Fracture Scaling and Safety of Quasibrittle Structures: Atomistic Basis, Computational Challenges and New Advances

Z. P. Bažant[1*], M. H. Hubler[1], M. Salviato[1], K. Kirane[2], J.-L. Le[3]

[1] Department of Civil and Environmental Engineering, Northwestern University, 2145 Sheridan Road, CEE/A135, Evanston, Il. 60208, z-bazant@northwestern.edu
[2] Department of Mechanical Engineering, Northwestern University, 2145 Sheridan Road, ME/B214, Evanston, Il. 60208
[3] Department of Civil Engineering, University of Minnesota, 500 Pillsbury Dr. S.E., Minneapolis, MN. 55455

The main objective of structural analysis is safety. This happens to be a particularly tricky problem for quasibrittle materials, i.e. heterogeneous materials with brittle constituents in which the inhomogeneity size, and thus the fracture process zone (FPZ), is not negligible compared to the structural dimension D.

The problem is that the type of strength distribution must be known up to the tail of failure probability 10^{-6}, which is the maximum failure probability tolerable in engineering design. Such a small probability is beyond direct experimental verification by repeated tests (since at least 10^8 of identical structures would have to be tested). So the distribution type must be based on a physically justified theory, verified indirectly in other ways.

For ductile (or plastic) failure, the distribution must be Gaussian (normal) because the failure load is a weighted sum of contributions of random strength values of representative volume elements (RVE) of material along the failure surface, all of which fail simultaneously. For perfectly brittle failure, in which the structural failure is caused by failure of one negligibly small RVE, the structural strength distribution must be Weibullian [16].

So, in these extreme cases, the load of failure probability 10^{-6} can be determined from the mean and standard deviation, which are easily measured or computationally simulated. Not, however, for quasibrittle structures, made of quasibrittle materials such as concrete, tough ceramics, fiber composites, rigid foams, many geomaterials, bio-materials, and all brittle materials on the micrometer scale [6, 2].

The analysis of interatomic bond breaks and mustiscale transitions to the RVE has shown that the strength of one RVE must have a Gaussian distribution transiting to a power law in the tail of probability $< 10^{-3}$. For the typical case of Type 1 size effect, which occurs in structures with the so-called positive geometry (i.e., the geometry for which the energy release rate at constant load increases with the crack extension), the quasi-brittleness means that a larger structure fails if a single RVE fails, in other words, if it behaves as a chain of links, each representing one RVE. It was shown that as the structure's size D (or the number N of RVEs in the chain) increases, the failure load distribution gradually changes from Gaussian to Weibullian in such a way that a Weibull tail gradually grows into the Gaussian core.

In quasibrittle structures, N is not large enough (not $> 10^5$) to make the distribution completely Weibullian. So one has a Gauss-Weibull graft, for which the mean and the coefficient of variation do not suffice to locate the tail probability of 10^{-6}. Since, for the Weibull distribution, the ratio of the load of 10^{-6} failure probability to the standard deviation is almost double the ratio for the Gaussian distribution, determining the Gauss-Weibull graft is crucially important. This means that the structure must be probabilistically modeled as a *finite* (rather than infinite) chain of RVEs [5]. Unlike ductile or perfectly brittle structures, the number of RVEs and their weighting becomes very important for safety assessments [13, 14, 15].

The present lecture reviews the nano-mechanical argument for fracture on the atomistic scale, the multiscale transition to the RVE level, and the computational challenges in calculating the failure probability tail. Some typical comparisons with test data for concrete and tough ceramics, documenting the applicability of the theory, are displayed (Fig. 1). Extension to the size effect on the lifetime distribution of quasibrittle structures subjected to static or cyclic fatigue is also explained [9, 10].

Although the present theory can be calibrated from histograms of strength of many identical specimens of different sizes, it is argued that test data on the mean size effect of a broad enough range allow a much more effective and simpler calibration. Then, the recently developed boundary-layer nonlocal method for computing the failure probabilities and the size effect on the probability distribution are briefly explained and their application illustrated [11].

In the second part of the lecture, two new advances are briefly presented. One involves the computation of failure probability of residual strength under sudden overload after a sustained period under constant stress.

The other involves the probability distribution of failures after large stable crack growth, which exhibit the so-called Type 2 size effect. Since this is a venture into an unexplored realm, let us now explain it briefly.

