Stochastická nelineární analýza betonových konstrukcí: spolehlivost, vliv velikosti, inverzní analýza

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Praha, 12. 4. 2007
Outline

• Stochastic techniques for uncertainties simulation
• Software FReET – Feasible Reliability Engineering Tool

Development stimulations:
• Long-term focus of Brno reliability group (Vořechovský, Rusina, Lehký …)
• SARA project (Bergmeister, Pukl, Červenka, Strauss …)
  - To combine efficient methods of reliability and nonlinear analysis
  - Software ATENA+FReET=SARA
  - To provide an advanced tool for assessment of real behavior of concrete structures
• Selected types of applications (stochastic nonlinear analysis)
Stochastic techniques for uncertainties simulation

• Introduction – computational demands
• Small-sample simulation of Monte Carlo type
• Imposing statistical correlation
• Simulation of random fields
• Sensitivity analysis
• Reliability analysis
• Inverse analysis
Two main categories of stochastic tasks/approaches

- Approaches focused on the calculation of statistical moments of response quantities (means, variances, etc.)
  \[ R = g_R(x_1, x_2, \ldots x_i, \ldots x_n) \]
  \[ \rightarrow \text{response function} \]

- Approaches aiming at the calculation of theoretical probability of failure
  \[ Z = g_z(x_1, x_2, \ldots x_i, \ldots x_n) \]
  \[ p_f = P(Z \leq 0) \]
Reliability analysis - computational demands:
Number of evaluation of limit state function

- Crude Monte Carlo: $1\,000\,000\,000 \ldots$
- Importance sampling: 1,000 - 10,000
- Approximation FORM, SORM - design point calculation: 100 - 1,000
- Response surface: 100 - 1,000
- Cornell safety index, Curve fitting: 10 - 100
  - MC
  - LHS

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The range \((0; 1)\) of PDF \(\Phi(Y_i)\) of each random variable \(Y_i\) is divided into \(N\) non-overlapping intervals of equal probability \(1/N\) (McKay et al. 1979. Iman & Conover 1980, Iman & Shortencarier 1984).

- The centroids are selected randomly based on random permutations of integers.
- Every interval of each variable is used only once during the simulation process.
Huntington & Lyrintzis (1998):

Improvements to and limitations of LHS

\[ x_{i,k} = N_{Sim} \cdot \int_{a}^{b} x \cdot f(x) \, dx \]

where

\[ a = F^{-1}\left(\frac{k-1}{N}\right) \]
\[ b = F^{-1}\left(\frac{k}{N}\right) \]

- Mean value: accurately
- Stand. deviation: significant improvement
**LHS: Step 2 – imposing statistical correlation**

<table>
<thead>
<tr>
<th>Simulation variable</th>
<th>( y_1 )</th>
<th>( \ldots )</th>
<th>( z_1 )</th>
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<td>( y_{NSim} )</td>
<td>( \ldots )</td>
<td>( z_{NSim} )</td>
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- Simulated annealing: Probability to escape from local minima
- Cooling - decreasing of system excitation
- Boltzmann PDF, energetic analogy

\[ P_r(E) \approx e^{\frac{-\Delta E}{k_b T}} \]
Best ordering (all possible rank combinations). Is it possible to find the *global minimum*?

One column remains stable. Others permute.

There exist \( (N_{Var})^{N_{Sim} - 1} \) possibilities.

In case of 6 simulations with 5 variables:

\[
(6!)^{5-1} = 2.6874 \cdot 10^{11}
\] possibilities.
LHS: Step 2 – imposing statistical correlation

- Simulated annealing: Probability to escape from local minima
- Cooling - decreasing of system excitation
- Boltzmann PDF, energetic analogy

\[
P_r(E) \approx e^{-\frac{\Delta E}{k_b T}}
\]
Statistical correlation in LHS - optimization problem

\[ \min \sum_{i,j}^{N,k} (a_{i,j} - b_{i,j})^2 \]

