Convergent Erosion Schemes for Three-Dimensional Fracture and Fragmentation

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Brittle fracture can be formulated as a *free-discontinuity problem*, i. e., a problem where the solution can have discontinuities and, in addition, the jump sets of the solution are *a priori* unknown. Within this variational framework, [1] have proposed an approximation scheme, which they term *eigenfracture*, based on the notion of *eigendeformation*. Eigendeformations are widely used in mechanics to describe deformation modes that cost no local energy. In the eigenfracture scheme, the energy functional depends on two fields: the displacement field u and an eigendeformation field ε^* that describes such cracks as may be present in the body. Specifically, eigendeformations allow the displacement field to develop jumps at no cost in local elastic energy. In addition, in the eigenfracture scheme the fracture energy is set to be proportional to the volume of the ϵ-neighborhood of the support of the eigendeformation field, suitably scaled by $1/\epsilon$. The optimal crack set is obtained by minimizing the resulting energy functional with respect to both the displacement and the eigendeformation fields, subject to irreversibility constraints.

The eigenfracture scheme of Schmidt *et al.* [1] has been specialized to element erosion by Pandolfi and Ortiz [2], and to material-point erosion by Li *et al.* [3], and referred to the resulting schemes as *eigenerosion*. Eigenerosion is derived from the general eigenfracture formulation by restricting the eigendeformations in a binary sense: they can be either zero, in which case the local behavior is elastic; or they can be equal to the local displacement gradient, in which case the corresponding material neighborhood is failed, or *eroded*. When combined with a finite-element approximation, this scheme gives rise to *element erosion*, i.e., the elements or material points can be either intact, in which case their behavior is elastic, or be completely failed — or eroded — and have no load bearing capacity. The implementation of the method, included the all-important ϵ-neighborhood construction, is exceedingly simple and applies to general situations, possibly involving complex three-dimensional fracture patterns such as branching and fragmentation. The accuracy and convergence of the eigenerosion approach is comparable — at a much reduced implementation cost and complexity — to that of other numerical fracture schemes.

Element erosion has been extensively used to simulate fracture in a number of areas of application, including terminal ballistics. However, some of these methods fail to converge or converge to the wrong limit. By contrast, the eigenfracture scheme is known to properly converge to Griffith fracture in the limit of vanishingly small mesh sizes [1]. In particular, the local-neighborhood averaging of the energy which underlies the calculation of the effective energy-release has the effect of eliminating spurious mesh-dependencies. The accuracy and fidelity of the eigenerosion approach has been assessed through convergence studies for Mode I crack growth, both in two and three dimensions and for structured and random meshes, and validated against quasistatic and dynamic experimental data.

References

[1] B. Schmidt, F. Fraternali, M. Ortiz, Eigenfracture: An eigendeformation approach to variational fracture, Multiscale Model Sim 7 (2009) 1237–1266.

[2] A. Pandolfi, M. Ortiz, An eigenerosion approach to brittle fracture, Int J Numer Meth Eng 8 (2012) 694–714.

[3] B. Li, A. Kidane, G. Ravichandran, M. Ortiz, Verification and validation of the Optimal Transportation Meshfree (OTM) simulation of terminal ballistics, Int J Impact Eng 42 (2012) 25–36.