These failures, whose statistics has apparently not yet been subjected to fundamental probabilistic modeling, are typical of reinforced concrete structures as well as some unreinforced ones, which typically have an initially negative geometry and fail once the crack growth switches the geometry to positive. They are known for a strong energetic (or deterministic) size effect roughly following Bažant's size effect law. The statistical part of size effect has been considered as negligible, mainly because the mechanics of fracture dictates the stable crack growth in structures of different sizes to follow nearly homologous paths despite the randomness of the material. Nevertheless, the coefficient of variation of strength is likely to depend on the structure size, and there might be some size effect on the mean as well.

Like in all studies of strength statistics, we assume that the tip of the dominant crack may lie, at the maximum load state, at various locations—particularly at the centers of square elements $i = 1, 2, ...N_D$ in a square grid, as exemplified for shear failure of a reinforced concrete beam in Fig. 2. Each square element is assumed to represent an RVE of the material of characteristic size l_0 (which implies an autocorrelated random strength field). Based on Bažant's size effect law as [6, 2] it can be shown that

$$\sigma_{Ni,\beta} = \frac{B_{i,\beta}}{\sqrt{1 + D/D_{0i,\beta}}} f'_t \tag{1}$$

$$\text{where} \quad D_{0i,\beta} = \frac{g'_{i,\beta}}{g_{i,\beta}} c_f \tag{2}$$

$$B_{i,\beta} = \sqrt{\frac{l_0}{g'_{i,\beta} c_f}}, \quad l_0 = \frac{E' G_f}{f'^2_t} \tag{3}$$

Here F = given load (maximum load), b = beam thickness, E = elastic modulus, G_f = fracture energy, l_0 = Irwin's characteristic material length roughly equal to the length of the FPZ, c_f = material characteristic length for size effect ($c_f/l_0 \approx 0.44$ for 3PB tests [7]); $g_{i,\beta}$ = dimensionless energy release rate for crack extension from element i in the direction β, and $g'_{i,\beta}$ = derivative with respect to the crack extension length in direction β; and $\sigma_{Ni,\beta}$ = nominal strength of structure of size D when the failure is due to crack extending in direction β from the i-th RVE with tensile strength f'_t (this means that we consider the crack growth to be governed by the cohesive crack model or crack band model, or some nonlocal damage model, in which f'_t is one basic material fracture characteristic, which is considered random). Before undertaking the failure analysis, the values of $g_{i,\beta}$ and $g'_{i,\beta}$ are evaluated by J-integrals and finite elements for different directions β of crack propagation. The direction β that gives the overall maximum energy release rate is determined and fixed (for the sake of simplicity, with same value for all tips i). The energy release rate also depends on the shape of the crack (which itself is a result of Markov random process of crack growth) and on the location of its starting point, but these effects appear to be secondary and are here neglected, for the sake of simplicity.

What greatly simplifies the problem is the fact that the deterministic size effect law in Eq. (1), which is already well established, can be imposed as the input, rather than being solved as part of the analysis. It is through this law that the energetic aspects of fracture mechanics are conveniently introduced.

The probability that the structure of size D fails due to a crack extending in direction β from the i-th RVE of tensile strength f'_t may be written as

$$P_{f\,i,\beta} = \text{Prob}(\,\sigma_{Ni,\beta} < \sigma^L_{ND}\,) \quad \text{where} \quad \sigma^L_{ND} = F/bD \tag{4}$$

where σ^L_{ND} is the nominal stress of structure under given load F.Then, since:

$$P_{f_{i,\beta}} = \text{Prob}\left(f'_t \leq \frac{\sqrt{1 + D/D_{0i,\beta}}}{B_{i,\beta}}\sigma^L_{ND}\right) \quad (5)$$

The following basic statistical hypothesis now appears logical:

$$P_{f_{i,\beta}} = \Phi_{GW}\left(\frac{\sqrt{1 + D/D_{0i,\beta}}}{B_{i,\beta}}\sigma^L_{ND}\right) \quad (6)$$

where Φ_{GW} = cumulative probability distribution function (cdf) characterizing the tensile strength of one RVE. Based on previous work [5, 9], it must have the form of a grafted Gauss-Weibull probability distribution, which was derived theoretically from nano-mechanics and multiscale transition of probability tail, and was extensively verified and calibrated by Type 1 tests of size effect and histograms.

In deterministic analysis, only one specific crack tip location corresponds to failure. But if the material is random, every location could correspond to failure, albeit with a very different probability $P_{f_{i,\beta}}$. For a random material, the structure of size D will survive under load F if none of the crack tips i leads to failure. So, according to the joint probability theorem:

$$1-P_{f_D} = \prod_{i=1}^{N_D}\left[1 - \Phi_{GW}\left(\frac{\sqrt{1 + D/D_{0i,\beta}}}{B_{i,\beta}}\sigma^L_{ND}\right)\right] \quad (7)$$

where P_{f_D} = failure probability of the structure of size D; and Φ_{GW} = cumulative grafted Gauss-Weibull probability distribution of the strength of one RVE of the material, derived theoretically from nano-mechanics and multiscale transition of probability tail, and calibrated experimentally by Type 1 size effect tests.