- \( a_{i,j} \) - the target correlation matrix
- \( b_{i,j} \) - the actual correlation matrix
Simulated annealing

\[ E_i = Err_i - Err_{i-1} \]

\[ k_b = 1 \]

\[ P_r(E) \approx e^\left( \frac{-E}{k_b \cdot T} \right) \]
Numerical test 1:
diminish spurious correlation

- 5 variables and 6 simulations.
  1. ULHS, iterations (Spearman)
  2. Simulated annealing, (PC 400MHz 3 sec )
Diminish spurious correlation comparison

**Cholesky decomp. iterative ULHS (Spearman)**

\[ E^2_{\text{overall}} = 0.22 \]

**Proposed algorithm (Simulated Annealing)**

\[ E^2_{\text{overall}} = 0.04 \]

**Difference:**

- \(| . | < 0.1\)
- \(| . | < 0.2\)
- \(| . | < 0.3\)

**Samples:**

- 1.4080
- 0.6867
- 0.2142
- 1.4080
- -0.2142

- 0.2142
- 1.4080
- -0.6867
- -0.2142
- 0.2142

**Target correlation matrix**
Numerical test– imposition of target statistical correlation

- 5 variables and 6 simulations.
- Number of simulation - great influence.
- All possibilities 50 minutes
- Simulated annealing 3 sec (always finds local minima)
Simulated annealing - results

Starting sampling matrix

\[ E_{overall}^2 = 3.79 \]

<table>
<thead>
<tr>
<th>1.4080</th>
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Samples:

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Difference:

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<td>0.5</td>
<td>0.2</td>
<td>0.5</td>
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</table>

Proposed genetic algorithm (Simulated Annealing)

\[ E_{overall}^2 = 0.004 \]

<table>
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<td>0.6</td>
<td>1</td>
<td>0.503</td>
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<tr>
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<td>0.5</td>
<td>0.2</td>
<td>0.5</td>
<td>1</td>
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LHS: Simulated annealing - weighted

- Resulting correlation matrix is positive definite and error is uniformly distributed among all coefficients - compromise

Positive definiteness of $K$
• Resulting correlation matrix is positive definite and error is uniformly distributed among all coefficients
• Weighted method: suppression of selected coefficients

Positive definiteness of $K$
Simulation of random fields

- Essential topic in stochastic continuum mechanics.
- The need for accurate representation and simulation in SFEM.
- Various methods ...
- Orthogonal transformation of covariance matrix
  (Schuëller et al. 1990, Liu et al. 1995)
  - Small number of random variables to represent random fields.

- Latin Hypercube Sampling (LHS)
  - Small number of simulations.
  - Combination: A new alternative method
Simulation of random fields

Orthogonal transformation of covariance matrix and LHS

\[ C_{XX} = \Phi \Lambda \Phi^T \]

\[ C_{YY} = \Lambda \]

\( \Phi \) - eigenvector matrix

\( \Lambda \) - Cov. matrix in uncorrelated space (diagonal) : eigenvalues \( \lambda_1, \lambda_2, \ldots, \lambda_{nd} \)

Simulation - uncorrelated

Gaussian random variables:

\[ Y^T = [Y_1, Y_2, \ldots, Y_{nr}] \]

\[ X = \Phi Y \]

\[ Y \xleftarrow{\text{MCS}} \text{LHS} \]

Vořechovský, Novák - Icossar 2005

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Comparison of convergence to target fields statistics

Number of simulations $n$

![Graphs showing comparison of convergence to target fields statistics.](image)


- SFEM model with 13000 DOF
- random field to describe geometrical imperfections
- 1500 random variables

\[
\begin{array}{c|cc}
\text{Method} & \text{Mean Value [MNm]} & \text{Coeff. of Variation [%]} \\
\hline
\text{LHS (32 samples)} & 22.3 & 0.087 \\
\text{MCS (200 samples)} & 21.9 & 0.076 \\
\end{array}
\]

