A Stabilization Technique for the Extended Finite Element Simulation of Arbitrary Crack Geometries in 3D

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The mesh independent simulation of fracture processes in three dimensions using the extended finite element method (XFEM) often leads to badly conditioned equation systems due to the almost linear dependence of standard and enriched degrees of freedom. Especially for arbitrary crack geometries and crack propagation simulations it frequently happens that a crack barely intersects an element which leads to this undesired effect. In such cases different techniques can be applied. The geometry of the crack can be changed slightly by pushing it onto the node that is cut off the rest of the element by the crack. Alternatively the crack can be pushed away from the affected node or the node can be pushed onto the crack or away from the crack [1]. All these techniques lead to a slight modification of the crack geometry which is undesirable.

To avoid a change of the geometry and possibly the mesh, stabilization techniques can be applied. In [2] recently a global stabilization technique for the generalized finite element method (GFEM) is presented for one dimensional problems. This method promises to overcome such problems. Stabilization techniques on the element level have been applied in the context of standard and underintegrated finite elements for a long time already. Most of these methods are based on the works of Flanagan and Belytschko [3].

In this contribution we present a stabilization technique for the regularization of almost singular element stiffness matrices that occur within the extended finite element method. The method is based on an eigenvalue decomposition of the element stiffness matrix and a corresponding modification of the matrix and the right hand side for those eigenmodes with small or even zero eigenvalues. Physically or numerically reasonable zero eigenmodes like rigid body translations or rotations remain untouched. This technique has the advantage that neither the mesh nor the crack geometry is modified. Additionally it is very efficient. The stabilization leads to a significantly improved convergence behaviour of iterative equation solvers used for large scale three dimensional problems, while the modification of the stiffness matrix and the righthand side does not affect the accuracy of the result significantly and the additional numerical effort on the element level is low. This is demonstrated with multiple examples of two and three dimensional fracture problems with arbitrary crack geometries.

References

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