The point to note is that, like in Type 1 strength analysis, the number of independently contributing material elements is greater in a large structure than it is in a small one; see Fig. 2. This intuitively explains the cause of the statistical size effect in Type 2 failures.

What allows us to base the failure probability analysis on the strength (rather than the critical energy release rate) of one RVE is that fracture of a quasibrittle material is properly analyzed in terms of the cohesive crack model or crack band model, for which the material strength is one essential material property characterizing the distributed damage in the FPZ. The lecture presents and reviews the results of this analysis for typical reinforced concrete beams of various sizes, including the statistical effect on the mean nominal strength and on the coefficient of variation of nominal strength.

Final comment: The results of computer analysis must be subjected to various safety factors, particularly the overload factors and the understrength factors dictated by the design code. At present, these factors are mostly empirical and highly uncertain, mainly because they do not properly reflect the tails of probability distributions and the size effect. Thus, in fact, *the design code provisions accounting for uncertainty are more uncertain than anything else in design*. Therefore, to benefit from accurate computer analysis of structures, these tails and the size effect must be realistically incorporated into the system of safety factors. Until that happens, a highly accurate computer simulation of the strength of concrete structures has only little practical value.

References

[1] O. H. Basquin, The Exponential Law of Endurance Tests, in: Proc., Amer. Soc. Testing and Mater., ASTEA 10 (1910) 625–630.

[2] Z. P. Bažant, Scaling of Structural Strength, 2nd Ed., Elsevier, London, 2005.

[3] Z. P. Bažant, J.-L. Le, Nano-mechanics based modeling of lifetime distribution of quasibrittle structures, Eng Fail Anal 16 (2009) 2521–2529.

[4] Z. P. Bažant, J.-L. Le, C. G. Hoover, Nonlocal boundary layer (NBL) model: Overcoming boundary condition problems in strength statistics and fracture analysis of quasibrittle materials, in: Proc., 7th International Conference on Fracture Mechanics of Concrete and Concrete Structures, Melbourne, 135–143, 2010.

[5] Z. P. Bažant, Activation energy based extreme value statistics and size effect in brittle and quasibrittle fracture, J Mech Phys Solids 55 (2007) 91–134.

[6] Z. P. Bažant, J. Planas, Fracture and Size Effect in Concrete and Other Quasibrittle Materials, CRC, 1998.

[7] G. Cusatis, E. A. Schauffert, Cohesive crack

analysis of size effect, Eng Fract Mech 76 (2009) 2163–2173.

[8] T. Fett, D. Munz, Static and cyclic fatigue of ceramic materials, Ceramics Today – Tomorrow's Ceramics, ed. P. Vincenzini, Elsevier Science (1991) 1827–1835.

[9] J.-L. Le, Z. P. Bažant, M. Z. Bazant, Unified nano-mechanics based probabilistic theory of quasibrittle and brittle structures: I. Strength, static crack growth, lifetime and scaling, J Mech Phys Solids 59 (2011) 1291–1321.

[10] J.-L. Le, Z. P. Bažant, M. Z. Bazant, Unified nano-mechanics based probabilistic theory of quasibrittle and brittle structures: II. Fatigue crack growth, lifetime and scaling, J Mech Phys Solids 59 (2011) 1322–1337.

[11] J.-L. Le, J. Elias, Z. P. Bažant, Computation of probability distribution of strength of quasibrittle structures failing at macro-crack initiation, J Eng Mech-ASCE 138(7) (2012) 888–899.

[12] P. C. Paris, F. Erdogan, A critical analysis of crack propagation law, J Basic Eng-T ASME 85 (1963) 528–534.

[13] S. L. Phoenix, The asymptotic time to failure of a mechanical system of parallel members, J Appl Maths-SIAM 34 (2) (1978a) 227–246.

[14] S. L. Phoenix, Stochastic strength and fatigue of fiber bundles, Int J Frac 14(3) (1978b) 327–344.

[15] R. L. Smith, The asymptotic distribution of the strength of a series-parallel system with equal load sharing, Ann Probab 10(1) (1982) 137–171.

[16] W. Weibull, The phenomenon of rupture in solids, in: Proc., Royal Swedish Inst. Eng. Res. (Ingenioersvetenskaps Akad. Handl.) 153 (1939) 1–55.