Statistics of ultimate bending moment

finite element model of flange

Realization of random field

1500 \text{ reduction} \rightarrow 128
Sensitivity analysis

Nonparametric rank-order correlation between input variables and output response variable

- Kendall tau \( \tau_i = \tau(q_{ji}, p_j), \ j = 1, 2, \ldots, N \)
- Spearman

\[
r^s = 1 - \frac{6 \sum_{i=1}^{n} d_i^2}{n(n-1)(n+1)}
\]

- Robust - uses only orders
- Additional result of LHS simulation, no extra effort
- Bigger correlation coefficient = high sensitivity
- Relative measure of sensitivity \((-1, 1)\)
Reliability analysis

- Simplified – rough estimates, as constrained by **extremely small number of simulations (10-100)!**
- Cornell safety index $\beta = \frac{\mu_Z}{\sigma_Z}$
- Curve fitting
- FORM, importance sampling response surface…
The method of Hasofer and Lind, 1974

- Hasofer and Lind, 1974 - important step
- Transformation of the limit state function into so-called standard space

\[ U_1 = \frac{R - \mu_R}{\sigma_R} \quad U_2 = \frac{S - \mu_S}{\sigma_S} \]

- New variables with mean value 0 and standard deviation 1
- In the new coordinate system the line \( G=R-S \) no longer passes through origin
- HL safety index - the distance from the design point to origin
- correct in case of normally distributed variables, for non-normally - a good approximation

\[
\left. \begin{array}{c}
\beta = \sqrt{u^T u} \\
\text{Subject to} \quad g(X) = 0, \\
G = R - S = (U_1 \cdot \sigma_R + \mu_R) - (U_2 \cdot \sigma_S + \mu_S) \\
p_f = \Phi(-\beta)
\end{array} \right.
\]
Identification of material parameters

Numerical model of structure

appropriate material model

many material parameters

Information about parameters

- experimental data
- recommended formulas
- engineering estimation
Identification of material parameters

Primary calculation

Correction of parameters:

• „trial – and – error“ method
• sofisticated identification methods
  – artificial neural network + stochastic calculations (LHS)
Artificial neural network

Modeling of processes in brain
(1943 - McCulloch-Pitts Perceptron)

Various fields of technical practice

Neural network type – Multi-layer perceptron:
- set of neurons arranged in several layers
- all neurons in one layer are connected with all neurons of the following layer
Artificial neural network

**NEURON:**

Output from 1 neuron:

\[
y = f(x) = f\left( \sum_{k} (w_k \cdot p_k) + b \right)
\]

- \(k\) – number of input impulses (1,...,K)
- \(w_k\) – weight coefficient of connecting path from \(k\)-th neuron of previous layer
- \(p_k\) – impulse from \(k\)-th neuron previous layer
- \(b\) – bias of neuron
- \(f\) – transfer function of neuron
Artificial neural network

Two phases:
- active period (simulation of process)
- adaptive period (training)

Training of network:
- training set, i.e. ordered pair \([p_i, y_i]\)

Minimization of criterion:

\[
E = \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{K} \left( y_{ik}^v - y_{ik}^* \right)^2
\]

- \(N\) – number of ordered pairs input - output in training set;
- \(y_{ik}^*\) – required output value of \(k\)-th output neuron at \(i\)-th input;
- \(y_{ik}^v\) – real output value (at same input).
Principle of identification method

Stochastic calculation (LHS) – training set for calibration of synaptic weights and biases

Material model parameters

Structural response

Prague, 12.4.2007
• **Stand alone module** - definition of reliability problem (user-defined limit state/response function) in programming language (C++, FORTRAN) – DLL function or by equation interpreter

• **Integration with software ATENA** - nonlinear fracture mechanics of reinforced concrete structures (Červenka Consulting) – SARA software shell
Software Freet

• **Freet version 1.1 – December 2004**
  – Enhanced set of PDF
  – a possibility to add new comparative values without need to perform simulation
  – outputs organization and printing possibilities
  – USB hasp

