Mixed Finite Elements with Prescribed Primary and Secondary Variables

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In this paper, three mixed finite element formulations for the solution of elastic and nonlinear problems are approximated, implemented and validated. The mixed approximations are formulated using two fields: displacement-stress, corresponding to the Hellinger-Reissner formulation, the displacement-strain formulation and the three field formulation using displacements, strains and stresses also referred as Veubeke-Hu-Washizu. For these formulations each of the independent fields considered, displacement, strain and stress, is approximated with linear interpolation functions. The finite element implementations of these formulations have additional strain and stress fields depending of displacements, which emerge in natural way from the finite element approximation [1]. It is shown that the proposed approximations do not present numerical problems as others do which need the stabilization of the resulting system of linear algebraic equations to avoid them.

In the approximations proposed, not only primary variables, displacements, are prescribed on the boundary, but also secondary variables as strain and stress are constrained where they are intrinsically known, although these are not conventional restriction variables. The formulation of finite elements with additional restriction variables lead to improved approximations of the displacement, strain and stress fields, compared with what standard displacement finite elements give.

In the formulation of nonlinear problems involving damage a continuous damage model equipped with two internal variables at each integration point is used. The finite element implementation of the displacement-stress formulation presents numerical problems when the material is completely damaged, thus the continuous damage model was only implanted in the displacement-strain and the three fields formulations as these do not present numerical problems.

Numerical results of elastic problems are compared against analytic solutions, showing that the mixed approach provides better approximation on the strain and stress fields than a standard formulation. In addition, problems showing the locking problem as those involving incompressible solids are solved and the problem overcome with the two and three field formulations, having better results with the last one. To show the effectiveness of the mixed formulations with the continuous damage model, some numerical results of the representing numerical examples are presented.

References

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