[17] A. Wöhler, summary of Versuche über die Festigkeit der Eisenbahnwagenachsen, in Zeitschrift für Bauwesen, (1860) 10, Engineering 4 (1867) 160–161.

[18] J. N. Yang, S. D. Manning, A simple second order approximation for stochastic crack growth analysis, Eng Fract Mech 51(5) (1991) 677–686.

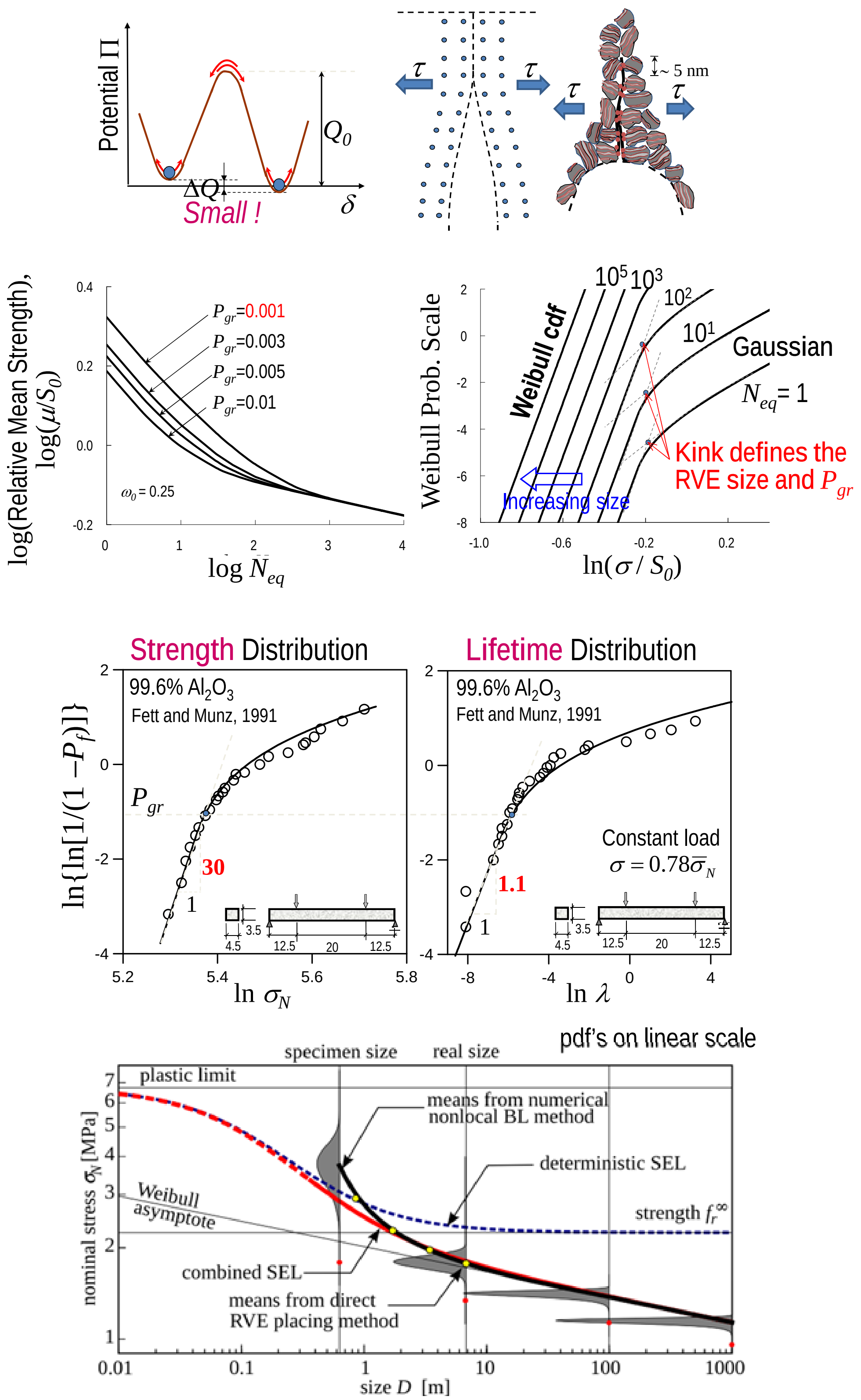

Figure 1: Illustrations of basic concepts of probability of quasibrittle failure. First row: Small change of activation energy barrier due to failure of an interatomic bond in an atomic lattice of nano-scale element; Second row: calibration of the Gauss-Weibull grafted distribution parameters using size effect data; Third row: optimum fits of strength and lifetime histograms of 99.99% Al_2O_3; Last row: Type 1 size effect on the mean strength and on the strength distribution evolving from mostly Gaussian toward Weibullian, calculated for collapsed Malpasset Dam.