• **Freet version 1.2 – February 2005**
  – Net version (BOKU computer lab installation)
  – 1 Hasp for SARA, ATENA, FREET
  – sensitivity graphical output enhanced (also what-if-studies, parametric study)

• **Freet version 1.3 – June 2005**
  – Weighting for correlation matrix input
  – Response Surface basics
  – File->New clearing results and input
  – Graphics enhancement and checking
  – Random fields basics

• **Freet version 1.4 – May 2006, new features:**
  – Graphics enhancement and checking
  – Random fields implementation – verified
  – Possibility to define a parameter for easy parametric study with graphical output
  – New type of probability distribution: Bounded normal PDF
  – Automatic running of FREET from command line

• **Freet version 1.5 – January/February 2007, new features:**
  – More general interface to third-parties programs – now DLL and BAT, EXE files communication via text input/output files
  – FORM – First Order Reliability Methods

13.4.2007
FREET - Feasible Reliability Engineering Tool
Z=R-E
Stochastic NLFEM – SARA Studio

Probabilistic software FReET

Software for nonlinear fracture mechanics analysis ATENA
Non-linear techniques and material models for concrete: ATENA software

Numerical core – advanced nonlinear material models

cracked concrete in tension

tensile cracks
post-peak behavior

smeared crack approach

Crack band method
fracture energy

Crack band size:  $L$

$\varepsilon = \frac{W}{L}$

fixed or rotated cracks

crack localization
size-effect is captured
Software ATENA

Well-balanced approach for practical applications of advanced FEM in civil engineering

Numerical core – state-of-art background + user friendly Graphical user environment – visualization + interaction

Equilibrium:

\[ K \Delta U = P - R \]
Numerical core – advanced nonlinear material models

Concrete in tension

tensile cracks
post-peak behavior
smeared crack approach
crack band method
fracture energy

fixed or rotated cracks
-crack localization
size-effect is captured
- Run-time
- histogram
- of results
Software tools

Software FReET: statistical, sensitivity and reliability analyses
http://www.freet.cz

Software DLNNET: neural networks
Software communication for inverse analysis

- FEM model
  - random response
  - experimental response
- FREEET
  - random parameters
  - identified parameters

ADAPTIVE PHASE
- TRAINING

ACTIVE PHASE
Selected types of applications

Example of FReET stand-alone application:
  • Statistical analysis of concrete subway tunnel under Vltava river

SARA - classes of tasks:
  • Probabilistic analyses of concrete structures
  • Statistical size effect studies
  • Verification of (code) design formulas
  • Identification of material model parameters (inverse analysis)
Large concrete subway tunnel under Vltava river in Prague (2002)

- Weight of tunnel
- Uplift force
- 211 random variables
- Imperfection of geometry, 14 segments
- Target: risk minimization
- Updating segments - convergence to required uplift force

Praha, 12. 4. 2007
Static scheme, forces acting on the tube
Statistical simulation of uplift force

Uplift force [t/m]

Segments

Praha, 12. 4. 2007
Statistical simulation and measurement

Uplift force [t/m] vs Segments

- Mean - simulation
- 5% percentile
- Reality
- 95% percentile
Statistical simulation and measurement

Uplift force [t/m]

Segments

- Mean - simulation
- 5% percentile
- Reality
- 95% percentile
Forces - barrels with water

Absolute Frequency Histogram - [metro]

Number

0.0000
2.8000
5.6000
8.4000
11.2000
14.0000
16.8000
19.6000
22.4000
25.2000
28.0000


0 t

64 t

A

x1

x2

B

0 1 2 3 4 5

Praha, 12. 4. 2007
Probabilistic analyses of concrete structures:
Box-girder prestressed bridge in Vienna
### Concrete grade B500

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<thead>
<tr>
<th>Random variable description</th>
<th>Symbol</th>
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<th>Mean value</th>
<th>COV</th>
<th>Distribution type</th>
<th>Reference</th>
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<td>m</td>
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<td>MN/m³</td>
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### Prestressing strands

