}}_c^i + \sum_{j\in J} \tilde{H}_S \psi^j \hat{\boldsymbol{\varphi}}_d^j}_{\boldsymbol{\varphi}_M^h} + \underbrace{\sum_{k\in K} \tilde{H}_S \psi^k \hat{\boldsymbol{\varphi}}_d^{f,k}}_{\boldsymbol{\varphi}_f^h} \quad (1)$$

where $\tilde{H}_S$ is the shifted discontinuity function and where, referring to Figure 1, I is the set of edge (master) nodes of the crack tip element, $J \subset I$ is the set of edge nodes enriched with discontinuous degrees of freedom (black squares) and K is the set of internal nodes enriched with fluctuation (discontinuous) degrees of freedom (white circles). In fact, in this non-standard interpolation of the discontinuity field, additional internal nodes are utilised for the interpolation. In Eq. (2), we therefore introduced N^i and ψ^i to distinguish between shape functions that have support over the entire crack tip element and only partially over a subset of the subscale elements respectively.

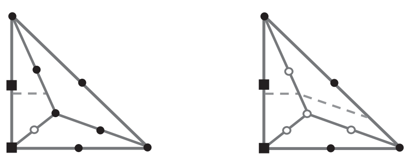

Figure 1: 6-noded shell element with quadratic interpolation for the discontinuity field $\boldsymbol{\varphi}_d$ enhanced with internal degrees of freedom for $\boldsymbol{\varphi}_d^f$.

Adopting an explicit time integration scheme and utilising that the fluctuation field vanishes along the crack tip element edges, the fluctuation field can be implicitly solved for in terms of the macroscopic field. By considering the discretised form of the momentum balance for the subscale problem

$$\begin{bmatrix} \mathbf{M}_{MM} & \mathbf{M}_{Mf} \\ \mathbf{M}_{fM} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{a}}_M^h \\ \hat{\boldsymbol{a}}_f^h \end{bmatrix} = \begin{bmatrix} \mathbf{f}_M^{ext} - \mathbf{f}_M^{int} \\ -\mathbf{f}_f^{int} \end{bmatrix} \quad (2)$$

it is clear that the discretised macroscopic form of the momentum balance for the crack tip element reduces to

$$\mathbf{M}_M \hat{\boldsymbol{a}}_M^h = \mathbf{f}_M^{ext} - \bar{\mathbf{f}}_M^{int} \quad (3)$$

with

$$\mathbf{M}_M = \mathbf{M}_{MM} - \mathbf{M}_{Mf}\mathbf{M}_{ff}^{-1}\mathbf{M}_{fM} \quad (4)$$

$$\bar{\mathbf{f}}_M^{int} = \mathbf{f}_M^{int} - \mathbf{M}_{Mf}\mathbf{M}_{ff}^{-1}\mathbf{f}_f^{int} \quad (5)$$

The current methodology is applied to ductile crack propagation in shells loaded at high strain rates, extending previous developments in [1] to allow for not only crack segments through the entire shell elements (edge to edge).

Acknowledgement This research is carried out under project number M41.2.10378 in the framework of the Research Program of the Materials innovation institute M2i (www.m2i.nl).

References

[1] S. Mostofizadeh, M. Fagerström, R. Larsson, Dynamic crack propagation in elastoplastic thinwalled structures: Modelling and validation, Int J Numer Meth Eng (*submitted 2012*).