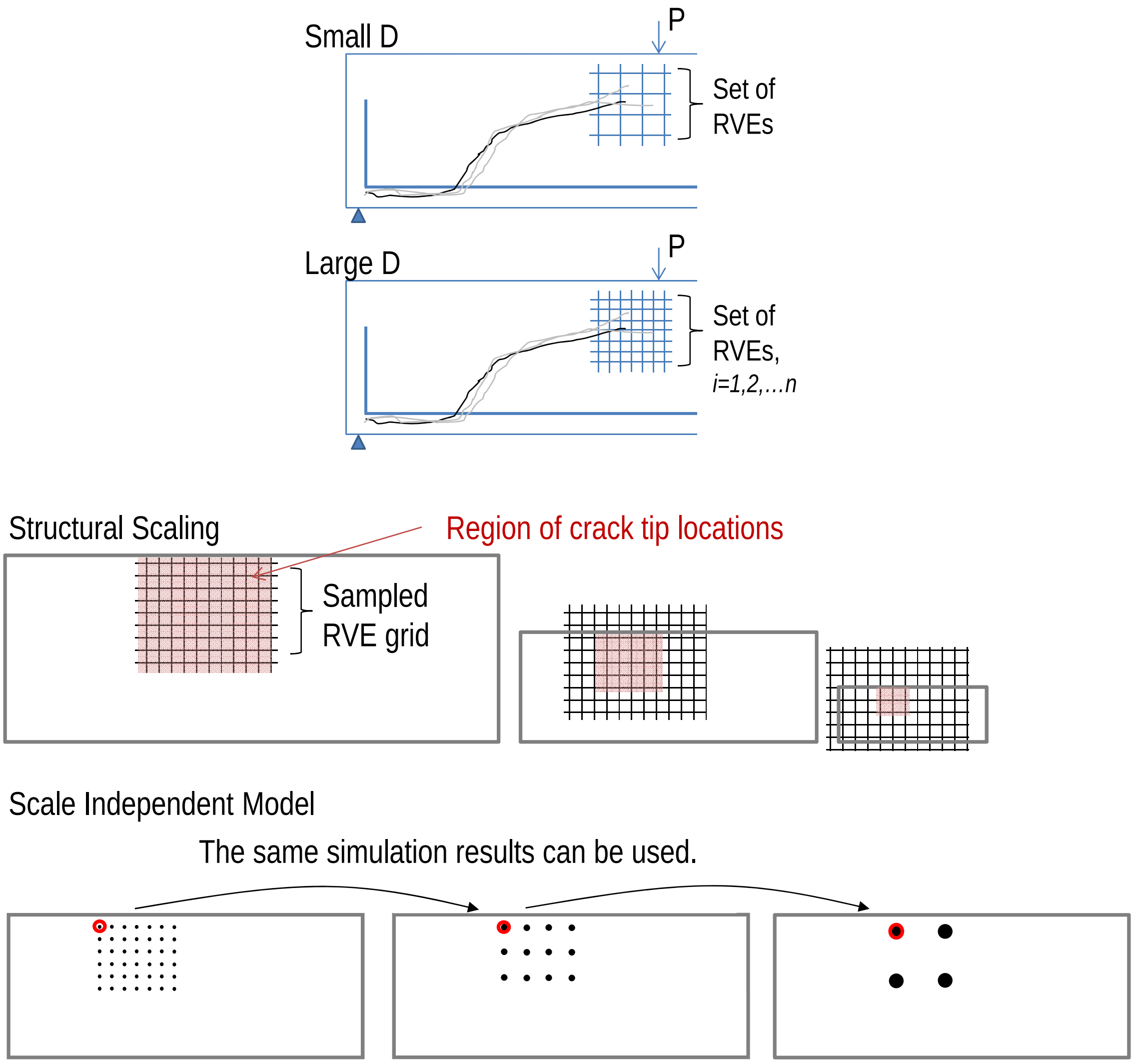

Figure 2: Top two rows: Schematic of half of a 4-point bending shear failure illustrating the approach used to determine the probability of quasibrittle failure of Type 2. A number of gray geometrically scaled crack paths from size effect tests are averaged to obtain an average crack path shown in black. The crack tip of the averaged crack path is placed in each RVE within the zone of possible locations to compute geometric fracture parameters. Bottom row: Schematic of the scaling of the set of RVEs employed to utilize a single set of strain energy release rate calculations to determine the size effect on the CoV of the structural strength. In a scaled down structure (on the right), there are fewer RVEs that influence the maximum load since their spacing is governed by the ratio of l_0/D.