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![Graph showing load vs. deflection and reliability index vs. load](image-url)
Probabilistic analyses of concrete structures: Cantilever beam bridge in Italy

Colle d’Isarco bridge. Brennero highway, Italy

Reliability index vs. load
Typical failures

-59.00 m

Shear

-59.00 m

107.00 m

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Probabilistic analyses of concrete structures:
Soil-structure interaction, spatial variability

- Stability of concrete tunnel tube in complicated geological conditions
- Influence of spatial variability of Young modulus and material constants of Drucker-Prager criterion (based on cohesion and angle of internal friction)
- Analyzed part 50 x 60m, diameter of tunnel 11m, wall thickness 0.5m
- Plain strain state, 5000 finite elements
Statistical size effect studies: **Four-point bending - different bending span**

- Koide at al. Experiments on 4PB
- Statistical size effect!
- Cannot be captured at deterministic level
Alt. I: No correlation between tensile strength and fracture energy

Alt. II, III: High correlation between tensile strength and fracture energy
Statistical size effect studies: **Four-point bending - different bending span**

Four-point bending and patterns of random fields

Random load – deflection curves (red curve – deterministic calculation).

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**Statistical size effect studies**: Dog-bone shaped concrete specimens in uniaxial tension

### EXPERIMENT - Van Vliet and Van Mier, 1997

<table>
<thead>
<tr>
<th>Size</th>
<th>D [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
</tr>
<tr>
<td>B</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>200</td>
</tr>
<tr>
<td>D</td>
<td>400</td>
</tr>
<tr>
<td>E</td>
<td>800</td>
</tr>
<tr>
<td>F</td>
<td>1600</td>
</tr>
</tbody>
</table>
Statistical size effect studies: Malpasset dam (failed 1959)

Calculation for different sizes, microplane M4 model
Bažant, Vořechovský, Novák - Icossar 2005
Size effect formulae and verification by statistical simulation

computation (microplane model-ATENA) ○
deterministic formula, $D_b, r$ and $f_r^0$ fitted

deterministic-statistical formula

nominal strength $\sigma_N$ [MPa]

normalized nominal strength $\sigma_N/f_r$ [-]

nominal strength $f_r^0$

size $D$ [m]

Weibull PDF

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Verification of (code) design formulas: Shear failure of reinforced concrete beams

Nominal strength vs. size for different design alternatives: formulas, experiment and simulation

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**Inverse analysis**

Training, stochastic preparation of training sets: classical Monte Carlo vs. Latin Hypercube Sampling methods

Failure surface approximation

\[ g(X) = aX_2^3 + bX_2^2 + cX_2 - X_1 + d \]

\[ a = -0.36355, \quad b = 1.18046, \quad c = -1.0892988, \quad d = 4.2042064 \]

\[ X_1 = Ha/m_p, \quad X_2 = Va/m_p \]
Inverse analysis

Aim: identification of parameters $a, b, c, d$

Parametric study for small numbers of simulations – 20, 30, 40 and 50

Same initial conditions (scatter of parameters, neural network type, same initiation of synaptic weights and biases to start training of network)
Inverse analysis

Training set (20 random realizations)
resulting failure surfaces
errors of identifications

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Experiments by Maier and Thürliman, 1985

Identification of material parameters: Shear wall test

Reality

Simulation

$F_v = 1653 \text{ kN}$

$F_{h, \text{max}} = 933 \text{ kN}$
### Identification of material parameters:

#### Shear wall test

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Unit</th>
<th>Mean value</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity</td>
<td>$E$</td>
<td>GPa</td>
<td>30</td>
<td>0.10</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>$f_t$</td>
<td>MPa</td>
<td>2.5</td>
<td>0.10</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>$f_c$</td>
<td>MPa</td>
<td>30</td>
<td>0.10</td>
</tr>
<tr>
<td>Fracture energy</td>
<td>$G_F$</td>
<td>N/m</td>
<td>75</td>
<td>0.20</td>
</tr>
<tr>
<td>Compressive strain</td>
<td>$\varepsilon_c$</td>
<td>-</td>
<td>0.0025</td>
<td>0.20</td>
</tr>
<tr>
<td>Max. comp. displacement</td>
<td>$w_d$</td>
<td>m</td>
<td>0.003</td>
<td>0.30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Unit</th>
<th>Mean value</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilinear diagram of steel for smeared reinforcement</td>
<td>$f_x$</td>
<td>kN</td>
<td>574</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>$x_2$</td>
<td>m</td>
<td>0.015</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>$f_{x1}$</td>
<td>kN</td>
<td>764</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Randomization of material parameters – preparation of training set

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Identification of material parameters:

Shear wall test

FEM model in software ATENA
Identification of material parameters

20 random realizations – training set

Horizontal force [kN] vs. Horizontal deformation [mm]
Identification of material parameters

24 points on l-d diagram

12 and 10 neurons in 2 hidden layers – nonlinear transfer function

6 or 10 neurons in output layer – linear transfer function

input 2 hidden layers output
### Identification of material parameters

<table>
<thead>
<tr>
<th>Spearman</th>
<th>E</th>
<th>$f_t$</th>
<th>$f_c$</th>
<th>$G_t$</th>
<th>$\varepsilon_c$</th>
<th>$w_d$</th>
<th>$x_1$</th>
<th>$f_{x_1}$</th>
<th>$x_2$</th>
<th>$f_{x_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>0,753</td>
<td>0,123</td>
<td>0,453</td>
<td>0,045</td>
<td>-0,335</td>
<td>-0,108</td>
<td>-0,167</td>
<td>0,015</td>
<td>-0,087</td>
<td>-0,107</td>
</tr>
<tr>
<td>$F_5$</td>
<td>0,262</td>
<td>0,513</td>
<td>0,460</td>
<td>0,014</td>
<td>-0,263</td>
<td>-0,081</td>
<td>-0,516</td>
<td>0,311</td>
<td>-0,051</td>
<td>0,045</td>
</tr>
<tr>
<td>$F_{10}$</td>
<td>0,158</td>
<td>0,382</td>
<td>0,608</td>
<td>0,081</td>
<td>-0,080</td>
<td>-0,027</td>
<td>-0,344</td>
<td>0,490</td>
<td>0,005</td>
<td>0,104</td>
</tr>
<tr>
<td>$F_{\text{max}}$</td>
<td>0,129</td>
<td>0,341</td>
<td>0,636</td>
<td>0,054</td>
<td>-0,042</td>
<td>-0,053</td>
<td>-0,307</td>
<td>0,537</td>
<td>-0,009</td>
<td>0,171</td>
</tr>
</tbody>
</table>

Sensitivity of material model parameters:

- 6 parameters identified
- 10 parameters identified

Parameters obtained from simulation of neural network:

<table>
<thead>
<tr>
<th>DLNNET</th>
<th>6 par.</th>
<th>10 par.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ [MPa]</td>
<td>29,9</td>
<td>33,0</td>
</tr>
<tr>
<td>$f_t$ [MPa]</td>
<td>2,47</td>
<td>2,47</td>
</tr>
<tr>
<td>$f_c$ [MPa]</td>
<td>34,51</td>
<td>35,3</td>
</tr>
<tr>
<td>$G_f$ [MN/m]</td>
<td>75,0</td>
<td>77,85</td>
</tr>
<tr>
<td>$\varepsilon_c$ [-]</td>
<td>2,51E-03</td>
<td>2,57E-03</td>
</tr>
<tr>
<td>$w_d$ [m]</td>
<td>3,00E-03</td>
<td>3,10E-03</td>
</tr>
<tr>
<td>$x_1$</td>
<td>2,72E-03</td>
<td>2,74E-03</td>
</tr>
<tr>
<td>$f_{x_1}$</td>
<td>566,9</td>
<td>570,7</td>
</tr>
<tr>
<td>$x_2$</td>
<td>1,50E-02</td>
<td>1,47E-02</td>
</tr>
<tr>
<td>$f_{x_2}$</td>
<td>764</td>
<td>768,8</td>
</tr>
</tbody>
</table>

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Identification of material parameters:
Shear wall test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DLNNET 6 par.</th>
<th>DLNNET 10 par.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E [MPa]</td>
<td>29,9</td>
<td>33,0</td>
</tr>
<tr>
<td>$f_t$ [MPa]</td>
<td>2,47</td>
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<td>34,51</td>
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<td>75,0</td>
<td>77,85</td>
</tr>
<tr>
<td>$e_c$ [-]</td>
<td>2,51E-03</td>
<td>2,57E-03</td>
</tr>
<tr>
<td>$w_d$ [m]</td>
<td>3,00E-03</td>
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<td>$f_{x_1}$</td>
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<tr>
<td>$x_2$</td>
<td>1,50E-02</td>
<td>1,47E-02</td>
</tr>
<tr>
<td>$f_{x_2}$</td>
<td>764</td>
<td>768,8</td>
</tr>
</tbody>
</table>

L-d diagrams obtained with identified parameters

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Experimental works (VUSTAH)

3-point bending experiment of fibre-reinforced concrete notched beams

<table>
<thead>
<tr>
<th>Specimen parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of specimen</td>
<td>mm</td>
<td>200</td>
</tr>
<tr>
<td>Width of specimen</td>
<td>mm</td>
<td>40</td>
</tr>
<tr>
<td>Depth of specimen</td>
<td>mm</td>
<td>40</td>
</tr>
<tr>
<td>Depth of notch</td>
<td>mm</td>
<td>15</td>
</tr>
<tr>
<td>Weight</td>
<td>kg</td>
<td>0.67</td>
</tr>
<tr>
<td>Span</td>
<td>mm</td>
<td>180</td>
</tr>
</tbody>
</table>
Experimental works (VUSTAH)

9 experimental load-deflection curves
Virtual numerical simulation of experiment

Nonlinear analysis – software ATENA (Červenka Consulting)

Material model SBETA – nonlinear fracture mechanics:
- smeared cracks model (fixed or rotated cracks)
- crack band method (localization limiter)
- crack opening ⇔ fracture energy
- softening model for fibre-reinforced concrete
  (parameters $c_1$ and $c_2$)
Inverse analysis

Experimental + virtual (numerical) load-deflection curves

![Graph showing load-deflection curves]
Statistics of fracture-mechanical parameters

Inverse analysis based on neural networks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Mean value</th>
<th>Standard deviation</th>
<th>Coefficient of variation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum failureload</td>
<td>kN</td>
<td>1,49</td>
<td>0,36</td>
<td>24,1</td>
</tr>
<tr>
<td>Deflection at maximum load</td>
<td>mm</td>
<td>0,67</td>
<td>0,20</td>
<td>29,3</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>GPa</td>
<td>5,4</td>
<td>1,68</td>
<td>30,9</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>MPa</td>
<td>11,3</td>
<td>3,39</td>
<td>29,9</td>
</tr>
<tr>
<td>Fracture energy</td>
<td>J/m²</td>
<td>2134</td>
<td>673</td>
<td>31,5</td>
</tr>
<tr>
<td>Softening parameter $c_1$</td>
<td>-</td>
<td>0,9</td>
<td>0,02</td>
<td>2,5</td>
</tr>
<tr>
<td>Softening parameter $c_2$</td>
<td>-</td>
<td>0,1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Conclusions

- Methods for statistical, sensitivity and reliability analyses, suitable for analysis of computationally intensive problems (e.g. continuum mechanics, FEM)
- Software tools FREET and SARA - for the assessment of real behavior of concrete structures, can be applied for any problem of quasibrittle modeling of concrete structures
- A wide range of applicability both practical and theoretical - gives an opportunity for further intensive development of both methods and software